10 Circles

Seismograph *(p. 520)*

Saturn *(p. 515)*

Car *(p. 494)*

Dartboard *(p. 487)*

Bicycle Chain *(p. 479)*

Maintaining Mathematical Proficiency

Multiplying Binomials

Example 1 **Find the product $(x + 3)(2x - 1)$.**

$$\begin{aligned} (x+3)(2x-1) &= \overset{\text{First}}{x(2x)} + \overset{\text{Outer}}{x(-1)} + \overset{\text{Inner}}{3(2x)} + \overset{\text{Last}}{(3)(-1)} && \text{FOIL Method} \\ &= 2x^2 + (-x) + 6x + (-3) && \text{Multiply.} \\ &= 2x^2 + 5x - 3 && \text{Simplify.} \end{aligned}$$

▶ The product is $2x^2 + 5x - 3$.

Find the product.

1. $(x + 7)(x + 4)$

2. $(a + 1)(a - 5)$

3. $(q - 9)(3q - 4)$

4. $(2v - 7)(5v + 1)$

5. $(4h + 3)(2 + h)$

6. $(8 - 6b)(5 - 3b)$

Solving Quadratic Equations by Completing the Square

Example 2 **Solve $x^2 + 8x - 3 = 0$ by completing the square.**

$$\begin{aligned} x^2 + 8x - 3 &= 0 && \text{Write original equation.} \\ x^2 + 8x &= 3 && \text{Add 3 to each side.} \\ x^2 + 8x + 4^2 &= 3 + 4^2 && \text{Complete the square by adding } \left(\tfrac{8}{2}\right)^2\text{, or } 4^2\text{, to each side.} \\ (x + 4)^2 &= 19 && \text{Write the left side as a square of a binomial.} \\ x + 4 &= \pm\sqrt{19} && \text{Take the square root of each side.} \\ x &= -4 \pm\sqrt{19} && \text{Subtract 4 from each side.} \end{aligned}$$

▶ The solutions are $x = -4 + \sqrt{19} \approx 0.36$ and $x = -4 - \sqrt{19} \approx -8.36$.

Solve the equation by completing the square. Round your answer to the nearest hundredth, if necessary.

7. $x^2 - 2x = 5$

8. $r^2 + 10r = -7$

9. $w^2 - 8w = 9$

10. $p^2 + 10p - 4 = 0$

11. $k^2 - 4k - 7 = 0$

12. $-z^2 + 2z = 1$

13. **ABSTRACT REASONING** Write an expression that represents the product of two consecutive positive odd integers. Explain your reasoning.

Mathematical Processes

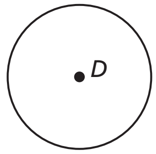
Mathematically proficient students make sense of problems and do not give up when faced with challenges.

Analyzing Relationships of Circles

Core Concept

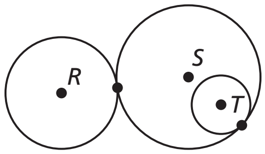
Circles and Tangent Circles

A **circle** is the set of all points in a plane that are equidistant from a given point called the **center** of the circle. A circle with center *D* is called "circle *D*" and can be written as ⊙*D*.



circle *D*, or ⊙*D*

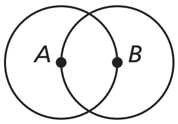
Coplanar circles that intersect in one point are called **tangent circles**.



⊙*R* and ⊙*S* are tangent circles.
⊙*S* and ⊙*T* are tangent circles.

EXAMPLE 1 Relationships of Circles and Tangent Circles

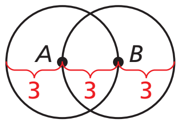
a. Each circle at the right consists of points that are 3 units from the center. What is the greatest distance from any point on ⊙*A* to any point on ⊙*B*?



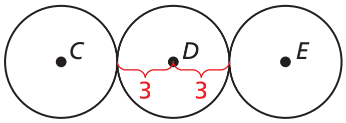
b. Three circles, ⊙*C*, ⊙*D*, and ⊙*E*, consist of points that are 3 units from their centers. The centers *C*, *D*, and *E* of the circles are collinear, ⊙*C* is tangent to ⊙*D*, and ⊙*D* is tangent to ⊙*E*. What is the distance from ⊙*C* to ⊙*E*?

SOLUTION

a. Because the points on each circle are 3 units from the center, the greatest distance from any point on ⊙*A* to any point on ⊙*B* is $3 + 3 + 3 = 9$ units.



