**8.3 Graphing** $f(x) = ax^2 + bx + c$

For use with Exploration 8.3

**Essential Question** How can you find the vertex of the graph of $f(x) = ax^2 + bx + c$?

**EXPLORATION: Comparing x-Intercepts with the Vertex**

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner.

a. Sketch the graphs of $y = 2x^2 - 8x$ and $y = 2x^2 - 8x + 6$.

![Graphs of y = 2x^2 - 8x and y = 2x^2 - 8x + 6]

b. What do you notice about the $x$-coordinate of the vertex of each graph?

c. Use the graph of $y = 2x^2 - 8x$ to find its $x$-intercepts. Verify your answer by solving $0 = 2x^2 - 8x$.

d. Compare the value of the $x$-coordinate of the vertex with the values of the $x$-intercepts.
**8.3** Graphing $f(x) = ax^2 + bx + c$ (continued)

### 2 EXPLORATION: Finding x-Intercepts

Work with a partner.

a. Solve $0 = ax^2 + bx$ for $x$ by factoring.

b. What are the $x$-intercepts of the graph of $y = ax^2 + bx$?

c. Complete the table to verify your answer.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = ax^2 + bx$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$-\frac{b}{a}$</td>
<td></td>
</tr>
</tbody>
</table>

### 3 EXPLORATION: Deductive Reasoning

Work with a partner. Complete the following logical argument.

The $x$-intercepts of the graph of $y = ax^2 + bx$ are 0 and $-\frac{b}{a}$.

The vertex of the graph of $y = ax^2 + bx$ occurs when $x = ________$.

The vertices of the graphs of $y = ax^2 + bx$ and $y = ax^2 + bx + c$ have the same $x$-coordinate.

The vertex of the graph of $y = ax^2 + bx + c$ occurs when $x = ________$.

**Communicate Your Answer**

4. How can you find the vertex of the graph of $f(x) = ax^2 + bx + c$?

5. Without graphing, find the vertex of the graph of $f(x) = x^2 - 4x + 3$.
   Check your result by graphing.
8.3 Notetaking with Vocabulary
For use after Lesson 8.3

In your own words, write the meaning of each vocabulary term.

maximum value

minimum value

Core Concepts

Graphing \( f(x) = ax^2 + bx + c \)
- The graph opens up when \( a > 0 \), and the graph opens down when \( a < 0 \).
- The \( y \)-intercept is \( c \).
- The \( x \)-coordinate of the vertex is \( x = \frac{-b}{2a} \).
- The axis of symmetry is \( x = \frac{-b}{2a} \).

Notes:
Maximum and Minimum Values

The \( y \)-coordinate of the vertex of the graph of \( f(x) = ax^2 + bx + c \) is the **maximum value** of the function when \( a < 0 \) or the **minimum value** of the function when \( a > 0 \).

\[
\begin{align*}
  f(x) &= ax^2 + bx + c, \quad a < 0 \\
  f(x) &= ax^2 + bx + c, \quad a > 0
\end{align*}
\]

Notes:

**Extra Practice**

In Exercises 1–4, find (a) the axis of symmetry and (b) the vertex of the graph of the function.

1. \( f(x) = x^2 - 10x + 2 \)
2. \( y = -4x^2 + 16x \)
3. \( y = -2x^2 - 8x + 5 \)
4. \( f(x) = -3x^2 + 6x + 1 \)
In Exercises 5–7, graph the function. Describe the domain and range.

5. \( f(x) = 3x^2 + 6x + 2 \)  

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\end{array}
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6. \( y = 2x^2 - 8x - 1 \)  

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& & & & & & \\
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\end{array}
\]

7. \( y = -\frac{1}{5}x^2 - x + 5 \)  

\[
\begin{array}{|c|c|c|c|c|c|c|}
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\end{array}
\]

In Exercises 8–13, tell whether the function has a minimum value or a maximum value. Then find the value.

8. \( y = -\frac{1}{2}x^2 - 5x + 2 \)  

9. \( y = 8x^2 + 16x - 2 \)  

10. \( y = -x^2 - 4x - 7 \)

11. \( y = -7x^2 + 7x + 5 \)  

12. \( y = 9x^2 + 6x + 4 \)  

13. \( y = -\frac{1}{4}x^2 + x - 6 \)

14. The function \( h = -16t^2 + 250t \) represents the height \( h \) (in feet) of a rocket \( t \) seconds after it is launched. The rocket explodes at its highest point.

a. When does the rocket explode?

b. At what height does the rocket explode?