Chapter 7  
Maintaining Mathematical Proficiency

Simplify the expression.

1. \(5x - 6 + 3x\)  
2. \(3t + 7 - 3t - 4\)  
3. \(8s - 4 + 4s - 6 - 5s\)

4. \(9m + 3 + m - 3 + 5m\)  
5. \(-4 - 3p - 7 - 3p - 4\)  
6. \(12(z - 1) + 4\)

7. \(-6(x + 2) - 4\)  
8. \(3(h + 4) - 3(h - 4)\)  
9. \(7(z + 4) - 3(z + 2) - 2(z - 3)\)

Find the greatest common factor.

10. \(24, 32\)  
11. \(30, 55\)  
12. \(48, 84\)

13. \(28, 72\)  
14. \(42, 60\)  
15. \(35, 99\)

16. Explain how to find the greatest common factor of 42, 70, and 84.
7.1 Adding and Subtracting Polynomials
For use with Exploration 7.1

Essential Question  How can you add and subtract polynomials?

1 EXPLORATION: Adding Polynomials

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Write the expression modeled by the algebra tiles in each step.

Step 1
\[(3x + 2) + (x - 5)\]

Step 2

Step 3

Step 4

2 EXPLORATION: Subtracting Polynomials

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Write the expression modeled by the algebra tiles in each step.

Step 1
\[(x^2 + 2x + 2) - (x - 1)\]

Step 2

Step 3

7.1 Adding and Subtracting Polynomials (continued)

2 EXPLORATION: Subtracting Polynomials (continued)

Communicate Your Answer

3. How can you add and subtract polynomials?

4. Use your methods in Question 3 to find each sum or difference.
   a. \((x^2 + 2x - 1) + (2x^2 - 2x + 1)\)  
   b. \((4x + 3) + (x - 2)\)  
   c. \((x^2 + 2) - (3x^2 + 2x + 5)\)  
   d. \((2x - 3x) - (x^2 - 2x + 4)\)
In your own words, write the meaning of each vocabulary term.

monomial

degree of a monomial

polynomial

binomial

trinomial

degree of a polynomial

standard form

leading coefficient

closed

Notes:
7.1 Notetaking with Vocabulary (continued)

Core Concepts

Polynomials

A polynomial is a monomial or a sum of monomials. Each monomial is called a term of the polynomial. A polynomial with two terms is a binomial. A polynomial with three terms is a trinomial.

<table>
<thead>
<tr>
<th>Binomial</th>
<th>Trinomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5x + 2$</td>
<td>$x^2 + 5x + 2$</td>
</tr>
</tbody>
</table>

The degree of a polynomial is the greatest degree of its terms. A polynomial in one variable is in standard form when the exponents of the terms decrease from left to right. When you write a polynomial in standard form, the coefficient of the first term is the leading coefficient.

Notes:

Extra Practice

In Exercises 1–8, find the degree of the monomial.

1. $-6s$
2. $w$
3. 8
4. $-2abc$
5. $7x^2y$
6. $4r^2s^3t$
7. $10mn^3$
8. $\frac{2}{3}$
In Exercises 9–12, write the polynomial in standard form. Identify the degree and leading coefficient of the polynomial. Then classify the polynomial by the number of terms.

9. \( x + 3x^2 + 5 \)  
10. \( \sqrt{5} y \)  
11. \( 3x^5 + 6x^8 \)  
12. \( f^2 - 2f + f^4 \)

In Exercises 13–16, find the sum.

13. \((-4x + 9) + (6x - 14)\)  
14. \((-3a - 2) + (7a + 5)\)

15. \((x^2 + 3x + 5) + (-x^2 + 6x - 4)\)  
16. \((t^2 + 3t^3 - 3) + (2t^2 + 7t - 2t^3)\)

In Exercises 17–20, find the difference.

17. \((g - 4) - (3g - 6)\)  
18. \((-5h - 2) - (7h + 6)\)

19. \((-x^2 - 5) - (-3x^2 - x - 8)\)  
20. \((k^2 + 6k^3 - 4) - (5k^3 + 7k - 3k^2)\)
7.2 Multiplying Polynomials

Essential Question  How can you multiply two polynomials?