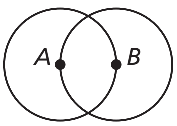
b. Because *C*, *D*, and *E* are collinear, ⊙*C* is tangent to ⊙*D*, and ⊙*D* is tangent to ⊙*E*, the circles are as shown. So, the distance from ⊙*C* to ⊙*E* is $3 + 3 = 6$ units.



Monitoring Progress

Let ⊙*A*, ⊙*B*, and ⊙*C* consist of points that are 3 units from the centers.

1. Draw ⊙*C* so that it passes through points *A* and *B* in the figure at the right. Explain your reasoning.
2. Draw ⊙*A*, ⊙*B*, and ⊙*C* so that each is tangent to the other two. Draw a larger circle, ⊙*D*, that is tangent to each of the other three circles. Is the distance from point *D* to a point on ⊙*D* *less than*, *greater than*, or *equal to* 6? Explain.



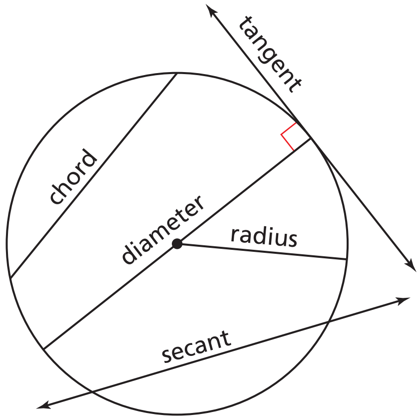
10.1 Lines and Segments That Intersect Circles

Essential Question

What are the definitions of the lines and segments that intersect a circle?

EXPLORATION 1 Lines and Line Segments That Intersect Circles

Work with a partner. The drawing at the right shows five lines or segments that intersect a circle. Use the relationships shown to write a definition for each type of line or segment. Then use the Internet or some other resource to verify your definitions.



Chord:

Secant:

Tangent:

Radius:

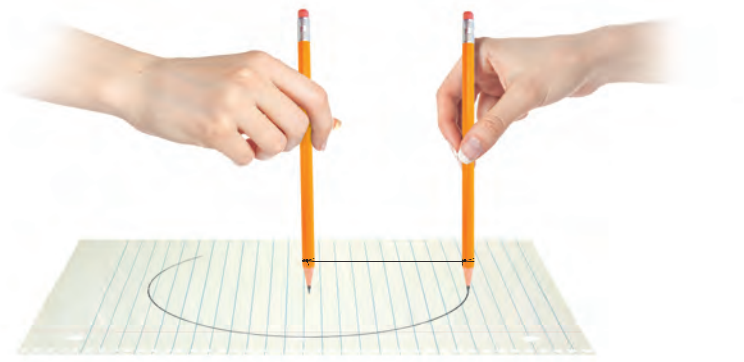
Diameter:

EXPLORATION 2 Using String to Draw a Circle

Work with a partner. Use two pencils, a piece of string, and a piece of paper.

a. Tie the two ends of the piece of string loosely around the two pencils.

b. Anchor one pencil on the paper at the center of the circle. Use the other pencil to draw a circle around the anchor point while using slight pressure to keep the string taut. Do not let the string wind around either pencil.



REASONING ABSTRACTLY

To be proficient in math, you need to know and flexibly use different properties of operations and objects.

c. Explain how the distance between the two pencil points as you draw the circle is related to two of the lines or line segments you defined in Exploration 1.

Communicate Your Answer

3. What are the definitions of the lines and segments that intersect a circle?

4. Of the five types of lines and segments in Exploration 1, which one is a subset of another? Explain.

5. Explain how to draw a circle with a diameter of 8 inches.

10.1 Lesson

Core Vocabulary

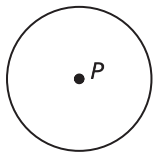
circle, *p. 474*
center, *p. 474*
radius, *p. 474*
chord, *p. 474*
diameter, *p. 474*
secant, *p. 474*
tangent, *p. 474*
point of tangency, *p. 474*
tangent circles, *p. 475*
concentric circles, *p. 475*
common tangent, *p. 475*

What You Will Learn

- Identify special segments and lines.
- Draw and identify common tangents.
- Use properties of tangents.

Identifying Special Segments and Lines

A **circle** is the set of all points in a plane that are equidistant from a given point called the **center** of the circle. A circle with center P is called "circle P" and can be written as $\odot P$.



