

**Chapter
6****Maintaining Mathematical Proficiency**

Evaluate the expression.

1. $(14 + 20 - 6) \div 4 - 6^2$ 2. $(8 + 4)^2 + (13 - 10 \div 5)$ 3. $8 \div 4 \cdot 19 + 18 + 13$

4. $3 \cdot 14 \cdot 11 + 4^2 + 19$ 5. $(21 + 2)(14 - 6) + 3^2$ 6. $7(3 \cdot 10 - 4^2) + 8$

Find the square root(s).

7. $\sqrt{36}$

8. $-\sqrt{49}$

9. $-\sqrt{225}$

10. $\sqrt{144}$

11. $\sqrt{169}$

12. $\sqrt{9}$

13. $-\sqrt{16}$

14. $\pm\sqrt{256}$

Write an equation for the n th term of the arithmetic sequence.

15. 1, 5, 9, 13, ...

16. 21, 15, 9, 3, ...

17. -2, 1, 4, 7, ...

18. 8, 6, 4, 2, ...

19. -10, -4, 2, 8, ...

20. 16, 8, 0, -8, ...

6.1**Properties of Exponents**

For use with Exploration 6.1

Essential Question How can you write general rules involving properties of exponents?

1 EXPLORATION: Writing Rules for Properties of Exponents

Work with a partner.

- a. What happens when you multiply two powers with the same base? Write the product of the two powers as a single power. Then write a *general rule* for finding the product of two powers with the same base.

i. $(2^2)(2^3) =$ _____

ii. $(4^1)(4^5) =$ _____

iii. $(5^3)(5^5) =$ _____

iv. $(x^2)(x^6) =$ _____

- b. What happens when you divide two powers with the same base? Write the quotient of the two powers as a single power. Then write a *general rule* for finding the quotient of two powers with the same base.

i. $\frac{4^3}{4^2} =$ _____

ii. $\frac{2^5}{2^2} =$ _____

iii. $\frac{x^6}{x^3} =$ _____

iv. $\frac{3^4}{3^4} =$ _____

- c. What happens when you find a power of a power? Write the expression as a single power. Then write a *general rule* for finding a power of a power.

i. $(2^2)^4 =$ _____

ii. $(7^3)^2 =$ _____

iii. $(y^3)^3 =$ _____

iv. $(x^4)^2 =$ _____

6.1 Properties of Exponents (continued)**1** **EXPLORATION:** Writing Rules for Properties of Exponents (continued)

- d. What happens when you find a power of a product? Write the expression as the product of two powers. Then write a *general rule* for finding a power of a product.

i. $(2 \cdot 5)^2 = \underline{\hspace{2cm}}$

ii. $(5 \cdot 4)^3 = \underline{\hspace{2cm}}$

iii. $(6a)^2 = \underline{\hspace{2cm}}$

iv. $(3x)^2 = \underline{\hspace{2cm}}$

- e. What happens when you find a power of a quotient? Write the expression as the quotient of two powers. Then write a *general rule* for finding a power of a quotient.

i. $\left(\frac{2}{3}\right)^2 = \underline{\hspace{2cm}}$

ii. $\left(\frac{4}{3}\right)^3 = \underline{\hspace{2cm}}$

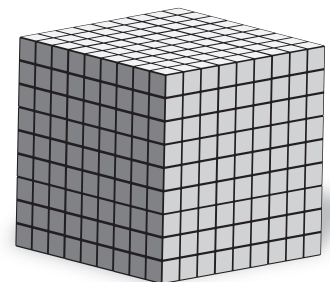
iii. $\left(\frac{x}{2}\right)^3 = \underline{\hspace{2cm}}$

iv. $\left(\frac{a}{b}\right)^4 = \underline{\hspace{2cm}}$

Communicate Your Answer

2. How can you write general rules involving properties of exponents?

3. There are 3^3 small cubes in the cube below. Write an expression for the number of small cubes in the large cube at the right.



6.1**Notetaking with Vocabulary**

For use after Lesson 6.1

In your own words, write the meaning of each vocabulary term.

power

exponent

base

scientific notation

Core Concepts**Zero Exponent**

Words For any nonzero number a , $a^0 = 1$. The power 0^0 is undefined.

Numbers $4^0 = 1$ **Algebra** $a^0 = 1$, where $a \neq 0$

Negative Exponents

Words For any integer n and any nonzero number a , a^{-n} is the reciprocal of a^n .

Numbers $4^{-2} = \frac{1}{4^2}$ **Algebra** $a^{-n} = \frac{1}{a^n}$, where $a \neq 0$

Notes:

6.1 Notetaking with Vocabulary (continued)**Product of Powers Property**

Let a be a real number, and let m and n be integers.

Words To multiply powers with the same base, add their exponents.

Numbers $4^6 \cdot 4^3 = 4^{6+3} = 4^9$ **Algebra** $a^m \cdot a^n = a^{m+n}$

Quotient of Powers Property

Let a be a nonzero real number, and let m and n be integers.

Words To divide powers with the same base, subtract their exponents.

Numbers $\frac{4^6}{4^3} = 4^{6-3} = 4^3$ **Algebra** $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$

Power of a Power Property

Let a be a real number, and let m and n be integers.

Words To find a power of a power, multiply the exponents.

