

3.6

Transformations of Graphs of Linear Functions

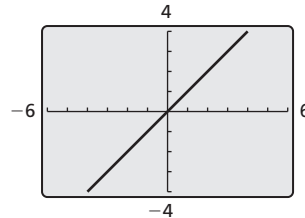
For use with Exploration 3.6

Essential Question How does the graph of the linear function $f(x) = x$ compare to the graphs of $g(x) = f(x) + c$ and $h(x) = f(cx)$?

1 EXPLORATION: Comparing Graphs of Functions

Work with a partner. The graph of $f(x) = x$ is shown.

Sketch the graph of each function, along with f , on the same set of coordinate axes. Use a graphing calculator to check your results. What can you conclude?

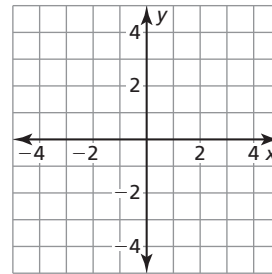
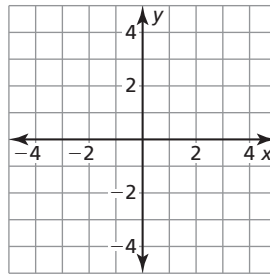
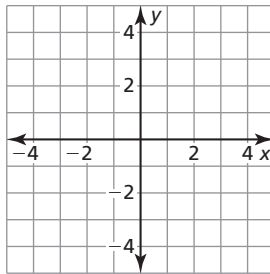
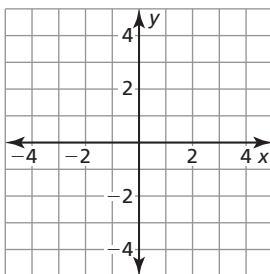


a. $g(x) = x + 4$

b. $g(x) = x + 2$

c. $g(x) = x - 2$

d. $g(x) = x - 4$



2 EXPLORATION: Comparing Graphs of Functions

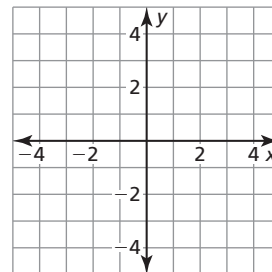
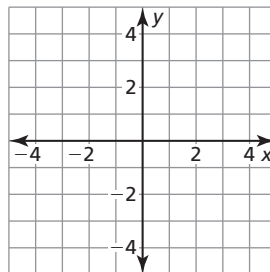
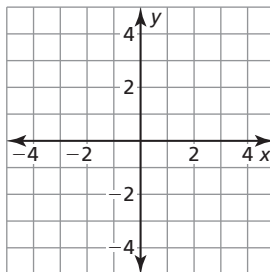
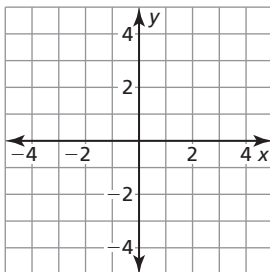
Work with a partner. Sketch the graph of each function, along with $f(x) = x$, on the same set of coordinate axes. Use a graphing calculator to check your results. What can you conclude?