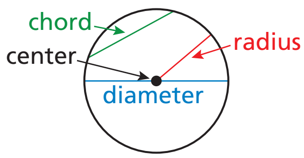
circle *P*, or $\odot P$

Core Concept

Lines and Segments That Intersect Circles

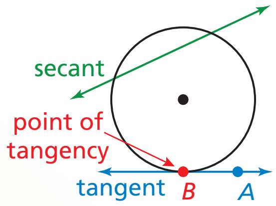
A segment whose endpoints are the center and any point on a circle is a **radius**.

A **chord** is a segment whose endpoints are on a circle. A **diameter** is a chord that contains the center of the circle.



A **secant** is a line that intersects a circle in two points.

A **tangent** is a line in the plane of a circle that intersects the circle in exactly one point, the **point of tangency**. The *tangent ray* $\overrightarrow{AB}$ and the *tangent segment* $\overline{AB}$ are also called tangents.

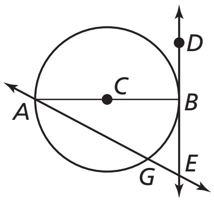


READING

The words "radius" and "diameter" refer to lengths as well as segments. For a given circle, think of *a* radius and *a* diameter as segments and *the* radius and *the* diameter as lengths.

EXAMPLE 1 Identifying Special Segments and Lines

Tell whether the line, ray, or segment is best described as a *radius*, *chord*, *diameter*, *secant*, or *tangent* of $\odot C$.



a. $\overline{AC}$ **b.** $\overline{AB}$

c. $\overrightarrow{DE}$ **d.** $\overleftrightarrow{AE}$

SOLUTION

a. $\overline{AC}$ is a radius because C is the center and A is a point on the circle.

b. $\overline{AB}$ is a diameter because it is a chord that contains the center C.

c. $\overrightarrow{DE}$ is a tangent ray because it is contained in a line that intersects the circle in exactly one point.

d. $\overleftrightarrow{AE}$ is a secant because it is a line that intersects the circle in two points.

STUDY TIP

In this book, assume that all segments, rays, or lines that appear to be tangent to a circle are tangents.

Monitoring Progress

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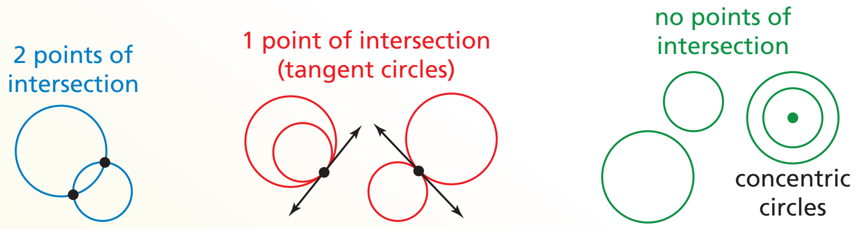
1. In Example 1, what word best describes $\overline{AG}$? $\overline{CB}$?
2. In Example 1, name a tangent and a tangent segment.

Drawing and Identifying Common Tangents

Core Concept

Coplanar Circles and Common Tangents

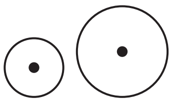
In a plane, two circles can intersect in two points, one point, or no points. Coplanar circles that intersect in one point are called **tangent circles**. Coplanar circles that have a common center are called **concentric circles**.

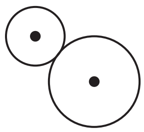


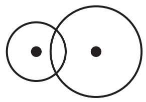
A line or segment that is tangent to two coplanar circles is called a **common tangent**. A *common internal tangent* intersects the segment that joins the centers of the two circles. A *common external tangent* does not intersect the segment that joins the centers of the two circles.

EXAMPLE 2 **Drawing and Identifying Common Tangents**

Tell how many common tangents the circles have and draw them. Use blue to indicate common external tangents and red to indicate common internal tangents.

a. 

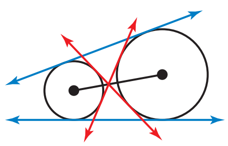
b. 

c. 

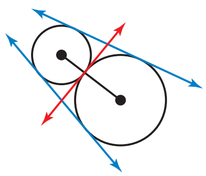
SOLUTION

Draw the segment that joins the centers of the two circles. Then draw the common tangents. Use blue to indicate lines that do not intersect the segment joining the centers and red to indicate lines that intersect the segment joining the centers.

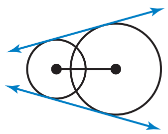
a. 4 common tangents



b. 3 common tangents



c. 2 common tangents



Monitoring Progress

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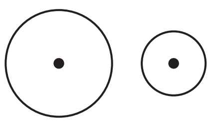
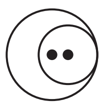
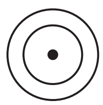
Tell how many common tangents the circles have and draw them. State whether the tangents are external tangents or internal tangents.

