Name ____ Date __________

Chapter 8 Maintaining Mathematical Proficiency

Graph the linear equation.

1. $y = 4x - 5$

2. $y = -2x + 3$

3. $y = \frac{1}{2}x + 3$

4. $y = -x + 2$

Evaluate the expression when $x = -4$.

5. $2x^2 + 8$

6. $-x^2 + 3x - 4$

7. $-3x^2 - 4$

8. $5x^2 - x + 8$

9. $4x^2 - 8x$

10. $6x^2 - 5x + 3$

11. $-2x^2 + 4x + 4$

12. $3x^2 + 2x + 2$

Name____ Date__________

8.1 Graphing $f(x) = ax^2$

For use with Exploration 8.1

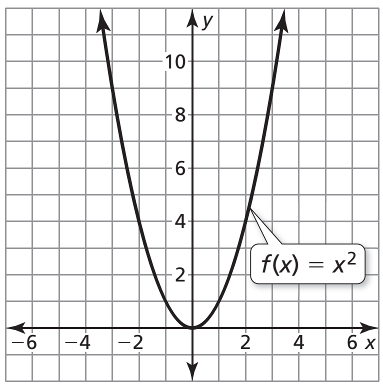
Essential Question What are some of the characteristics of the graph of a quadratic function of the form $f(x) = ax^2$?

1 EXPLORATION: Graphing Quadratic Functions

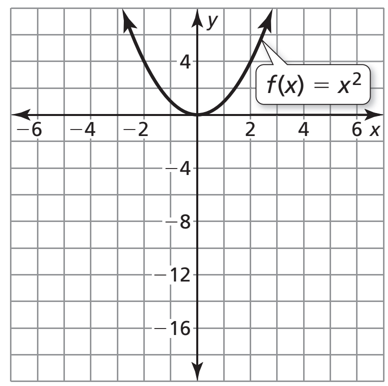
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Graph each quadratic function. Compare each graph to the graph of $f(x) = x^2$.

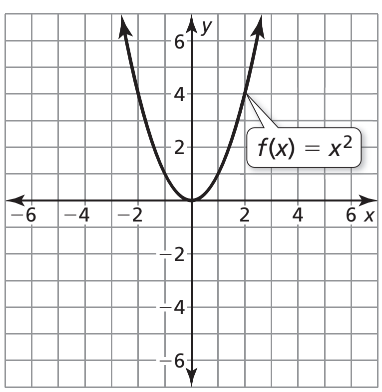
a. $g(x) = 3x^2$



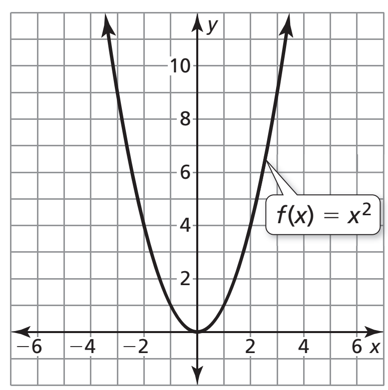
b. $g(x) = -5x^2$



c. $g(x) = -0.2x^2$



d. $g(x) = \frac{1}{10}x^2$

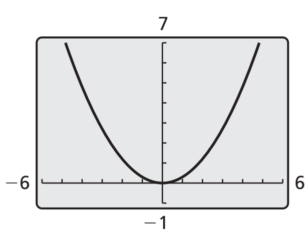


Communicate Your Answer

2. What are some of the characteristics of the graph of a quadratic function of the form $f(x) = ax^2$?

3. How does the value of a affect the graph of $f(x) = ax^2$? Consider $0 < a < 1$, $a > 1$, $-1 < a < 0$, and $a < -1$. Use a graphing calculator to verify your answers.

4. The figure shows the graph of a quadratic function of the form $y = ax^2$. Which of the intervals in Question 3 describes the value of a? Explain your reasoning.



Name____ Date__________

8.1 Notetaking with Vocabulary
For use after Lesson 8.1

In your own words, write the meaning of each vocabulary term.

quadratic function

parabola

vertex

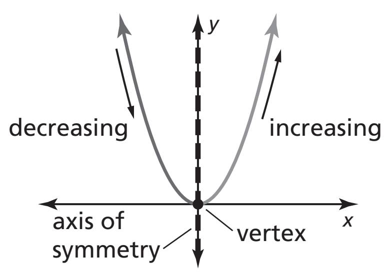
axis of symmetry

Core Concepts

Characteristics of Quadratic Functions

The *parent quadratic function* is $f(x) = x^2$. The graphs of all other quadratic functions are *transformations* of the graph of the parent quadratic function.

The lowest point on a parabola that opens up or the highest point on a parabola that opens down is the **vertex.** The vertex of the graph of $f(x) = x^2$ is (0, 0).



The vertical line that divides the parabola into two symmetric parts is the **axis of symmetry.** The axis of symmetry passes through the vertex. For the graph of $f(x) = x^2$, the axis of symmetry is the y-axis, or $x = 0$.

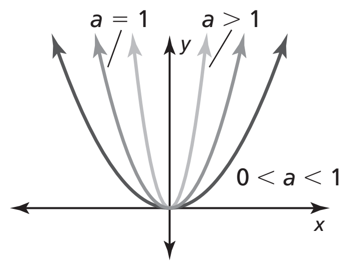
Notes:

Name ____ Date __________

8.1 Notetaking with Vocabulary (continued)

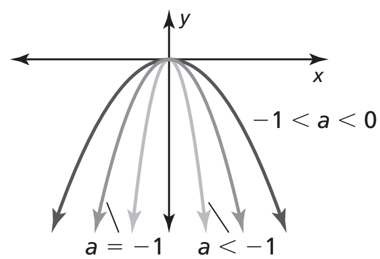
Graphing $f(x) = ax^2$ When $a > 0$

- When $0 < a < 1$, the graph of $f(x) = ax^2$ is a vertical shrink of the graph of $f(x) = x^2$.
- When $a > 1$, the graph of $f(x) = ax^2$ is a vertical stretch of the graph of $f(x) = x^2$.



