Name____ Date__________

Chapter 3 Parallel and Perpendicular Lines

Dear Family,

Parallel and perpendicular lines are used in building bridges all over the world. Your student may not notice, but every bridge they see or cross is made up of these two types of lines.

With your student, look up the following images on the Internet.

Pratt Bridge

Pennsylvania Bridge

Parker Bridge

Notice how these bridges are structurally different and have different ways of supporting transportation. Take time to discuss parallel and perpendicular lines that make up the bridge structure or support system.

- Where are parallel lines present in each of the bridges?
- Where are perpendicular lines present in each of the bridges?

Take a trip to a few bridges in your area. Although these bridges may be simpler in design, parallel and perpendicular lines are most likely present. Take pictures of the bridges you see to bring home with you.

Using the images that you found on the Internet and the photographs that you took, come up with a creative design for a new bridge. Make use of parallel and perpendicular lines in the structure and support system. Use the space below to draw a diagram. Be sure to label at least one pair each of parallel and perpendicular lines.

Be creative!

Name ____________________ Date ________

Chapter 3 Parallel and Perpendicular Lines (continued)

	Learning Target	Success Criteria
Chapter 3 Parallel and Perpendicular Lines	Understand parallel and perpendicular lines.	• I can identify lines and angles. • I can describe angle relationships formed by parallel lines and a transversal. • I can prove theorems involving parallel and perpendicular lines. • I can write equations of parallel and perpendicular lines.
3.1 Pairs of Lines and Angles	Understand lines, planes, and pairs of angles.	• I can identify lines and planes. • I can identify parallel and perpendicular lines. • I can identify pairs of angles formed by transversals.
3.2 Parallel Lines and Transversals	Prove and use theorems about parallel lines.	• I can use properties of parallel lines to find angle measures. • I can prove theorems about parallel lines.
3.3 Proofs with Parallel Lines	Prove and use theorems about identifying parallel lines.	• I can use theorems to identify parallel lines. • I can construct parallel lines. • I can prove theorems about identifying parallel lines.
3.4 Proofs with Perpendicular Lines	Prove and use theorems about perpendicular lines.	• I can find the distance from a point to a line. • I can construct perpendicular lines and perpendicular bisectors. • I can prove theorems about perpendicular lines.
3.5 Equations of Parallel and Perpendicular Lines	Partition a directed line segment and understand slopes of parallel and perpendicular lines.	• I can partition directed line segments using slope. • I can use slopes to identify parallel and perpendicular lines. • I can write equations of parallel and perpendicular lines. • I can find the distance from a point to a line.

Nombre ______________________________ Fecha __________

Capítulo 3

Rectas paralelas y perpendiculares

Estimada familia:

Las rectas paralelas y perpendiculares se usan para construir puentes en todo el mundo. Su hijo tal vez no lo note, pero cada puente que ve o cruza está formado por estas dos clases de rectas.

Con su hijo, busquen las siguientes imágenes en Internet.

Puente Pratt

Puente de Pensilvania

Puente Parker

Observen cómo estos puentes tienen distintas estructuras y diferentes maneras de soportar el transporte. Tómense un tiempo para hablar sobre las rectas paralelas y perpendiculares que forman la estructura del puente o el sistema de soporte.

- ¿Dónde están las rectas paralelas en cada uno de los puentes?
- ¿Dónde están las rectas perpendiculares en cada uno de los puentes?

Vayan de ver algunos puentes de su zona. Aunque estos puentes tengan un diseño más simple, es muy probable que las rectas paralelas y perpendiculares estén presentes. Tomen fotografías de los puentes que vean para llevarse a casa.

Con las imágenes que encontraron en Internet y las fotografías que tomaron, piensen en un diseño creativo para un puente nuevo. Usen rectas paralelas y perpendiculares en la estructura y en el sistema de soporte. Usen el siguiente espacio para dibujar un diagrama. Asegúrense de rotular, al menos, un par de rectas paralelas y perpendiculares.

¡Sean creativos!

Capítulo 3

Rectas paralelas y perpendiculares (continuación)

	Objetivo de aprendizaje	Criterios de éxito
Capítulo 3 Rectas paralelas y perpendiculares	Comprender rectas paralelas y perpendiculares.	• Puedo identificar rectas y ángulos. • Puedo describir relaciones entre ángulos formados por rectas paralelas y una transversal. • Puedo probar teoremas que involucran rectas paralelas y perpendiculares. • Puedo escribir ecuaciones de rectas paralelas y perpendiculares.
3.1 Pares de rectas y ángulos	Comprender rectas, planos y pares de ángulos.	• Puedo identificar rectas y planos. • Puedo identificar rectas paralelas y perpendiculares. • Puedo identificar pares de ángulos formados por transversales.
3.2 Rectas paralelas y transversales	Probar y usar teoremas sobre rectas paralelas.	• Puedo usar las propiedades de las rectas paralelas para encontrar las medidas de los ángulos. • Puedo probar teoremas sobre rectas paralelas.
3.3 Pruebas con rectas paralelas	Probar y usar teoremas sobre cómo identificar rectas paralelas.	• Puedo usar teoremas para identificar rectas paralelas. • Puedo construir rectas paralelas. • Puedo probar teoremas sobre cómo identificar rectas paralelas.
3.4 Pruebas con rectas perpendiculares	Probar y usar teoremas sobre rectas perpendiculares.	• Puedo encontrar la distancia desde un punto a una recta. • Puedo construir rectas perpendiculares y bisectrices perpendiculares. • Puedo probar teoremas sobre rectas perpendiculares.
3.5 Ecuaciones de rectas paralelas y perpendiculares	Dividir un segmento orientado y comprender las pendientes de rectas paralelas y perpendiculares.	• Puedo dividir segmentos orientados usando una pendiente. • Puedo usar pendientes para identificar rectas paralelas y perpendiculares. • Puedo escribir ecuaciones de rectas paralelas y perpendiculares. • Puedo encontrar la distancia desde un punto a una recta.

Name____ Date __________

3.1 Cumulative Practice
For use before Lesson 3.1

The endpoints of $\overline{AB}$ are given. Find the coordinates of the midpoint *M*.

1. $A(2, -9)$ and $B(3, -2)$

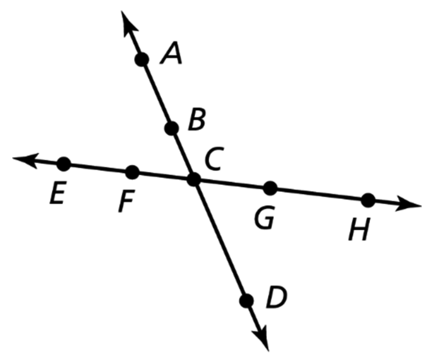
2. $A(6, 2)$ and $B(5, -1)$

3.1 Prerequisite Skills Practice
For use before Lesson 3.1

Use the diagram.

1. What is another name for $\overleftrightarrow{BD}$?

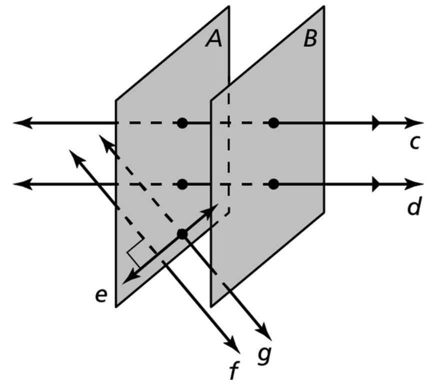
2. What is another name for $\overrightarrow{CH}$?



Name ____ Date __________

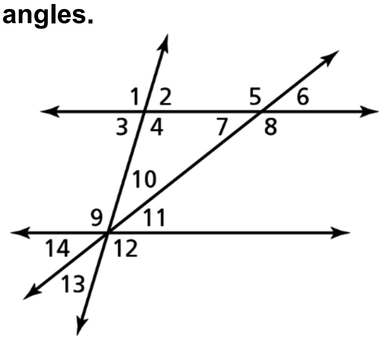
3.1 Extra Practice

In Exercises 1–6, use the diagram.



1. Name a pair of parallel lines.
2. Name a pair of perpendicular lines.
3. Name a pair of skew lines.
4. Name a pair of parallel planes.
5. Is line f parallel to line g? Explain.
6. Is line e perpendicular to line g? Explain.

In Exercises 7–11, classify the angle pair as corresponding, *alternate interior*, *alternate exterior*, or *consecutive interior* angles.



7. $\angle 4$ and $\angle 9$
8. $\angle 1$ and $\angle 9$
9. $\angle 1$ and $\angle 12$
10. $\angle 6$ and $\angle 11$
11. $\angle 4$ and $\angle 7$

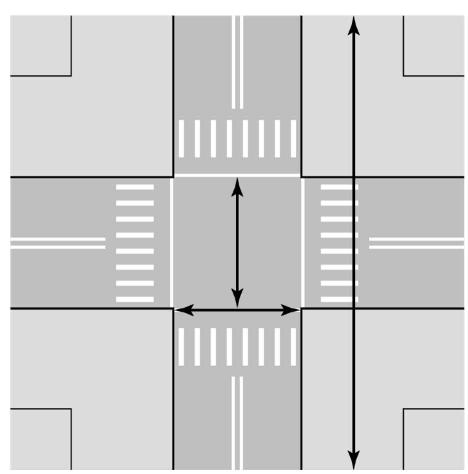
12. Two planes are parallel and each plane contains a line. Are the two lines skew? Explain your reasoning.

13. Use the figure to decide whether each statement is true or false. Explain your reasoning.

    a. The line containing the sidewalk and the line containing the center of the road are parallel to each other.

    b. The line containing the center of the road is skew to the line containing the crosswalk.

    c. The plane containing a stop sign is perpendicular to the plane containing the ground.