3. 

4. 

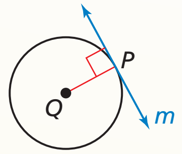
5. 

Using Properties of Tangents

Theorems

Theorem 10.1 Tangent Line to Circle Theorem

In a plane, a line is tangent to a circle if and only if the line is perpendicular to a radius of the circle at its endpoint on the circle.

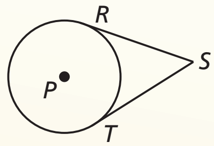


Line m is tangent to $\odot Q$ if and only if $m \perp \overline{QP}$.

Proof Ex. 47, p. 480

Theorem 10.2 External Tangent Congruence Theorem

Tangent segments from a common external point are congruent.

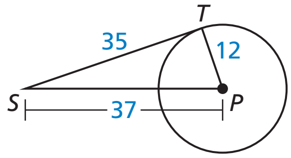


If $\overline{SR}$ and $\overline{ST}$ are tangent segments, then $\overline{SR} \cong \overline{ST}$.

Proof Ex. 46, p. 480

EXAMPLE 3 Verifying a Tangent to a Circle

Is $\overline{ST}$ tangent to $\odot P$?



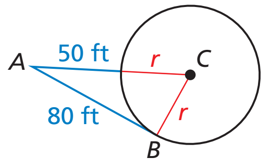
SOLUTION

Use the Converse of the Pythagorean Theorem (Theorem 9.2). Because $12^2 + 35^2 = 37^2$, $\triangle PTS$ is a right triangle and $\overline{ST} \perp \overline{PT}$. So, $\overline{ST}$ is perpendicular to a radius of $\odot P$ at its endpoint on $\odot P$.

▶ By the Tangent Line to Circle Theorem, $\overline{ST}$ is tangent to $\odot P$.

EXAMPLE 4 Finding the Radius of a Circle

In the diagram, point B is a point of tangency. Find the radius r of $\odot C$.



SOLUTION

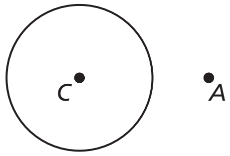
You know from the Tangent Line to Circle Theorem that $\overline{AB} \perp \overline{BC}$, so $\triangle ABC$ is a right triangle. You can use the Pythagorean Theorem (Theorem 9.1).

$AC^2 = BC^2 + AB^2$	Pythagorean Theorem
$(r + 50)^2 = r^2 + 80^2$	Substitute.
$r^2 + 100r + 2500 = r^2 + 6400$	Multiply.
$100r = 3900$	Subtract r^2 and 2500 from each side.
$r = 39$	Divide each side by 100.

▶ The radius is 39 feet.

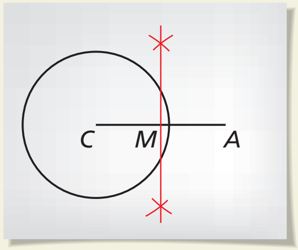
CONSTRUCTION Constructing a Tangent to a Circle

Given $\odot C$ and point A, construct a line tangent to $\odot C$ that passes through A. Use a compass and straightedge.



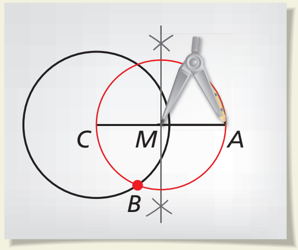
SOLUTION

Step 1



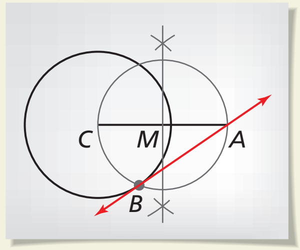
Find a midpoint
Draw $\overline{AC}$. Construct the bisector of the segment and label the midpoint M.

Step 2



Draw a circle
Construct $\odot M$ with radius MA. Label one of the points where $\odot M$ intersects $\odot C$ as point B.

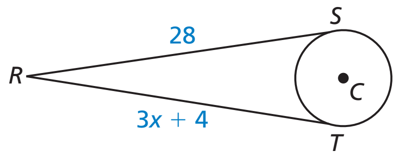
Step 3



Construct a tangent line
Draw $\overleftrightarrow{AB}$. It is a tangent to $\odot C$ that passes through A.

EXAMPLE 5 Using Properties of Tangents

$\overline{RS}$ is tangent to $\odot C$ at S, and $\overline{RT}$ is tangent to $\odot C$ at T. Find the value of x.