1 EXPLORATION: Multiplying Monomials Using Algebra Tiles

Work with a partner. Write each product. Explain your reasoning.

a. $\square \cdot \square = \phantom{}$

b. $\square \cdot \square = \phantom{}$

c. $\square \cdot \square = \phantom{}$

d. $\square \cdot \square = \phantom{}$

e. $\square \cdot \square = \phantom{}$

f. $\square \cdot \square = \phantom{}$

g. $\square \cdot \square = \phantom{}$

h. $\square \cdot \square = \phantom{}$

i. $\square \cdot \square = \phantom{}$

j. $\square \cdot \square = \phantom{}$
2 EXPLORATION: Multiplying Binomials Using Algebra Tiles

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Write the product of two binomials modeled by each rectangular array of algebra tiles. In parts (c) and (d), first draw the rectangular array of algebra tiles that models each product.

a. \((x + 3)(x - 2) = \) _________

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\cdot & + & - & - & - \\
+ & + & + & - & - \\
+ & + & + & - & - \\
+ & + & + & - & - \\
+ & + & + & - & - \\
\end{array}
\]

b. \((2x - 1)(2x + 1) = \) _________

\[
\begin{array}{c|cccc}
\cdot & + & + & + & + \\
+ & + & + & + & + \\
+ & + & + & + & + \\
\end{array}
\]

c. \((x + 2)(2x - 1) = \) _________

\[
\begin{array}{c|cccc}
\cdot & + & + & + & - \\
+ & + & + & - & - \\
+ & + & + & - & - \\
\end{array}
\]

d. \((-x - 2)(x - 3) = \) _________

\[
\begin{array}{c|cccc}
\cdot & + & + & + & + \\
+ & + & + & + & + \\
+ & + & + & + & + \\
\end{array}
\]

Communicate Your Answer

3. How can you multiply two polynomials?

4. Give another example of multiplying two binomials using algebra tiles that is similar to those in Exploration 2.
7.2 Notetaking with Vocabulary
For use after Lesson 7.2

In your own words, write the meaning of each vocabulary term.

**FOIL Method**

**Core Concepts**

**FOIL Method**

To multiply two binomials using the FOIL Method, find the sum of the products of the

First terms, \((x + 1)(x + 2)\) \(\Rightarrow\) \(x(x) = x^2\)

Outer terms, \((x + 1)(x + 2)\) \(\Rightarrow\) \(x(2) = 2x\)

Inner terms, and \((x + 1)(x + 2)\) \(\Rightarrow\) \(1(x) = x\)

Last terms, \((x + 1)(x + 2)\) \(\Rightarrow\) \(1(2) = 2\)

\((x + 1)(x + 2) = x^2 + 2x + x + 2 = x^2 + 3x + 2\)

**Notes:**
Extra Practice

In Exercises 1–6, use the Distributive Property to find the product.

1. \((x - 2)(x - 1)\)  
2. \((b - 3)(b + 2)\)  
3. \((g + 2)(g + 4)\)  
4. \((a - 1)(2a + 5)\)  
5. \((3n - 4)(n + 1)\)  
6. \((r + 3)(3r + 2)\)

In Exercises 7–12, use a table to find the product.

7. \((x - 3)(x - 2)\)  
8. \((y + 1)(y - 6)\)  
9. \((q + 3)(q + 7)\)  
10. \((2w - 5)(w - 3)\)  
11. \((6h - 2)(-3 - 2h)\)  
12. \((-3 + 4j)(3j + 4)\)
In Exercises 13–18, use the FOIL Method to find the product.

13. \((x + 2)(x - 3)\)  
14. \((z + 3)(z + 2)\)  
15. \((h - 2)(h + 4)\)  
16. \((2m - 1)(m + 2)\)  
17. \((4n - 1)(3n + 4)\)  
18. \((-q - 1)(q + 1)\)  

In Exercises 19–24, find the product.

