

6.7

Recursively Defined Sequences




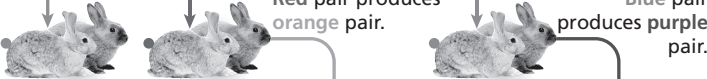

For use with Exploration 6.7

Essential Question How can you define a sequence recursively?

A **recursive rule** gives the beginning term(s) of a sequence and a *recursive equation* that tells how a_n is related to one or more preceding terms

1 EXPLORATION: Describing a Pattern

Work with a partner. Consider a hypothetical population of rabbits. Start with one breeding pair. After each month, each breeding pair produces another breeding pair. The total number of rabbits each month follows the exponential pattern 2, 4, 8, 16, 32, Now suppose that in the first month after each pair is born, the pair is too young to reproduce. Each pair produces another pair after it is 2 months old. Find the total number of pairs in months 6, 7, and 8.

| Month | | Number of pairs |
|-------|---|-----------------|
| 1 |  <p>Red pair is too young to produce.</p> | 1 |
| 2 |  <p>Red pair produces blue pair.</p> | 1 |
| 3 |  <p>Red pair produces green pair.</p> | 2 |
| 4 |  <p>Red pair produces orange pair.</p> <p>Blue pair produces purple pair.</p> | 3 |
| 5 |  | 5 |
| 6 | | |
| 7 | | |
| 8 | | |

6.7 Recursively Defined Sequences (continued)**2 EXPLORATION:** Using a Recursive Equation

Work with a partner. Consider the following recursive equation.

$$a_n = a_{n-1} + a_{n-2}$$

Each term in the sequence is the sum of the two preceding terms.

Complete the table. Compare the results with the sequence of the number of pairs in Exploration 1.

| a_1 | a_2 | a_3 | a_4 | a_5 | a_6 | a_7 | a_8 |
|-------|-------|-------|-------|-------|-------|-------|-------|
| 1 | 1 | | | | | | |

Communicate Your Answer

- How can you define a sequence recursively?
- Use the Internet or some other reference to determine the mathematician who first described the sequences in Explorations 1 and 2.

6.7**Notetaking with Vocabulary**

For use after Lesson 6.7

In your own words, write the meaning of each vocabulary term.

explicit rule

recursive rule

Core Concepts**Recursive Equation for an Arithmetic Sequence**

$a_n = a_{n-1} + d$, where d is the common difference

Recursive Equation for a Geometric Sequence

$a_n = r \cdot a_{n-1}$, where r is the common ratio

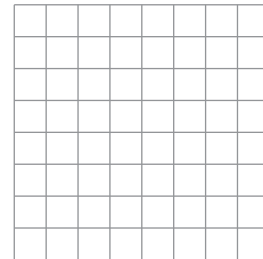
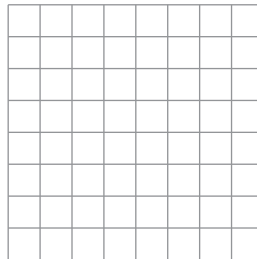
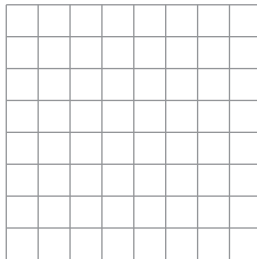
Notes:

6.7 Notetaking with Vocabulary (continued)

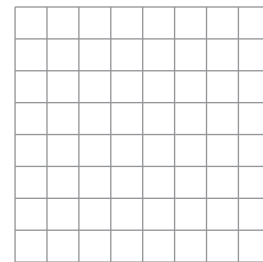
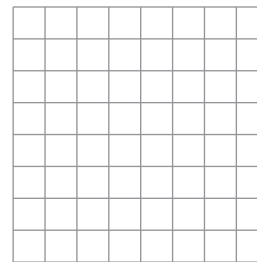
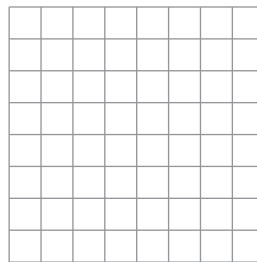
Extra Practice

In Exercises 1–6, write the first six terms of the sequence. Then graph the sequence.

1. $a_1 = -2; a_n = -2a_{n-1}$ 2. $a_1 = -4; a_n = a_{n-1} + 3$ 3. $a_1 = 4; a_n = 1.5a_{n-1}$



4. $a_1 = 14; a_n = a_{n-1} - 4$ 5. $a_1 = -\frac{1}{2}; a_n = -2a_{n-1}$ 6. $a_1 = -3; a_n = a_{n-1} + 2$



In Exercises 7 and 8, write a recursive rule for the sequence.

7.

| | | | | |
|-------|-----|-----|----|----|
| n | 1 | 2 | 3 | 4 |
| a_n | 324 | 108 | 36 | 12 |

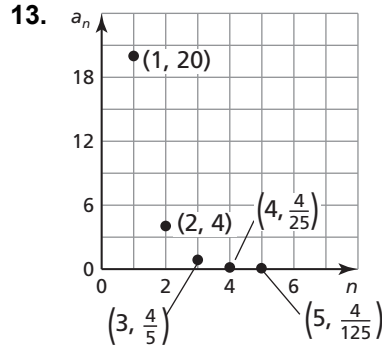
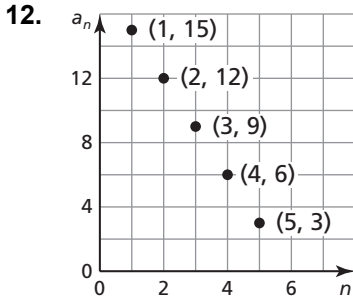
8.

| | | | | |
|-------|---|----|----|----|
| n | 1 | 2 | 3 | 4 |
| a_n | 9 | 14 | 19 | 24 |

6.7 Notetaking with Vocabulary (continued)

In Exercises 9–13, write a recursive rule for the sequence.

9. 3125, 625, 125, 25, ... 10. 8, -24, 72, -216, ... 11. 7, 13, 19, 25, ...



In Exercises 14–16, write an explicit rule for the recursive rule.

14. $a_1 = 4; a_n = 3a_{n-1}$ 15. $a_1 = 6; a_n = a_{n-1} + 11$ 16. $a_1 = -1; a_n = 5a_{n-1}$

In Exercises 17–19, write a recursive rule for the explicit rule.

17. $a_n = 6n + 2$ 18. $a_n = (-3)^{n-1}$ 19. $a_n = -2n + 1$

In Exercises 20–22, write a recursive rule for the sequence. Then write the next two terms of the sequence.

20. 2, 4, 6, 10, 16, 26, ... 21. 1, 3, -2, 5, -7, 12, ... 22. 1, 2, 2, 4, 8, 32, ...