SOLUTION

$RS = RT$	External Tangent Congruence Theorem
$28 = 3x + 4$	Substitute.
$8 = x$	Solve for x.

▶ The value of x is 8.

Monitoring Progress

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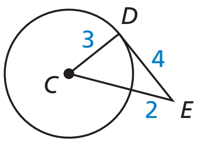
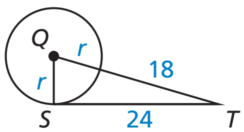
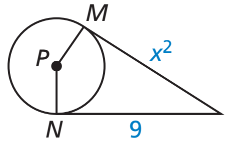
6. Is $\overline{DE}$ tangent to $\odot C$?



7. $\overline{ST}$ is tangent to $\odot Q$. Find the radius of $\odot Q$.



8. Points M and N are points of tangency. Find the value(s) of x.



10.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** How are chords and secants alike? How are they different?

2. **WRITING** Explain how you can determine from the context whether the words *radius* and *diameter* are referring to segments or lengths.

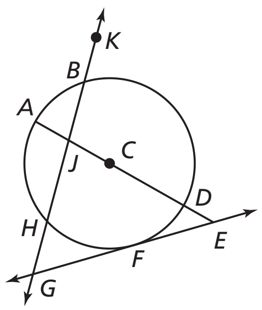
3. **COMPLETE THE SENTENCE** Coplanar circles that have a common center are called ________.

4. **WHICH ONE DOESN'T BELONG?** Which segment does *not* belong with the other three? Explain your reasoning.

chord	radius	tangent	diameter

Monitoring Progress and Modeling with Mathematics

In Exercises 5–10, use the diagram. *(See Example 1.)*



5. Name the circle.

6. Name two radii.

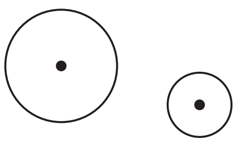
7. Name two chords.

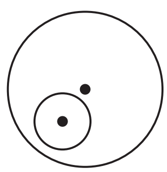
8. Name a diameter.

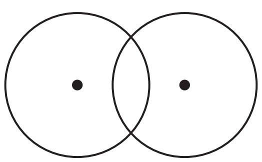
9. Name a secant.

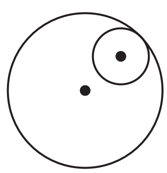
10. Name a tangent and a point of tangency.

In Exercises 11–14, copy the diagram. Tell how many common tangents the circles have and draw them. *(See Example 2.)*

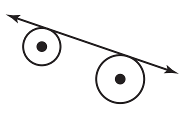
11. 

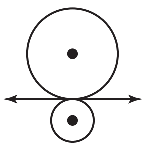
12. 

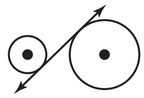
13. 

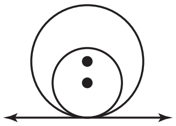
14. 

In Exercises 15–18, tell whether the common tangent is *internal* or *external*.

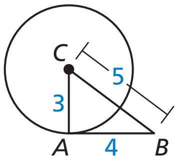
15. 

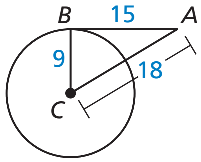
16. 

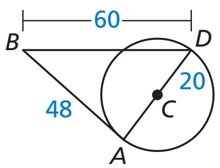
17. 

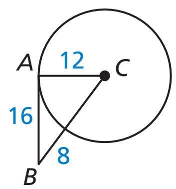
18. 

In Exercises 19–22, tell whether $\overline{AB}$ is tangent to $\odot C$. Explain your reasoning. *(See Example 3.)*

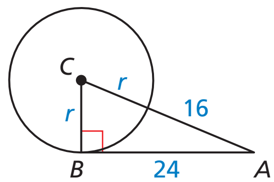
19. 

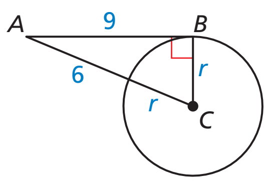
20. 

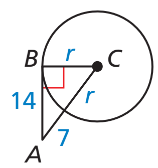
21. 

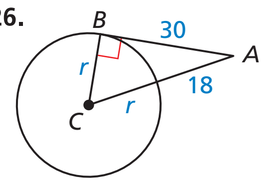
22. 

In Exercises 23–26, point B is a point of tangency. Find the radius r of $\odot C$. *(See Example 4.)*

23. 

24. 

25. 

26. 