a. $h(x) = \frac{1}{2}x$

b. $h(x) = 2x$

c. $h(x) = -\frac{1}{2}x$

d. $h(x) = -2x$



3.6 Transformations of Graphs of Linear Functions (continued)

3 EXPLORATION: Matching Functions with Their Graphs

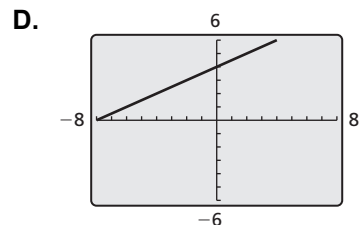
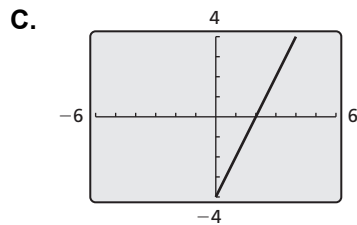
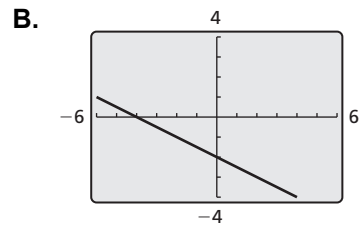
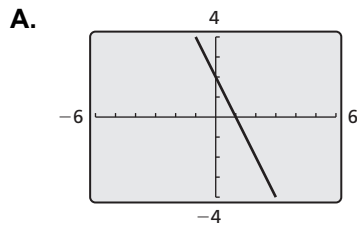
Work with a partner. Match each function with its graph. Use a graphing calculator to check your results. Then use the results of Explorations 1 and 2 to compare the graph of k to the graph of $f(x) = x$.

a. $k(x) = 2x - 4$

b. $k(x) = -2x + 2$

c. $k(x) = \frac{1}{2}x + 4$

d. $k(x) = -\frac{1}{2}x - 2$



Communicate Your Answer

4. How does the graph of the linear function $f(x) = x$ compare to the graphs of $g(x) = f(x) + c$ and $h(x) = f(cx)$?

3.6

Notetaking with Vocabulary

For use after Lesson 3.6

In your own words, write the meaning of each vocabulary term.

family of functions

parent function

transformation

translation

reflection

horizontal shrink

horizontal stretch

vertical stretch

vertical shrink

Notes:

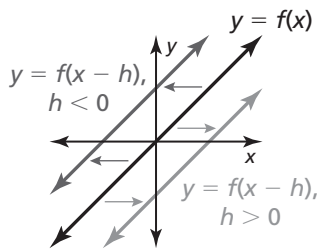
3.6 Notetaking with Vocabulary (continued)

Core Concepts

A **translation** is a transformation that shifts a graph horizontally or vertically but does not change the size, shape, or orientation of the graph.

Horizontal Translations

The graph of $y = f(x - h)$ is a horizontal translation of the graph of $y = f(x)$, where $h \neq 0$.

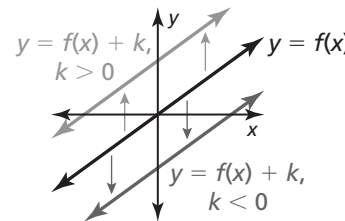


Subtracting h from the *inputs* before evaluating the function shifts the graph left when $h < 0$ and right when $h > 0$.

Notes:

Vertical Translations

The graph of $y = f(x) + k$ is a vertical translation of the graph of $y = f(x)$, where $k \neq 0$.

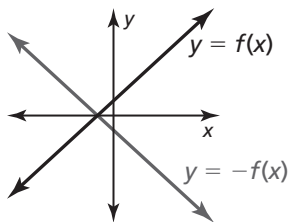


Adding k to the *outputs* shifts the graph down when $k < 0$ and up when $k > 0$.

A **reflection** is a transformation that flips a graph over a line called the *line of reflection*.

Reflections in the x-axis

The graph of $y = -f(x)$ is a reflection in the x -axis of the graph of $y = f(x)$.

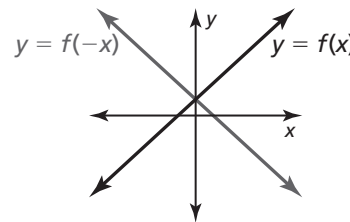


Multiplying the outputs by -1 changes their signs.

Notes:

Reflections in the y-axis

The graph of $y = f(-x)$ is a reflection in the y -axis of the graph of $y = f(x)$.



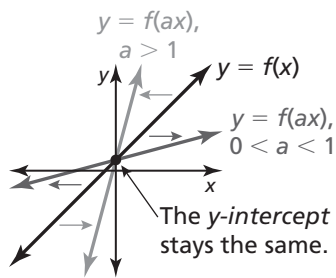
Multiplying the inputs by -1 changes their signs.