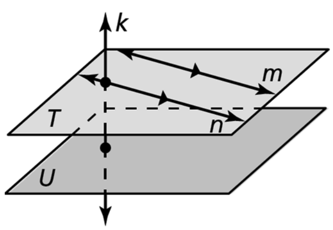
Name____ Date__________

3.1 Reteach

Key Ideas

Parallel Lines, Skew Lines, and Parallel Planes

Two lines that do not intersect are either *parallel lines* or *skew lines*. Two lines are **parallel lines** when they do not intersect and are coplanar. Two lines are **skew lines** when they do not intersect and are not coplanar. Two planes that do not intersect are **parallel planes**.



Lines m and n are parallel lines $(m \parallel n)$.

Lines m and k are skew lines.

Planes T and U are parallel planes $(T \parallel U)$.

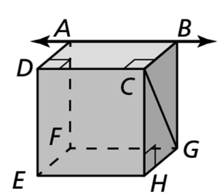
Lines k and n are intersecting lines, and there is a plane (not shown) containing them.

Small directed arrows, as shown on lines m and n above, are used to show that lines are parallel. The symbol $\parallel$ means "is parallel to," as in $m \parallel n$.

Segments and rays are parallel when they lie in parallel lines. A line is parallel to a plane when the line is in a plane parallel to the given plane. In the diagram above, line n is parallel to plane U.

EXAMPLE Identifying Lines and Planes

Consider the lines that contain the segments in the figure and the planes that contain the faces of the figure. Which line(s) or plane(s) appear to fit each description?



a. line(s) parallel to $\overleftrightarrow{AB}$ and containing point C

b. line(s) skew to $\overleftrightarrow{AB}$ and containing point C

c. line(s) perpendicular to $\overleftrightarrow{AB}$ and containing point C

d. plane(s) parallel to plane EFG and containing point C

SOLUTION

a. $\overleftrightarrow{CD}$, $\overleftrightarrow{EH}$, and $\overleftrightarrow{FG}$ all appear parallel to $\overleftrightarrow{AB}$, but only $\overleftrightarrow{CD}$ contains point C.

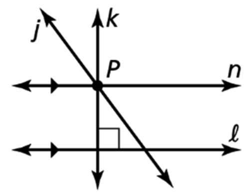
b. Both $\overleftrightarrow{CH}$ and $\overleftrightarrow{CG}$ appear skew to $\overleftrightarrow{AB}$ and contain point C.

c. $\overleftrightarrow{BC}$, $\overleftrightarrow{AD}$, $\overleftrightarrow{AF}$, and $\overleftrightarrow{BG}$ all appear perpendicular to $\overleftrightarrow{AB}$, but only $\overleftrightarrow{BC}$ contains point C.

d. Plane ABC appears parallel to plane EFG and contains point C.

Name ______________________________ Date __________

3.1 Reteach (continued)



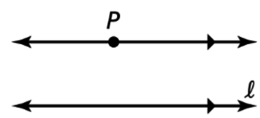
Two lines in the same plane are either parallel, like line ℓ and line n, or intersect in a point, like line j and line n.

Through a point not on a given line, there are infinitely many lines. Exactly one of these lines is parallel to the given line, and exactly one of them is perpendicular to the given line. For example, line k is the line through point P perpendicular to line ℓ, and line n is the line through point P parallel to line ℓ.

Postulates

3.1 Parallel Postulate

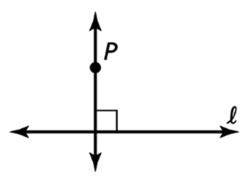
If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line.



There is exactly one line through P parallel to ℓ.

3.2 Perpendicular Postulate

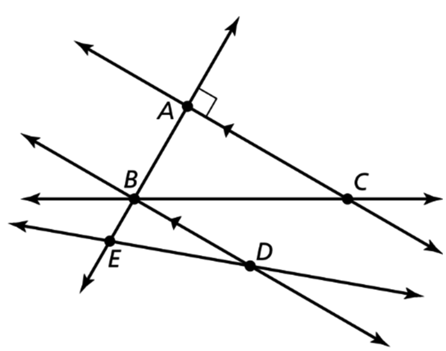
If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line.



There is exactly one line through P perpendicular to ℓ.

EXAMPLE Identifying Parallel and Perpendicular Lines

Use the diagram.



a. Name a pair of parallel lines.

b. Name a pair of perpendicular lines.

c. Is $\overleftrightarrow{ED} \parallel \overleftrightarrow{AC}$? Explain.

SOLUTION

a. $\overleftrightarrow{AC} \parallel \overleftrightarrow{BD}$

b. $\overleftrightarrow{AB} \perp \overleftrightarrow{AC}$

c. $\overleftrightarrow{ED}$ is not parallel to $\overleftrightarrow{AC}$, because $\overleftrightarrow{BD}$ is parallel to $\overleftrightarrow{AC}$, and by the Parallel Postulate, there is exactly one line parallel to $\overleftrightarrow{AC}$ through point D.

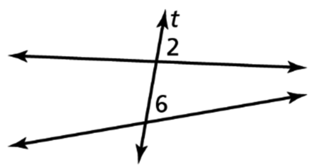
Name____ Date ___________

3.1 Reteach (continued)

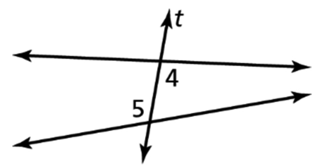
A **transversal** is a line that intersects two or more coplanar lines at different points.

Key Idea

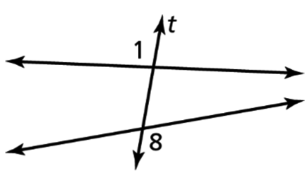
Angles Formed by Transversals



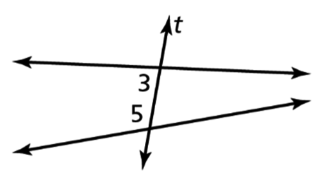
Two angles are **corresponding angles** when they have corresponding positions. For example, ∠2 and ∠6 are above the lines and to the right of the transversal *t*.



Two angles are **alternate interior angles** when they lie between the two lines and on opposite sides of the transversal *t*.



Two angles are **alternate exterior angles** when they lie outside the two lines and on opposite sides of the transversal *t*.



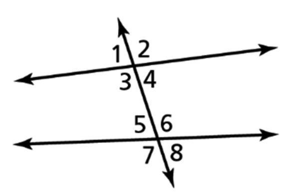
Two angles are **consecutive interior angles** when they lie between the two lines and on the same side of the transversal *t*.

EXAMPLE Identifying Pairs of Angles

Identify all pairs of angles of the given type.

a. corresponding **b.** alternate interior

c. alternate exterior **d.** consecutive interior



SOLUTION

a. ∠1 and ∠5
∠2 and ∠6
∠3 and ∠7
∠4 and ∠8

b. ∠3 and ∠6
∠4 and ∠5

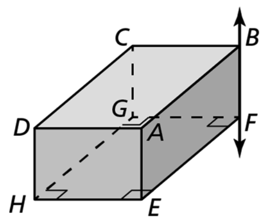
c. ∠1 and ∠8
∠2 and ∠7

d. ∠3 and ∠5
∠4 and ∠6

Name ____ Date _________

3.1 Reteach (continued)

In Exercises 1–4, consider the lines that contain the segments in the figure and the planes that contain the faces of the figure. All angles are right angles. Which line(s) or plane(s) contain point *A* and appear to fit the description.



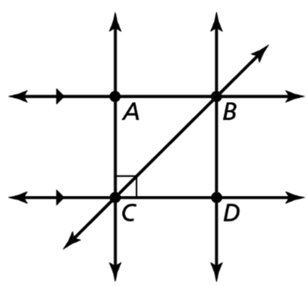
1. line(s) parallel to $\overleftrightarrow{BF}$

2. line(s) skew to $\overleftrightarrow{BF}$

3. line(s) perpendicular to $\overleftrightarrow{BF}$

4. plane(s) parallel to plane EFG

In Exercises 5–8, use the diagram.



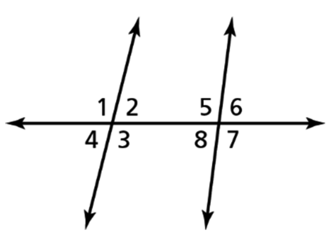
5. Name a pair of parallel lines.

6. Name a pair of perpendicular lines.

7. Is $\overleftrightarrow{AB} \parallel \overleftrightarrow{BC}$? Explain.

8. Is $\overleftrightarrow{BD} \perp \overleftrightarrow{CD}$? Explain.

In Exercises 9–12, identify all pairs of angles of the given type.



9. alternate interior

10. alternate exterior

11. corresponding

12. consecutive interior

Name____ Date__________

3.1 Enrichment and Extension

Planes, Lines, and Angles

1. If two parallel planes are cut by a third plane, are the lines of intersection parallel? Explain your reasoning and include a drawing.

2. Draw line a parallel to line b. Draw line c parallel to line b. What relationship appears to exist between lines a and c? Make a conjecture about two lines that are parallel to the same line.

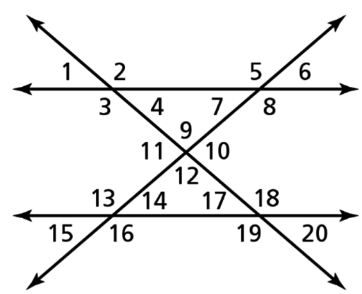
3. Draw line ℓ perpendicular to a line m. Draw a line n perpendicular to line m. What relationship appears to exist between line ℓ and line n? Make a conjecture about two lines that are perpendicular to the same line.