Graphing $f(x) = ax^2$ When $a < 0$

- When $-1 < a < 0$, the graph of $f(x) = ax^2$ is a vertical shrink with a reflection in the x-axis of the graph of $f(x) = x^2$.
- When $a < -1$, the graph of $f(x) = ax^2$ is a vertical stretch with a reflection in the x-axis of the graph of $f(x) = x^2$.

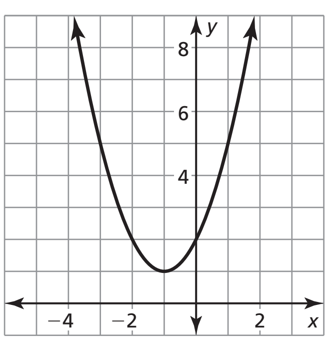


Notes:

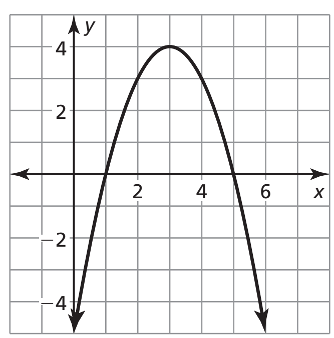
Extra Practice

In Exercises 1 and 2, identify characteristics of the quadratic function and its graph.

1.



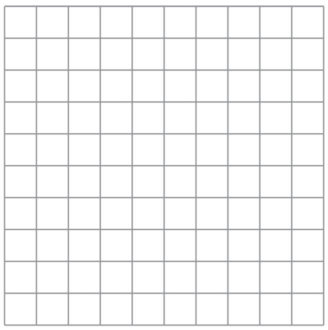
2.



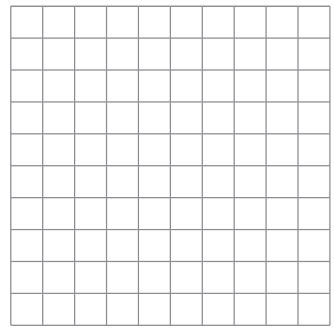
8.1 Notetaking with Vocabulary (continued)

In Exercises 3–8, graph the function. Compare the graph to the graph of $f(x) = x^2$.

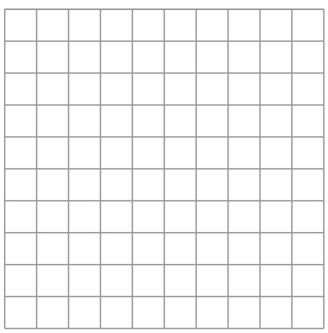
3. $g(x) = 5x^2$



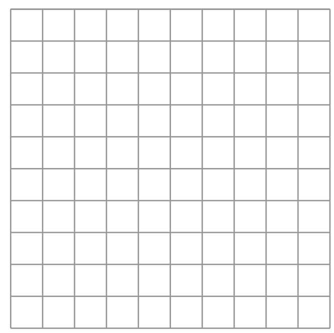
4. $m(x) = -4x^2$



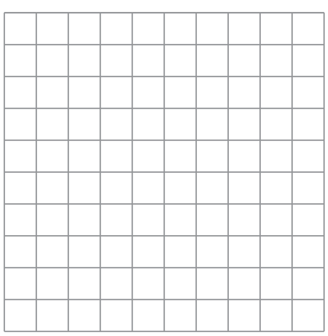
5. $k(x) = -x^2$



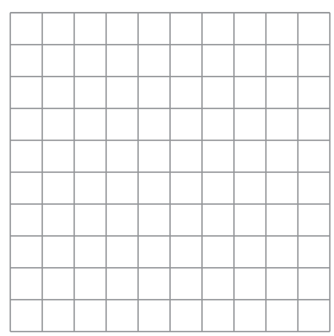
6. $l(x) = -7x^2$



7. $n(x) = -\frac{1}{5}x^2$



8. $p(x) = 0.6x^2$



In Exercises 9 and 10, determine whether the statement is *always, sometimes,* or *never* true. Explain your reasoning.

9. The graph of $g(x) = ax^2$ is wider than the graph of $f(x) = x^2$ when $a > 0$.

10. The graph of $g(x) = ax^2$ is narrower than the graph of $f(x) = x^2$ when $|a| < 1$.

Name ____ Date _________

8.2 Graphing $f(x) = ax^2 + c$

For use with Exploration 8.2

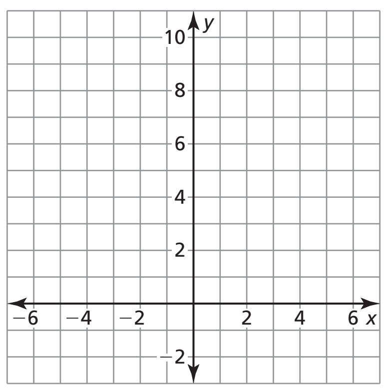
Essential Question How does the value of c affect the graph of $f(x) = ax^2 + c$?