19. \((x - 2)(x^2 + x - 1)\)  
20. \((2 - a)(3a^2 + 3a - 5)\)  
21. \((h + 1)(h^2 - h - 1)\)  
22. \((d + 3)(d^2 - 4d + 1)\)  
23. \((3n^2 + 2n - 5)(2n + 1)\)  
24. \((2p^2 + p - 3)(3p - 1)\)
7.3 Special Products of Polynomials
For use with Exploration 7.3

**Essential Question**  What are the patterns in the special products

(a + b)(a - b), (a + b)^2, and (a - b)^2?

1 **EXPLORATION:** Finding a Sum and Difference Pattern

*Work with a partner.* Write the product of two binomials modeled by each rectangular array of algebra tiles.

a. \((x + 2)(x - 2) = \) 

\[
\begin{array}{c|c|c|c}
\cdot & + & - & - \\
+ & + & + & - \\
+ & + & + & - \\
+ & + & + & - \\
\end{array}
\]

b. \((2x - 1)(2x + 1) = \) 

\[
\begin{array}{c|c|c|c}
\cdot & + & - & - \\
+ & + & + & + \\
+ & + & + & + \\
+ & + & + & + \\
\end{array}
\]

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

2 **EXPLORATION:** Finding the Square of a Binomial Pattern

*Work with a partner.* Draw the rectangular array of algebra tiles that models each product of two binomials. Write the product.

a. \((x + 2)^2 = \) 

\[
\begin{array}{c|c|c|c}
\cdot & + & - & - \\
+ & + & + & - \\
+ & + & + & - \\
+ & + & + & - \\
\end{array}
\]

b. \((2x - 1)^2 = \) 

\[
\begin{array}{c|c|c|c}
\cdot & + & - & - \\
+ & + & + & + \\
+ & + & + & + \\
+ & + & + & + \\
\end{array}
\]
Communicate Your Answer

3. What are the patterns in the special products \((a + b)(a - b), (a + b)^2,\)
   and \((a - b)^2\)?

4. Use the appropriate special product pattern to find each product. Check your
   answers using algebra tiles.
   
   a. \((x + 3)(x - 3)\)   b. \((x - 4)(x + 4)\)   c. \((3x + 1)(3x - 1)\)
   
   d. \((x + 3)^2\)   e. \((x - 2)^2\)   f. \((3x + 1)^2\)
7.3 Notetaking with Vocabulary
For use after Lesson 7.3

In your own words, write the meaning of each vocabulary term.

binomial

Core Concepts
Square of a Binomial Pattern

Algebra

\[(a + b)^2 = a^2 + 2ab + b^2\]

\[(a - b)^2 = a^2 - 2ab + b^2\]

Example

\[(x + 5)^2 = (x)^2 + 2(x)(5) + (5)^2\]
\[= x^2 + 10x + 25\]

\[(2x - 3)^2 = (2x)^2 - 2(2x)(3) + (3)^2\]
\[= 4x^2 - 12x + 9\]

Notes:

Sum and Difference Pattern

Algebra

\[(a + b)(a - b) = a^2 - b^2\]

Example

\[(x + 3)(x - 3) = x^2 - 9\]

Notes:
7.3 Notetaking with Vocabulary (continued)

Extra Practice

In Exercises 1–18, find the product.

1. \((a + 3)^2\)  
2. \((b - 2)^2\)  
3. \((c + 4)^2\)  

4. \((-2x + 1)^2\)  
5. \((3x - 2)^2\)  
6. \((-4p - 3)^2\)  

7. \((3x + 2y)^2\)  
8. \((2a - 3b)^2\)  
9. \((-4c + 5d)^2\)  

10. \((x - 3)(x + 3)\)  
11. \((q + 5)(q - 5)\)  
12. \((t - 11)(t + 11)\)
7.3 Notetaking with Vocabulary (continued)

13. 

14. 

15. 

16. 

17. 

18. 

In Exercises 19–24, use special product patterns to find the product.