In Exercises 4 and 5, sketch a diagram of the description.

4. Lines ℓ and m are skew, lines ℓ and n are skew, and lines m and n are parallel.

5. Line ℓ is parallel to plane A, plane A is parallel to plane B, and line ℓ is not parallel to plane B.

6. List all possible answers for each.



a. $\angle 1$ and _____ are corresponding angles.

b. $\angle 13$ and _____ are corresponding angles.

c. $\angle 14$ and _____ are consective interior angles.

d. $\angle 4$ and _____ are consective interior angles.

e. $\angle 7$ and _____ are alternate interior angles.

f. $\angle 17$ and _____ are alternate interior angles.

g. $\angle 6$ and _____ are alterior exterior angles.

h. $\angle 18$ and _____ are alternate exterior angles.

Name ____ Date __________

3.1 Puzzle Time

What Has A Foot On Each End And One In The Middle?

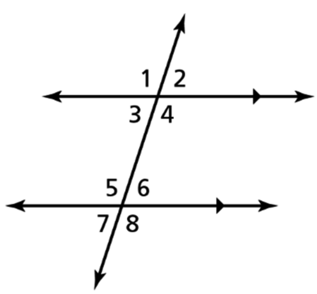
Write the letter of each answer in the box containing the exercise number.

Fill in the blank.

1. Two lines are ______ if and only if they are both vertical lines or they both have the same slope.
2. Two lines are ______ if and only if one is vertical and the other is horizontal or the slopes of the lines are negative reciprocals of each other.
3. Two lines are ______ if and only if their equations are equivalent.
4. Two lines are ______ lines when they do not intersect and are not coplanar.
5. A(n) ______ is a line that intersects two or more coplanar lines at different points.

Identify the type of the pairs of angles.

6. $\angle 3$ and $\angle 5$
7. $\angle 1$ and $\angle 8$
8. $\angle 2$ and $\angle 6$
9. $\angle 1$ and $\angle 4$
10. $\angle 4$ and $\angle 5$



1		10	8	2	6	4	9	5	7	3

Answers

G. unskew
K. coincident
H. conditional
C. alternate exterior angles
I. transversal
T. angular
U. straight
S. skew
L. horizontal
R. perpendicular
N. lined angles
T. vertical angles
P. inverse angles
A. parallel
D. consecutive interior angles
B. revolving angles
L. converse angles
Y. alternate interior angles
M. intersecting angles
A. corresponding angles

Name____________________ Date__________

3.2 Cumulative Practice

For use before Lesson 3.2

Find the distance between the two points.

1. $A(2, 3)$ and $B(3, 8)$

2. $A(6, 5)$ and $B(13, 10)$

3.2 Prerequisite Skills Practice

For use before Lesson 3.2

Find the angle measure.

1. $(3x + 22)^\circ = (10x - 6)^\circ$

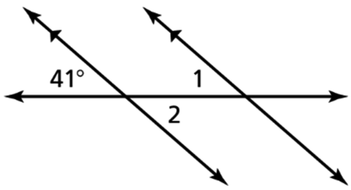
2. $(7x - 46)^\circ = (9x - 64)^\circ$

Name ____ Date __________

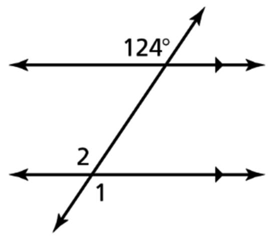
3.2 Extra Practice

In Exercises 1 and 2, find $m\angle 1$ and $m\angle 2$. Tell which theorem you used in each case.

1.

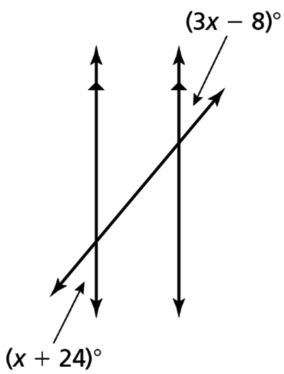


2.

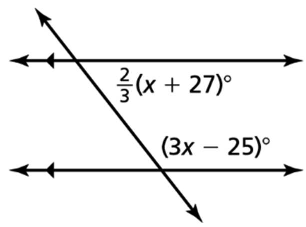


In Exercises 3 and 4, find the value of *x*. Show your steps.

3.

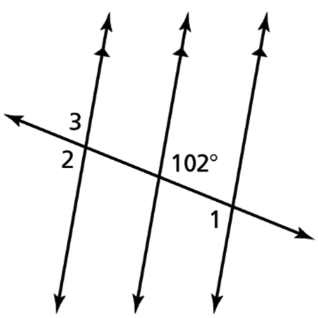


4.

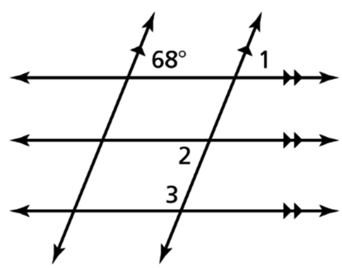


In Exercises 5 and 6, find $m\angle 1$, $m\angle 2$, and $m\angle 3$. Explain your reasoning.

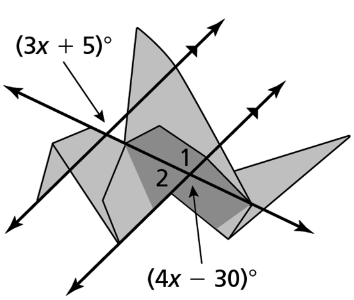
5.



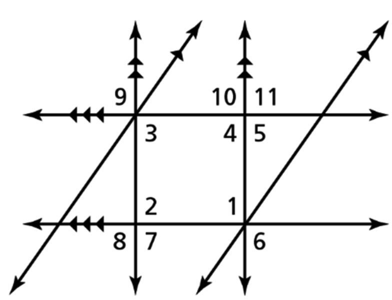
6.



7. The figure shows a two-dimensional representation of a bird made out of origami paper. Find $m\angle 1$ and $m\angle 2$. Explain your reasoning.



8. The figure shows three pairs of parallel lines. Which angles are congruent to $\angle 1$? Tell which theorem you used in each case.

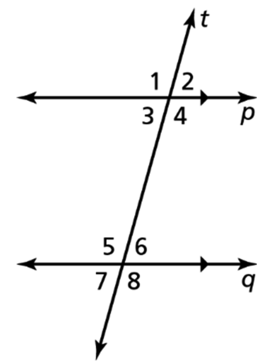


Name______________________________ Date __________

3.2 Reteach

Theorems

3.1 Corresponding Angles Theorem



If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.

Examples In the diagram at the right, $\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$, $\angle 3 \cong \angle 7$, and $\angle 4 \cong \angle 8$

3.2 Alternate Interior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

Examples In the diagram at the right, $\angle 3 \cong \angle 6$ and $\angle 4 \cong \angle 5$.

3.3 Alternate Exterior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.

Examples In the diagram at the right, $\angle 1 \cong \angle 8$ and $\angle 2 \cong \angle 7$.

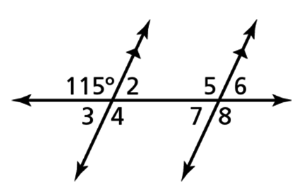
3.4 Consecutive Interior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.

Examples In the diagram at the right, $\angle 3$ and $\angle 5$ are supplementary, and $\angle 4$ and $\angle 6$ are supplementary.

EXAMPLE Identifying Angles

The measures of three of the numbered angles are 115°. Identify the other angles. Explain your reasoning.



SOLUTION

Using the Corresponding Angles Theorem, $m\angle 5 = 115°$.

$\angle 5$ and $\angle 8$ are vertical angles. Using the Vertical Angles Congruence Theorem, $m\angle 8 = 115°$.

$\angle 5$ and $\angle 4$ are alternate interior angles. Using the Alternate Interior Angles Theorem, $m\angle 4 = 115°$.

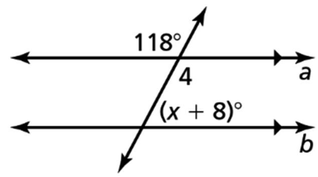
▶ So, the three angles that each have a measure of 115° are $\angle 4$, $\angle 5$, and $\angle 8$.

Name ______________________________ Date __________

3.2 Reteach (continued)

EXAMPLE Using Properties of Parallel Lines

Find the value of x.



SOLUTION

By the Vertical Angles Congruence Theorem, $m\angle 4 = 118°$. Lines a and b are parallel, so you can use theorems about parallel lines.

$m\angle 4 + (x + 8)° = 180°$	Consecutive Interior Angles Theorem
$118° + (x + 8)° = 180°$	Substitute 118° for $m\angle 4$.
$x + 126 = 180$	Combine like terms.
$x = 54$	Subtract 126 from each side.

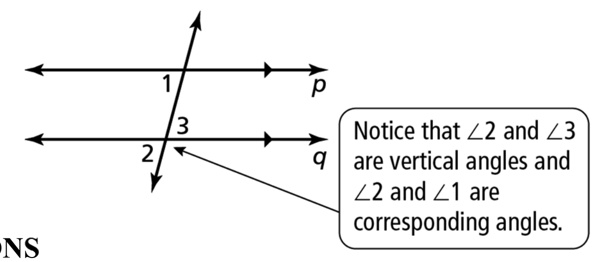
▶ So, the value of x is 54.

