Name_____ Date _____

# Chapter 11 Maintaining Mathematical Proficiency

**The table shows the results of a survey. Display the data in a histogram.**

**1.**

| Movies attended last month | Frequency |
|---|---|
| 0–1 | 16 |
| 2–3 | 12 |
| 4–5 | 8 |

**2.**

| Hours of homework | Frequency |
|---|---|
| 0–1 | 8 |
| 2–3 | 15 |
| 4–5 | 4 |
| 6–7 | 1 |

**The table shows the results of a survey. Display the data in a circle graph.**

**3.**

| Favorite ice cream flavor | Vanilla | Chocolate | Strawberry | Butter Pecan |
|---|---|---|---|---|
| Students | 5 | 6 | 4 | 3 |

**4.**

| Favorite Sport | Baseball | Tennis | Basketball | Soccer | Golf |
|---|---|---|---|---|---|
| Students | 10 | 4 | 8 | 7 | 2 |

Name _____ Date _____

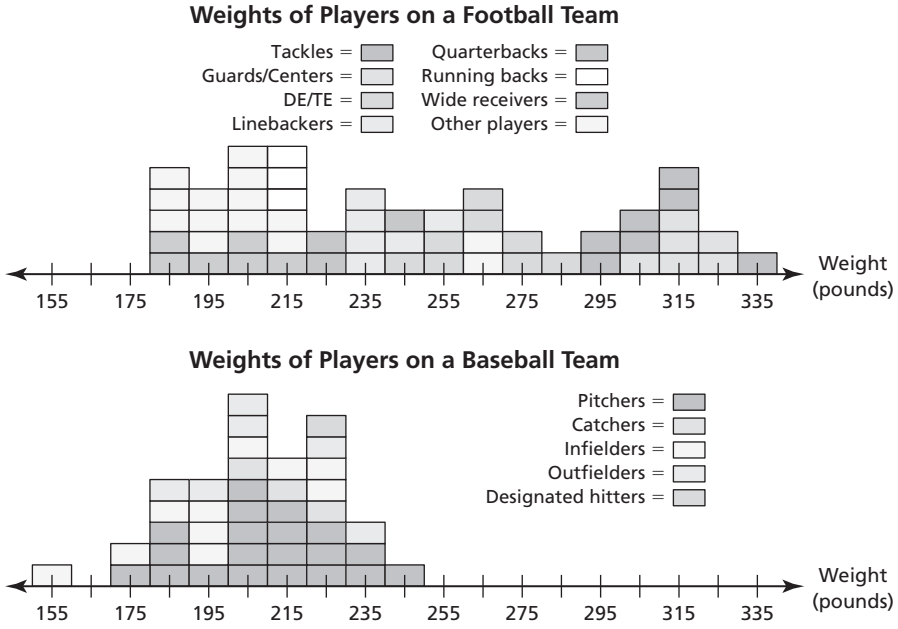
# 11.1 Measures of Center and Variation

**For use with Exploration 11.1**

**Essential Question** How can you describe the variation of a data set?

## 1 EXPLORATION: Describing the Variation of Data

**Work with a partner.** The graphs show the weights of the players on a professional football team and a professional baseball team.




**a.** Describe the data in each graph in terms of how much the weights vary from the mean. Explain your reasoning.

**b.** Compare how much the weights of the players on the football team vary from the mean to how much the weights of the players on the baseball team vary from the mean.

**c.** Does there appear to be a correlation between the body weights and the positions of players in professional football? in professional baseball? Explain.

## 11.1 Measures of Center and Variation (continued)

### 2 EXPLORATION: Describing the Variation of Data

**Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.**

**Work with a partner.** The weights (in pounds) of the players on a professional basketball team by position are as follows.

Power forwards: 235, 255, 295, 245; small forwards: 235, 235; centers: 255, 245, 325; point guards: 205, 185, 205; shooting guards: 205, 215, 185.

Make a graph that represents the weights and positions of the players. Does there appear to be a correlation between the body weights and the positions of players in professional basketball? Explain your reasoning

### Communicate Your Answer

**3.** How can you describe the variation of a data set?

Name _____ Date _____

# 11.1 Notetaking with Vocabulary

**For use after Lesson 11.1**

**In your own words, write the meaning of each vocabulary term.**

measure of center

mean

median

mode

outlier

measure of variation

range

standard deviation

data transformation

**Notes:**

Name_____ Date _____

## 11.1 Notetaking with Vocabulary (continued)

## Core Concepts

### Mean

The **mean** of a numerical data set is the sum of the data divided by the number of data values. The symbol $\bar{x}$ represents the mean. It is read as "$x$-bar."