Numbers $(4^6)^3 = 4^{6 \cdot 3} = 4^{18}$ **Algebra** $(a^m)^n = a^{mn}$

Notes:

Power of a Product Property

Let a and b be real numbers, and let m be an integer.

Words To find a power of a product, find the power of each factor and multiply.

Numbers $(3 \cdot 2)^5 = 3^5 \cdot 2^5$ **Algebra** $(ab)^m = a^m b^m$

Power of a Quotient Property

Let a and b be real numbers with $b \neq 0$, and let m be an integer.

Words To find the power of a quotient, find the power of the numerator and the power of the denominator and divide.

Numbers $\left(\frac{3}{2}\right)^5 = \frac{3^5}{2^5}$ **Algebra** $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, where $b \neq 0$

Notes:

6.1 Notetaking with Vocabulary (continued)**Extra Practice**

In Exercises 1–8, evaluate the expression.

1. 3^0

2. $(-2)^0$

3. 3^{-4}

4. $(-4)^{-3}$

5. $\frac{2^{-3}}{5^0}$

6. $\frac{-3^{-2}}{2^{-3}}$

7. $\frac{4^{-1}}{-7^0}$

8. $\frac{3^{-1}}{(-5)^0}$

In Exercises 9–23, simplify the expression. Write your answer using only positive exponents.

9. z^0

10. a^{-8}

11. $6a^0b^{-2}$

12. $14m^{-4}n^0$

13. $\frac{3^{-2}r^{-3}}{s^0}$

14. $\frac{2^3a^{-3}}{8^{-1}b^{-5}c^0}$

15. $\frac{3^5}{3^3}$

16. $\frac{(-2)^7}{(-2)^5}$

17. $(-5)^3 \cdot (-5)^3$

18. $(q^5)^3$

19. $(a^{-4})^2$

20. $\frac{c^4 \cdot c^3}{c^6}$

21. $(-4d)^4$

22. $(-3f)^{-3}$

23. $\left(\frac{4}{x}\right)^{-3}$

24. A rectangular prism has length x , width $\frac{x}{2}$, and height $\frac{x}{3}$. Which of the expressions represent the volume of the prism? Select all that apply.

A. $6^{-1}x^3$

B. $6^{-1}x^{-3}$

C. $(6x^{-3})^{-1}$

D. $2^{-1} \cdot 3^{-1} \cdot x^3$

6.2**Radicals and Rational Exponents**

For use with Exploration 6.2

Essential Question How can you write and evaluate an n th root of a number?

Recall that you cube a number as follows.

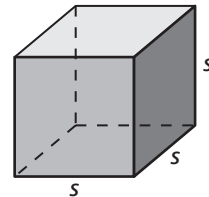
3rd power \rightarrow $2^3 = 2 \cdot 2 \cdot 2 = 8$ 2 cubed is 8.

To “undo” cubing a number, take the cube root of the number.

Symbol for cube root is $\sqrt[3]{\quad}$. \rightarrow $\sqrt[3]{8} = \sqrt[3]{2^3} = 2$ The cube root of 8 is 2.

1 EXPLORATION: Finding Cube Roots

Work with a partner. Use a cube root symbol to write the side length of each cube. Then find the cube root. Check your answers by multiplying. Which cube is the largest? Which two cubes are the same size? Explain your reasoning.



a. Volume = 27 ft^3

b. Volume = 125 cm^3

c. Volume = 3375 in.^3

d. Volume = 3.375 m^3

e. Volume = 1 yd^3

f. Volume = $\frac{125}{8} \text{ mm}^3$

6.2 Radicals and Rational Exponents (continued)**2 EXPLORATION:** Estimating n th Roots

Work with a partner. Estimate each positive n th root. Then match each n th root with the point on the number line. Justify your answers.

a. $\sqrt[4]{25}$

b. $\sqrt{0.5}$

c. $\sqrt[5]{2.5}$

d. $\sqrt[3]{65}$

e. $\sqrt[3]{55}$

f. $\sqrt[6]{20,000}$

**Communicate Your Answer**

3. How can you write and evaluate an n th root of a number?

4. The body mass m (in kilograms) of a dinosaur that walked on two feet can be modeled by

$$m = (0.00016)C^{2.73}$$

where C is the circumference (in millimeters) of the dinosaur's femur. The mass of a *Tyrannosaurus rex* was 4000 kilograms. Use a calculator to approximate the circumference of its femur.

6.2**Notetaking with Vocabulary**

For use after Lesson 6.2

In your own words, write the meaning of each vocabulary term.

n th root of a

radical

index of a radical

Core Concepts**Real n th Roots of a**

Let n be an integer greater than 1, and let a be a real number.

- If n is odd, then a has one real n th root: $\sqrt[n]{a} = a^{1/n}$
- If n is even and $a > 0$, then a has two real n th roots: $\pm\sqrt[n]{a} = \pm a^{1/n}$
- If n is even and $a = 0$, then a has one real n th root: $\sqrt[n]{0} = 0$
- If n is even and $a < 0$, then a has no real n th roots.

Notes:

6.2 Notetaking with Vocabulary (continued)**Rational Exponents**

Let $a^{1/n}$ be an n th root of a , and let m be a positive integer.