19. 18 • 22

20. 49 • 51

21. \( \frac{19}{5} \) • \( \frac{2}{5} \)

22. \((31)^2\)

23. \((20.7)^2\)

24. \((109)^2\)

25. Find \(k\) so that \(kx^2 - 12x + 9\) is the square of a binomial.
**7.4 Solving Polynomial Equations in Factored Form**  
For use with Exploration 7.4

**Essential Question**  How can you solve a polynomial equation?

**1 EXPLORATION: Matching Equivalent Forms of an Equation**

**Work with a partner.** An equation is considered to be in *factored form* when the product of the factors is equal to 0. Match each factored form of the equation with its equivalent standard form and nonstandard form.

<table>
<thead>
<tr>
<th>Factored Form</th>
<th>Standard Form</th>
<th>Nonstandard Form</th>
</tr>
</thead>
</table>
| a. 

**2 EXPLORATION: Writing a Conjecture**

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

**Work with a partner.** Substitute 1, 2, 3, 4, 5, and 6 for *x* in each equation and determine whether the equation is true. Organize your results in the table. Write a conjecture describing what you discovered.

<table>
<thead>
<tr>
<th>Equation</th>
<th><em>x</em> = 1</th>
<th><em>x</em> = 2</th>
<th><em>x</em> = 3</th>
<th><em>x</em> = 4</th>
<th><em>x</em> = 5</th>
<th><em>x</em> = 6</th>
</tr>
</thead>
</table>
| a. 

| (x - 1)(x - 2) = 0      |
| b. 

| (x - 2)(x - 3) = 0      |
| c. 

| (x - 3)(x - 4) = 0      |
| d. 

| (x - 4)(x - 5) = 0      |
| e. 

| (x - 5)(x - 6) = 0      |
| f. 

| (x - 6)(x - 1) = 0      |
Work with a partner. The numbers 0 and 1 have special properties that are shared by no other numbers. For each of the following, decide whether the property is true for 0, 1, both, or neither. Explain your reasoning.

a. When you add ____ to a number $n$, you get $n$.

b. If the product of two numbers is ____ , then at least one of the numbers is 0.

c. The square of ____ is equal to itself.

d. When you multiply a number $n$ by ____ , you get $n$.

e. When you multiply a number $n$ by ____ , you get 0.

f. The opposite of ____ is equal to itself.

Communicate Your Answer

4. How can you solve a polynomial equation?

5. One of the properties in Exploration 3 is called the Zero-Product Property. It is one of the most important properties in all of algebra. Which property is it? Why do you think it is called the Zero-Product Property? Explain how it is used in algebra and why it so important.
7.4 Notetaking with Vocabulary
For use after Lesson 7.4

In your own words, write the meaning of each vocabulary term.

factored form

Zero-Product Property

roots

repeated roots

Core Concepts

Zero-Product Property

Words If the product of two real numbers is 0, then at least one of the numbers is 0.

Algebra If \(a\) and \(b\) are real numbers and \(ab = 0\), then \(a = 0\) or \(b = 0\).
7.4 Notetaking with Vocabulary (continued)

Extra Practice

In Exercises 1–12, solve the equation.

1. \( x(x + 5) = 0 \)
2. \( a(a - 12) = 0 \)
3. \( 5p(p - 2) = 0 \)

4. \( (c - 2)(c + 1) = 0 \)
5. \( (2b - 6)(3b + 18) = 0 \)
6. \( (3 - 5s)(-3 + 5s) = 0 \)

7. \( (x - 3)^2 = 0 \)
8. \( (3d + 7)(5d - 6) = 0 \)
9. \( (2t + 8)(2t - 8) = 0 \)

10. \( (w + 4)^2(w + 1) = 0 \)
11. \( g(6 - 3g)(6 + 3g) = 0 \)
12. \( (4 - m)(8 + \frac{2}{3}m)(-2 - 3m) = 0 \)
In Exercises 13–18, factor the polynomial.

13. $6x^2 + 3x$
14. $4y^4 - 20y^3$
15. $18u^4 - 6u$

16. $7z^7 + 2z^6$
17. $24h^3 + 8h$
18. $15f^4 - 45f$

In Exercises 19–24, solve the equation.