### Median

The **median** of a numerical data set is the middle number when the values are written in numerical order. When a data set has an even number of values, the median is the mean of the two middle values.

### Mode

The **mode** of a data set is the value or values that occur most often. There may be one mode, no mode, or more than one mode.

### Notes:

### Standard Deviation

The **standard deviation** of a numerical data set is a measure of how much a typical value in the data set differs from the mean. The symbol $\sigma$ represents the standard deviation. It is read as "sigma." It is given by

$$\sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}}$$

where $n$ is the number of values in the data set. The deviation of a data value $x$ is the difference of the data value and the mean of the data set, $x - \bar{x}$.

**Step 1** Find the mean, $\bar{x}$.

**Step 2** Find the deviation of each data value, $x - \bar{x}$.

**Step 3** Square each deviation, $(x - \bar{x})^2$.

**Step 4** Find the mean of the squared deviations. This is called the *variance*.

**Step 5** Take the square root of the variance.

### Notes:

Name _____ Date _____

## 11.1 Notetaking with Vocabulary (continued)

### Data Transformations Using Addition

When a real number $k$ is added to each value in a numerical data set

- the measures of center of the new data set can be found by adding $k$ to the original measures of center.
- the measures of variation of the new data set are the *same* as the original measures of variation.

### Data Transformations Using Multiplication

When each value in a numerical data set is multiplied by a real number $k$, where $k > 0$, the measures of center and variation can be found by multiplying the original measures by $k$.

### Notes:

## Extra Practice

**1.** Consider the data set: 2, 5, 16, 2, 2, 7, 3, 4, 4.

**a.** Find the mean, median, and mode of the data set.

**b.** Determine which measure of center best represents the data. Explain.

**2.** The table shows the masses of eight gorillas.

| Masses (kilograms) | | | | | | | |
|---|---|---|---|---|---|---|---|
| 160 | 157 | 162 | 158 | 44 | 160 | 159 | 161 |

**a.** Identify the outlier. How does the outlier affect the mean, median, and mode?

**b.** Describe one possible explanation for the outlier.

Name_____ Date _____

## 11.1 Notetaking with Vocabulary (continued)

**3.** The heights of the members of two girls' basketball teams are shown. Find the range of the heights for each team. Compare your results.

| Team A Heights (inches) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 58 | 75 | 60 | 48 | 56 | 78 | 60 | 57 | 54 | 59 |

| Team B Heights (inches) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 49 | 50 | 70 | 56 | 58 | 66 | 64 | 57 | 62 | 63 |

**4.** Consider the data in Exercise 3.

**a.** Find the standard deviation of the heights of Team A. Interpret your result.

**b.** Find the standard deviation of the heights of Team B. Interpret your result.

**c.** Compare the standard deviations for Team A and Team B. What can you conclude?

**5.** Find the values of the measures shown when each value in the data set increases by 8.

Mean: 42 Median: 40 Mode: 38
Range: 15 Standard deviation: 4.9

Name _____ Date _____

# 11.2 Box-and-Whisker Plots

**For use with Exploration 11.2**

**Essential Question** How can you use a box-and-whisker plot to describe a data set?

## 1 EXPLORATION: Drawing a Box-and-Whisker Plot

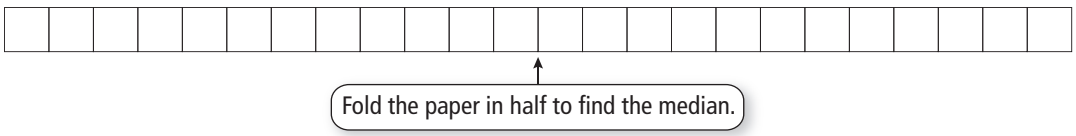
**Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.**

**Work with a partner.** The numbers of first cousins of the students in a ninth-grade class are shown. A *box-and-whisker plot* is one way to represent the data visually.

