6.5 Solving Exponential Equations
For use with Exploration 6.5

Essential Question  How can you solve an exponential equation graphically?

1 EXPLORATION: Solving an Exponential Equation Graphically

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Use a graphing calculator to solve the exponential equation $2.5^{x-3} = 6.25$ graphically. Describe your process and explain how you determined the solution.

2 EXPLORATION: The Number of Solutions of an Exponential Equation

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner.

a. Use a graphing calculator to graph the equation $y = 2^x$.

b. In the same viewing window, graph a linear equation (if possible) that does not intersect the graph of $y = 2^x$.

c. In the same viewing window, graph a linear equation (if possible) that intersects the graph of $y = 2^x$ in more than one point.

d. Is it possible for an exponential equation to have no solution? more than one solution? Explain your reasoning.
6.5 Solving Exponential Equations (continued)

3 EXPLORATION: Solving Exponential Equations Graphically

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Use a graphing calculator to solve each equation.

a. $2^x = \frac{1}{2}$

b. $2^{x+1} = 0$

c. $2^x = \sqrt{2}$

d. $3^x = 9$

e. $3^{x-1} = 0$

f. $4^{2x} = 2$

g. $2^{x/2} = \frac{1}{4}$

h. $3^{x+2} = \frac{1}{9}$

i. $2^{x-2} = \frac{3}{2^x} - 2$

Communicate Your Answer

4. How can you solve an exponential equation graphically?

5. A population of 30 mice is expected to double each year. The number $p$ of mice in the population each year is given by $p = 30(2^n)$. In how many years will there be 960 mice in the population?
In your own words, write the meaning of each vocabulary term.

exponential equation

Core Concepts

Property of Equality for Exponential Equations

Words Two powers with the same positive base \( b \), where \( b \neq 1 \), are equal if and only if their exponents are equal.

Numbers If \( 2^x = 2^5 \), then \( x = 5 \). If \( x = 5 \), then \( 2^x = 2^5 \).

Algebra If \( b > 0 \) and \( b \neq 1 \), then \( b^x = b^y \) if and only if \( x = y \).
6.5 Notetaking with Vocabulary (continued)

Extra Practice

In Exercises 1–15, solve the equation. Check your solution.

1. \(3^{4x} = 3^{12}\)
2. \(8^{x+5} = 8^{20}\)
3. \(6^{4x-5} = 6^{2x}\)

4. \(5^{6x-3} = 5^{-3+4x}\)
5. \(4^{2x+11} = 1024\)
6. \(8^{3-2x} = 512\)

7. \(4^{7-x} = 256\)
8. \(49^{x-2} = 343\)
9. \(36^{6x-1} = 6^{5x}\)

10. \(9^{x-4} = 81^{3x}\)
11. \(64^{x+1} = 512^x\)
12. \(6^{2x} = 36^{2x+1}\)
6.5 Notetaking with Vocabulary (continued)

13. \( \left( \frac{1}{7} \right)^x = 2401 \)
14. \( \frac{1}{512} = 2^{3x-1} \)
15. \( 25^{2-2x} = \left( \frac{1}{625} \right)^{x+1} \)

In Exercises 16–21, use a graphing calculator to solve the equation.

16. \( 3^{x+3} = -9 \)  
17. \( \left( \frac{1}{4} \right)^{x-1} = 18 \)  
18. \( 3^x = -2^{-x+1} \)

19. \( 2^{x+2} = 5^{x-3} \)  
20. \( 7^{-x+1} = -4^{x-1} \)  
21. \( \frac{1}{4}x + 1 = \left( \frac{2}{3} \right)^{2x-1} \)

22. You deposit $1000 in a savings account that earns 5% annual interest compounded yearly.
   
a. Write an exponential equation to determine when the balance of the account will be $1500.
   
b. Solve the equation.