Algebra $a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$

Numbers $27^{2/3} = (27^{1/3})^2 = (\sqrt[3]{27})^2$

Notes:

Extra Practice

In Exercises 1–6, find the indicated real n th root(s) of a .

1. $n = 2, a = 64$

2. $n = 3, a = 27$

3. $n = 4, a = 256$

4. $n = 5, a = 243$

5. $n = 8, a = 256$

6. $n = 4, a = 10,000$

In Exercises 7–12, evaluate the expression.

7. $\sqrt[4]{625}$

8. $\sqrt[3]{-512}$

9. $\sqrt[3]{-216}$

10. $\sqrt[5]{-243}$

11. $729^{1/6}$

12. $(-81)^{1/2}$

6.2 Notetaking with Vocabulary (continued)

In Exercises 13–15, rewrite the expression in rational exponent form.

13. $(\sqrt[5]{4})^3$

14. $(\sqrt[3]{-8})^2$

15. $(\sqrt[4]{15})^7$

In Exercises 16–18, rewrite the expression in radical form.

16. $(-3)^{2/5}$

17. $6^{3/2}$

18. $12^{3/4}$

In Exercises 19–24, evaluate the expression.

19. $32^{2/5}$

20. $(-64)^{3/2}$

21. $343^{2/3}$

22. $256^{7/8}$

23. $-729^{5/6}$

24. $(-625)^{3/4}$

25. The radius r of a sphere is given by the equation

$$r = \left(\frac{A}{4\pi} \right)^{1/2}$$

where A is the surface area of the sphere. The surface area of a sphere is 1493 square meters. Find the radius of the sphere to the nearest tenth of a meter. Use 3.14 for π .

6.3

Exponential Functions

For use with Exploration 6.3

Essential Question What are some of the characteristics of the graph of an exponential function?

1 EXPLORATION: Exploring an Exponential Function

Work with a partner. Complete each table for the exponential function $y = 16(2)^x$. In each table, what do you notice about the values of x ? What do you notice about the values of y ?

x	$y = 16(2)^x$
0	
1	
2	
3	
4	
5	

x	$y = 16(2)^x$
0	
2	
4	
6	
8	
10	

2 EXPLORATION: Exploring an Exponential Function

Work with a partner. Repeat Exploration 1 for the exponential function $y = 16\left(\frac{1}{2}\right)^x$.

x	$y = 16\left(\frac{1}{2}\right)^x$
0	
1	
2	
3	
4	
5	

x	$y = 16\left(\frac{1}{2}\right)^x$
0	
2	
4	
6	
8	
10	

Do you think the statement below is true for *any* exponential function? Justify your answer.

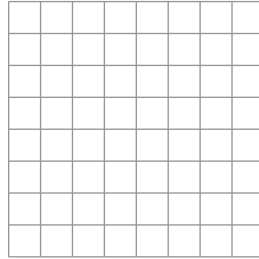
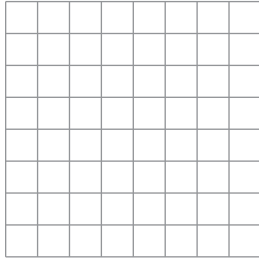
“As the independent variable x changes by a constant amount, the dependent variable y is multiplied by a constant factor.”

6.3 Exponential Functions (continued)

3 EXPLORATION: Graphing Exponential Functions

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Sketch the graphs of the functions given in Explorations 1 and 2. How are the graphs similar? How are they different?

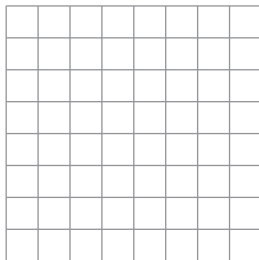


Communicate Your Answer

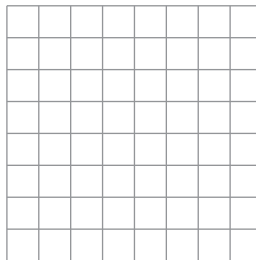
4. What are some of the characteristics of the graph of an exponential function?

5. Sketch the graph of each exponential function. Does each graph have the characteristics you described in Question 4? Explain your reasoning.

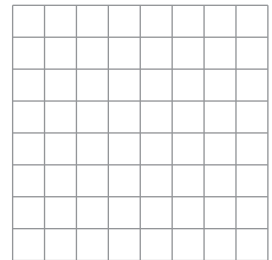
a. $y = 2^x$



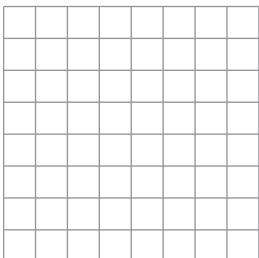
b. $y = 2(3)^x$



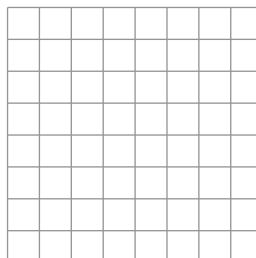
c. $y = 3(1.5)^x$



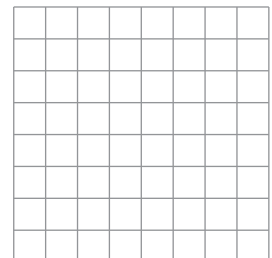
d. $y = \left(\frac{1}{2}\right)^x$



e. $y = 3\left(\frac{1}{2}\right)^x$



f. $y = 2\left(\frac{3}{4}\right)^x$

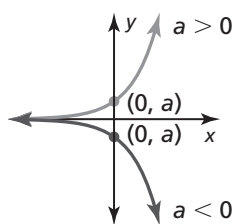
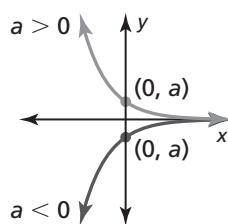