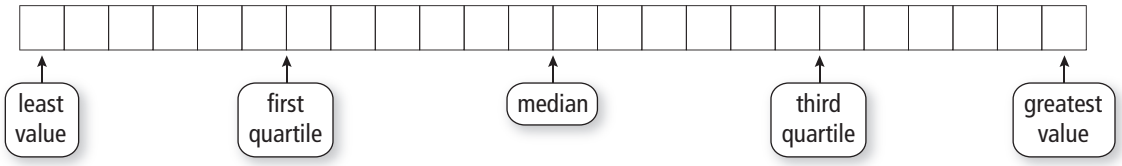
**Numbers of First Cousins**

| | | | |
|---|---|---|---|
| 3 | 10 | 18 | 8 |
| 9 | 3 | 0 | 32 |
| 23 | 19 | 13 | 8 |
| 6 | 3 | 3 | 10 |
| 12 | 45 | 1 | 5 |
| 13 | 24 | 16 | 14 |

**a.** Order the data on a strip of grid paper with 24 equally spaced boxes.



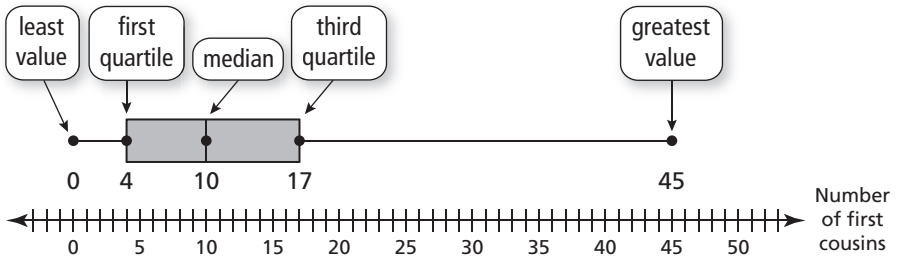

**b.** Fold the paper in half again to divide the data into four groups. Because there are 24 numbers in the data set, each group should have 6 numbers. Find the least value, the greatest value, the first quartile, and the third quartile.



## 11.2 Box-and-Whisker Plots (continued)

### 1 EXPLORATION: Drawing a Box-and-Whisker Plot (continued)

**c.** Explain how the box-and-whisker plot shown represents the data set.

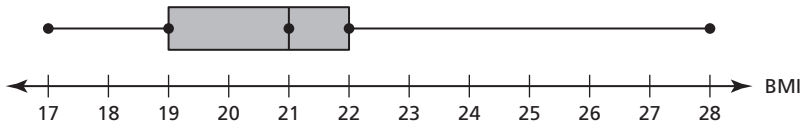



## Communicate Your Answer

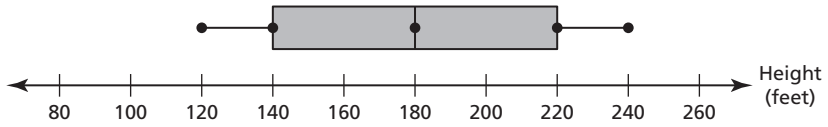
**2.** How can you use a box-and-whisker plot to describe a data set?

**3.** Interpret each box-and-whisker plot.

**a.** body mass indices (BMI) of students in a ninth-grade class




**b.** heights of roller coasters at an amusement park



Name _____ Date _____

# 11.2 Notetaking with Vocabulary

**For use after Lesson 11.2**

**In your own words, write the meaning of each vocabulary term.**

box-and-whisker plot

quartile

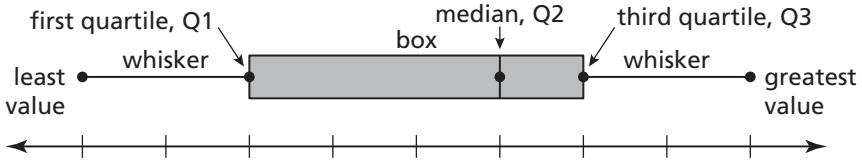
five-number summary

interquartile range

## Core Concepts

### Box-and-Whisker Plot

A **box-and-whisker plot** shows the variability of a data set along a number line using the least value, the greatest value, and the *quartiles* of the data. **Quartiles** divide the data set into four equal parts. The median (second quartile, Q2) divides the data set into two halves. The median of the lower half is the first quartile, Q1. The median of the upper half is the third quartile, Q3.