1 EXPLORATION: Graphing $y = ax^2 + c$

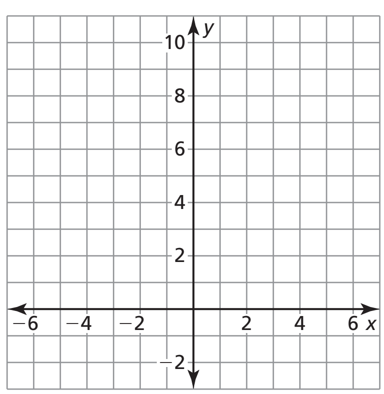
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Sketch the graphs of the functions in the same coordinate plane. What do you notice?

a. $f(x) = x^2$ and $g(x) = x^2 + 2$



b. $f(x) = 2x^2$ and $g(x) = 2x^2 - 2$



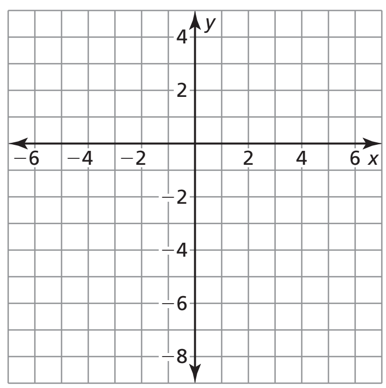
Name____ Date__________

2 **EXPLORATION:** Finding x-Intercepts of Graphs

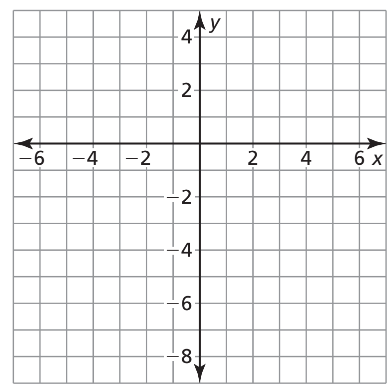
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Graph each function. Find the x-intercepts of the graph. Explain how you found the x-intercepts.

a. $y = x^2 - 7$



b. $y = -x^2 + 1$

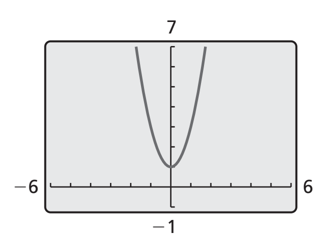


Communicate Your Answer

3. How does the value of c affect the graph of $f(x) = ax^2 + c$?

4. Use a graphing calculator to verify your answers to Question 3.

5. The figure shows the graph of a quadratic function of the form $y = ax^2 + c$. Describe possible values of a and c. Explain your reasoning.



Name ____ Date __________

8.2 Notetaking with Vocabulary

For use after Lesson 8.2

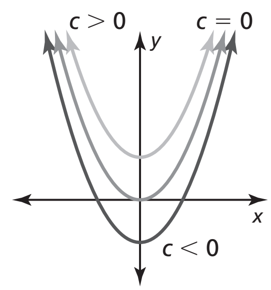
In your own words, write the meaning of each vocabulary term.

zero of a function

Core Concepts

Graphing $f(x) = ax^2 + c$

- When $c > 0$, the graph of $f(x) = ax^2 + c$ is a vertical translation c units up of the graph of $f(x) = ax^2$.
- When $c < 0$, the graph of $f(x) = ax^2 + c$ is a vertical translation $|c|$ units down of the graph of $f(x) = ax^2$.



The vertex of the graph of $f(x) = ax^2 + c$ is $(0, c)$, and the axis of symmetry is $x = 0$.

Notes:

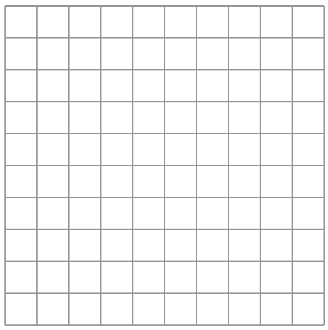
Name____ Date__________

8.2 Notetaking with Vocabulary (continued)

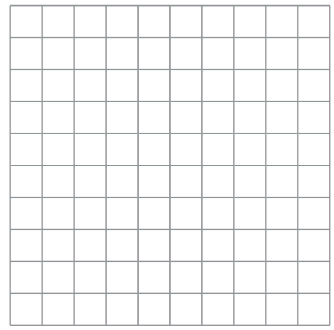
Extra Practice

In Exercises 1–4, graph the function. Compare the graph to the graph of $f(x) = x^2$.

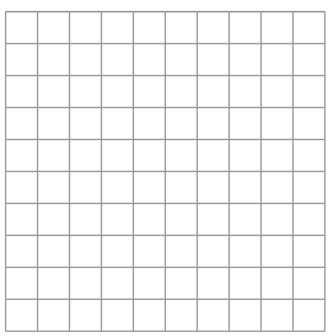
1. $g(x) = x^2 + 5$



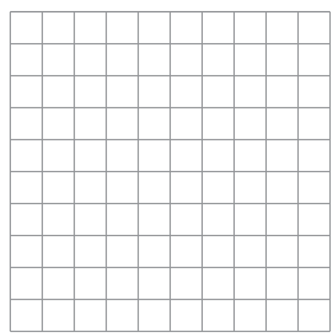
2. $m(x) = x^2 - 3$



3. $n(x) = -3x^2 - 2$



4. $q(x) = \frac{1}{2}x^2 - 4$



8.2 Notetaking with Vocabulary (continued)

In Exercises 5–8, find the zeros of the function.