3.6 Notetaking with Vocabulary (continued)**Horizontal Stretches and Shrinks**

The graph of $y = f(ax)$ is a horizontal stretch

or shrink by a factor of $\frac{1}{a}$ of the graph of

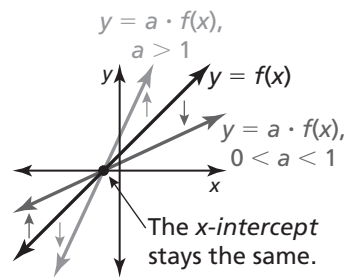
$y = f(x)$, where $a > 0$ and $a \neq 1$.

**Vertical Stretches and Shrinks**

The graph of $y = a \cdot f(x)$ is a vertical stretch

or shrink by a factor of a of the graph of

$y = f(x)$, where $a > 0$ and $a \neq 1$.

**Notes:****Transformations of Graphs**

The graph of $y = a \cdot f(x - h) + k$ or the graph of $y = f(ax - h) + k$ can be obtained from the graph of $y = f(x)$ by performing these steps.

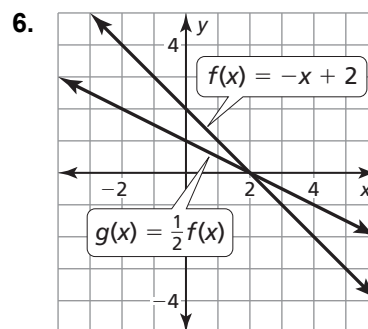
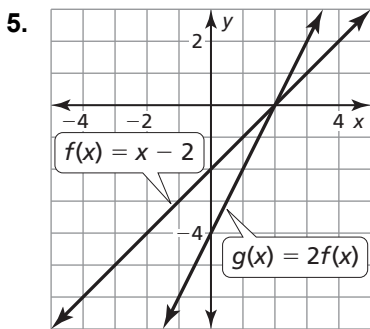
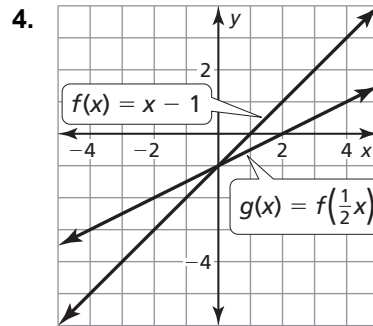
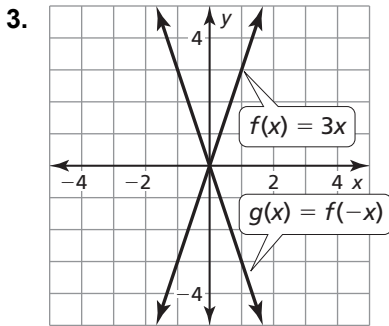
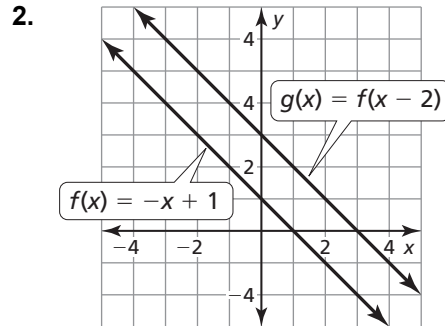
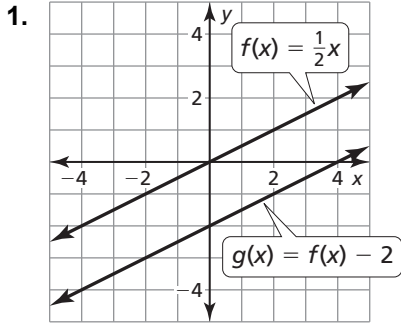
- Step 1** Translate the graph of $y = f(x)$ horizontally h units.
- Step 2** Use a to stretch or shrink the resulting graph from Step 1.
- Step 3** Reflect the resulting graph from Step 2 when $a < 0$.
- Step 4** Translate the resulting graph from Step 3 vertically k units.

Notes:

3.6 Notetaking with Vocabulary (continued)

Extra Practice

In Exercises 1–6, use the graphs of f and g to describe the transformation from the graph of f to the graph of g .



7. Graph $f(x) = x$ and $g(x) = 3x - 2$. Describe the transformations from the graph of f to the graph of g .

