Evaluate.

1. \(7 \cdot 3^2 + 11\)
2. \(10 - 3(3 + 1)^3\)
3. \(64 + 4^2 + \frac{1}{2}\)

4. \(-99 + 3^2 \cdot 5\)
5. \(\frac{1}{7}(7^2 + 28)\)
6. \(-\frac{1}{8}(8 + 24) - 2^2\)

Graph the transformation of the figure.

7. Translate the rectangle 3 units left and 4 units up.

8. Reflect the right triangle in the y-axis. Then translate 3 units down.

9. Translate the trapezoid 2 units up. Then reflect in the x-axis.

10. The point \((1, 1)\) is on \(f(x)\). After a series of 3 transformations, \((1, 1)\) has been moved to \((2, -7)\). Write a function \(g(x)\) that represents the transformations on \(f(x)\).
1.1 Parent Functions and Transformations
For use with Exploration 1.1

Essential Question  What are the characteristics of some of the basic parent functions?

1  EXPLORATION: Identifying Basic Parent Functions

Work with a partner. Graphs of eight basic parent functions are shown below. Classify each function as constant, linear, absolute value, quadratic, square root, cubic, reciprocal, or exponential. Justify your reasoning.

a.  

b.  

c.  

d.  

e.  

f.  

1.1 Parent Functions and Transformations (continued)

EXPLORATION: Identifying Basic Parent Functions (continued)

2. What are the characteristics of some of the basic parent functions?

3. Write an equation for each function whose graph is shown in Exploration 1. Then use a graphing calculator to verify that your equations are correct.
In your own words, write the meaning of each vocabulary term.

**parent function**

**transformation**

**translation**

**reflection**

**vertical stretch**

**vertical shrink**

### Core Concepts

#### Parent Functions

<table>
<thead>
<tr>
<th>Family</th>
<th>Constant</th>
<th>Linear</th>
<th>Absolute Value</th>
<th>Quadratic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rule</td>
<td>$f(x) = 1$</td>
<td>$f(x) = x$</td>
<td>$f(x) =</td>
<td>x</td>
</tr>
<tr>
<td>Graph</td>
<td><img src="constant_graph" alt="Graph" /></td>
<td><img src="linear_graph" alt="Graph" /></td>
<td><img src="absolute_value_graph" alt="Graph" /></td>
<td><img src="quadratic_graph" alt="Graph" /></td>
</tr>
<tr>
<td>Domain</td>
<td>All real numbers</td>
<td>All real numbers</td>
<td>All real numbers</td>
<td>All real numbers</td>
</tr>
<tr>
<td>Range</td>
<td>$y = 1$</td>
<td>All real numbers</td>
<td>$y \geq 0$</td>
<td>$y \geq 0$</td>
</tr>
</tbody>
</table>

**Notes:**
Extra Practice

In Exercises 1–4, identify the function family to which \( f \) belongs. Compare the graph of \( f \) to the graph of its parent function.

1. \[ f(x) = -\frac{1}{4}x + 8 \]
2. \[ f(x) = -3 \]
3. \[ f(x) = 5|x| \]
4. \[ f(x) = (x - 1)^2 + 4 \]

In Exercises 5–10, graph the function and its parent function. Then describe the transformation.

5. \[ f(x) = x - 7 \]
6. \[ f(x) = -9 \]
7. \[ f(x) = |x| + 1 \]
11. Identify the function family of \( f(x) = \frac{1}{3}|x| + 4 \) and describe the domain and range. Use a graphing calculator to verify your answer.

12. The table shows the distance a biker rides in his first team relay competition.

<table>
<thead>
<tr>
<th>Time (hours), ( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (miles), ( y )</td>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
</tr>
</tbody>
</table>

a. What type of function can you use to model the data? Explain.

b. If the biker's teammate rides at the same pace but leaves 1 hour later, what type of transformation does this represent?
Transformations of Linear and Absolute Value Functions

For use with Exploration 1.2

**Essential Question**  How do the graphs of \( y = f(x) + k \), \( y = f(x - h) \), and \( y = -f(x) \) compare to the graph of the parent function \( f \)?