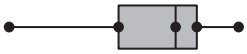

The five numbers that make up a box-and-whisker plot are called the **five-number summary** of the data set.

### Notes:

Name_____ Date _____

## 11.2 Notetaking with Vocabulary (continued)

### Shapes of Box-and-Whisker Plots

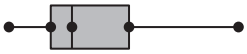


**Skewed left**

- The left whisker is longer than the right whisker.
- Most of the data are on the right side of the plot.

**Symmetric**

- The whiskers are about the same length.
- The median is in the middle of the plot.



**Skewed right**

- The right whisker is longer than the left whisker.
- Most of the data are on the left side of the plot.

### Notes:

## Extra Practice

**In Exercises 1 and 2, make a box-and-whisker plot that represents the data.**

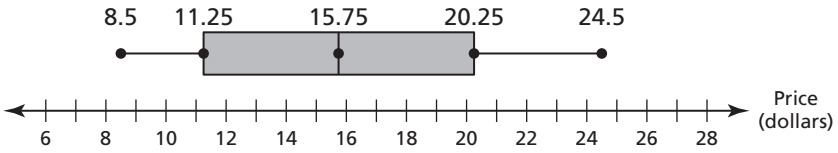
**1.** Hours of sleep: 7, 9, 8, 8, 8, 6, 6, 5, 4

**2.** Algebra test scores: 71, 92, 84, 76, 88, 96, 84, 63, 82

Name _____ Date _____

## 11.2 Notetaking with Vocabulary (continued)

**3.** The box-and-whisker plot represents the prices (in dollars) of soccer balls at different sporting goods stores.



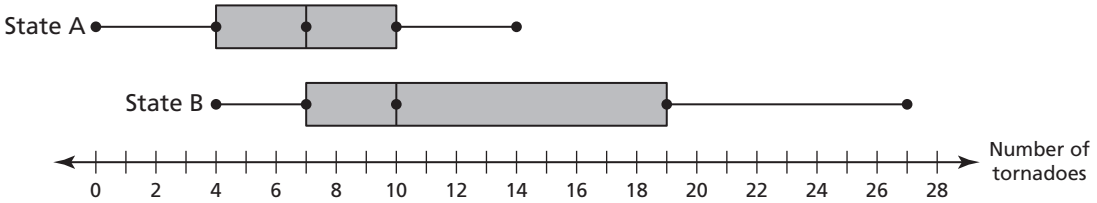

**a.** Find and interpret the range of the data.

**b.** Describe the distribution of the data.

**c.** Find and interpret the interquartile range of the data.

**d.** Are the data more spread out below Q1 or above Q3? Explain.

**4.** The double box-and-whisker plot represents the number of tornados per month for a year for two states.




**a.** Identify the shape of each distribution.

**b.** Which state's tornadoes are more spread out? Explain.

**c.** Which state had the single least number of tornadoes in a month during the year? Explain.

Name_____ Date _____

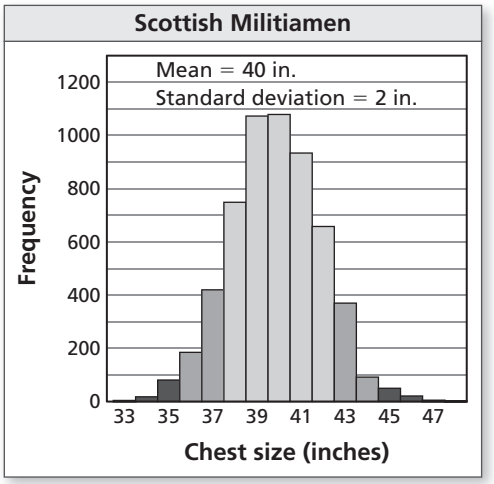
# 11.3 Shapes of Distributions

**For use with Exploration 11.3**

**Essential Question** How can you use a histogram to characterize the basic shape of a distribution?

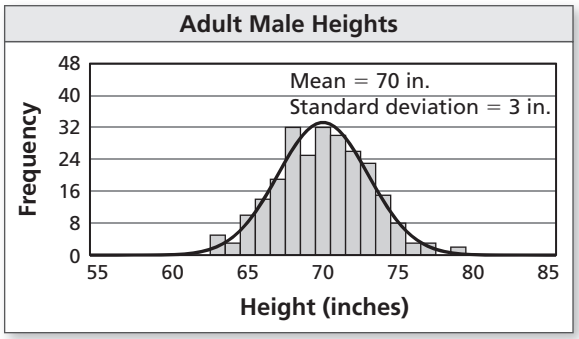
## 1 EXPLORATION: Analyzing a Famous Symmetric Distribution