EXAMPLE Proving the Alternate Interior Angles Theorem

Use the diagram to prove that if two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

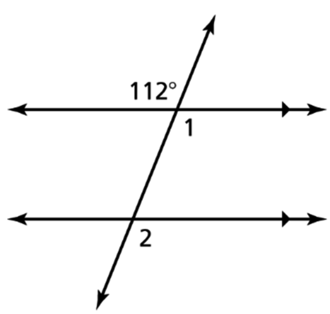
Given $p \parallel q$

Prove $\angle 1 \cong \angle 3$

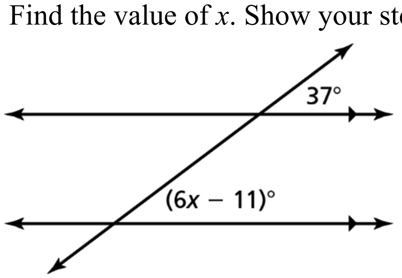


STATEMENTS	REASONS
1. $p \parallel q$	**1.** Given
2. $\angle 1 \cong \angle 2$	**2.** Corresponding Angles Theorem
3. $\angle 2 \cong \angle 3$	**3.** Vertical Angles Congruence Theorem
4. $\angle 1 \cong \angle 3$	**4.** Transitive Property of Angle Congruence

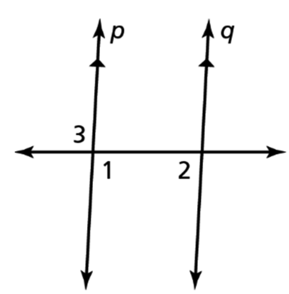
1. Find $m\angle 1$ and $m\angle 2$. Tell which theorem you use in each case.



2. Find the value of x. Show your steps.



3. Use the diagram to prove that if $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 3$. What is $m\angle 1$? Explain.

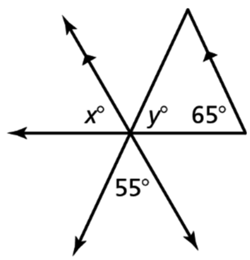


3.2 Enrichment and Extension

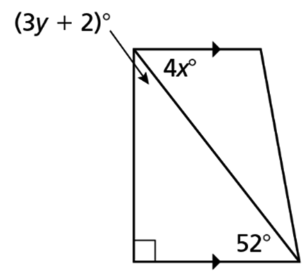
Parallel Lines and Transversals

In Exercises 1 and 2, find the values of *x* and *y*.

1.



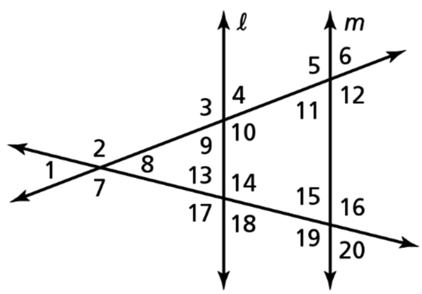
2.



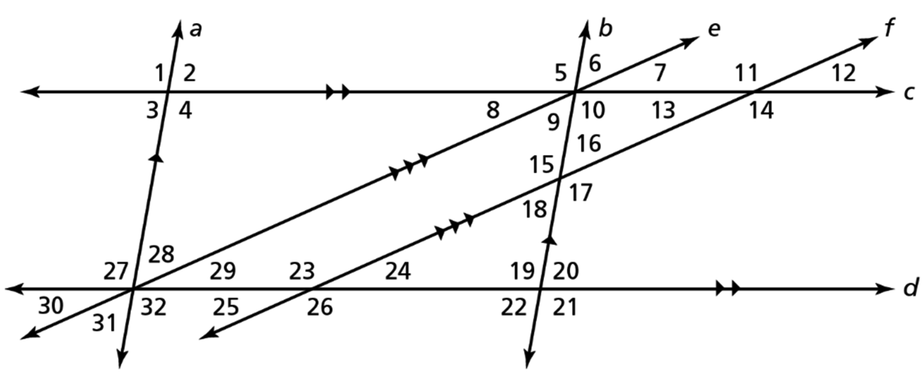
3. Draw a four-sided figure in which $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$. Prove $\angle A \cong \angle C$.

In Exercises 4 and 5, find the measures of all angles in the diagram.

4. **Given** $\ell \parallel m$, $m\angle 1 = 35°$, and $m\angle 12 = 111°$



5. **Given** $a \parallel b$, $c \parallel d$, $e \parallel f$, $m\angle 7 = 24°$, and $m\angle 20 = 80°$



Name ____ Date __________

3.2 Puzzle Time

What Did The Acorn Say When It Grew Up?

Circle the letter of each correct answer in the boxes below. The circled letters will spell out the answer to the riddle.

Complete the sentence.

1. If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are ____________.

2. If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are ____________.

Using the properties of parallel lines, find the angle measure.

3. $m\angle 2 = 74°$; Find $m\angle 1$.

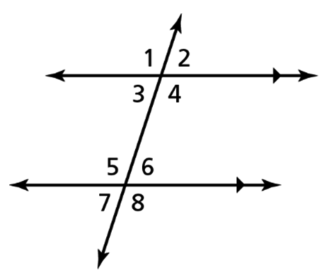
4. $m\angle 2 = 74°$; Find $m\angle 3$.

5. $m\angle 1 = 114°$; Find $m\angle 8$.

6. $m\angle 4 = 56°$; Find $m\angle 6$.

7. $m\angle 1 = 84°$; Find $m\angle 7$.

8. $m\angle 8 = 116°$; Find $m\angle 2$.

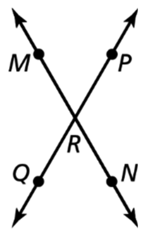


G	**E**	**I**	**F**	**O**	**A**	**E**	**M**
64°	124°	116°	66°	106°	transitive	complementary	congruent
T	**E**	**I**	**T**	**R**	**Y**	**M**	**E**
84°	74°	34°	96°	114°	supplementary	56°	116°

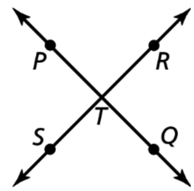
Name______________________________ Date__________

3.3 Cumulative Practice
For use before Lesson 3.3

1. What is another name for $\overline{NM}$?



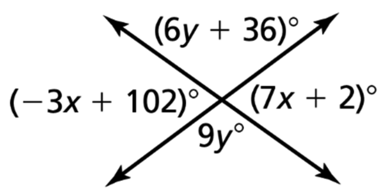
2. What is another name for $\overline{RS}$?



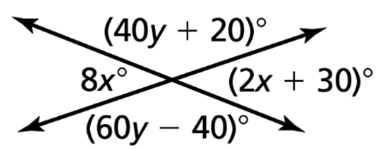
3.3 Prerequisite Skills Practice
For use before Lesson 3.3

Find the values of *x* and *y*.

1.



2.

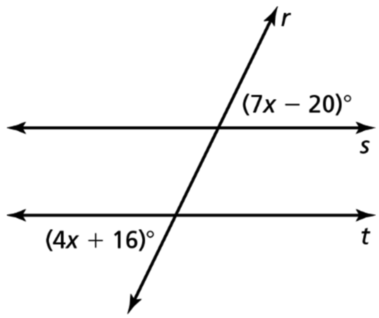


Name ____ Date __________

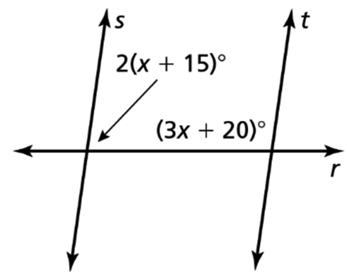
3.3 Extra Practice

In Exercises 1 and 2, find the value of *x* that makes $s \parallel t$. Explain your reasoning.

1.

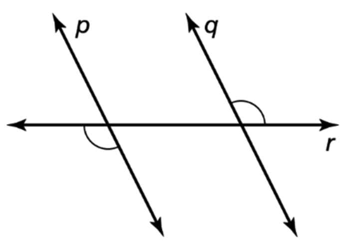


2.

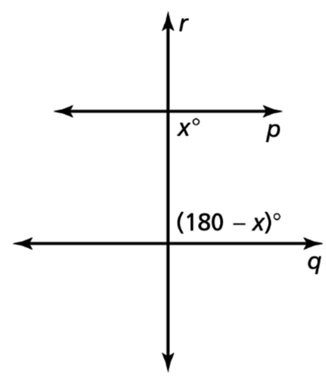


In Exercises 3 and 4, decide whether there is enough information to prove that $p \parallel q$. If so, state the theorem you can use.

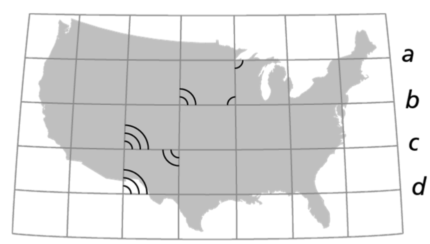
3.



4.



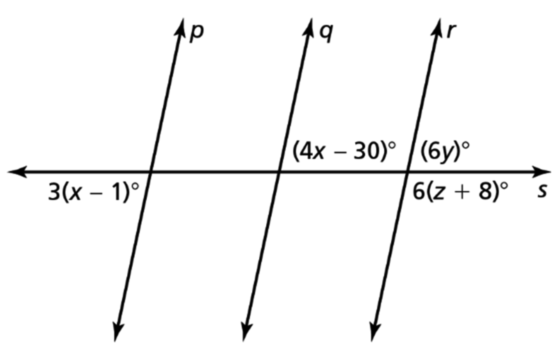
5. The map of the United States shows the lines of latitude and longitude. The lines of latitude run horizontally and the lines of longitude run vertically.



a. Are the lines of latitude parallel? Explain.

b. Are the lines of longitude parallel? Explain.