6.3**Notetaking with Vocabulary**

For use after Lesson 6.3

In your own words, write the meaning of each vocabulary term.

exponential function

Core ConceptsGraphing $y = ab^x$ When $b > 1$ Graphing $y = ab^x$ When $0 < b < 1$ **Notes:**

6.3 Notetaking with Vocabulary (continued)

Extra Practice

In Exercises 1–4, determine whether the table represents a *linear* or an *exponential* function. Explain.

1.

x	y
1	8
2	4
3	2
4	1

2.

x	y
1	3
2	7
3	11
4	15

3.

x	y
-1	12
0	9
1	6
2	3

4.

x	y
-1	0.125
0	0.5
1	2
2	8

In Exercises 5–7, evaluate the function for the given value of x.

5. $y = 3^x; x = 5$

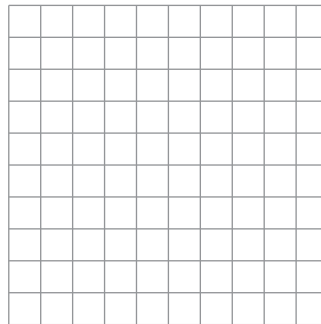
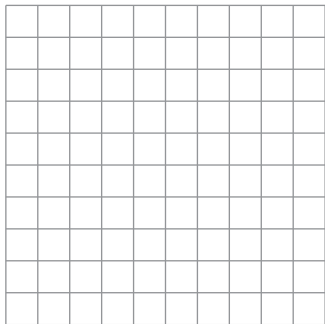
6. $y = \left(\frac{1}{4}\right)^x; x = 3$

7. $y = 3(4)^x; x = 4$

In Exercises 8 and 9, graph the function. Compare the graph to the graph of the parent function. Describe the domain and range of f.

8. $f(x) = -2^x$

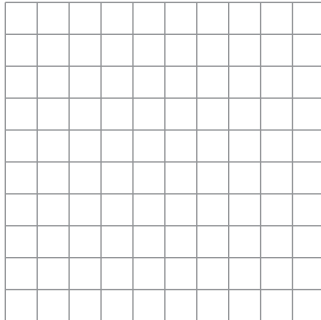
9. $f(x) = 2\left(\frac{1}{4}\right)^x$



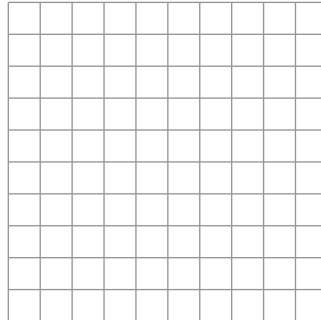
6.3 Notetaking with Vocabulary (continued)

In Exercises 10 and 11, graph the function. Describe the domain and range.

10. $f(x) = 4^x - 2$



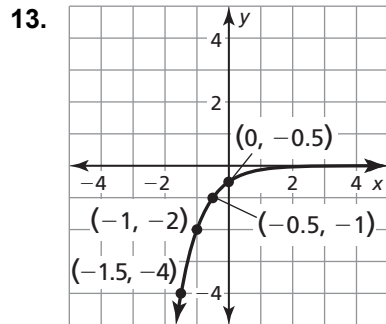
11. $f(x) = 4\left(\frac{1}{2}\right)^{x+1}$



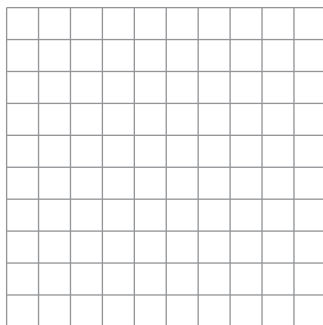
In Exercises 12 and 13, write an exponential function represented by the table or graph.

12.

x	0	1	2	3
$f(x)$	3	18	108	648



14. Graph the function $f(x) = 2^x$. Then graph $g(x) = 2^x + 3$. How are the y -intercept, domain, and range affected by the translation?



6.4

Exponential Growth and Decay

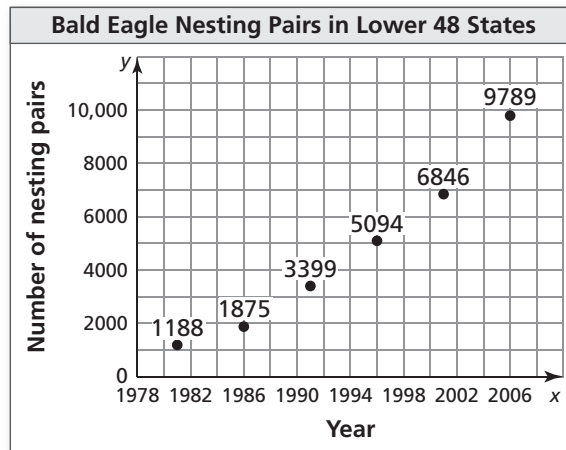
For use with Exploration 6.4

Essential Question What are some of the characteristics of exponential growth and exponential decay functions?