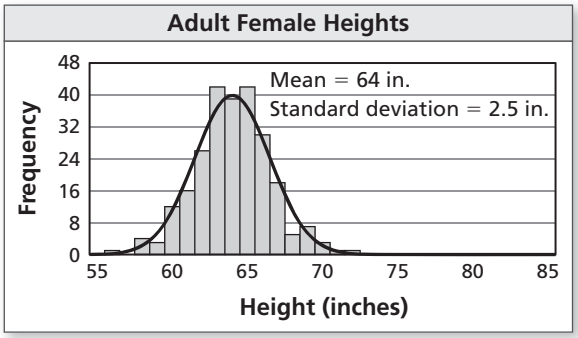
**Work with a partner.** A famous data set was collected in Scotland in the mid-1800s. It contains the chest sizes, measured in inches, of 5738 men in the Scottish Militia. Estimate the percent of the chest sizes that lie within (a) 1 standard deviation of the mean, (b) 2 standard deviations of the mean, and (c) 3 standard deviations of the mean. Explain your reasoning.



## 2 EXPLORATION: Comparing Two Symmetric Distributions

**Work with a partner.** The graphs show the distributions of the heights of 250 adult American males and 250 adult American females.




**a.** Which data set has a smaller standard deviation? Explain what this means in the context of the problem.

**b.** Estimate the percent of male heights between 67 inches and 73 inches.

## Communicate Your Answer

**3.** How can you use a histogram to characterize the basic shape of a distribution?

**4.** All three distributions in Explorations 1 and 2 are roughly symmetric. The histograms are called "bell-shaped."

**a.** What are the characteristics of a symmetric distribution?

**b.** Why is a symmetric distribution called "bell-shaped?"

**c.** Give two other real-life examples of symmetric distributions.

Name_____ Date _____

# 11.3 Notetaking with Vocabulary

**For use after Lesson 11.3**

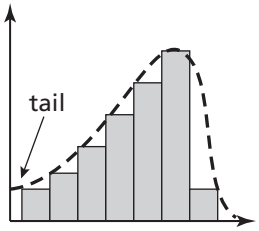
**In your own words, write the meaning of each vocabulary term.**

histogram

frequency table

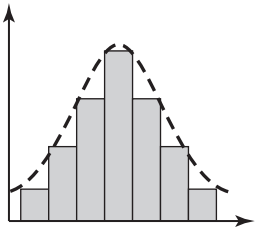
## Core Concepts

### Symmetric and Skewed Distributions



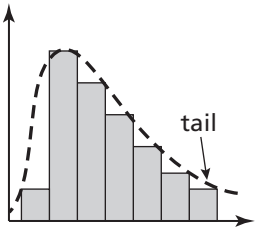

**Skewed left**

- The "tail" of the graph extends to the left.
- Most of the data are on the right.



**Symmetric**

- The data on the right of the distribution are approximately a mirror image of the data on the left of the distribution.




**Skewed right**

- The "tail" of the graph extends to the right.
- Most of the data are on the left.

### Notes:

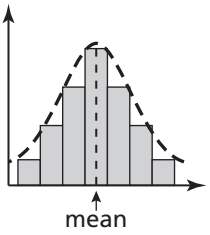
Name _____ Date _____

## 11.3 Notetaking with Vocabulary (continued)

### Choosing Appropriate Measures

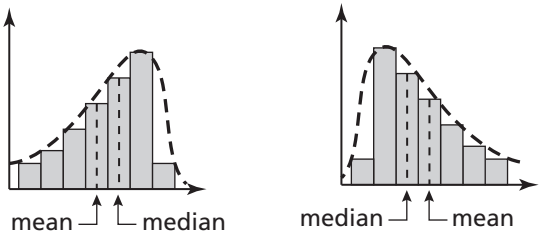
When a data distribution is symmetric,

- use the mean to describe the center and
- use the standard deviation to describe the variation.




When a data distribution is skewed,

- use the median to describe the center and
- use the five-number summary to describe the variation.