6. Use the diagram to answer the following.



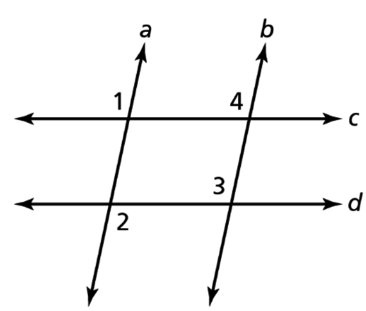
a. Find the values of *x*, *y*, and *z* that makes $p \parallel q$ and $q \parallel r$. Explain your reasoning.

b. Is $p \parallel r$? Explain your reasoning.

7. Write a proof.

Given $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$

Prove $\angle 1 \cong \angle 4$



Name____ Date __________

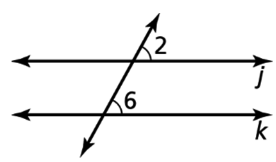
3.3 Reteach

Theorem 3.5 below is the converse of the Corresponding Angles Theorem. Similarly, the other theorems about angles formed when parallel lines are cut by a transversal have true converses. Remember that the converse of a true conditional statement is not necessarily true, so you must prove each converse of a theorem.

Theorem

3.5 Corresponding Angles Converse

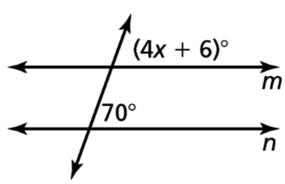
If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are parallel.



$j \parallel k$

EXAMPLE Using the Corresponding Angles Converse

Find the value of x that makes $m \parallel n$.



SOLUTION

Lines m and n are parallel when the marked corresponding angles are congruent.

$(4x + 6)° = 70°$	Use the Corresponding Angles Converse to write an equation.
$4x = 64$	Subtract 6 from each side.
$x = 16$	Divide each side by 4.

▶ So, lines m and n are parallel when $x = 16$.

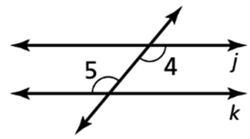
Name ______________________________ Date __________

3.3 Reteach (continued)

Theorems

3.6 Alternate Interior Angles Converse

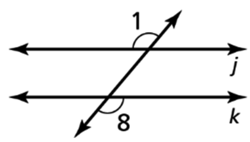
If two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.



$j \parallel k$

3.7 Alternate Exterior Angles Converse

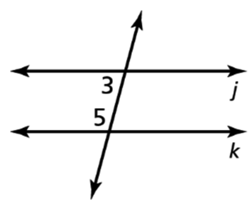
If two lines are cut by a transversal so the alternate exterior angles are congruent, then the lines are parallel.



$j \parallel k$

3.8 Consecutive Interior Angles Converse

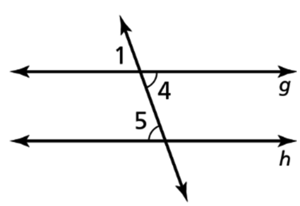
If two lines are cut by a transversal so the consecutive interior angles are supplementary, then the lines are parallel.



If $\angle 3$ and $\angle 5$ are supplementary, then $j \parallel k$.

EXAMPLE Proving the Alternate Interior Angles Converse

Prove that if two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.



SOLUTION

Given $\angle 4 \cong \angle 5$

Prove $g \parallel h$

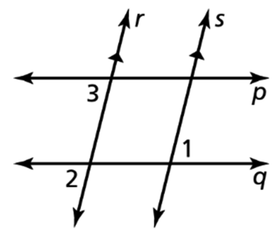
Notice that $\angle 1$ and $\angle 4$ are vertical angles and $\angle 1$ and $\angle 5$ are corresponding angles.

STATEMENTS	REASONS
1. $\angle 4 \cong \angle 5$	**1.** Given
2. $\angle 1 \cong \angle 4$	**2.** Vertical Angles Congruence Theorem
3. $\angle 1 \cong \angle 5$	**3.** Transitive Property of Congruence
4. $g \parallel h$	**4.** Corresponding Angles Converse

3.3 Reteach (continued)

EXAMPLE Determining Whether Lines Are Parallel

In the diagram, $r \parallel s$ and $\angle 1$ is congruent to $\angle 3$. Prove $p \parallel q$.



SOLUTION

Look at the diagram to make a plan. The diagram suggests that you look at angles 1, 2, and 3. Also, you may find it helpful to focus on one pair of lines and one transversal at a time.

Plan for Proof **a.** Look at $\angle 1$ and $\angle 2$. $\angle 1 \cong \angle 2$ because $r \parallel s$.

b. Look at $\angle 2$ and $\angle 3$. If $\angle 2 \cong \angle 3$, then $p \parallel q$.

Plan in Action **a.** It is given that $r \parallel s$, so by the Alternate Exterior Angles Theorem, $\angle 1 \cong \angle 2$.

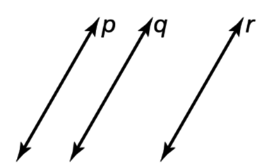
b. It is also given that $\angle 1 \cong \angle 3$. Then $\angle 2 \cong \angle 3$ by the Transitive Property of Congruence.

▶ So, by the Corresponding Angles Converse, $p \parallel q$.

Theorem

3.9 Transitive Property of Parallel Lines

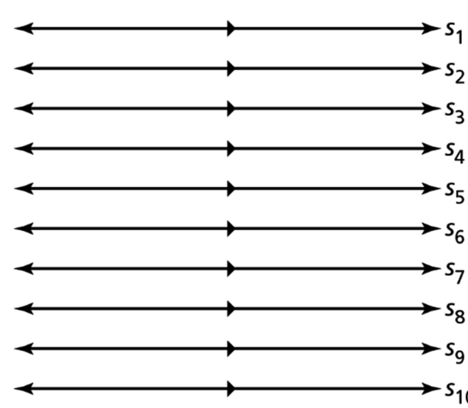
If two lines are parallel to the same line, then they are parallel to each other.



If $p \parallel q$ and $q \parallel r$, then $p \parallel r$.

EXAMPLE Using the Transitive Property of Parallel Lines

Each line is parallel to the line immediately below it. Explain why the top line is parallel to the bottom line.



SOLUTION

Each line is parallel to the one immediately below it, so $s_1 \parallel s_2$, $s_2 \parallel s_3$, and so on. Then $s_1 \parallel s_3$ by the Transitive Property of Parallel Lines. Similarly, because $s_3 \parallel s_4$, it follows that $s_1 \parallel s_4$. By continuing this reasoning, $s_1 \parallel s_{10}$.

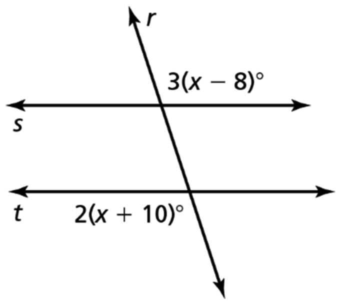
▶ So, the top line is parallel to the bottom line by the Transitive Property of Parallel Lines.

Name ____ Date __________

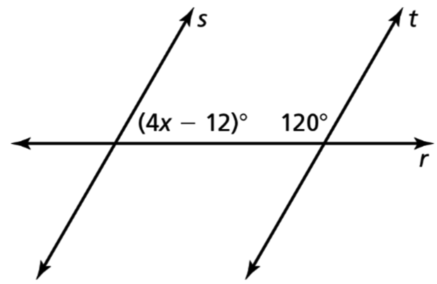
3.3 Reteach (continued)

In Exercises 1 and 2, find the value of *x* that makes $s \parallel t$.

1.

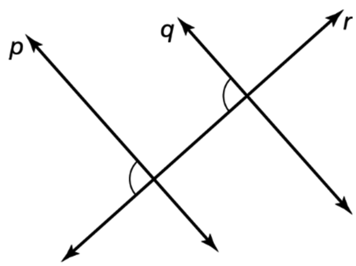


2.



In Exercises 3 and 4, decide whether there is enough information to prove that $p \parallel q$. If so, state the theorem you can use.

3.



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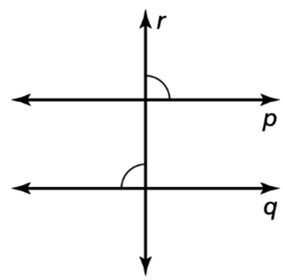
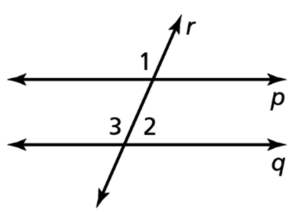


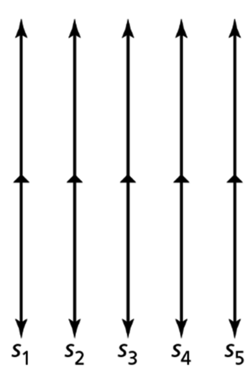
5. Write a proof.

Given $\angle 1$ and $\angle 2$ are supplementary.

Prove $p \parallel q$



6. Each line is parallel to the line immediately to its right. Explain why the leftmost line is parallel to the rightmost line.

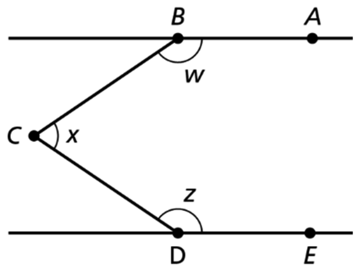


Name____ Date ___________

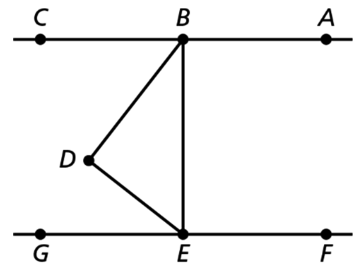
3.3 Enrichment and Extension

Using Theorems About Parallel Lines

1. $\overline{AB}$ is parallel to $\overline{DE}$, $m\angle w = 135°$, and $m\angle z = 147°$. Find $m\angle BCD$.