1 **EXPLORATION:** Transformations of the Absolute Value Function

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

*Work with a partner.* Compare the graph of the function

\[
y = |x| + k
\]

Transformation

to the graph of the parent function

\[
f(x) = |x|.
\]

Parent function

2 **EXPLORATION:** Transformations of the Absolute Value Function

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

*Work with a partner.* Compare the graph of the function

\[
y = |x - h|
\]

Transformation

to the graph of the parent function

\[
f(x) = |x|.
\]

Parent function
1.2 Transformations of Linear and Absolute Value Functions (continued)

3 EXPLORATION: Transformation of the Absolute Value Function

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Compare the graph of the function

\[ y = -|x| \]  Transformation

...to the graph of the parent function

\[ f(x) = |x| \]  Parent function

Communicate Your Answer

4. How do the graphs of \( y = f(x) + k \), \( y = f(x - h) \), and \( y = -f(x) \) compare to the graph of the parent function \( f \)?

5. Compare the graph of each function to the graph of its parent function \( f \). Use a graphing calculator to verify your answers are correct.

- a. \( y = \sqrt{x} - 4 \)
- b. \( y = \sqrt{x} + 4 \)
- c. \( y = -\sqrt{x} \)

- d. \( y = x^2 + 1 \)
- e. \( y = (x - 1)^2 \)
- f. \( y = -x^2 \)
1.2 Notetaking with Vocabulary

For use after Lesson 1.2

Core Concepts

Horizontal Translations
The graph of \( y = f(x - h) \) is a horizontal translation of the graph of \( y = f(x) \), where \( h \neq 0 \).

Vertical Translations
The graph of \( y = f(x) + k \) is a vertical translation of the graph of \( y = f(x) \), where \( k \neq 0 \).

Subtracting \( h \) from the inputs before evaluating the function shifts the graph left when \( h < 0 \) and right when \( h > 0 \).

Adding \( k \) to the outputs shifts the graph down when \( k < 0 \) and up when \( k > 0 \).

Notes:
1.2 Notetaking with Vocabulary (continued)

**Reflections in the x-axis**
The graph of \( y = -f(x) \) is a reflection in the x-axis of the graph of \( y = f(x) \).

\[
\begin{align*}
\text{y} & = f(x) \\
\text{y} & = -f(x)
\end{align*}
\]

Multiplying the **outputs** by \(-1\) changes their signs.

**Notes:**

**Reflections in the y-axis**
The graph of \( y = f(-x) \) is a reflection in the y-axis of the graph of \( y = f(x) \).

\[
\begin{align*}
\text{y} & = f(-x) \\
\text{y} & = f(x)
\end{align*}
\]

Multiplying the **inputs** by \(-1\) changes their signs.

**Notes:**

**Horizontal Stretches and Shrinks**
The graph of \( y = f(ax) \) is a horizontal stretch or shrink by a factor of \( \frac{1}{a} \) of the graph of \( y = f(x) \), where \( a > 0 \) and \( a \neq 1 \).

\[
\begin{align*}
\text{y} & = f(ax), \quad a > 1 \\
\text{y} & = f(ax), \quad 0 < a < 1
\end{align*}
\]

The **y-intercept** stays the same.

Multiplying the **inputs** by \( a \) before evaluating the function stretches the graph horizontally (away from the y-axis) when \( 0 < a < 1 \), and shrinks the graph horizontally (toward the y-axis) when \( a > 1 \).

**Notes:**
Vertical Stretches and Shrinks

The graph of \( y = a \cdot f(x) \) is a vertical stretch or shrink by a factor of \( a \) of the graph of \( y = f(x) \), where \( a > 0 \) and \( a \neq 1 \).

Multiplying the outputs by \( a \) stretches the graph vertically (away from the \( x \)-axis) when \( a > 1 \), and shrinks the graph vertically (toward the \( x \)-axis) when \( 0 < a < 1 \).

Notes:

Extra Practice

In Exercises 1–9, write a function \( g \) whose graph represents the indicated transformation of the graph of \( f \). Use a graphing calculator to check your answer.

1. \( f(x) = \frac{1}{3}x \); translation 2 units to the left

2. \( f(x) = -|x + 9| - 1 \); translation 6 units down

3. \( f(x) = -2x + 2 \); translation 7 units down
Notetaking with Vocabulary (continued)

4. \( f(x) = \frac{1}{2}x + 8 \); reflection in the x-axis

5. \( f(x) = 4 + |x + 1| \); reflection in the y-axis

6. \( f(x) = -5x \); vertical shrink by a factor of \( \frac{1}{5} \)

7. \( f(x) = |x + 3| + 2 \); vertical stretch by a factor of 4

8. \( f(x) = 3x - 9 \); horizontal stretch by a factor of 6

9. \( f(x) = -|8x| - 4 \); horizontal shrink by a factor of \( \frac{1}{4} \)

10. Consider the function \( f(x) = |x| \). Write a function \( g \) whose graph represents a reflection in the x-axis followed by a horizontal stretch by a factor of 3 and a translation 5 units down of the graph of \( f \).