**Notes:**

## Extra Practice

**1.** The table shows the average annual snowfall (in inches) of 26 cities.

| Average Annual Snowfall (inches) | | |
|---|---|---|
| 22 | 68 | 33 |
| 15 | 28 | 31 |
| 20 | 18 | 30 |
| 15 | 54 | 16 |
| 44 | 43 | 17 |
| 95 | 41 | 30 |
| 29 | 23 | 47 |
| 37 | 26 | 54 |
| 16 | 30 | |

**a.** Display the data in a histogram using six intervals beginning with 15–28.

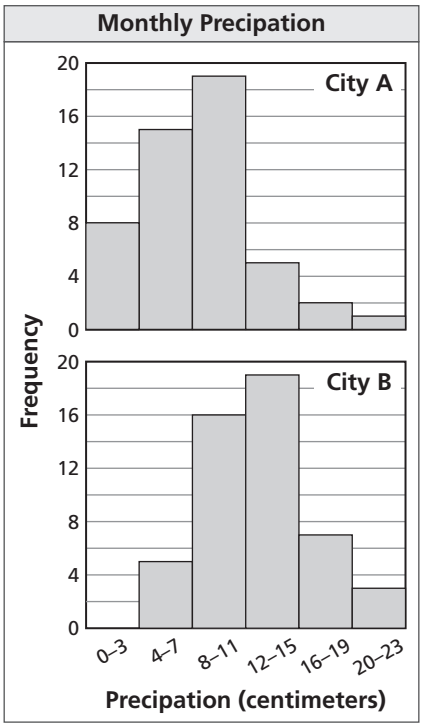
**b.** Which measures of center and variation best represent the data? Explain.

**c.** A weather station lists the top 20 snowiest major cities. The city in 20th place had 51 inches of snow. How would you interpret the data?

Name_____ Date _____

## 11.3 Notetaking with Vocabulary (continued)

**2.** The double histogram shows the distributions of monthly precipitation for two towns over a 50-month period. Compare the distributions using their shapes and appropriate measures of center and variation.




**3.** The table shows the results of a survey that asked high school students how many hours a week they listen to music.

| | Females | Males |
|---|---|---|
| **Survey size** | 50 | 58 |
| **Minimum** | 16 | 18 |
| **Maximum** | 40 | 52 |
| **1st Quartile** | 24 | 30 |
| **Median** | 28 | 38 |
| **3rd Quartile** | 32 | 46 |
| **Mean** | 28 | 30 |
| **Standard deviation** | 6 | 12 |

**a.** Make a double box-and-whisker plot that represents the data. Describe the shape of each distribution.

**b.** Compare the number of hours of music listened to by females to the number of hours of music listened to by males.

**c.** About how many females surveyed would you expect to listen to music between 22 and 34 hours per week?

**d.** If you survey 100 more females, about how many would you expect to listen to music between 16 and 40 hours per week?

Name _____ Date _____

# 11.4 Two-Way Tables

**For use with Exploration 11.4**

**Essential Question** How can you read and make a two-way table?

## 1 EXPLORATION: Reading a Two-Way Table

**Work with a partner.** You are the manager of a sports shop. The two-way tables show the numbers of soccer T-shirts in stock at your shop at the beginning and end of the selling season. (a) Complete the totals for the rows and columns in each table. (b) How would you alter the number of T-shirts you order for next season? Explain your reasoning.

**Beginning of season**

| | | T-Shirt Size | | | | | |
|---|---|---|---|---|---|---|---|
| | | S | M | L | XL | XXL | Total |
| Color | blue/white | 5 | 6 | 7 | 6 | 5 | |
| | blue/gold | 5 | 6 | 7 | 6 | 5 | |
| | red/white | 5 | 6 | 7 | 6 | 5 | |
| | black/white | 5 | 6 | 7 | 6 | 5 | |
| | black/gold | 5 | 6 | 7 | 6 | 5 | |
| | Total | | | | | | 145 |