2. $\overline{AC}$ is parallel to $\overline{FG}$. $\overline{BD}$ is the bisector of $\angle CBE$ and $\overline{DE}$ is the bisector of $\angle BEG$. Write a two-column proof that shows $m\angle BDE = 90°$.



3. Point R is not in plane ABC.

a. How many lines through R are perpendicular to plane ABC?

b. How many lines through R are parallel to plane ABC?

c. How many planes through R are parallel to plane ABC?

4. In the diagram to the right, $e \parallel d$, $g \parallel f$, and $a \parallel b \parallel c$. Find the following.

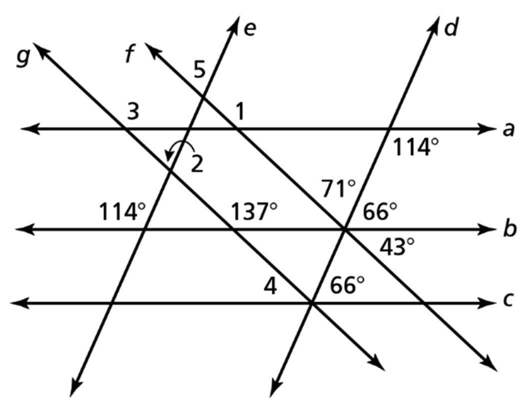
a. $m\angle 1$

b. $m\angle 2$

c. $m\angle 3$

d. $m\angle 4$

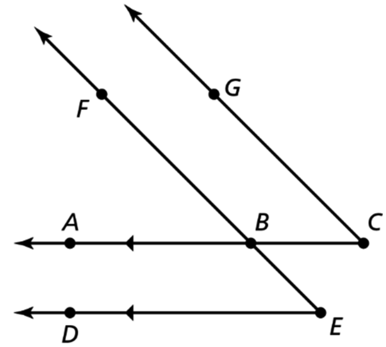
e. $m\angle 5$



5. Write a two-column proof.

Given $\overrightarrow{CA} \parallel \overrightarrow{ED}$, $m\angle FED = m\angle GCA = 45°$

Prove $\overrightarrow{EF} \parallel \overrightarrow{CG}$



Name ______________________________ Date __________

3.3 Puzzle Time

Why Did The Boy Throw His Clock Out The Window?

A	B	C	D	E	F
G					

Complete each exercise. Find the answer in the answer column. Write the word under the answer in the box containing the exercise letter.

11	TO
13	PLANE
77	BREAK
6	SEE
4	AN
5	BIRD
70	THE
12	HE

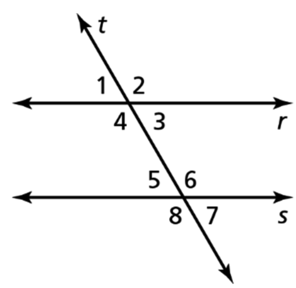
Using the diagram, find the value of *x* that makes *r* parallel to *s*.

A. $m\angle 1 = 30^\circ$ and $m\angle 7 = (2x + 10)^\circ$

B. $m\angle 4 = 135^\circ$ and $m\angle 5 = (4x - 3)^\circ$

C. $m\angle 2 = 124^\circ$ and $m\angle 6 = (4x + 4)^\circ$

D. $m\angle 3 = 24^\circ$ and $m\angle 5 = (2x + 2)^\circ$



Use the diagram. Complete the proof using the correct reason below.

Given $\angle 2 \cong \angle 8$ **Prove** $r \parallel s$

STATEMENTS	REASONS
$\angle 2 \cong \angle 8$	Given
$\angle 4 \cong \angle 2$	**E.**
$\angle 4 \cong \angle 8$	**F.**
$r \parallel s$	**G.**

1. Consecutive Interior Angles Converse
2. Alternate Interior Angles Converse
3. Transitive Property of Congruence
4. Transitive Property of Parallel Lines
5. Alternate Exterior Angles Converse
6. Vertical Angles Congruence Theorem
7. Corresponding Angles Converse

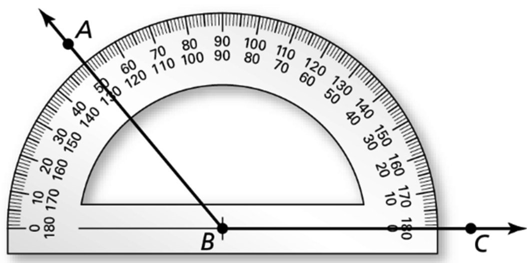
7	FLY
3	TIME
30	WANTED
1	TAKE
2	FOREVER
10	BECAUSE
$34\frac{1}{2}$	SOUND
9	HOLD

Name________________________________ Date __________

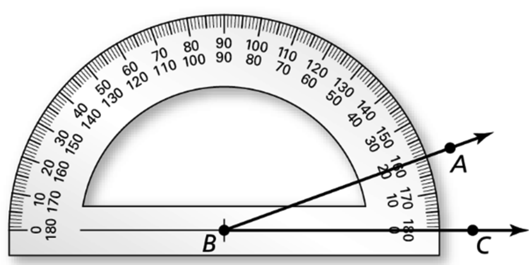
3.4 Cumulative Practice
For use before Lesson 3.4

Find the angle measure. Then classify the angle.

1. $m\angle ABC$



2. $m\angle ABC$



3.4 Prerequisite Skills Practice
For use before Lesson 3.4

Find the distance between the two points.

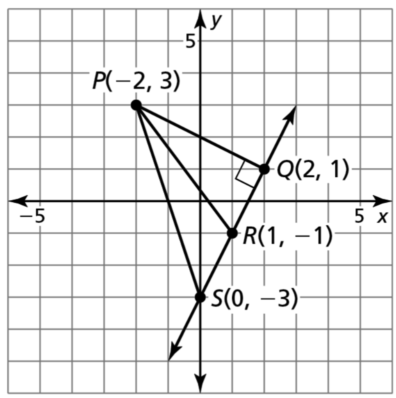
1. $P(-3, 2)$ and $Q(1, -3)$

2. $S(-5, -4)$ and $T(2, -1)$

Name ______________________________ Date __________

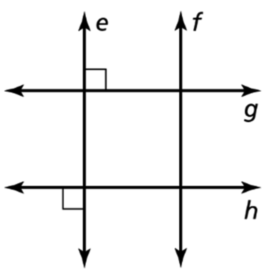
3.4 Extra Practice

1. Find the distance from point P to $\overleftrightarrow{QS}$.



In Exercises 2 and 3, determine which lines, if any, must be parallel. Explain your reasoning.

2.



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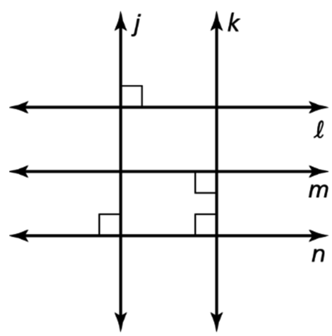
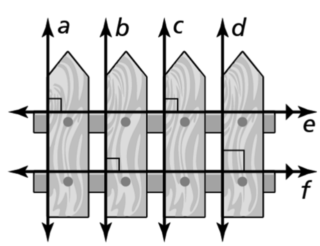
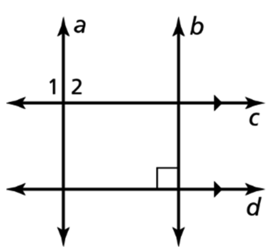


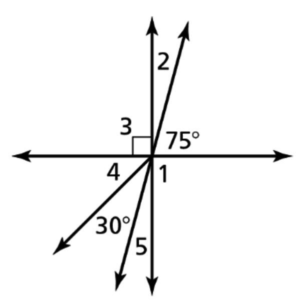
4. Your friend claims that you have enough information to determine that all of the vertical panels are parallel to each other. Is your friend correct? Explain your reasoning.



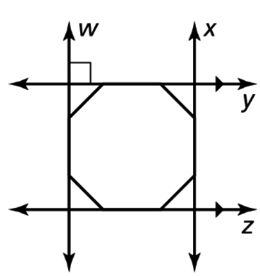
5. Write a proof.
Given $\angle 1 \cong \angle 2$, $c \parallel d$, and $b \perp d$
Prove $a \parallel b$



6. Find all the unknown angle measures in the diagram. Justify your answer for each angle measure.



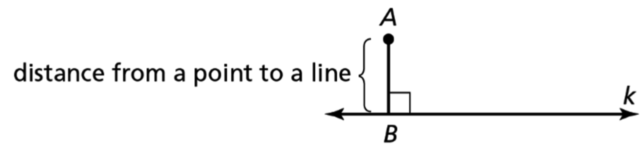
7. You extend the sides of a regular octagon as shown in the figure. You are given that $w \perp y$ and $y \parallel z$. Do you have enough information to conclude that $x \perp z$? Explain.



Name____ Date __________

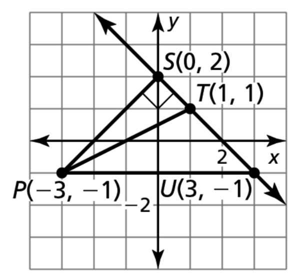
3.4 Reteach

The **distance from a point to a line** is the length of the perpendicular segment from the point to the line. The length of this segment is the shortest distance between the point and the line. For example, the distance between point *A* and line *k* is *AB*.