1 EXPLORATION: Predicting a Future Event

Work with a partner. It is estimated, that in 1782, there were about 100,000 nesting pairs of bald eagles in the United States. By the 1960s, this number had dropped to about 500 nesting pairs. In 1967, the bald eagle was declared an endangered species in the United States. With protection, the nesting pair population began to increase. Finally, in 2007, the bald eagle was removed from the list of endangered and threatened species.

Describe the pattern shown in the graph. Is it exponential growth? Assume the pattern continues. When will the population return to that of the late 1700s? Explain your reasoning.



6.4 Exponential Growth and Decay (continued)**2 EXPLORATION: Describing a Decay Pattern**

Work with a partner. A forensic pathologist was called to estimate the time of death of a person. At midnight, the body temperature was 80.5°F and the room temperature was a constant 60°F . One hour later, the body temperature was 78.5°F .

- By what percent did the difference between the body temperature and the room temperature drop during the hour?
- Assume that the original body temperature was 98.6°F . Use the percent decrease found in part (a) to make a table showing the decreases in body temperature. Use the table to estimate the time of death.

Time (h)								
Temperature difference ($^{\circ}\text{F}$)								
Body temperature ($^{\circ}\text{F}$)								

Communicate Your Answer

- What are some of the characteristics of exponential growth and exponential decay functions?
- Use the Internet or some other reference to find an example of each type of function. Your examples should be different than those given in Explorations 1 and 2.
 - exponential growth
 - exponential decay

6.4**Notetaking with Vocabulary**

For use after Lesson 6.4

In your own words, write the meaning of each vocabulary term.

exponential growth

exponential growth function

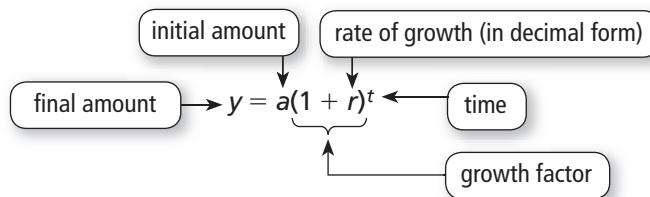
exponential decay

exponential decay function

compound interest

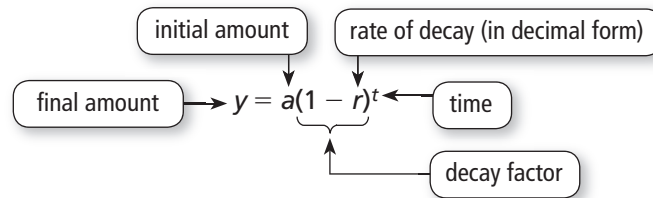
Core Concepts**Exponential Growth Functions**

A function of the form $y = a(1 + r)^t$, where $a > 0$ and $r > 0$, is an **exponential growth function**.

**Notes:**

6.4 Notetaking with Vocabulary (continued)**Exponential Decay Functions**

A function of the form $y = a(1 - r)^t$, where $a > 0$ and $0 < r < 1$, is an **exponential decay function**.

**Notes:****Compound Interest**

Compound interest is the interest earned on the principal *and* on previously earned interest. The balance y of an account earning compound interest is

$$y = P\left(1 + \frac{r}{n}\right)^{nt}$$

P = principal (initial amount)

r = annual interest rate (in decimal form)

t = time (in years)

n = number of times interest is compounded per year

Notes:

6.4 Notetaking with Vocabulary (continued)**Extra Practice**

1. In 2005, there were 100 rabbits in Polygon Park. The population increased by 11% each year.
 - a. Write an exponential growth function that represents the population t years after 2005.
 - b. What will the population be in 2025? Round your answer to the nearest whole number.

In Exercises 2–5, determine whether the table represents an *exponential growth function*, an *exponential decay function*, or *neither*. Explain.

2.

x	y
0	20
1	30
2	45
3	67.5

3.

x	y
-1	160
0	40
1	10
2	2.5

4.

x	y
1	32
2	22
3	12
4	2

5.

x	y
-1	4
0	10
1	25
2	62.5

In Exercises 6–8, determine whether each function represents *exponential growth* or *exponential decay*. Identify the percent rate of change.

6. $y = 4(0.95)^t$

7. $y = 500(1.08)^t$

8. $w(t) = \left(\frac{3}{4}\right)^t$

In Exercises 9 and 10, write a function that represents the balance after t years.

9. \$3000 deposit that earns 6% annual interest compounded quarterly.
10. \$5000 deposit that earns 7.2% annual interest compounded monthly.

6.5**Solving Exponential Equations**

For use with Exploration 6.5

Essential Question How can you solve an exponential equation graphically?