5. $y = -x^2 + 1$

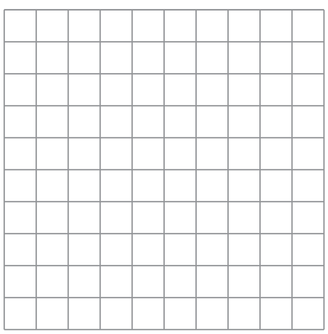
6. $y = -4x^2 + 16$

7. $n(x) = -x^2 + 64$

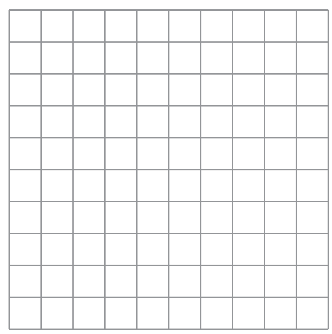
8. $p(x) = -9x^2 + 1$

In Exercises 9 and 10, sketch a parabola with the given characteristics.

9. The parabola opens down, and the vertex is (0, 5).



10. The lowest point on the parabola is (0, 4).



11. The function $f(t) = -16t^2 + s_0$ represents the approximate height (in feet) of a falling object t seconds after it is dropped from an initial height s_0 (in feet). A tennis ball falls from a height of 400 feet.

a. After how many seconds does the tennis ball hit the ground?

b. Suppose the initial height is decreased by 384 feet. After how many seconds does the ball hit the ground?

Name____ Date__________

8.3 Graphing $f(x) = ax^2 + bx + c$

For use with Exploration 8.3

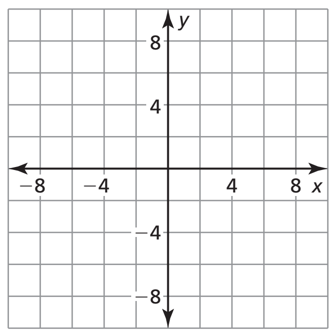
Essential Question How can you find the vertex of the graph of $f(x) = ax^2 + bx + c$?

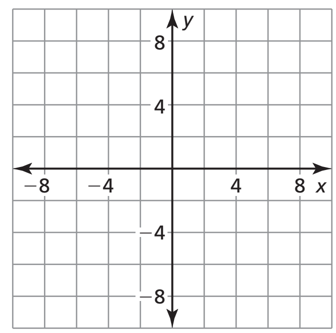
1 EXPLORATION: Comparing x-Intercepts with the Vertex

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

a. Sketch the graphs of $y = 2x^2 - 8x$ and $y = 2x^2 - 8x + 6$.





b. What do you notice about the x-coordinate of the vertex of each graph?

c. Use the graph of $y = 2x^2 - 8x$ to find its x-intercepts. Verify your answer by solving $0 = 2x^2 - 8x$.

d. Compare the value of the x-coordinate of the vertex with the values of the x-intercepts.

8.3 Graphing $f(x) = ax^2 + bx + c$ (continued)

2 EXPLORATION: Finding x-Intercepts

Work with a partner.

a. Solve $0 = ax^2 + bx$ for x by factoring.

b. What are the x-intercepts of the graph of $y = ax^2 + bx$?

c. Complete the table to verify your answer.

x	$y = ax^2 + bx$
0	
$-\frac{b}{a}$	

3 EXPLORATION: Deductive Reasoning

Work with a partner. Complete the following logical argument.

The x-intercepts of the graph of $y = ax^2 + bx$ are 0 and $-\frac{b}{a}$.

The vertex of the graph of $y = ax^2 + bx$ occurs when $x =$ _________.

The vertices of the graphs of $y = ax^2 + bx$ and $y = ax^2 + bx + c$ have the same x-coordinate.

The vertex of the graph of $y = ax^2 + bx + c$ occurs when $x =$ _________.

Communicate Your Answer

4. How can you find the vertex of the graph of $f(x) = ax^2 + bx + c$?

5. Without graphing, find the vertex of the graph of $f(x) = x^2 - 4x + 3$. Check your result by graphing.

Name____ Date__________

8.3 Notetaking with Vocabulary
For use after Lesson 8.3

In your own words, write the meaning of each vocabulary term.

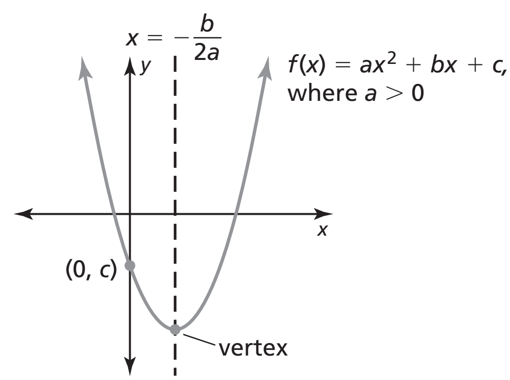
maximum value

minimum value

Core Concepts

Graphing $f(x) = ax^2 + bx + c$

- The graph opens up when $a > 0$, and the graph opens down when $a < 0$.
- The y-intercept is c.
- The x-coordinate of the vertex is $-\frac{b}{2a}$.
- The axis of symmetry is $x = -\frac{b}{2a}$.



Notes:

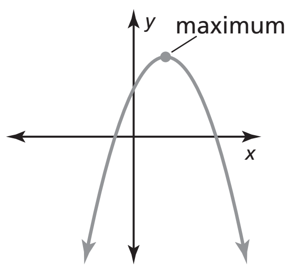
Name ______________________________ Date __________

8.3 **Notetaking with Vocabulary (continued)**

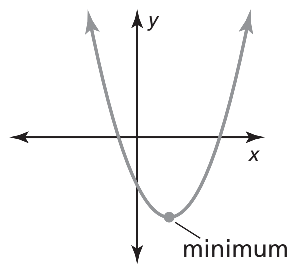
Maximum and Minimum Values

The y-coordinate of the vertex of the graph of $f(x) = ax^2 + bx + c$ is the **maximum value** of the function when $a < 0$ or the **minimum value** of the function when $a > 0$.