**End of season**

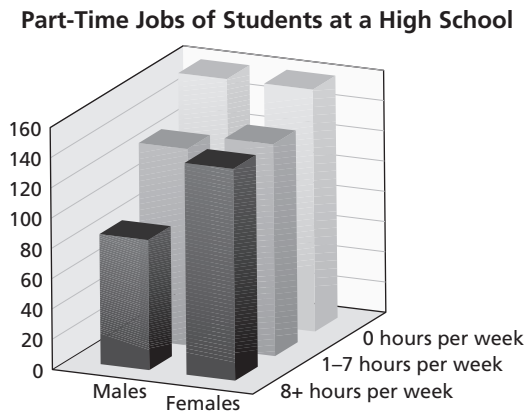
| | | T-Shirt Size | | | | | |
|---|---|---|---|---|---|---|---|
| | | S | M | L | XL | XXL | Total |
| Color | blue/white | 5 | 4 | 1 | 0 | 2 | |
| | blue/gold | 3 | 6 | 5 | 2 | 0 | |
| | red/white | 4 | 2 | 4 | 1 | 3 | |
| | black/white | 3 | 4 | 1 | 2 | 1 | |
| | black/gold | 5 | 2 | 3 | 0 | 2 | |
| | Total | | | | | | |

## 11.4 Two-Way Tables (continued)

### 2 EXPLORATION: Making a Two-Way Table

**Work with a partner.** The three-dimensional bar graph shows the numbers of hours students work at part-time jobs.

**a.** Make a two-way table showing the data. Use estimation to find the entries in your table.




**b.** Write two observations that summarize the data in your table.

## Communicate Your Answer

**3.** How can you read and make a two-way table?

Name _____ Date _____

# 11.4 Notetaking with Vocabulary

**For use after Lesson 11.4**

**In your own words, write the meaning of each vocabulary term.**

two-way table

joint frequency

marginal frequency

joint relative frequency

marginal relative frequency

conditional relative frequency

## Core Concepts

### Relative Frequencies

A **joint relative frequency** is the ratio of a frequency that is not in the "total" row or the "total" column to the total number of values or observations.

A **marginal relative frequency** is the sum of the joint relative frequencies in a row or column.

When finding relative frequencies in a two-way table, you can use the corresponding decimals or percents.

### Notes:

Name_____ Date _____

## 11.4 Notetaking with Vocabulary (continued)

### Conditional Relative Frequencies

A **conditional relative frequency** is the ratio of a joint relative frequency to the marginal relative frequency. You can find a conditional relative frequency using a row total or a column total of a two-way table.

### Notes:

### Extra Practice

**In Exercises 1 and 2, find and interpret the marginal frequencies.**

**1.**

| | | Attend College | |
|---|---|---|---|
| | | **Yes** | **No** |
| **Gender** | **Male** | 98 | 132 |
| | **Female** | 120 | 88 |

**2.**

| | | Own a Car | |
|---|---|---|---|
| | | **Yes** | **No** |
| **Gender** | **Male** | 54 | 136 |
| | **Female** | 45 | 137 |

**3.** You conduct a survey that asks 85 students in your school whether they are in Math Club or Chess Club. Thirty-five of the students are in Math Club, and 20 of those students are also in Chess Club. Thirty-eight of the students are not in Math Club or Chess Club. Organize the results in a two-way table. Include the marginal frequencies.

Name _____ Date _____

## 11.4 Notetaking with Vocabulary (continued)

**4.** Make a two-way table that shows the joint and marginal relative frequencies.

| | | Read *Catcher in the Rye* | |
|---|---|---|---|
| | | **Yes** | **No** |
| **Gender** | **Male** | 96 | 80 |
| | **Female** | 54 | 88 |

**5.** A company is organizing a baseball game for their employees. The employees are asked whether they prefer to attend a day game or a night game. They are also asked whether they prefer to sit in the upper deck or lower deck. The results are shown in a two-way table. Make a two-way table that shows the conditional relative frequencies based on the row totals. Given that an employee prefers to go to a day game, what is the conditional relative frequency that he or she prefers to sit in the lower deck?

| | | Seat | |
|---|---|---|---|
| | | **Upper** | **Lower** |
| **Game Time** | **Day** | 28 | 34 |
| | **Night** | 22 | 52 |

Name_____ Date _____

# 11.5 Choosing a Data Display

**For use with Exploration 11.5**

**Essential Question** How can you display data in a way that helps you make decisions?

## 1 EXPLORATION: Displaying Data

**Work with a partner.** Analyze the data and then create a display that best represents the data. Explain your choice of data display.