1 EXPLORATION: Solving an Exponential Equation Graphically

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

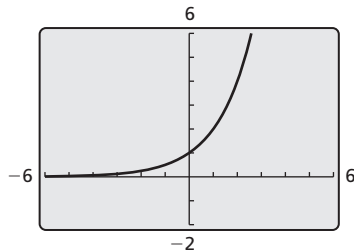
Work with a partner. Use a graphing calculator to solve the exponential equation $2.5^{x-3} = 6.25$ graphically. Describe your process and explain how you determined the solution.

2 EXPLORATION: The Number of Solutions of an Exponential Equation

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

- a. Use a graphing calculator to graph the equation $y = 2^x$.



- b. In the same viewing window, graph a linear equation (if possible) that does not intersect the graph of $y = 2^x$.
- c. In the same viewing window, graph a linear equation (if possible) that intersects the graph of $y = 2^x$ in more than one point.
- d. Is it possible for an exponential equation to have no solution? more than one solution? Explain your reasoning.

6.5 Solving Exponential Equations (continued)**3 EXPLORATION: Solving Exponential Equations Graphically**

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use a graphing calculator to solve each equation.

a. $2^x = \frac{1}{2}$

b. $2^{x+1} = 0$

c. $2^x = \sqrt{2}$

d. $3^x = 9$

e. $3^{x-1} = 0$

f. $4^{2x} = 2$

g. $2^{x/2} = \frac{1}{4}$

h. $3^{x+2} = \frac{1}{9}$

i. $2^{x-2} = \frac{3}{2}x - 2$

Communicate Your Answer

- How can you solve an exponential equation graphically?
- A population of 30 mice is expected to double each year. The number p of mice in the population each year is given by $p = 30(2^n)$. In how many years will there be 960 mice in the population?

6.5**Notetaking with Vocabulary**

For use after Lesson 6.5

In your own words, write the meaning of each vocabulary term.

exponential equation

Core Concepts**Property of Equality for Exponential Equations**

Words Two powers with the *same positive base* b , where $b \neq 1$, are equal if and only if their exponents are equal.

Numbers If $2^x = 2^5$, then $x = 5$. If $x = 5$, then $2^x = 2^5$.

Algebra If $b > 0$ and $b \neq 1$, then $b^x = b^y$ if and only if $x = y$.

Notes:

6.5 Notetaking with Vocabulary (continued)**Extra Practice**

In Exercises 1–15, solve the equation. Check your solution.

1. $3^{4x} = 3^{12}$

2. $8^{x+5} = 8^{20}$

3. $6^{4x-5} = 6^{2x}$

4. $5^{6x-3} = 5^{-3+4x}$

5. $4^{2x+11} = 1024$

6. $8^{3-2x} = 512$

7. $4^{7-x} = 256$

8. $49^{x-2} = 343$

9. $36^{6x-1} = 6^{5x}$

10. $9^{x-4} = 81^{3x}$

11. $64^{x+1} = 512^x$

12. $6^{2x} = 36^{2x+1}$

6.5 Notetaking with Vocabulary (continued)

13. $\left(\frac{1}{7}\right)^x = 2401$

14. $\frac{1}{512} = 2^{3x-1}$

15. $25^{2-2x} = \left(\frac{1}{625}\right)^{x+1}$

In Exercises 16–21, use a graphing calculator to solve the equation.

16. $3^{x+3} = -9$

17. $\left(\frac{1}{4}\right)^{-x-1} = 18$

18. $3^x = -2^{-x+1}$

19. $2^{x+2} = 5^{x-3}$

20. $7^{-x+1} = -4^{x-1}$

21. $\frac{1}{4}x + 1 = \left(\frac{2}{3}\right)^{2x-1}$

22. You deposit \$1000 in a savings account that earns 5% annual interest compounded yearly.

a. Write an exponential equation to determine when the balance of the account will be \$1500.

b. Solve the equation.

6.6

Geometric Sequences

For use with Exploration 6.6

Essential Question How can you use a geometric sequence to describe a pattern?

In a **geometric sequence**, the ratio between each pair of consecutive terms is the same. This ratio is called the **common ratio**.

1 EXPLORATION: Describing Calculator Patterns

Work with a partner. Enter the keystrokes on a calculator and record the results in the table. Describe the pattern.

a. Step 1 2 =

Step 2 × 2 =

Step 3 × 2 =

Step 4 × 2 =

Step 5 × 2 =

Step	1	2	3	4	5
Calculator display					

b. Step 1 6 4 =

Step 2 × . 5 =

Step 3 × . 5 =

Step 4 × . 5 =

Step 5 × . 5 =

Step	1	2	3	4	5
Calculator display					

c. Use a calculator to make your own sequence. Start with any number and multiply by 3 each time. Record your results in the table.

Step	1	2	3	4	5
Calculator display					

d. Part (a) involves a geometric sequence with a common ratio of 2. What is the common ratio in part (b)? part (c)?

6.6 Geometric Sequences (continued)**2 EXPLORATION: Folding a Sheet of Paper**

Work with a partner. A sheet of paper is about 0.1 millimeter thick.

- a. How thick will it be when you fold it in half once? twice? three times?



- b. What is the greatest number of times you can fold a piece of paper in half? How thick is the result?



- c. Do you agree with the statement below? Explain your reasoning.

“If it were possible to fold the paper in half 15 times, it would be taller than you.”

Communicate Your Answer

3. How can you use a geometric sequence to describe a pattern?
4. Give an example of a geometric sequence from real life other than paper folding.