19. $6k^2 + k = 0$
20. $35n - 49n^2 = 0$
21. $4z^2 + 52z = 0$

22. $6x^2 = -72x$
23. $22s = 11s^2$
24. $7p^2 = 21p$

25. A boy kicks a ball in the air. The height $y$ (in feet) above the ground of the ball is modeled by the equation $y = -16x^2 + 80x$, where $x$ is the time (in seconds) since the ball was kicked. Find the roots of the equation when $y = 0$. Explain what the roots mean in this situation.
Essential Question  How can you use algebra tiles to factor the trinomial  \( x^2 + bx + c \) into the product of two binomials?

1 EXPLORATION: Finding Binomial Factors

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Use algebra tiles to write each polynomial as the product of two binomials. Check your answer by multiplying.

Sample  \( x^2 + 5x + 6 \)

Step 1  Arrange algebra tiles that model  \( x^2 + 5x + 6 \) into a rectangular array.

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Step 3  Write the polynomial in factored form using the dimensions of the rectangle.

\[
\text{Area} = x^2 + 5x + 6 = (x + 2)(x + 3)
\]

\begin{enumerate}
\item \( x^2 - 3x + 2 = \) \ \\
\item \( x^2 + 5x + 4 = \)
\end{enumerate}
7.5 Factoring $x^2 + bx + c$ (continued)

1 EXPLORATION: Finding Binomial Factors (continued)

- c. $x^2 - 7x + 12 = \underline{\hspace{2cm}}$
- d. $x^2 + 7x + 12 = \underline{\hspace{2cm}}$

Communicate Your Answer

2. How can you use algebra tiles to factor the trinomial $x^2 + bx + c$ into the product of two binomials?

3. Describe a strategy for factoring the trinomial $x^2 + bx + c$ that does not use algebra tiles.
In your own words, write the meaning of each vocabulary term.

**polynomial**

**FOIL Method**

**Zero-Product Property**

**Core Concepts**

### Factoring $x^2 + bx + c$ When $c$ is Positive

**Algebra**  
$x^2 + bx + c = (x + p)(x + q)$ when $p + q = b$ and $pq = c$.  
When $c$ is positive, $p$ and $q$ have the same sign as $b$.

**Examples**
1. $x^2 + 6x + 5 = (x + 1)(x + 5)$
2. $x^2 - 6x + 5 = (x - 1)(x - 5)$

**Notes:**

### Factoring $x^2 + bx + c$ When $c$ is Negative

**Algebra**  
$x^2 + bx + c = (x + p)(x + q)$ when $p + q = b$ and $pq = c$.  
When $c$ is negative, $p$ and $q$ have different signs.

**Example**  
$x^2 - 4x - 5 = (x + 1)(x - 5)$

**Notes:**
Extra Practice

In Exercises 1–12, factor the polynomial.

1. \( c^2 + 8c + 7 \)

2. \( a^2 + 16a + 64 \)

3. \( x^2 + 11x + 18 \)

4. \( d^2 + 6d + 8 \)

5. \( s^2 + 11s + 10 \)

6. \( u^2 + 10u + 9 \)

7. \( b^2 + 3b - 54 \)

8. \( y^2 - y - 2 \)

9. \( u^2 + 3u - 18 \)

10. \( z^2 - z - 56 \)

11. \( h^2 + 2h - 24 \)

12. \( f^2 - 3f - 40 \)
7.5 Notetaking with Vocabulary (continued)

In Exercises 13–18, solve the equation.

13. \(g^2 - 13g + 40 = 0\)
14. \(k^2 - 5k + 6 = 0\)
15. \(w^2 - 7w + 10 = 0\)
16. \(x^2 - x = 30\)
17. \(r^2 - 3r = -2\)
18. \(t^2 - 7t = 8\)

19. The area of a right triangle is 16 square miles. One leg of the triangle is 4 miles longer than the other leg. Find the length of each leg.

