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# 9.5 Solving Quadratic Equations Using the Quadratic Formula

**For use with Exploration 9.5**

**Essential Question** How can you derive a formula that can be used to write the solutions of any quadratic equation in standard form?

## 1 EXPLORATION: Deriving the Quadratic Formula

**Work with a partner.** The following steps show a method of solving $ax^2 + bx + c = 0$. Explain what was done in each step.

$ax^2 + bx + c = 0$ 1. Write the equation.

$4a^2x^2 + 4abx + 4ac = 0$ 2. _____

$4a^2x^2 + 4abx + 4ac + b^2 = b^2$ 3. _____

$4a^2x^2 + 4abx + b^2 = b^2 - 4ac$ 4. _____

$(2ax + b)^2 = b^2 - 4ac$ 5. _____

$2ax + b = \pm\sqrt{b^2 - 4ac}$ 6. _____

$2ax = -b \pm \sqrt{b^2 - 4ac}$ 7. _____

**Quadratic Formula:** $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ 8. _____

## 2 EXPLORATION: Deriving the Quadratic Formula by Completing the Square

**Work with a partner.**

**a.** Solve $ax^2 + bx + c = 0$ by completing the square. (*Hint:* Subtract $c$ from each side, divide each side by $a$, and then proceed by completing the square.)

**b.** Compare this method with the method in Exploration 1. Explain why you think $4a$ and $b^2$ were chosen in Steps 2 and 3 of Exploration 1.

## Communicate Your Answer

**3.** How can you derive a formula that can be used to write the solutions of any quadratic equation in standard form?

**4.** Use the Quadratic Formula to solve each quadratic equation.

**a.** $x^2 + 2x - 3 = 0$ **b.** $x^2 - 4x + 4 = 0$ **c.** $x^2 + 4x + 5 = 0$

**5.** Use the Internet to research *imaginary numbers*. How are they related to quadratic equations?

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# 9.5 Notetaking with Vocabulary

**For use after Lesson 9.5**

**In your own words, write the meaning of each vocabulary term.**

Quadratic Formula

discriminant

## Core Concepts

### Quadratic Formula

The real solutions of the quadratic equation $ax^2 + bx + c = 0$ are

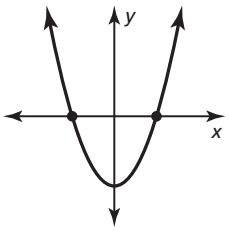
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

where $a \neq 0$ and $b^2 - 4ac \geq 0$.

**Notes:**

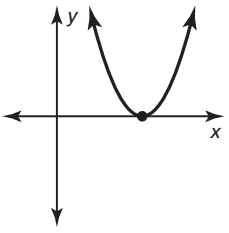
### Interpreting the Discriminant

$b^2 - 4ac > 0$



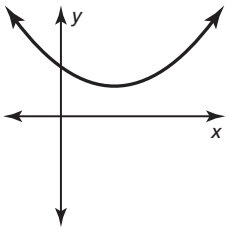

- two real solutions
- two $x$-intercepts

$b^2 - 4ac = 0$




- one real solution
- one $x$-intercept

$b^2 - 4ac < 0$




- no real solutions
- no $x$-intercepts

**Notes:**

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## 9.5 Notetaking with Vocabulary (continued)

### Methods for Solving Quadratic Equations

| Method | Advantages | Disadvantages |
|---|---|---|
| Factoring (*Lessons 7.5–7.8*) | • Straightforward when the equation can be factored easily | • Some equations are not factorable. |
| Graphing (*Lesson 9.2*) | • Can easily see the number of solutions<br>• Use when approximate solutions are sufficient.<br>• Can use a graphing calculator | • May not give exact solutions |
| Using Square Roots (*Lesson 9.3*) | • Used to solve equations of the form $x^2 = d$. | • Can only be used for certain equations |
| Completing the Square (*Lesson 9.4*) | • Best used when $a = 1$ and $b$ is even | • May involve difficult calculations |
| Quadratic Formula (*Lesson 9.5*) | • Can be used for any quadratic equation<br>• Gives exact solutions | • Takes time to do calculations |

### Notes:

## Extra Practice

**In Exercises 1–6, solve the equation using the Quadratic Formula. Round your solutions to the nearest tenth, if necessary.**

**1.** $x^2 - 10x + 16 = 0$

**2.** $x^2 + 2x - 8 = 0$

**3.** $3x^2 - x - 2 = 0$

**4.** $x^2 + 6x = -13$

**5.** $-3x^2 + 5x - 1 = -7$

**6.** $-4x^2 + 8x + 12 = 6$

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## 9.5 Notetaking with Vocabulary (continued)

**7.** A square pool has a side length of $x$ feet. A uniform border around the pool is 1 foot wide. The total area of the pool and the border is 361 square feet. What is the area of the pool?

**In Exercises 8–10, determine the number of real solutions of the equation.**

**8.** $-x^2 + 6x + 3 = 0$ **9.** $x^2 + 6x + 9 = 0$ **10.** $x^2 + 3x + 8 = 0$

**In Exercises 11–13 find the number of *x*-intercepts of the graph of the function.**

**11.** $y = -x^2 + 4x + 3$ **12.** $y = x^2 + 14x + 49$ **13.** $y = -x^2 - 8x - 18$

**In Exercises 14–16, solve the equation using any method. Explain your choice of method.**

**14.** $x^2 - 4x + 4 = 16$ **15.** $x^2 - 8x + 7 = 0$ **16.** $3x^2 + x - 5 = 0$