10.4 Inverse of a Function
For use with Exploration 10.4

Essential Question  How are a function and its inverse related?

1 EXPLORATION: Exploring Inverse Functions

Work with a partner. The functions \( f \) and \( g \) are inverses of each other. Compare the tables of values of the two functions. How are the functions related?

\[
\begin{array}{c|cccccccc}
 x & 0 & 0.5 & 1 & 1.5 & 2 & 2.5 & 3 & 3.5 \\
f(x) & 0 & 0.25 & 1 & 2.25 & 4 & 6.25 & 9 & 12.25 \\
\end{array}
\]

\[
\begin{array}{c|cccccccc}
 x & 0 & 0.25 & 1 & 2.25 & 4 & 6.25 & 9 & 12.25 \\
g(x) & 0 & 0.5 & 1 & 1.5 & 2 & 2.5 & 3 & 3.5 \\
\end{array}
\]

2 EXPLORATION: Exploring Inverse Functions

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner.

a. Plot the two sets of points represented by the tables in Exploration 1. Use the coordinate plane below.

b. Connect each set of points with a smooth curve.

c. Describe the relationship between the two graphs.

d. Write an equation for each function.
Communicate Your Answer

3. How are a function and its inverse related?

4. A table of values for a function $f$ is given. Create a table of values for a function $g$, the inverse of $f$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x$</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$g(x)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Sketch the graphs of $f(x) = x + 4$ and its inverse in the same coordinate plane. Then write an equation of the inverse of $f$. Explain your reasoning.
Notetaking with Vocabulary

For use after Lesson 10.4

In your own words, write the meaning of each vocabulary term.

inverse relation

inverse function

Core Concepts

Inverse Relation

When a relation contains \((a, b)\), the inverse relation contains \((b, a)\).

Notes:

Finding Inverses of Functions Algebraically

Step 1 Set \(y\) equal to \(f(x)\).

Step 2 Switch \(x\) and \(y\) in the equation.

Step 3 Solve the equation for \(y\).

Notes:

Horizontal Line Test

The inverse of a function \(f\) is also a function if and only if no horizontal line intersects the graph of \(f\) more than once.

Notes:
Extra Practice

In Exercises 1 and 2, find the inverse of the relation.

1. \((1, -1), (2, 5), (4, -2), (6, 8), (8, 9)\)

2. 

<table>
<thead>
<tr>
<th>Input</th>
<th>-3</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

In Exercises 3–5, solve \(y = f(x)\) for \(x\). Then find the input when the output is 3.

3. \(f(x) = x + 3\)

4. \(f(x) = 3x - 2\)

5. \(f(x) = 4x^2\)

In Exercises 6–11, find the inverse of the function. Then graph the function and its inverse.

6. \(f(x) = 3x - 1\)

7. \(f(x) = -3x + 2\)

8. \(f(x) = \frac{1}{2}x + 2\)
10.4 Notetaking with Vocabulary (continued)

9. \(f(x) = 2x^2, x \geq 0\)  
10. \(f(x) = -x^2 + 5, x \leq 0\)  
11. \(f(x) = 16x^2 + 3, x \geq 0\)

In Exercises 12–17, determine whether the inverse of \(f\) is a function. Then find the inverse.

12. \(f(x) = \sqrt{x} + 4\)  
13. \(f(x) = \sqrt{3x} - 9\)  
14. \(f(x) = 2\sqrt{x} - 4\)

15. \(f(x) = 3x^2\)  
16. \(f(x) = 5x^2 - 1\)  
17. \(f(x) = -\sqrt{2x + 3} - 5\)