20. You have two circular flower beds, as shown. The sum of the areas of the two flower beds is \(136\pi\) square feet. Find the radius of each bed.
7.6 Factoring $ax^2 + bx + c$

For use with Exploration 7.6

**Essential Question**  How can you use algebra tiles to factor the trinomial $ax^2 + bx + c$ into the product of two binomials?

**1 EXPLORATION: Finding Binomial Factors**

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

**Work with a partner.** Use algebra tiles to write each polynomial as the product of two binomials. Check your answer by multiplying.

**Sample** $2x^2 + 5x + 2$

**Step 1** Arrange algebra tiles that model $2x^2 + 5x + 2$ into a rectangular array.

**Step 2** Use additional algebra tiles to model the dimensions of the rectangle.

**Step 3** Write the polynomial in factored form using the dimensions of the rectangle.

$\text{Area} = 2x^2 + 5x + 2 = (x + 2)(2x + 1)$

**a.** $3x^2 + 5x + 2 = \underline{\phantom{0}}$
7.6 Factoring $ax^2 + bx + c$ (continued)

1 EXPLORATION: Finding Binomial Factors (continued)

b. $4x^2 + 4x - 3 = \underline{_____}$

c. $2x^2 - 11x + 5 = \underline{_____}$

Communicate Your Answer

2. How can you use algebra tiles to factor the trinomial $ax^2 + bx + c$ into the product of two binomials?

3. Is it possible to factor the trinomial $2x^2 + 2x + 1$? Explain your reasoning.
7.6 Notetaking with Vocabulary
For use after Lesson 7.6

In your own words, write the meaning of each vocabulary term.

polynomial

greatest common factor (GCF)

Zero-Product Property

Notes:
7.6 Notetaking with Vocabulary (continued)

Extra Practice

In Exercises 1–18, factor the polynomial.

1. \(2c^2 - 14c - 36\)  
2. \(4a^2 + 8a - 140\)  
3. \(3x^2 - 6x - 24\)

4. \(2d^2 - 2d - 60\)  
5. \(5s^2 + 55s + 50\)  
6. \(3q^2 + 30q + 27\)

7. \(12g^2 - 37g + 28\)  
8. \(6k^2 - 11k + 4\)  
9. \(9w^2 + 9w + 2\)

10. \(12a^2 + 5a - 2\)  
11. \(15b^2 + 14b - 8\)  
12. \(5t^2 + 12t - 9\)
Notetaking with Vocabulary (continued)

13. \(-12b^2 + 5b + 2\)  
14. \(-6x^2 + x + 15\)  
15. \(-60g^2 - 11g + 1\)

16. \(-2d^2 - d + 6\)  
17. \(-3r^2 - 4r - 1\)  
18. \(-8x^2 + 14x - 5\)

19. The length of a rectangular shaped park is \((3x + 5)\) miles. The width is \((2x + 8)\) miles. The area of the park is 360 square miles. What are the dimensions of the park?

20. The sum of two numbers is 8. The sum of the squares of the two numbers is 34. What are the two numbers?
**7.7 Factoring Special Products**
For use with Exploration 7.7

**Essential Question**  How can you recognize and factor special products?

1 **EXPLORATION:** Factoring Special Products

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

**Work with a partner.** Use algebra tiles to write each polynomial as the product of two binomials. Check your answer by multiplying. State whether the product is a “special product” that you studied in Section 7.3.

a. $4x^2 - 1 = \underline{\quad}$

b. $4x^2 - 4x + 1 = \underline{\quad}$

c. $4x^2 + 4x + 1 = \underline{\quad}$

d. $4x^2 - 6x + 2 = \underline{\quad}$
**7.7 Factoring Special Products (continued)**

**EXPLORATION:** Factoring Special Products

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Use algebra tiles to complete the rectangular arrays in three different ways, so that each way represents a different special product. Write each special product in standard form and in factored form.

![Algebra Tiles Arrays](image)

**Communicate Your Answer**

3. How can you recognize and factor special products? Describe a strategy for recognizing which polynomials can be factored as special products.

