Essential Question  How can you write and evaluate an \( n \)th root of a number?

Recall that you cube a number as follows.

\[
2^3 = 2 \cdot 2 \cdot 2 = 8
\]

2 cubed is 8.

To “undo” cubing a number, take the cube root of the number.

\[
\sqrt[3]{8} = \sqrt[3]{2^3} = 2
\]

The cube root of 8 is 2.

EXPLORATION: Finding Cube Roots

Work with a partner. Use a cube root symbol to write the side length of each cube. Then find the cube root. Check your answers by multiplying. Which cube is the largest? Which two cubes are the same size? Explain your reasoning.

\begin{align*}
\text{a. Volume} & = 27 \text{ ft}^3 \\
\text{b. Volume} & = 125 \text{ cm}^3 \\
\text{c. Volume} & = 3375 \text{ in.}^3 \\
\text{d. Volume} & = 3.375 \text{ m}^3 \\
\text{e. Volume} & = 1 \text{ yd}^3 \\
\text{f. Volume} & = \frac{125}{8} \text{ mm}^3
\end{align*}
6.2 Radicals and Rational Exponents (continued)

2 EXPLORATION: Estimating $n$th Roots

Work with a partner. Estimate each positive $n$th root. Then match each $n$th root with the point on the number line. Justify your answers.

a. $\sqrt[4]{25}$  
b. $\sqrt{0.5}$  
c. $\sqrt[5]{2.5}$

d. $\sqrt[3]{65}$  
e. $\sqrt[5]{55}$  
f. $\sqrt[6]{20,000}$

Communicate Your Answer

3. How can you write and evaluate an $n$th root of a number?

4. The body mass $m$ (in kilograms) of a dinosaur that walked on two feet can be modeled by

$$m = (0.00016)C^{2.73}$$

where $C$ is the circumference (in millimeters) of the dinosaur’s femur. The mass of a *Tyrannosaurus rex* was 4000 kilograms. Use a calculator to approximate the circumference of its femur.
6.2 Notetaking with Vocabulary
For use after Lesson 6.2

In your own words, write the meaning of each vocabulary term.

nth root of $a$

radical

index of a radical

Core Concepts

Real $n$th Roots of $a$

Let $n$ be an integer greater than 1, and let $a$ be a real number.

- If $n$ is odd, then $a$ has one real $n$th root: $\sqrt[n]{a} = a^{\frac{1}{n}}$
- If $n$ is even and $a > 0$, then $a$ has two real $n$th roots: $\pm \sqrt[n]{a} = \pm a^{\frac{1}{n}}$
- If $n$ is even and $a = 0$, then $a$ has one real $n$th root: $\sqrt[n]{0} = 0$
- If $n$ is even and $a < 0$, then $a$ has no real $n$th roots.

Notes:
6.2 Notetaking with Vocabulary (continued)

Rational Exponents
Let $a^{1/n}$ be an $n$th root of $a$, and let $m$ be a positive integer.

Algebra $a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$

Numbers $27^{2/3} = (27^{1/3})^2 = (\sqrt[3]{27})^2$

Notes:

Extra Practice
In Exercises 1–6, find the indicated real $n$th root(s) of $a$.

1. $n = 2, a = 64$  
2. $n = 3, a = 27$  
3. $n = 4, a = 256$

4. $n = 5, a = 243$  
5. $n = 8, a = 256$  
6. $n = 4, a = 10,000$

In Exercises 7–12, evaluate the expression.

7. $\sqrt[4]{625}$  
8. $\sqrt[3]{-512}$  
9. $\sqrt[3]{-216}$

10. $\sqrt[5]{-243}$  
11. $729^{1/6}$  
12. $(-81)^{1/2}$
In Exercises 13–15, rewrite the expression in rational exponent form.

13. \((\sqrt[5]{4})^3\)  
14. \((\sqrt[2]{-8})^2\)  
15. \((\sqrt[3]{15})^7\)

In Exercises 16–18, rewrite the expression in radical form.

16. \((-3)^{2/5}\)  
17. \(6^{1/2}\)  
18. \(12^{3/4}\)

In Exercises 19–24, evaluate the expression.

19. \(32^{2/5}\)  
20. \((-64)^{1/2}\)  
21. \(343^{2/3}\)

22. \(256^{7/8}\)  
23. \(-729^{5/6}\)  
24. \((-625)^{3/4}\)

25. The radius \(r\) of a sphere is given by the equation

\[ r = \left( \frac{A}{4\pi} \right)^{1/2} \]

where \(A\) is the surface area of the sphere. The surface area of a sphere is 1493 square meters. Find the radius of the sphere to the nearest tenth of a meter. Use 3.14 for \(\pi\).