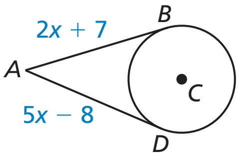
CONSTRUCTION **In Exercises 27 and 28, construct ⊙C with the given radius and point A outside of ⊙C. Then construct a line tangent to ⊙C that passes through A.**

27. $r = 2$ in.

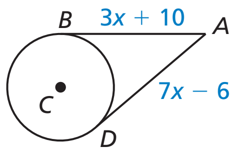
28. $r = 4.5$ cm

In Exercises 29–32, points B and D are points of tangency. Find the value(s) of x. *(See Example 5.)*

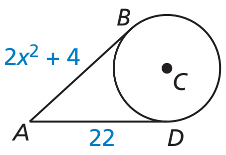
29.



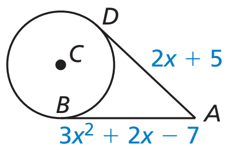
30.



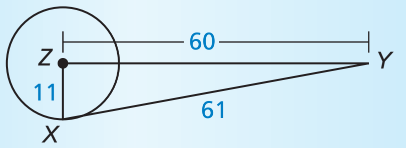
31.



32.

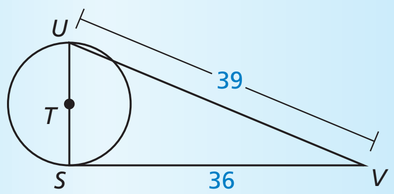


33. **ERROR ANALYSIS** Describe and correct the error in determining whether $\overline{XY}$ is tangent to ⊙Z.



Because $11^2 + 60^2 = 61^2$, △XYZ is a right triangle. So, $\overline{XY}$ is tangent to ⊙Z.

34. **ERROR ANALYSIS** Describe and correct the error in finding the radius of ⊙T.



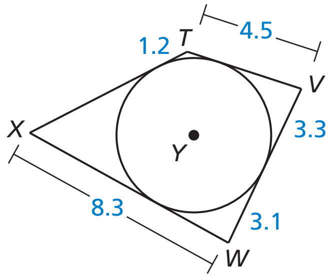
$39^2 - 36^2 = 15^2$

So, the radius is 15.

35. **ABSTRACT REASONING** For a point outside of a circle, how many lines exist tangent to the circle that pass through the point? How many such lines exist for a point on the circle? inside the circle? Explain your reasoning.

36. **CRITICAL THINKING** When will two lines tangent to the same circle not intersect? Justify your answer.

37. **USING STRUCTURE** Each side of quadrilateral $TVWX$ is tangent to ⊙Y. Find the perimeter of the quadrilateral.

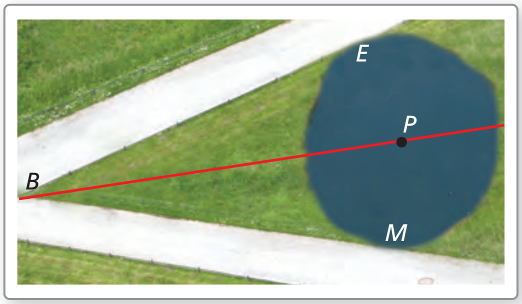


38. **LOGIC** In ⊙C, radii $\overline{CA}$ and $\overline{CB}$ are perpendicular. $\overleftrightarrow{BD}$ and $\overleftrightarrow{AD}$ are tangent to ⊙C.

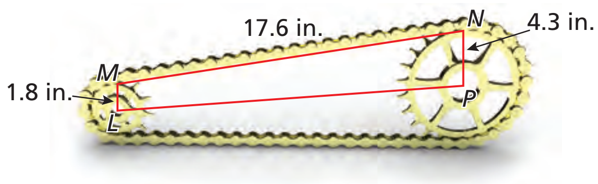
a. Sketch ⊙C, $\overline{CA}$, $\overline{CB}$, $\overleftrightarrow{BD}$, and $\overleftrightarrow{AD}$.

b. What type of quadrilateral is $CADB$? Explain your reasoning.

39. **MAKING AN ARGUMENT** Two bike paths are tangent to an approximately circular pond. Your class is building a nature trail that begins at the intersection B of the bike paths and runs between the bike paths and over a bridge through the center P of the pond. Your classmate uses the Converse of the Angle Bisector Theorem (Theorem 6.4) to conclude that the trail must bisect the angle formed by the bike paths. Is your classmate correct? Explain your reasoning.