6.6**Notetaking with Vocabulary**

For use after Lesson 6.6

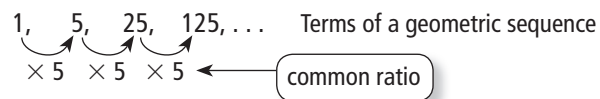
In your own words, write the meaning of each vocabulary term.

geometric sequence

common ratio

Core Concepts**Geometric Sequence**

In a **geometric sequence**, the ratio between each pair of consecutive terms is the same. This ratio is called the **common ratio**. Each term is found by multiplying the previous term by the common ratio.

**Notes:****Equation for a Geometric Sequence**

Let a_n be the n th term of a geometric sequence with first term a_1 and common ratio r . The n th term is given by

$$a_n = a_1 r^{n-1}.$$

Notes:

6.6 Notetaking with Vocabulary (continued)**Extra Practice**

In Exercises 1–6, determine whether the sequence is *arithmetic*, *geometric*, or *neither*. Explain your reasoning.

1. $1, -4, 16, -64, \dots$ 2. $3, 7, 11, 15, \dots$ 3. $2, 4, 8, 32, \dots$

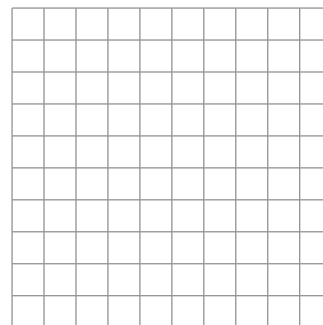
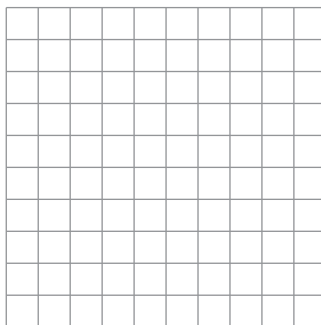
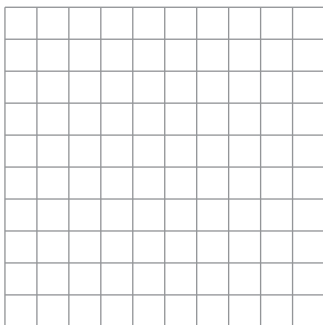
4. $12, 9, 7, 5, \dots$ 5. $6, 18, 54, 162, \dots$ 6. $11, 19, 27, 35, \dots$

In Exercises 7–9, write the next three terms of the geometric sequence.

7. $7, 21, 63, 189, \dots$ 8. $576, 288, 144, 72, \dots$ 9. $5, -10, 20, -40, \dots$

In Exercises 10–12, write the next three terms of the geometric sequence. Then graph the sequence.

10. $12, 6, 3, \frac{3}{2}, \dots$ 11. $3, 12, 48, 192, \dots$ 12. $0.008, 0.04, 0.2, 1, \dots$



6.6 Notetaking with Vocabulary (continued)

In Exercises 13–20, write an equation for the n th term of the geometric sequence. Then find a_6 .

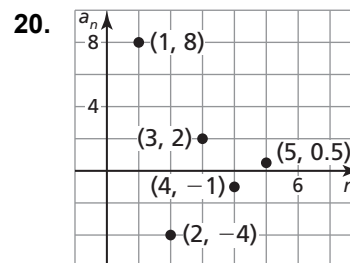
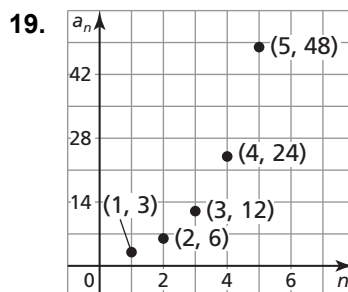
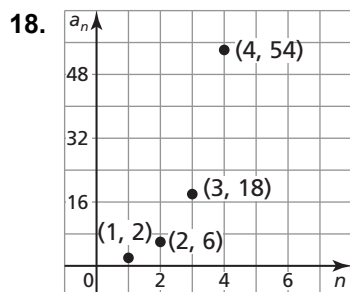
13. 6561, 2187, 729, 243, ... 14. 8, -24, 72, -216, ... 15. 3, 15, 75, 375, ...

16.

n	1	2	3	4
a_n	2916	972	324	108

17.

n	1	2	3	4
a_n	11	44	176	704



6.7

Recursively Defined Sequences






For use with Exploration 6.7

Essential Question How can you define a sequence recursively?

A **recursive rule** gives the beginning term(s) of a sequence and a *recursive equation* that tells how a_n is related to one or more preceding terms

1 EXPLORATION: Describing a Pattern

Work with a partner. Consider a hypothetical population of rabbits. Start with one breeding pair. After each month, each breeding pair produces another breeding pair. The total number of rabbits each month follows the exponential pattern 2, 4, 8, 16, 32, Now suppose that in the first month after each pair is born, the pair is too young to reproduce. Each pair produces another pair after it is 2 months old. Find the total number of pairs in months 6, 7, and 8.