EXAMPLE Finding the Distance from a Point to a Line

Find the distance from point *P* to $\overleftrightarrow{ST}$.



SOLUTION

Because $\overline{PS} \perp \overleftrightarrow{ST}$, the distance from point *P* to $\overleftrightarrow{ST}$ is *PS*. Use the Distance Formula.

$$PS = \sqrt{[0-(-3)]^2 + [2-(-1)]^2} = \sqrt{3^2 + 3^2} = \sqrt{18} \approx 4.2$$

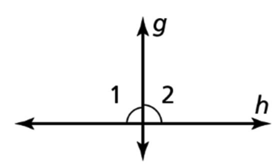
▶ So, the distance from point *P* to $\overleftrightarrow{ST}$ is about 4.2 units.

Theorems

3.10 Linear Pair Perpendicular Theorem

If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.

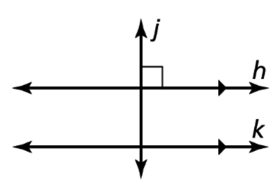
If $\angle 1 \cong \angle 2$, then $g \perp h$.



3.11 Perpendicular Transversal Theorem

In a plane, if a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other line.

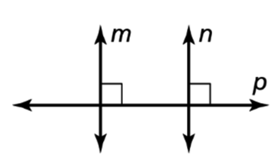
If $h \parallel k$ and $j \perp h$, then $j \perp k$.



3.12 Lines Perpendicular to a Transversal Theorem

In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

If $m \perp p$ and $n \perp p$, then $m \parallel n$.

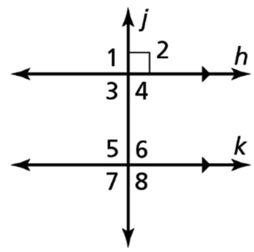


Name ____ Date __________

3.4 Reteach (continued)

EXAMPLE Proving the Perpendicular Transversal Theorem

Use the diagram to prove the Perpendicular Transversal Theorem.



SOLUTION

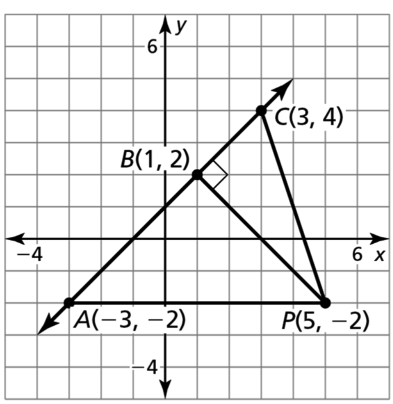
Given $h \parallel k, j \perp h$

Prove $j \perp k$

To prove that $j \perp k$, prove that $m\angle 5 = 90°$ or $m\angle 6 = 90°$.

STATEMENTS	REASONS
1. $h \parallel k, j \perp h$	**1.** Given
2. $m\angle 2 = 90°$	**2.** Definition of perpendicular lines
3. $\angle 2 \cong \angle 6$	**3.** Corresponding Angles Theorem
4. $m\angle 2 = m\angle 6$	**4.** Definition of congruent angles
5. $m\angle 6 = 90°$	**5.** Transitive Property of Equality
6. $j \perp k$	**6.** Definition of perpendicular lines

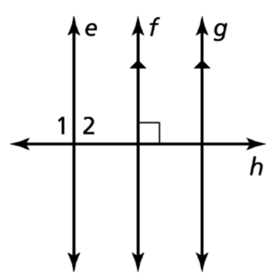
1. Find the distance from point P to $\overleftrightarrow{AB}$.



2. Use the diagram to write a proof.

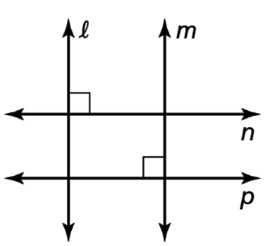
Given $\angle 1 \cong \angle 2$, $f \perp h$, and $f \parallel g$

Prove $e \parallel g$



In Exercises 3 and 4, determine which lines, if any, must be parallel. Explain your reasoning.

3.



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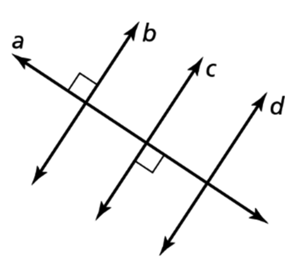


Name____ Date__________

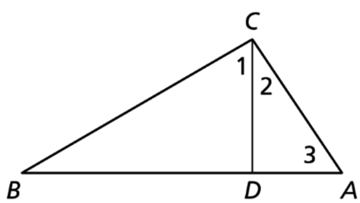
3.4 Enrichment and Extension

Proofs with Perpendicular Lines and Distance Between a Point and a Line

In Exercises 1–4, refer to the diagram to write a two-column proof.

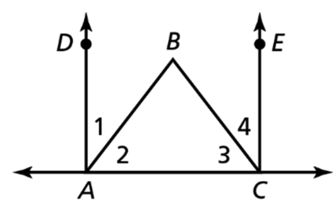
1. Given $\overline{AC} \perp \overline{BC}$; $\angle 3$ is complementary to $\angle 1$.

Prove $\angle 3 \cong \angle 2$



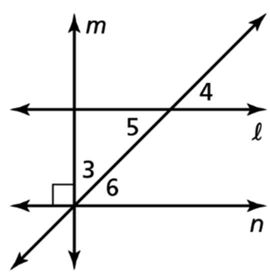
2. Given $\overline{AB}$ bisects $\angle DAC$; $\overline{CB}$ bisects $\angle ECA$; $m\angle 2 = 45°$; $m\angle 3 = 45°$

Prove $\overrightarrow{AD}$ is parallel to $\overrightarrow{CE}$.



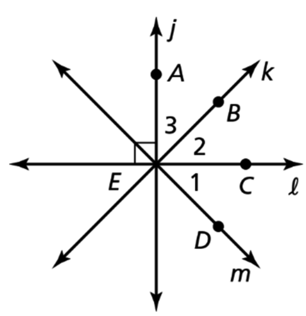
3. Given $m \perp n$; $\angle 3$ and $\angle 4$ are complementary.

Prove $\angle 5 \cong \angle 6$



4. Given $j \perp \ell$; $\angle 1 \cong \angle 3$

Prove $k \perp m$



In Exercises 5–8, find the distance between the point and the line. The distance d between the point (x_1, y_1) and the line $Ax + By = C$ is $d = \dfrac{|Ax_1 + By_1 - C|}{\sqrt{A^2 + B^2}}$.

5. $(3, 6)$; $3x + 4y = -2$

6. $(-2, 1)$; $x - y = 2$

7. $(8, 6)$; $-3x + 5y = -2$

8. $(5, -2)$; $2x + 3y = 1$

Name ______________________________ Date __________

3.4 Puzzle Time

What Snake Is The Best Mathematician?

Write the letter of each answer in the box containing the exercise number.

Complete the sentence.

1. The distance from a point to a line is the length of the ________ segment from the point to the line.
2. If two lines intersect to form a(n) ________ of congruent angles, then the lines are perpendicular.
3. In a plane, if a transversal is perpendicular to one of two ________ lines, then it is perpendicular to the other line.
4. In a(n) ________ , if two lines are perpendicular to the same line, then they are parallel to each other.

Indicate the distance of the segment using the given information. Round to the nearest tenth.

5. Find AX. $A(-4, 5)$, $X(1, -2)$
6. Find CX. $C(6, -4)$, $X(1, -2)$
7. Find DX. $D(-7, 3)$, $X(3, 4)$
8. Find BX. $B(5, 2)$, $X(3, 4)$

Answers

R. 5.4
D. perpendicular
I. vertical pair
P. longest segment
A. plane
A. 9.8
M. straight
D. 11.6
E. linear pair
A. graph
E. 1.9
H. 8.6
V. 5.3
D. 10.0
A. 3.6
T. parallel
M. 4.5
E. 2.8

3	5	8		4	1	7	2	6

Name____ Date __________

3.5 Cumulative Practice
For use before Lesson 3.5

Find the area of the polygon with the given vertices.

1. $A(-4, 3), B(1, 3), C(4, -5), D(-1, -5)$

2. $A(-3, -4), B(3, -4), C(5, -14)$

3.5 Prerequisite Skills Practice
For use before Lesson 3.5

Graph the line in a coordinate plane

1. $y = \frac{2}{3}x - 2$

2. $y = -\frac{4}{3}x + 3$

Name ______________________________ Date __________

3.5 Extra Practice

In Exercises 1 and 2, find the coordinates of point *Q* along the directed line segment *LM* so that *LQ* to *QM* is the given ratio.

1. $L(-1, -2), M(3, 6);$ 5 to 3

2. $L(2, 7), M(-1, 1);$ 2 to 1

3. Tell whether the lines through the given points are parallel, perpendicular, or neither. Justify your answer.

Line 1: $(2.5, -2), (9.5, 12)$ Line 2: $(-4, -2), (8, -4)$

4. Write an equation of the line passing through point $P(-1, -4)$ that is parallel to $y = -6x + 8$.

5. Write an equation of the line passing through point $P(-1, 3)$ that is perpendicular to $y = 4x - 7$.

In Exercises 6 and 7, find the distance from point *P* to the given line.

6. $P(4, 8), 6 = y + 2x$

7. $P(-2, 1), y = \frac{1}{4}x - 3$

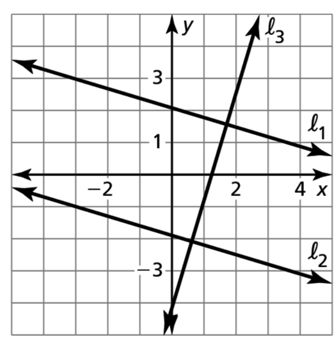
8. A line through $(-1, b)$ and $(c, 8)$ is parallel to a line through $(-6, 3)$ and $(0, 12)$. Find values of b and c that make the above statement true.