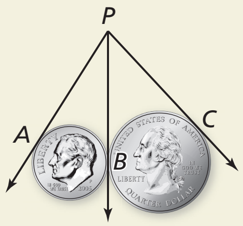


40. **MODELING WITH MATHEMATICS** A bicycle chain is pulled tightly so that $\overline{MN}$ is a common tangent of the gears. Find the distance between the centers of the gears.

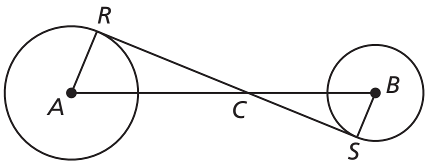


41. **WRITING** Explain why the diameter of a circle is the longest chord of the circle.

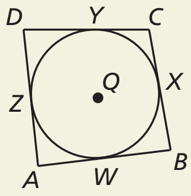
42. HOW DO YOU SEE IT? In the figure, $\overrightarrow{PA}$ is tangent to the dime, $\overrightarrow{PC}$ is tangent to the quarter, and $\overrightarrow{PB}$ is a common internal tangent. How do you know that $\overline{PA} \cong \overline{PB} \cong \overline{PC}$?



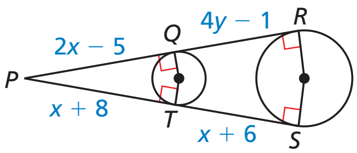
43. PROOF In the diagram, $\overline{RS}$ is a common internal tangent to $\odot A$ and $\odot B$. Prove that $\frac{AC}{BC} = \frac{RC}{SC}$.



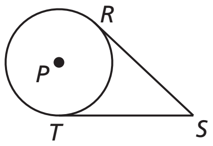
44. THOUGHT PROVOKING A polygon is *circumscribed* about a circle when every side of the polygon is tangent to the circle. In the diagram, quadrilateral *ABCD* is circumscribed about $\odot Q$. Is it always true that $AB + CD = AD + BC$? Justify your answer.



45. MATHEMATICAL CONNECTIONS Find the values of *x* and *y*. Justify your answer.



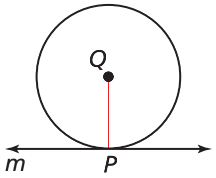
46. PROVING A THEOREM Prove the External Tangent Congruence Theorem (Theorem 10.2).



Given $\overline{SR}$ and $\overline{ST}$ are tangent to $\odot P$.

Prove $\overline{SR} \cong \overline{ST}$

47. PROVING A THEOREM Use the diagram to prove each part of the biconditional in the Tangent Line to Circle Theorem (Theorem 10.1).



a. Prove indirectly that if a line is tangent to a circle, then it is perpendicular to a radius. (*Hint*: If you assume line *m* is not perpendicular to $\overline{QP}$, then the perpendicular segment from point *Q* to line *m* must intersect line *m* at some other point *R*.)

Given Line *m* is tangent to $\odot Q$ at point *P*.

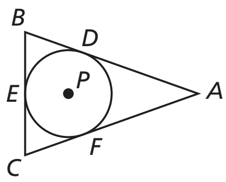
Prove $m \perp \overline{QP}$

b. Prove indirectly that if a line is perpendicular to a radius at its endpoint, then the line is tangent to the circle.

Given $m \perp \overline{QP}$

Prove Line *m* is tangent to $\odot Q$.

48. REASONING In the diagram, $AB = AC = 12$, $BC = 8$, and all three segments are tangent to $\odot P$. What is the radius of $\odot P$? Justify your answer.

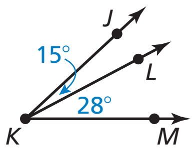


Maintaining Mathematical Proficiency

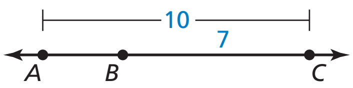
Reviewing what you learned in previous grades and lessons

Find the indicated measure.

49. $m\angle JKM$



50. *AB*



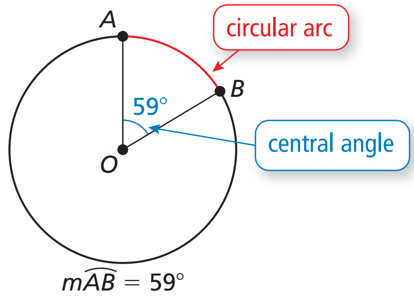
10.2 Finding Arc Measures

Essential Question

How are circular arcs measured?

A **central angle** of a circle is an angle whose vertex is the center of the circle. A *circular arc* is a portion of a circle, as shown below. The measure of a circular arc is the measure of its central angle.

