5.4 Solving Special Systems of Linear Equations
For use with Exploration 5.4

Essential Question  Can a system of linear equations have no solution or infinitely many solutions?

1 EXPLORATION: Using a Table to Solve a System

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. You invest $450 for equipment to make skateboards. The materials for each skateboard cost $20. You sell each skateboard for $20.

a. Write the cost and revenue equations. Then complete the table for your cost $C$ and your revenue $R$.

<table>
<thead>
<tr>
<th>$x$ (skateboards)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$ (dollars)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R$ (dollars)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

b. When will your company break even? What is wrong?

2 EXPLORATION: Writing and Analyzing a System

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. A necklace and matching bracelet have two types of beads. The necklace has 40 small beads and 6 large beads and weighs 10 grams. The bracelet has 20 small beads and 3 large beads and weighs 5 grams. The threads holding the beads have no significant weight.

a. Write a system of linear equations that represents the situation. Let $x$ be the weight (in grams) of a small bead and let $y$ be the weight (in grams) of a large bead.

b. Graph the system in the coordinate plane shown. What do you notice about the two lines?

c. Can you find the weight of each type of bead? Explain your reasoning.
Communicate Your Answer

3. Can a system of linear equations have no solution or infinitely many solutions?
   Give examples to support your answers.

4. Does the system of linear equations represented by each graph have no solution, one solution, or infinitely many solutions? Explain.
   a. 
   b. 
   c. 
In your own words, write the meaning of each vocabulary term.

parallel

Core Concepts

Solutions of Systems of Linear Equations

A system of linear equations can have one solution, no solution, or infinitely many solutions.

<table>
<thead>
<tr>
<th>One solution</th>
<th>No solution</th>
<th>Infinitely many solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="https://via.placeholder.com/150" alt="Graph" /></td>
<td><img src="https://via.placeholder.com/150" alt="Graph" /></td>
<td><img src="https://via.placeholder.com/150" alt="Graph" /></td>
</tr>
<tr>
<td>The lines intersect.</td>
<td>The lines are parallel.</td>
<td>The lines are the same.</td>
</tr>
</tbody>
</table>

Notes:
5.4 Notetaking with Vocabulary (continued)

Extra Practice

In Exercises 1–18, solve the system of linear equations.

1. \( y = 3x - 7 \) \( y = 3x + 4 \)
2. \( y = 5x - 1 \) \( y = -5x + 5 \)
3. \( 2x - 3y = 10 \)
4. \( x + 3y = 6 \) \( -x - 3y = 3 \)
5. \( 6x + 6y = -3 \) \( -6x - 6y = 3 \)
6. \( 2x - 5y = -3 \) \( 3x + 5y = 8 \)
7. \( 2x + 3y = 1 \) \( -2x + 3y = -7 \)
8. \( 4x + 3y = 17 \) \( -8x - 6y = 34 \)
9. \( 3x - 2y = 6 \) \( -9x + 6y = -18 \)
10. \(-2x + 5y = -21\) \(2x - 5y = 21\)
11. \(3x - 8y = 3\) \(8x - 3y = 8\)
12. \(18x + 12y = 24\) \(3x + 2y = 6\)
13. \(15x - 6y = 9\) \(5x - 2y = 27\)
14. \(-3x - 5y = 8\) \(6x + 10y = -16\)
15. \(2x - 4y = 2\) \(-2x - 4y = 6\)
16. \(5x + 7y = 7\) \(7x + 5y = 5\)
17. \(y = \frac{2}{3}x + 7\) \(y = \frac{2}{3}x - 5\)
18. \(-3x + 5y = 15\) \(9x - 15y = -45\)

19. You have $15 in savings. Your friend has $25 in savings. You both start saving $5 per week. Write a system of linear equations that represents this situation. Will you ever have the same amount of savings as your friend? Explain.