$f(x) = ax^2 + bx + c, a < 0$



$f(x) = ax^2 + bx + c, a > 0$



Notes:

Extra Practice

In Exercises 1–4, find (a) the axis of symmetry and (b) the vertex of the graph of the function.

1. $f(x) = x^2 - 10x + 2$

2. $y = -4x^2 + 16x$

3. $y = -2x^2 - 8x + 5$

4. $f(x) = -3x^2 + 6x + 1$

8.3 Notetaking with Vocabulary (continued)

In Exercises 5–7, graph the function. Describe the domain and range.

5. $f(x) = 3x^2 + 6x + 2$

6. $y = 2x^2 - 8x - 1$

7. $y = -\frac{1}{5}x^2 - x + 5$

In Exercises 8–13, tell whether the function has a minimum value or a maximum value. Then find the value.

8. $y = -\frac{1}{2}x^2 - 5x + 2$

9. $y = 8x^2 + 16x - 2$

10. $y = -x^2 - 4x - 7$

11. $y = -7x^2 + 7x + 5$

12. $y = 9x^2 + 6x + 4$

13. $y = -\frac{1}{4}x^2 + x - 6$

14. The function $h = -16t^2 + 250t$ represents the height h (in feet) of a rocket t seconds after it is launched. The rocket explodes at its highest point.

a. When does the rocket explode?

b. At what height does the rocket explode?

Name ____ Date _________

8.4 Graphing $f(x) = a(x - h)^2 + k$

For use with Exploration 8.4

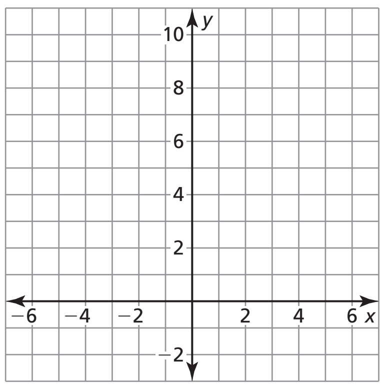
Essential Question How can you describe the graph of $f(x) = a(x - h)^2$?

1 EXPLORATION: Graphing $y = a(x - h)^2$ When $h > 0$

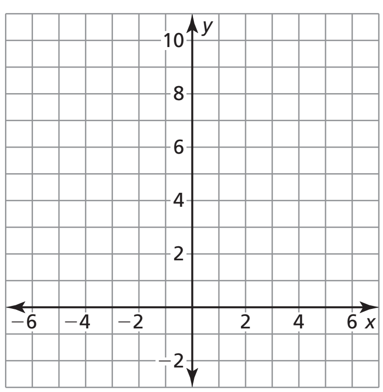
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Sketch the graphs of the functions in the same coordinate plane. How does the value of h affect the graph of $y = a(x - h)^2$?

a. $f(x) = x^2$ and $g(x) = (x - 2)^2$



b. $f(x) = 2x^2$ and $g(x) = 2(x - 2)^2$



Name_______________________________ Date__________

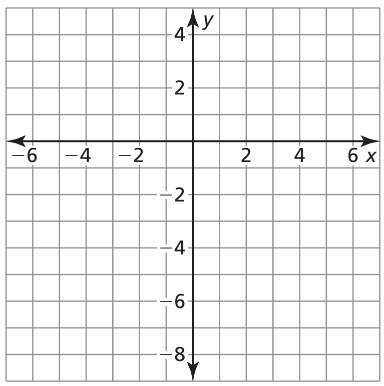
8.4 **Graphing $f(x) = a(x - h)^2 + k$ (continued)**

2 EXPLORATION: Graphing $y = a(x - h)^2$ When $h < 0$

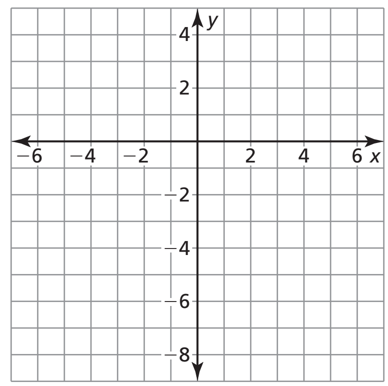
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Sketch the graphs of the functions in the same coordinate plane. How does the value of h affect the graph of $y = a(x - h)^2$?

a. $f(x) = -x^2$ and $g(x) = -(x + 2)^2$



b. $f(x) = -2x^2$ and $g(x) = -2(x + 2)^2$



Communicate Your Answer

3. How can you describe the graph of $f(x) = a(x - h)^2$?

4. Without graphing, describe the graph of each function. Use a graphing calculator to check your answer.

a. $y = (x - 3)^2$

b. $y = (x + 3)^2$

c. $y = -(x - 3)^2$

Name ____ Date _________

8.4 Notetaking with Vocabulary
For use after Lesson 8.4

In your own words, write the meaning of each vocabulary term.

even function

odd function

vertex form (of a quadratic function)

Core Concepts

Even and Odd Functions

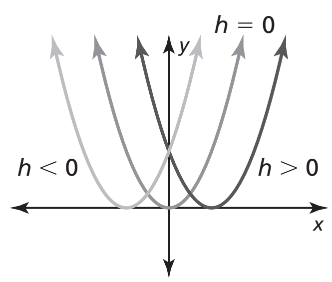
A function $y = f(x)$ is **even** when $f(-x) = f(x)$ for each x in the domain of f. The graph of an even function is symmetric about the y-axis.

A function $y = f(x)$ is **odd** when $f(-x) = -f(x)$ for each x in the domain of f. The graph of an odd function is symmetric about the origin. A graph is *symmetric about the origin* when it looks the same after reflections in the x-axis and then in the y-axis.