11. Which of the transformation(s) in Section 1.2 will not change the y-intercept of \( f(x) = |x| + 3 \)?
1.3 Modeling with Linear Functions

For use with Exploration 1.3

Essential Question  How can you use a linear function to model and analyze a real-life situation?

1 EXPLORATION: Modeling with a Linear Function

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. A company purchases a copier for $12,000. The spreadsheet shows how the copier depreciates over an 8-year period.

a. Write a linear function to represent the value \( V \) of the copier as a function of the number \( t \) of years.

\[
\begin{array}{|c|c|}
\hline
A & B \\
\hline
1 & Year, \( t \) \\
2 & \text{Value, } V \\
3 & 0 \$12,000 \\
4 & 1 \$10,750 \\
5 & 2 \$9,500 \\
6 & 3 \$8,250 \\
7 & 4 \$7,000 \\
8 & 5 \$5,750 \\
9 & 6 \$4,500 \\
10 & 7 \$3,250 \\
11 & 8 \$2,000 \\
\hline
\end{array}
\]

b. Sketch a graph of the function. Explain why this type of depreciation is called straight line depreciation.

Interpret the slope of the graph in the context of the problem.
1.3 Modeling with Linear Functions (continued)

2 EXPLORATION: Modeling with Linear Functions

Work with a partner. Match each description of the situation with its corresponding graph. Explain your reasoning.

a. A person gives $20 per week to a friend to repay a $200 loan.

b. An employee receives $12.50 per hour plus $2 for each unit produced per hour.

c. A sales representative receives $30 per day for food plus $0.565 for each mile driven.

d. A computer that was purchased for $750 depreciates $100 per year.

Communicate Your Answer

3. How can you use a linear function to model and analyze a real-life situation?

4. Use the Internet or some other reference to find a real-life example of straight line depreciation.

   a. Use a spreadsheet to show the depreciation.

   b. Write a function that models the depreciation.

   c. Sketch a graph of the function.
1.3 Notetaking with Vocabulary
For use after Lesson 1.3

In your own words, write the meaning of each vocabulary term.

line of fit

line of best fit

correlation coefficient

Core Concepts
Writing an Equation of a Line

Given slope \(m\) and \(y\)-intercept \(b\)
Use slope-intercept form:

\[ y = mx + b \]

Given slope \(m\) and a point \((x_1, y_1)\)
Use point-slope form:

\[ y - y_1 = m(x - x_1) \]

Given points \((x_1, y_1)\) and \((x_2, y_2)\)
First use the slope formula to find \(m\).
Then use point-slope form with either given point.

Notes:
1.3 Notetaking with Vocabulary (continued)

Finding a Line of Fit

Step 1 Create a scatter plot of the data.

Step 2 Sketch the line that most closely appears to follow the trend given by the data points. There should be about as many points above the line as below it.

Step 3 Choose two points on the line and estimate the coordinates of each point. These points do not have to be original data points.

Step 4 Write an equation of the line that passes through the two points from Step 3. This equation is a model for the data.

Notes:

Extra Practice

In Exercises 1–3, use the graph to write an equation of the line and interpret the slope.

1. Cell Phone Costs

   - (0, 50)
   - (10, 51)

   Cost (dollars)

   - 0
   - 10
   - 20
   - 52

   Number of text messages sent

2. Travel Distance

   - (0, 360)
   - (6, 0)

   Distance (miles)

   - 0
   - 2
   - 6
   - 320

   Time (hours)

3. Math Homework Grade

   - (10, 85)
   - (20, 95)

   Homework grade (percent)

   - 0
   - 10
   - 20
   - 120

Completed assignments
4. The cost of parking in a parking garage in Chicago is represented by the equation 
   \[ y = 15x + 20 \]  
   where \( y \) is the total cost (in dollars) and \( x \) is the time (in hours). The table 
   shows the total cost to park in a parking garage in Denver. Which city’s parking garage 
   charges more per hour and by how much more? After how many hours would parking in 
   both cities cost the same?

   \[
   \begin{array}{c|cccc}
   \text{Hours, } x & 2 & 3 & 4 & 5 \\
   \text{Cost, } y & 43 & 51 & 59 & 67 \\
   \end{array}
   \]

   In Exercises 5–7, use the linear regression feature on a graphing calculator to find an 
   equation of the line of best fit for the data. Find and interpret the correlation coefficient.