9. The graph shows three lines. The slope of line ℓ_1 is m_1, where $-1 \leq m_1 < 0$.

a. Lines ℓ_1 and ℓ_2 are parallel. What do you know about the slope of line ℓ_2?

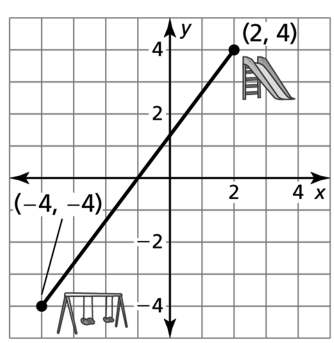
b. Lines ℓ_1 and ℓ_3 are perpendicular. What do you know about the slope of line ℓ_3?

c. What is the relationship between ℓ_2 and ℓ_3? Justify your answer.



10. Two lines are perpendicular. Is it possible for the lines to have the same y-intercept? Justify your answer.

11. The diagram shows a map of a playground. The water fountain lies directly between the swings and the slide. The distance from the swings to the water fountain is one-third the distance from the water fountain to the slide. What point on the graph represents the water fountain?



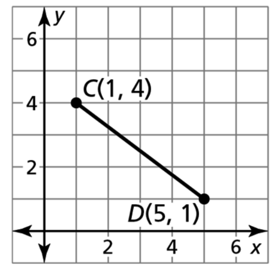
Name________________________________ Date__________

3.5 Reteach

A **directed line segment** AB is a segment that represents moving from point A to point B.

EXAMPLE Partitioning a Directed Line Segment

Find the coordinates of point Q along the directed line segment CD so that the ratio of CQ to QD is 1 to 3.



SOLUTION

In order to divide the segment in the ratio 1 to 3, think of dividing, or *partitioning*, the segment into 1 + 3, or 4 congruent pieces. Point Q is the point that is $\frac{1}{4}$ of the way from point C to point D.

Find the rise and run from point C to point D. Leave the slope in terms of rise and run, and do not simplify. Let $(x_1, y_1) = (1, 4)$ and $(x_2, y_2) = (5, 1)$.

$$\text{slope of } \overline{CD}\text{: } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-4}{5-1} = \frac{-3}{4} = \frac{\text{rise}}{\text{run}}$$

To find the coordinates of point Q, add $\frac{1}{4}$ of the run to the x-coordinate of C,

and add $\frac{1}{4}$ of the rise to the y-coordinate of C.

run: $\frac{1}{4}$ of $4 = \frac{1}{4} \bullet 4 = 1$ rise: $\frac{1}{4}$ of $-3 = \frac{1}{4} \bullet -3 = -0.75$

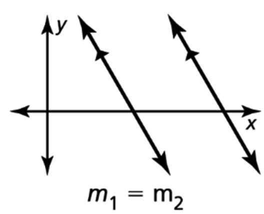
▶ So, the coordinates of Q are $(1 + 1, 4 + [-0.75]) = (2, 3.25)$.

Theorems

3.13 Slopes of Parallel Lines

In a coordinate plane, two nonvertical lines are parallel if and only if they have the same slope.

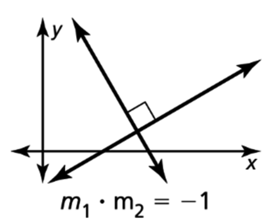
Any two vertical lines are parallel.



3.14 Slopes of Perpendicular Lines

In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

Horizontal lines are perpendicular to vertical lines.

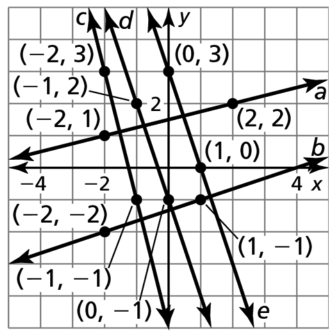


Name ______________________________ Date __________

3.5 Reteach (continued)

EXAMPLE Identifying Parallel and Perpendicular Lines

Determine which lines are parallel and which lines are perpendicular.



SOLUTION

Find the slope of each line.

$$\text{Line } a\text{: } m = \frac{2-1}{2-(-2)} = \frac{1}{4}$$

$$\text{Line } b\text{: } m = \frac{-1-(-2)}{1-(-2)} = \frac{1}{3}$$

$$\text{Line } c\text{: } m = \frac{-1-3}{-1-(-2)} = \frac{-4}{1} = -4$$

$$\text{Line } d\text{: } m = \frac{-1-2}{0-(-1)} = \frac{-3}{1} = -3$$

$$\text{Line } e\text{: } m = \frac{0-3}{1-0} = \frac{-3}{1} = -3$$

▶ Because lines *d* and *e* have the same slope, lines *d* and *e* are parallel. Because $\frac{1}{4}(-4) = -1$, lines *a* and *c* are perpendicular. Because $\frac{1}{3}(-3) = -1$, lines *b* and *d* are perpendicular and lines *b* and *e* are perpendicular.

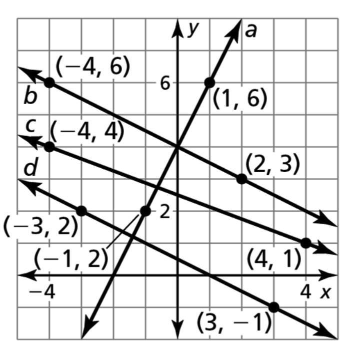
In Exercises 1 and 2, find the coordinates of point *P* along the directed line segment *ST* so that *SP* to *PT* is the given ratio.

1. $S(6, 4)$, $T(-4, -8)$; 1 to 3

2. $S(-6, 7)$, $T(9, 25)$; 2 to 3

In Exercises 3 and 4, determine which lines are parallel and which lines are perpendicular.

3.



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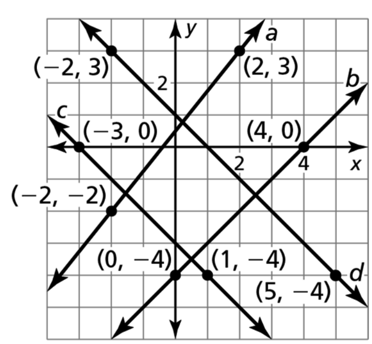

Name____________________ Date__________

3.5 Enrichment and Extension

Equations of Parallel and Perpendicular Lines

1. Write an equation of the perpendicular bisector for the line segment defined between points $A(2, 5)$ and $B(-6, -1)$.

2. Find the values of a and b in $ax + by = 90$ such that the line is perpendicular to $-20x + 12y = 36$ and has the same y-intercept.

3. Consider the linear equation $y = 3.62(x - 1.35) + 2.74$.

a. What is the slope of this line?

b. What is the value of y when $x = 1.35$?

c. Find an equation for the line through $(4.23, -2.58)$ that is parallel to this line.

d. Find an equation for the line through $(4.23, -2.58)$ that is perpendicular to this line. Round values to the nearest thousandth.

4. What is the slope of the line $ax + by = c$? Find an equation for the line through the origin that is parallel to the line $ax + by = c$. Find an equation for the line through the origin that is perpendicular to the line $ax + by = c$.

5. A line passes through the points $(k + 10, -2k - 1)$ and $(2, 9)$ and has a y-intercept of 10. Find the value of k and an equation of the line.

6. A line passes through the points $(3k, 6k - 5)$ and $(-1, -7)$ and has a y-intercept of -5. Find the value of k and an equation of the line.

7. Consider the two linear equations $ax + by = c$ and $dx + ey = f$.

a. Under what conditions will the graphs of the two equations intersect at one point?

b. Under what conditions will the graphs of the two equations be parallel?

8. Point F is located at $(0, 4)$.

a. Find coordinates of three points that are equidistant from F and the x-axis.

b. If possible, write the equations of the lines that are parallel or perpendicular to the line $x = 0$ and pass through the coordinates from part (a).

c. Consider $G(0, y)$. Find the coordinates of three points that are equidistant from G and the x-axis.

Name ______________________________ Date __________

3.5 Puzzle Time

How Do You Make Seven Even?

Circle the letter of each correct answer in the boxes below. The circled letters will spell out the answer to the riddle.

Complete the sentence.

1. A(n) ____________ line segment $\overline{AB}$ is a segment that represents moving from point A to point B.

2. In a coordinate plane, two nonvertical lines are parallel if and only if they have the ____________.

3. In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their ____________.

Tell whether the lines through the given points are (1) parallel, (2) perpendicular, (3) neutral, (4) directed, (5) undirected, (6) none of these.

4. Line 1: $(-7, -3), (1, 4)$; Line 2: $(-6, 6), (1, -2)$

5. Line 1: $(-4, -2), (4, 5)$; Line 2: $(-2, 3), (2, -3)$

6. Line 1: $(0, 4), (-6, 0)$; Line 2: $(3, 2), (-3, -2)$

Find the distance from point *A* to the given line. Round to the nearest tenth.

7. $A(-4, 4), y = 0.8x - 0.4$

8. $A(-3, -3), y = 0.5x + 6.5$

R	**D**	**N**	**R**	**P**	**O**	**M**	**A**
5	2	6.4	slopes is -1	slopes is $-\frac{1}{2}$	5.9	straight	3
P	**T**	**L**	**H**	**L**	**I**	**E**	**S**
7.2	1	6.7	same slope	slopes is 0	4	directed	6