4. Use the strategy you described in Question 3 to factor each polynomial.
   
   a. $25x^2 + 10x + 1$
   
   b. $25x^2 - 10x + 1$
   
   c. $25x^2 - 1$
In your own words, write the meaning of each vocabulary term.

polynomial

trinomial

### Core Concepts

#### Difference of Two Squares Pattern

<table>
<thead>
<tr>
<th>Algebra</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^2 - b^2 = (a + b)(a - b)$</td>
<td>$x^2 - 9 = x^2 - 3^2 = (x + 3)(x - 3)$</td>
</tr>
</tbody>
</table>

Notes:

#### Perfect Square Trinomial Pattern

<table>
<thead>
<tr>
<th>Algebra</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^2 + 2ab + b^2 = (a + b)^2$</td>
<td>$x^2 + 6x + 9 = x^2 + 2(x)(3) + 3^2$</td>
</tr>
<tr>
<td></td>
<td>$= (x + 3)^2$</td>
</tr>
<tr>
<td>$a^2 - 2ab + b^2 = (a - b)^2$</td>
<td>$x^2 - 6x + 9 = x^2 - 2(x)(3) + 3^2$</td>
</tr>
<tr>
<td></td>
<td>$= (x - 3)^2$</td>
</tr>
</tbody>
</table>

Notes:
Extra Practice

In Exercises 1–6, factor the polynomial.

1. \( s^2 - 49 \)  
2. \( t^2 - 81 \)  
3. \( 16 - x^2 \)  
4. \( 4g^2 - 25 \)  
5. \( 36h^2 - 121 \)  
6. \( 81 - 49h^2 \)  

In Exercises 7–12, use a special product pattern to evaluate the expression.

7. \( 57^2 - 53^2 \)  
8. \( 38^2 - 32^2 \)  
9. \( 68^2 - 64^2 \)  
10. \( 45^2 - 40^2 \)  
11. \( 79^2 - 71^2 \)  
12. \( 86^2 - 84^2 \)
In Exercises 13–18, factor the polynomial.

13. \(x^2 + 16x + 64\)  
14. \(p^2 + 28p + 196\)  
15. \(r^2 - 26r + 169\)

16. \(a^2 - 18a + 81\)  
17. \(36c^2 + 84c + 49\)  
18. \(100x^2 - 20x + 1\)

In Exercises 19–24, solve the equation.

19. \(x^2 - 144 = 0\)  
20. \(9y^2 = 49\)  
21. \(c^2 + 14c + 49 = 0\)

22. \(d^2 - 4d + 4 = 0\)  
23. \(n^2 + \frac{2}{3}n = -\frac{1}{9}\)  
24. \(-\frac{6}{5}k + \frac{9}{25} = -k^2\)

25. The dimensions of a rectangular prism are \((x + 1)\) feet by \((x + 2)\) feet by 4 feet. The volume of the prism is \((24x - 1)\) cubic feet. What is the value of \(x\)?
7.8 Factoring Polynomials Completely
For use with Exploration 7.8

Essential Question  How can you factor a polynomial completely?

1 EXPLORATION: Writing a Product of Linear Factors

Work with a partner. Write the product represented by the algebra tiles. Then multiply
to write the polynomial in standard form.

a. \((\square + \square)(\square + \square)(\square - \square)\)

b. \((\square + \square + \square)(\square + \square)(\square - \square)\)

c. \((\square + \square + \square + \square)(\square + \square)(\square + \square)\)

d. \((\square + \square)(\square + \square)(\square - \square)(\square + \square)\)

e. \((\square - \square)(\square + \square + \square)(\square - \square)\)

f. \((\square - \square)(\square + \square + \square)(\square - \square)\)

2 EXPLORATION: Matching Standard and Factored Forms

Work with a partner. Match the standard form of the polynomial with the equivalent
factored form on the next page. Explain your strategy.