Notes:

Graphing $f(x) = a(x - h)^2$

- When $h > 0$, the graph of $f(x) = a(x - h)^2$ is a horizontal translation h units right of the graph $f(x) = ax^2$.
- When $h < 0$, the graph of $f(x) = a(x - h)^2$ is a horizontal translation $|h|$ units left of the graph of $f(x) = ax^2$.



The vertex of the graph of $f(x) = a(x - h)^2$ is $(h, 0)$, and the axis of symmetry is $x = h$.

Notes:

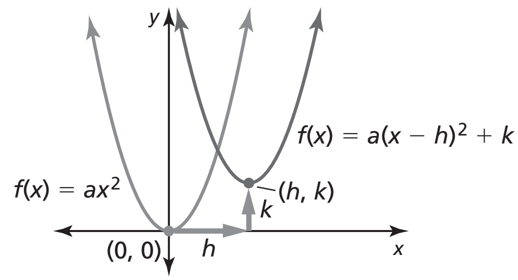
Name____ Date__________

8.4 **Notetaking with Vocabulary (continued)**

Graphing $f(x) = a(x - h)^2 + k$

The **vertex form** of a quadratic function is $f(x) = a(x - h)^2 + k$, where $a \neq 0$. The graph of $f(x) = a(x - h)^2 + k$ is a translation h units horizontally and k units vertically of the graph of $f(x) = ax^2$.

The vertex of the graph of $f(x) = a(x - h)^2 + k$ is (h, k), and the axis of symmetry is $x = h$.



Notes:

Extra Practice

In Exercises 1–4, determine whether the function is *even, odd,* or *neither.*

1. $f(x) = 5x$

2. $f(x) = -4x^2$

3. $h(x) = \frac{1}{2}x^2$

4. $f(x) = -3x^2 + 2x + 1$

In Exercises 5–8, find the vertex and the axis of symmetry of the graph of the function.

5. $f(x) = 5(x - 2)^2$

6. $f(x) = -4(x + 8)^2$

8.4 Notetaking with Vocabulary (continued)

7. $p(x) = -\frac{1}{2}(x - 1)^2 + 4$

8. $g(x) = -(x + 1)^2 - 5$

In Exercises 9 and 10, graph the function. Compare the graph to the graph of $f(x) = x^2$.

9. $m(x) = 3(x + 2)^2$

10. $g(x) = -\frac{1}{4}(x - 6)^2 + 4$

In Exercises 11 and 12, graph *g*.

11. $f(x) = 3(x + 1)^2 - 1;\ g(x) = f(x + 2)$

12. $f(x) = \frac{1}{2}(x - 3)^2 - 5;\ g(x) = -f(x)$

Name____ Date__________

8.5 Using Intercept Form

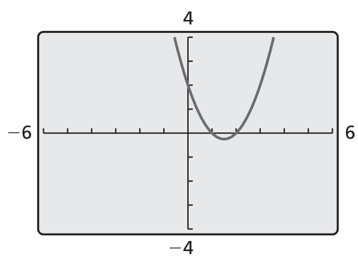
For use with Exploration 8.5

Essential Question What are some of the characteristics of the graph of $f(x) = a(x - p)(x - q)$?

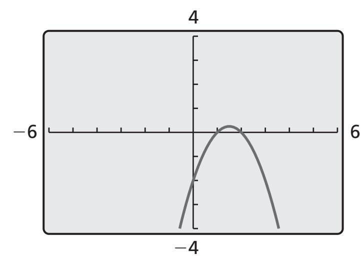
1 EXPLORATION: Using Zeros to Write Functions

Work with a partner. Each graph represents a function of the form $f(x) = (x - p)(x - q)$ or $f(x) = -(x - p)(x - q)$. Write the function represented by each graph. Explain your reasoning.

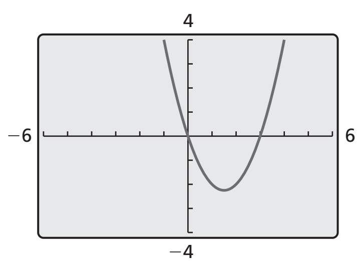
a.



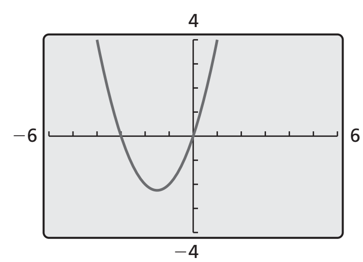
b.



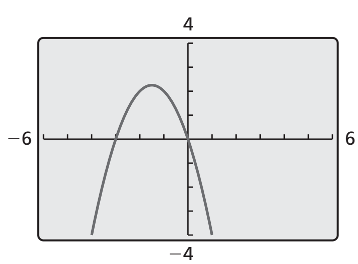
c.



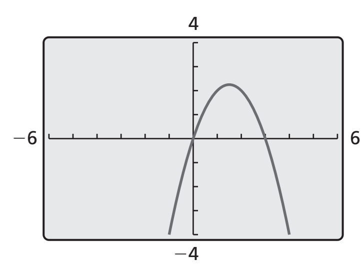
d.



e.

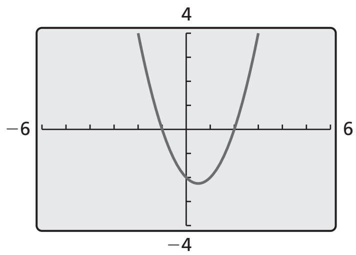


f.

