9.5 Solving Quadratic Equations Using the Quadratic Formula

For use with Exploration 9.5

Essential Question  How can you derive a formula that can be used to write the solutions of any quadratic equation in standard form?

1 EXPLORATION: Deriving the Quadratic Formula

Work with a partner. The following steps show a method of solving $ax^2 + bx + c = 0$. Explain what was done in each step.

1. Write the equation.

$$ax^2 + bx + c = 0$$

2. _____________________________

$$4a^2x^2 + 4abx + 4ac = 0$$

3. _____________________________

$$4a^2x^2 + 4abx + 4ac + b^2 = b^2$$

4. _____________________________

$$4a^2x^2 + 4abx + b^2 = b^2 - 4ac$$

5. _____________________________

$$(2ax + b)^2 = b^2 - 4ac$$

6. _____________________________

$$2ax + b = \pm \sqrt{b^2 - 4ac}$$

7. _____________________________

$$2ax = -b \pm \sqrt{b^2 - 4ac}$$

8. _____________________________

Quadratic Formula:  $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
9.5 Solving Quadratic Equations Using the Quadratic Formula (continued)

Work with a partner.

a. Solve \( ax^2 + bx + c = 0 \) by completing the square. (Hint: Subtract \( c \) from each side, divide each side by \( a \), and then proceed by completing the square.)

b. Compare this method with the method in Exploration 1. Explain why you think \( 4a \) and \( b^2 \) were chosen in Steps 2 and 3 of Exploration 1.

Communicate Your Answer

3. How can you derive a formula that can be used to write the solutions of any quadratic equation in standard form?

4. Use the Quadratic Formula to solve each quadratic equation.
   a. \( x^2 + 2x - 3 = 0 \)  
   b. \( x^2 - 4x + 4 = 0 \)  
   c. \( x^2 + 4x + 5 = 0 \)

5. Use the Internet to research imaginary numbers. How are they related to quadratic equations?
In your own words, write the meaning of each vocabulary term.

Quadratic Formula

discriminant

Core Concepts

Quadratic Formula

The real solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $a \neq 0$ and $b^2 - 4ac \geq 0$.

Notes:

Interpreting the Discriminant

- $b^2 - 4ac > 0$  
  - two real solutions
  - two $x$-intercepts

- $b^2 - 4ac = 0$  
  - one real solution
  - one $x$-intercept

- $b^2 - 4ac < 0$  
  - no real solutions
  - no $x$-intercepts

Notes:
Methods for Solving Quadratic Equations

<table>
<thead>
<tr>
<th>Method</th>
<th>Advantages</th>
<th>Disadvantages</th>
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<tbody>
<tr>
<td>Factoring (Lessons 7.5–7.8)</td>
<td>• Straightforward when the equation can be factored easily</td>
<td>• Some equations are not factorable.</td>
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<tr>
<td>Graphing (Lesson 9.2)</td>
<td>• Can easily see the number of solutions</td>
<td>• May not give exact solutions</td>
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<td></td>
<td>• Use when approximate solutions are sufficient.</td>
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<td></td>
<td>• Can use a graphing calculator</td>
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<tr>
<td>Using Square Roots (Lesson 9.3)</td>
<td>• Used to solve equations of the form $x^2 = d$.</td>
<td>• Can only be used for certain equations.</td>
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<tr>
<td>Completing the Square (Lesson 9.4)</td>
<td>• Best used when $a = 1$ and $b$ is even</td>
<td>• May involve difficult calculations.</td>
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<tr>
<td>Quadratic Formula (Lesson 9.5)</td>
<td>• Can be used for any quadratic equation</td>
<td>• Takes time to do calculations</td>
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Notes:

Extra Practice
In Exercises 1–6, solve the equation using the Quadratic Formula. Round your solutions to the nearest tenth, if necessary.

1. $x^2 - 10x + 16 = 0$
2. $x^2 + 2x - 8 = 0$
3. $3x^2 - x - 2 = 0$

4. $x^2 + 6x = -13$
5. $-3x^2 + 5x - 1 = -7$
6. $-4x^2 + 8x + 12 = 6$
7. A square pool has a side length of $x$ feet. A uniform border around the pool is 1 foot wide. The total area of the pool and the border is 361 square feet. What is the area of the pool?

In Exercises 8–10, determine the number of real solutions of the equation.

8. $-x^2 + 6x + 3 = 0$  
9. $x^2 + 6x + 9 = 0$  
10. $x^2 + 3x + 8 = 0$

In Exercises 11–13 find the number of $x$-intercepts of the graph of the function.

11. $y = -x^2 + 4x + 3$  
12. $y = x^2 + 14x + 49$  
13. $y = -x^2 - 8x - 18$

In Exercises 14–16, solve the equation using any method. Explain your choice of method.

14. $x^2 - 4x + 4 = 16$  
15. $x^2 - 8x + 7 = 0$  
16. $3x^2 + x - 5 = 0$