Essential Question  How can you factor a polynomial completely?

1 EXPLORATION: Writing a Product of Linear Factors

Work with a partner. Write the product represented by the algebra tiles. Then multiply to write the polynomial in standard form.

a. \((\underline{+} \quad +)(\underline{+} \quad +)(\underline{-} \quad -)\)

b. \((\underline{+} \quad + \quad +)(\underline{+} \quad +)(\underline{-} \quad -)\)

c. \((\underline{+} \quad +)(\underline{+})(\underline{+} \quad +)\)

d. \((\underline{+} \quad +)(\underline{+} \quad -)(\underline{+} \quad +)\)

e. \((\underline{-} \quad +)(\underline{+} \quad +)(\underline{-} \quad -)\)

f. \((\underline{-} \quad -)(\underline{+} \quad +)(\underline{-} \quad -)\)

2 EXPLORATION: Matching Standard and Factored Forms

Work with a partner. Match the standard form of the polynomial with the equivalent factored form on the next page. Explain your strategy.

a. \(x^3 + x^2\)

d. \(x^3 - 4x^2 + 4x\)

g. \(x^3 - 4x\)

j. \(x^3 - 3x^2 + 2x\)
m. \(x^3 - 2x^2\)

b. \(x^3 - x\)

e. \(x^3 - 2x^2 - 3x\)

h. \(x^3 + 2x^2\)

k. \(x^3 + 2x^2 - 3x\)

i. \(x^3 - x^2\)

l. \(x^3 - 4x^2 + 3x\)

o. \(x^3 + 2x^2 + x\)

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7.8 Factoring Polynomials Completely (continued)

2 EXPLORE: Matching Standard and Factored Forms (continued)

A. \((x + 1)(x - 1)\)  
B. \((x - 1)^2\)  
C. \((x + 1)^2\)  
D. \((x + 2)(x - 1)\)  
E. \((x - 1)(x - 2)\)  
F. \((x + 2)(x - 2)\)  
G. \((x - 2)^2\)  
H. \((x + 2)^2\)  
I. \(x^2(x - 1)\)  
J. \(x^2(x + 1)\)  
K. \(x^2(x - 2)\)  
L. \(x^2(x + 2)\)  
M. \((x + 3)(x - 1)\)  
N. \((x + 1)(x - 3)\)  
O. \((x - 1)(x - 3)\)  

Communicate Your Answer

3. How can you factor a polynomial completely?

4. Use your answer to Question 3 to factor each polynomial completely.
   a. \(x^3 + 4x^2 + 3x\)  
   b. \(x^3 - 6x^2 + 9x\)  
   c. \(x^3 + 6x^2 + 9x\)
In your own words, write the meaning of each vocabulary term.

factoring by grouping

factored completely

**Core Concepts**

**Factoring by Grouping**

To factor a polynomial with four terms, group the terms into pairs. Factor the GCF out of each pair of terms. Look for and factor out the common binomial factor. This process is called factoring by grouping.

**Guidelines for Factoring Polynomials Completely**

To factor a polynomial completely, you should try each of these steps.

1. Factor out the greatest common monomial factor.  
   \[ 3x^2 + 6x = 3x(x + 2) \]

2. Look for a difference of two squares or a perfect square trinomial.  
   \[ x^2 + 4x + 4 = (x + 2)^2 \]

3. Factor a trinomial of the form \( ax^2 + bx + c \) into a product of binomial factors.  
   \[ 3x^2 - 5x - 2 = (3x + 1)(x - 2) \]

4. Factor a polynomial with four terms by grouping.  
   \[ x^3 + x - 4x^2 - 4 = (x^2 + 1)(x - 4) \]
7.8 Notetaking with Vocabulary (continued)

Extra Practice

In Exercises 1–8, factor the polynomial by grouping.

1. \( b^3 - 4b^2 + b - 4 \)
2. \( ac + ad + bc + bd \)

3. \( d^2 + 2c + cd + 2d \)
4. \( 5t^3 + 6t^2 + 5t + 6 \)

5. \( 8s^3 + s - 64s^2 - 8 \)
6. \( 12a^3 + 2a^2 - 30a - 5 \)

7. \( 4x^3 - 12x^2 - 5x + 15 \)
8. \( 21h^3 + 18h^2 - 35h - 30 \)
7.8 Notetaking with Vocabulary (continued)

In Exercises 9–16, factor the polynomial completely.

9. $4c^3 - 4c$  
10. $100x^4 - 25x^2$  
11. $2a^2 + 3a - 2$

12. $9x^2 + 3x - 14$  
13. $20p^2 + 22p - 12$  
14. $12x^2 - 20x - 48$

15. $3s^3 + 2s^2 - 21s - 14$  
16. $2t^4 + t^3 - 10t - 5$

In Exercises 17–22, solve the equation.

17. $3x^2 - 21x + 30 = 0$  
18. $5y^2 - 5y - 30 = 0$  
19. $c^4 - 81c^2 = 0$

20. $9d + 9 = d^3 + d^2$  
21. $48n - 3n^2 = 0$  
22. $x^3 + 3x^2 = 16x + 48$