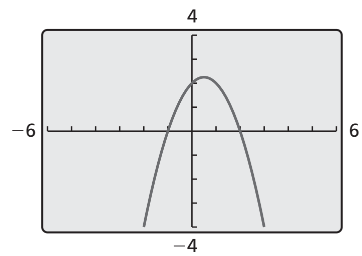


1 EXPLORATION: Using Zeros to Write Functions (continued)

g.



h.



Communicate Your Answer

2. What are some of the characteristics of the graph of $f(x) = a(x - p)(x - q)$?

3. Consider the graph of $f(x) = a(x - p)(x - q)$.

a. Does changing the sign of a change the x-intercepts? Does changing the sign of a change the y-intercept? Explain your reasoning.

b. Does changing the value of p change the x-intercepts? Does changing the value of p change the y-intercept? Explain your reasoning.

Name____ Date __________

8.5 Notetaking with Vocabulary
For use after Lesson 8.5

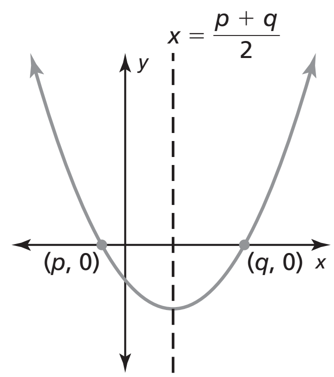
In your own words, write the meaning of each vocabulary term.

intercept form

Core Concepts

Graphing $f(x) = a(x - p)(x - q)$

- The x-intercepts are p and q.
- The axis of symmetry is halfway between $(p, 0)$ and $(q, 0)$. So, the axis of symmetry is $x = \frac{p + q}{2}$.
- The graph opens up when $a > 0$, and the graph opens down when $a < 0$.



Notes:

Factors and Zeros

For any factor $x - n$ of a polynomial, n is a zero of the function defined by the polynomial.

Notes:

Name ______________________________ Date ________

8.5 Notetaking with Vocabulary (continued)

Extra Practice

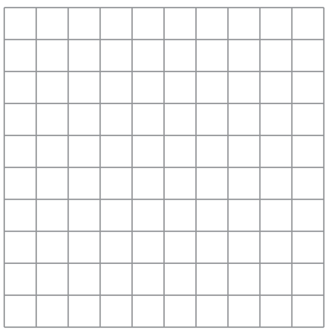
In Exercises 1 and 2, find the *x*-intercepts and axis of symmetry of the graph of the function.

1. $y = (x + 2)(x - 4)$

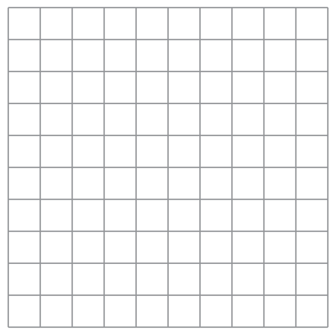
2. $y = -3(x - 2)(x - 3)$

In Exercises 3–6, graph the quadratic function. Label the vertex, axis of symmetry, and *x*-intercepts. Describe the domain and range of the function.

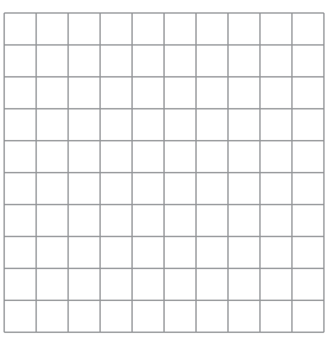
3. $m(x) = (x + 5)(x + 1)$



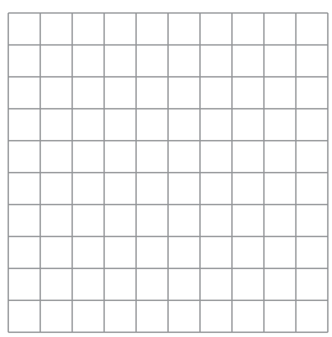
4. $y = -4(x - 3)(x - 1)$



5. $y = x^2 - 4$



6. $f(x) = x^2 + 2x - 15$



8.5 Notetaking with Vocabulary (continued)

In Exercises 7 and 8, find the zero(s) of the function.

7. $y = 6x^2 - 6$

8. $y = x^2 + 9x + 20$

In Exercises 9–12, use zeros to graph the function.

9. $f(x) = x^2 - 3x - 10$

10. $f(x) = -2(x + 3)(x - 1)$

11. $f(x) = x^3 - 9x$

12. $f(x) = 2x^3 - 12x^2 + 10x$

Name ____ Date __________

8.6 Comparing Linear, Exponential, and Quadratic Functions

For use with Exploration 8.6

Essential Question How can you compare the growth rates of linear, exponential, and quadratic functions?

1 EXPLORATION: Comparing Speeds

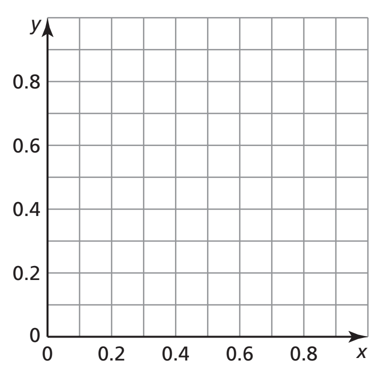
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Three cars start traveling at the same time. The distance traveled in t minutes is y miles. Complete each table and sketch all three graphs in the same coordinate plane. Compare the speeds of the three cars. Which car has a constant speed? Which car is accelerating the most? Explain your reasoning.