Month		Number of pairs
1	 <p>Red pair is too young to produce.</p>	1
2	 <p>Red pair produces blue pair.</p>	1
3	 <p>Red pair produces green pair.</p>	2
4	 <p>Red pair produces orange pair.</p> <p>Blue pair produces purple pair.</p>	3
5		5
6		
7		
8		

6.7 Recursively Defined Sequences (continued)**2 EXPLORATION:** Using a Recursive Equation

Work with a partner. Consider the following recursive equation.

$$a_n = a_{n-1} + a_{n-2}$$

Each term in the sequence is the sum of the two preceding terms.

Complete the table. Compare the results with the sequence of the number of pairs in Exploration 1.

a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
1	1						

Communicate Your Answer

- How can you define a sequence recursively?
- Use the Internet or some other reference to determine the mathematician who first described the sequences in Explorations 1 and 2.

6.7**Notetaking with Vocabulary**

For use after Lesson 6.7

In your own words, write the meaning of each vocabulary term.

explicit rule

recursive rule

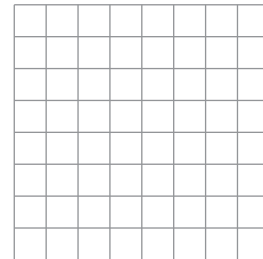
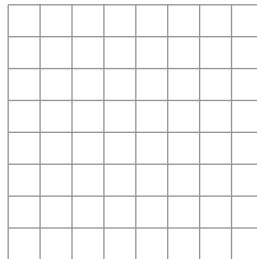
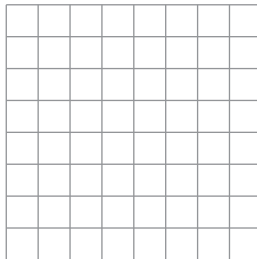
Core Concepts**Recursive Equation for an Arithmetic Sequence** $a_n = a_{n-1} + d$, where d is the common difference**Recursive Equation for a Geometric Sequence** $a_n = r \cdot a_{n-1}$, where r is the common ratio**Notes:**

6.7 Notetaking with Vocabulary (continued)

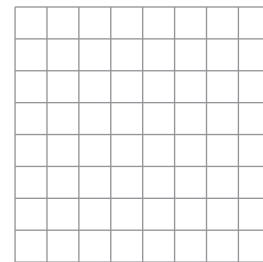
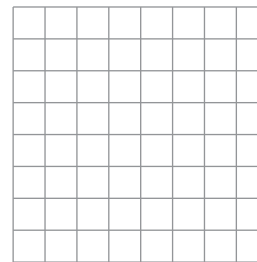
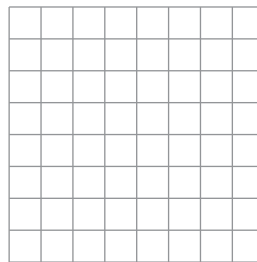
Extra Practice

In Exercises 1–6, write the first six terms of the sequence. Then graph the sequence.

1. $a_1 = -2; a_n = -2a_{n-1}$ 2. $a_1 = -4; a_n = a_{n-1} + 3$ 3. $a_1 = 4; a_n = 1.5a_{n-1}$



4. $a_1 = 14; a_n = a_{n-1} - 4$ 5. $a_1 = -\frac{1}{2}; a_n = -2a_{n-1}$ 6. $a_1 = -3; a_n = a_{n-1} + 2$



In Exercises 7 and 8, write a recursive rule for the sequence.

7.

n	1	2	3	4
a_n	324	108	36	12

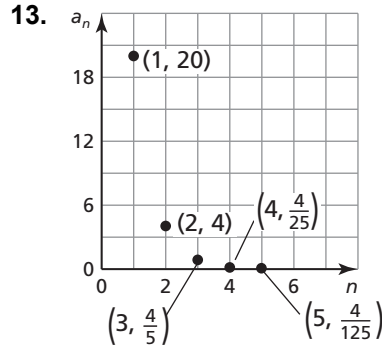
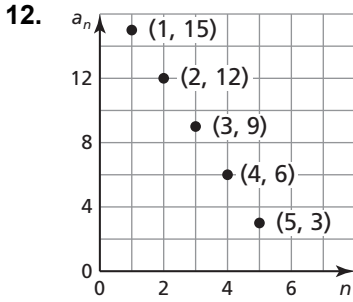
8.

n	1	2	3	4
a_n	9	14	19	24

6.7 Notetaking with Vocabulary (continued)

In Exercises 9–13, write a recursive rule for the sequence.

9. 3125, 625, 125, 25, ... 10. 8, -24, 72, -216, ... 11. 7, 13, 19, 25, ...



In Exercises 14–16, write an explicit rule for the recursive rule.

14. $a_1 = 4; a_n = 3a_{n-1}$ 15. $a_1 = 6; a_n = a_{n-1} + 11$ 16. $a_1 = -1; a_n = 5a_{n-1}$

In Exercises 17–19, write a recursive rule for the explicit rule.

17. $a_n = 6n + 2$ 18. $a_n = (-3)^{n-1}$ 19. $a_n = -2n + 1$

In Exercises 20–22, write a recursive rule for the sequence. Then write the next two terms of the sequence.

20. 2, 4, 6, 10, 16, 26, ... 21. 1, 3, -2, 5, -7, 12, ... 22. 1, 2, 2, 4, 8, 32, ...