**a.** A group of schools in New England participated in a 2-month study and reported 3962 animals found dead along roads.

birds: 307

mammals: 2746

amphibians: 145

reptiles: 75

unknown: 689

**b.** The data below show the numbers of black bears killed on a state's roads from 1993 to 2012.

| | |
|---|---|
| 1993: 30 | 2003: 74 |
| 1994: 37 | 2004: 88 |
| 1995: 46 | 2005: 82 |
| 1996: 33 | 2006: 109 |
| 1997: 43 | 2007: 99 |
| 1998: 35 | 2008: 129 |
| 1999: 43 | 2009: 111 |
| 2000: 47 | 2010: 127 |
| 2001: 49 | 2011: 141 |
| 2002: 61 | 2012: 135 |

**c.** A 1-week study along a 4-mile section of road found the following weights (in pounds) of raccoons that had been killed by vehicles.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 13.4 | 14.8 | 17.0 | 12.9 | 21.3 | 21.5 | 16.8 | 14.8 |
| 15.2 | 18.7 | 18.6 | 17.2 | 18.5 | 9.4 | 19.4 | 15.7 |
| 14.5 | 9.5 | 25.4 | 21.5 | 17.3 | 19.1 | 11.0 | 12.4 |
| 20.4 | 13.6 | 17.5 | 18.5 | 21.5 | 14.0 | 13.9 | 19.0 |

## 11.5 Choosing a Data Display (continued)

### 1 EXPLORATION: Displaying Data (continued)

**d.** A yearlong study by volunteers in California reported the following numbers of animals killed by motor vehicles.

| | |
|---|---|
| raccoons: 1693 | gray squirrels: 715 |
| skunks: 1372 | cottontail rabbits: 629 |
| ground squirrels: 845 | barn owls: 486 |
| opossum: 763 | jackrabbits: 466 |
| deer: 761 | gopher snakes: 363 |

## Communicate Your Answer

**2.** How can you display data in a way that helps you make decisions?

**3.** Use the Internet or some other reference to find examples of the following types of data displays.

| | | |
|---|---|---|
| bar graph | circle graph | scatter plot |
| stem-and-leaf plot | pictograph | line graph |
| box-and-whisker plot | histogram | dot plot |

Name_____ Date _____

# 11.5 Notetaking with Vocabulary

**For use after Lesson 11.5**

**In your own words, write the meaning of each vocabulary term.**

qualitative (categorical) data

quantitative data

misleading graph

## Core Concepts

### Types of Data

**Qualitative data,** or **categorical data,** consist of labels or nonnumerical entries that can be separated into different categories. When using qualitative data, operations such as adding or finding a mean do not make sense.

**Quantitative data** consist of numbers that represent counts or measurements.

### Notes:

Name _____ Date _____

## 11.5 Notetaking with Vocabulary (continued)

## Extra Practice

**In Exercises 1–4, tell whether the data are *qualitative* or *quantitative*. Explain your reasoning.**

**1.** bookmarks in your web browser

**2.** heights of players on a basketball team

**3.** the number of kilobytes in a downloaded file

**4.** FM radio station numbers

**In Exercises 5 and 6, analyze the data and then create a display that best represents the data. Explain your reasoning.**

**5.**

| Home Runs Each Year | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Babe Ruth | | | | | | Hank Aaron | | | | | | |
| 0 | 4 | 3 | 2 | 11 | 29 | 13 | 27 | 26 | 44 | 30 | 39 |
| 54 | 59 | 35 | 41 | 46 | 25 | 40 | 34 | 45 | 44 | 24 | 32 |
| 47 | 60 | 54 | 46 | 49 | 46 | 44 | 39 | 29 | 44 | 38 | 47 |
| 41 | 34 | 22 | 6 | | | 34 | 40 | 20 | 12 | 10 | |

## 11.5 Notetaking with Vocabulary (continued)

**6.**

| Total Points Scored by a Basketball Team for Each Game | | | | | |
|---|---|---|---|---|---|
| 48 | 56 | 49 | 52 | 40 | 65 |
| 30 | 47 | 62 | 40 | 59 | 37 |
| 45 | 41 | 44 | 33 | 44 | 30 |

**In Exercises 7 and 8, describe how the graph is misleading. Then explain how someone might misinterpret the graph.**

**7.**


**8.**