   5. 
   6. 
   7.
1.4 Solving Linear Systems
For use with Exploration 1.4

Essential Question  How can you determine the number of solutions of a linear system?

A linear system is consistent when it has at least one solution. A linear system is inconsistent when it has no solution.

1  EXPLORATION: Recognizing Graphs of Linear Systems

Work with a partner. Match each linear system with its corresponding graph. Explain your reasoning. Then classify the system as consistent or inconsistent.

a. \[2x - 3y = 3\]
   \[-4x + 6y = 6\]

b. \[2x - 3y = 3\]
   \[x + 2y = 5\]

c. \[2x - 3y = 3\]
   \[-4x + 6y = -6\]

A.

B.

C.

2  EXPLORATION: Solving Systems of Linear Equations

Work with a partner. Solve each linear system by substitution or elimination. Then use the graph of the system on the next page to check your solution.

a. \[2x + y = 5\]
   \[x - y = 1\]

b. \[x + 3y = 1\]
   \[-x + 2y = 4\]

c. \[x + y = 0\]
   \[3x + 2y = 1\]
Communicate Your Answer

3. How can you determine the number of solutions of a linear system?

4. Suppose you were given a system of three linear equations in three variables. Explain how you would approach solving such a system.

5. Apply your strategy in Question 4 to solve the linear system.

\[
\begin{align*}
x + y + z &= 1 \quad &\text{Equation 1} \\
x - y - z &= 3 \quad &\text{Equation 2} \\
-x - y + z &= -1 \quad &\text{Equation 3}
\end{align*}
\]
1.4 Notetaking with Vocabulary
For use after Lesson 1.4

In your own words, write the meaning of each vocabulary term.

linear equation in three variables

system of three linear equations

solution of a system of three linear equations

ordered triple

Core Concepts
Solving a Three-Variable System

Step 1  Rewrite the linear system in three variables as a linear system in two variables by using the substitution or elimination method.

Step 2  Solve the new linear system for both of its variables.

Step 3  Substitute the values found in Step 2 into one of the original equations and solve for the remaining variable.

When you obtain a false equation, such as $0 = 1$, in any of the steps, the system has no solution.

When you do not obtain a false equation, but obtain an identity such as $0 = 0$, the system has infinitely many solutions.

Notes:
Extra Practice

In Exercises 1–3, solve the system using the elimination method.

1. \[ \begin{align*}
    x + 2y - 3z &= 11 \\
    2x + y - 2z &= 9 \\
    4x + 3y + z &= 16
\end{align*} \]

2. \[ \begin{align*}
    x - y + 3z &= 19 \\
    -2x + 2y - 6z &= 9 \\
    3x + 5y - 2z &= 3
\end{align*} \]

3. \[ \begin{align*}
    x + y - z &= -9 \\
    2x - 3y + 2z &= 13 \\
    3x - 5y - 6z &= -15
\end{align*} \]

In Exercises 4–6, solve the system using the substitution method.

4. \[ \begin{align*}
    x + y + z &= 4 \\
    x + y - z &= 4 \\
    3x + 3y + z &= 12
\end{align*} \]

5. \[ \begin{align*}
    2x + 3y - z &= 9 \\
    x - 3y + z &= -6 \\
    3x + y - 4z &= 31
\end{align*} \]

6. \[ \begin{align*}
    x + 2y - 5z &= -12 \\
    2x + 2y - 3z &= -2 \\
    3x - 4y - z &= 11
\end{align*} \]

7. You found $6.60 on the ground at school, all in nickels, dimes, and quarters. You have twice as many quarters as dimes and 42 coins in all. How many of each type of coin do you have?
1.4 Notetaking with Vocabulary (continued)

8. Find the values of $a$, $b$, and $c$ so that the linear system below has $(3, -2, 1)$ as its only solution. Explain your reasoning.

\[
\begin{align*}
3x + 2y - 7z &= a \\
x + 3y + z &= b \\
4x - 2y - z &= c
\end{align*}
\]

9. Does the system of linear equations have more than one solution? Justify your answer.

\[
\begin{align*}
\frac{1}{2}x - \frac{3}{8}y + \frac{1}{8}z &= -\frac{5}{4} \\
\frac{1}{2}x + \frac{1}{4}y + \frac{3}{4}z &= 0 \\
-x + 2y - 5z &= 17
\end{align*}
\]

10. If $\angle A$ is three times as large as $\angle B$, and $\angle B$ is $30^\circ$ smaller than $\angle C$, what are the measures of angles $A$, $B$, and $C$?

$A$

$B$

$C$