t	$y = t$
0	
0.2	
0.4	
0.6	
0.8	
1.0	

t	$y = 2^t - 1$
0	
0.2	
0.4	
0.6	
0.8	
1.0	

t	$y = t^2$
0	
0.2	
0.4	
0.6	
0.8	
1.0	



2 EXPLORATION: Comparing Speeds

Work with a partner. Analyze the speeds of the three cars over the given time periods. The distance traveled in *t* minutes is *y* miles. Which car eventually overtakes the others?

t	$y = t$
1.0	
1.5	
2.0	
2.5	
3.0	
3.5	
4.0	
4.5	
5.0	

t	$y = 2^t - 1$
1.0	
1.5	
2.0	
2.5	
3.0	
3.5	
4.0	
4.5	
5.0	

t	$y = t^2$
1.0	
1.5	
2.0	
2.5	
3.0	
3.5	
4.0	
4.5	
5.0	

Communicate Your Answer

3. How can you compare the growth rates of linear, exponential, and quadratic functions?

4. Which function has a growth rate that is eventually much greater than the growth rates of the other two functions? Explain your reasoning.

Name ____ Date __________

8.6 Notetaking with Vocabulary
For use after Lesson 8.6

In your own words, write the meaning of each vocabulary term.

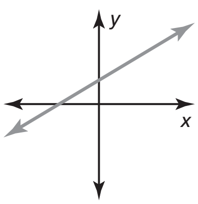
average rate of change

Core Concepts

Linear, Exponential, and Quadratic Functions

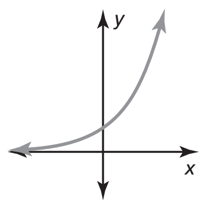
Linear Function

$y = mx + b$



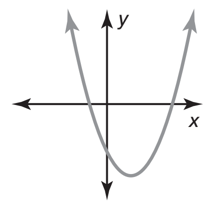
Exponential Function

$y = ab^x$



Quadratic Function

$y = ax^2 + bx + c$



Notes:

Differences and Ratios of Functions

You can use patterns between consecutive data pairs to determine which type of function models the data. The differences of consecutive y-values are called *first differences.* The differences of consecutive first differences are called *second differences*.

- **Linear Function** The first differences are constant.
- **Exponential Function** Consecutive y-values have a common *ratio*.
- **Quadratic Function** The second differences are constant.

In all cases, the differences of consecutive x-values need to be constant.

Notes:

Comparing Functions Using Average Rates of Change

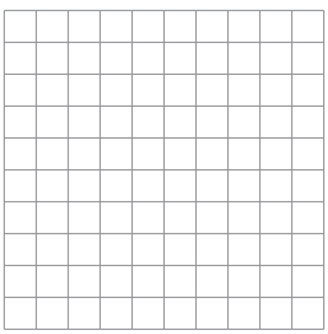
- Over the same interval, the average rate of change of a function increasing quadratically eventually exceeds the average rate of change of a function increasing linearly. So, the value of the quadratic function eventually exceeds the value of the linear function.
- Over the same interval, the average rate of change of a function increasing exponentially eventually exceeds the average rate of change of a function increasing linearly or quadratically. So, the value of the exponential function eventually exceeds the value of the linear or quadratic function.

Notes:

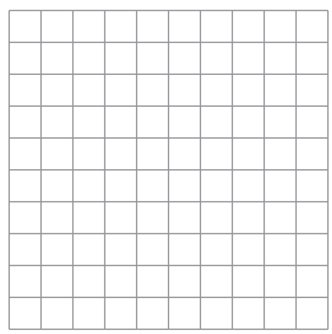
Extra Practice

In Exercises 1–4, plot the points. Tell whether the points appear to represent a *linear*, an *exponential*, or a *quadratic* function.

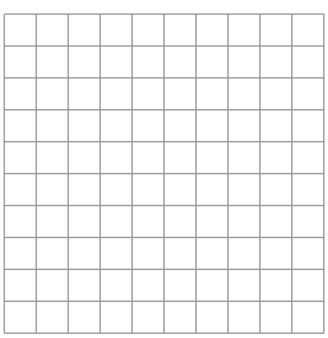
1. $(-3, 2), (-2, 4), (-4, 4), (-1, 8), (-5, 8)$



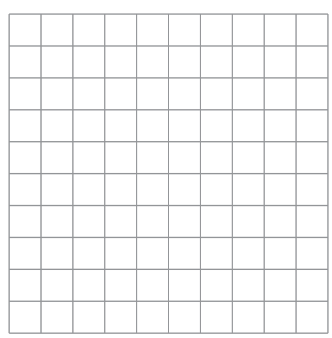
2. $(-3, 1), (-2, 2), (-1, 4), (0, 8), (2, 14)$



3. $(4, 0), (2, 1), (0, 3), (-1, 6), (-2, 10)$



4. $(2, -4), (0, -2), (-2, 0), (-4, 2), (-6, 4)$



8.6 Notetaking with Vocabulary (continued)

In Exercises 5 and 6, tell whether the table of values represents a *linear*, an *exponential*, or a *quadratic* function.

5.

x	−2	−1	0	1	2
y	7	4	1	−2	−5

6.

x	−2	−1	0	1	2
y	6	2	0	2	6

In Exercises 7 and 8, tell whether the data represent a *linear*, an *exponential*, or a *quadratic* function. Then write the function.

7. $(-2, -4), (-1, -1), (0, 2), (1, 5), (2, 8)$

8. $(-2, -9), (-1, 0), (0, 3), (1, 0), (2, -9)$

9. A ball is dropped from a height of 305 feet. The table shows the height h (in feet) of the ball t seconds after being dropped. Let the time t represent the independent variable. Tell whether the data can be modeled by a *linear*, an *exponential*, or a *quadratic* function. Explain.

Time, *t*	0	1	2	3	4
Height, *h*	305	289	241	161	49