a. \(x^3 + x^2\)  b. \(x^3 - x\)  c. \(x^3 + x^2 - 2x\)

d. \(x^3 - 4x^2 + 4x\)  e. \(x^3 - 2x^2 - 3x\)  f. \(x^3 - 2x^2 + x\)

g. \(x^3 - 4x\)  h. \(x^3 + 2x^2\)  i. \(x^3 - x^2\)

j. \(x^3 - 3x^2 + 2x\)  k. \(x^3 + 2x^2 - 3x\)  l. \(x^3 - 4x^2 + 3x\)

m. \(x^3 - 2x^2\)  n. \(x^3 + 4x^2 + 4x\)  o. \(x^3 + 2x^2 + x\)
7.8 Factoring Polynomials Completely (continued)

2 EXPLORATION: Matching Standard and Factored Forms (continued)

A. \(x(x + 1)(x - 1)\)  
B. \(x(x - 1)^2\)  
C. \(x(x + 1)^2\)  
D. \(x(x + 2)(x - 1)\)  
E. \(x(x - 1)(x - 2)\)  
F. \(x(x + 2)(x - 2)\)  
G. \(x(x - 2)^2\)  
H. \(x(x + 2)^2\)  
I. \(x^2(x - 1)\)  
J. \(x^2(x + 1)\)  
K. \(x^2(x - 2)\)  
L. \(x^2(x + 2)\)  
M. \(x(x + 3)(x - 1)\)  
N. \(x(x + 1)(x - 3)\)  
O. \(x(x - 1)(x - 3)\)

Communicate Your Answer

3. How can you factor a polynomial completely?

4. Use your answer to Question 3 to factor each polynomial completely.
   
a. \(x^3 + 4x^2 + 3x\)  
b. \(x^3 - 6x^2 + 9x\)  
c. \(x^3 + 6x^2 + 9x\)
In your own words, write the meaning of each vocabulary term.

factoring by grouping

factored completely

**Core Concepts**

**Factoring by Grouping**

To factor a polynomial with four terms, group the terms into pairs. Factor the GCF out of each pair of terms. Look for and factor out the common binomial factor. This process is called **factoring by grouping**.

**Guidelines for Factoring Polynomials Completely**

To factor a polynomial completely, you should try each of these steps.

1. Factor out the greatest common monomial factor.  
   \[3x^2 + 6x = 3(x + 2)\]

2. Look for a difference of two squares or a perfect square trinomial.  
   \[x^2 + 4x + 4 = (x + 2)^2\]

3. Factor a trinomial of the form \(ax^2 + bx + c\) into a product of binomial factors.  
   \[3x^2 - 5x - 2 = (3x + 1)(x - 2)\]

4. Factor a polynomial with four terms by grouping.  
   \[x^3 + x - 4x^2 - 4 = (x^2 + 1)(x - 4)\]
Extra Practice

In Exercises 1–8, factor the polynomial by grouping.

1. \( b^3 - 4b^2 + b - 4 \)
2. \( ac + ad + bc + bd \)

3. \( d^2 + 2c + cd + 2d \)
4. \( 5t^3 + 6t^2 + 5t + 6 \)

5. \( 8s^3 + s - 64s^2 - 8 \)
6. \( 12a^3 + 2a^2 - 30a - 5 \)

7. \( 4x^3 - 12x^2 - 5x + 15 \)
8. \( 21h^3 + 18h^2 - 35h - 30 \)
In Exercises 9–16, factor the polynomial completely.

9. $4c^3 - 4c$
10. $100x^4 - 25x^2$
11. $2a^2 + 3a - 2$

12. $9x^2 + 3x - 14$
13. $20p^2 + 22p - 12$
14. $12x^2 - 20x - 48$

15. $3s^3 + 2s^2 - 21s - 14$
16. $2t^4 + t^3 - 10t - 5$

In Exercises 17–22, solve the equation.

17. $3x^2 - 21x + 30 = 0$
18. $5y^2 - 5y - 30 = 0$
19. $c^4 - 81c^2 = 0$

20. $9d + 9 = d^3 + d^2$
21. $48n - 3n^2 = 0$
22. $x^3 + 3x^2 = 16x + 48$