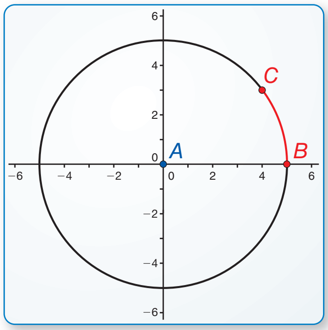
If $m\angle AOB < 180°$, then the circular arc is called a **minor arc** and is denoted by $\overset{\frown}{AB}$.



EXPLORATION 1 Measuring Circular Arcs

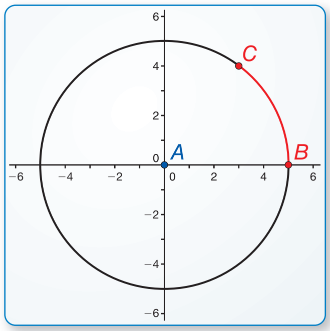
Work with a partner. Use dynamic geometry software to find the measure of $\overset{\frown}{BC}$. Verify your answers using trigonometry.

a.



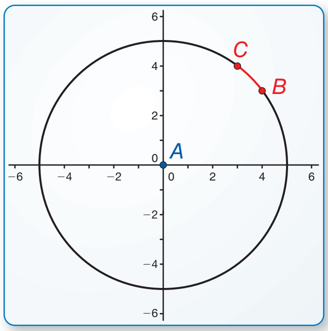
Points
$A(0, 0)$
$B(5, 0)$
$C(4, 3)$

b.



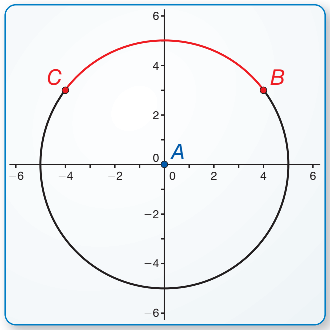
Points
$A(0, 0)$
$B(5, 0)$
$C(3, 4)$

c.



Points
$A(0, 0)$
$B(4, 3)$
$C(3, 4)$

d.



Points
$A(0, 0)$
$B(4, 3)$
$C(-4, 3)$

USING TOOLS STRATEGICALLY

To be proficient in math, you need to use technological tools to explore and deepen your understanding of concepts.

Communicate Your Answer

2. How are circular arcs measured?

3. Use dynamic geometry software to draw a circular arc with the given measure.

a. 30° **b.** 45°

c. 60° **d.** 90°

10.2 Lesson

Core Vocabulary

central angle, *p. 482*
minor arc, *p. 482*
major arc, *p. 482*
semicircle, *p. 482*
measure of a minor arc, *p. 482*
measure of a major arc, *p. 482*
adjacent arcs, *p. 483*
congruent circles, *p. 484*
congruent arcs, *p. 484*
similar arcs, *p. 485*

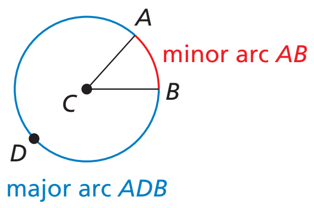
What You Will Learn

- Find arc measures.
- Identify congruent arcs.
- Prove circles are similar.

Finding Arc Measures

A **central angle** of a circle is an angle whose vertex is the center of the circle. In the diagram, $\angle ACB$ is a central angle of $\odot C$.

If $m\angle ACB$ is less than 180°, then the points on $\odot C$ that lie in the interior of $\angle ACB$ form a **minor arc** with endpoints A and B. The points on $\odot C$ that do not lie on the minor arc AB form a **major arc** with endpoints A and B. A **semicircle** is an arc with endpoints that are the endpoints of a diameter.



Minor arcs are named by their endpoints. The minor arc associated with $\angle ACB$ is named $\overset{\frown}{AB}$. Major arcs and semicircles are named by their endpoints and a point on the arc. The major arc associated with $\angle ACB$ can be named $\overset{\frown}{ADB}$.

STUDY TIP

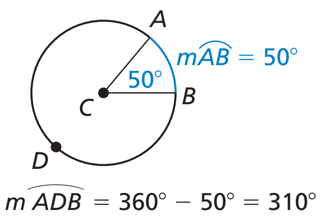
The measure of a minor arc is less than 180°. The measure of a major arc is greater than 180°.

Core Concept

Measuring Arcs

The **measure of a minor arc** is the measure of its central angle. The expression $m\overset{\frown}{AB}$ is read as "the measure of arc AB."

The measure of the entire circle is 360°. The **measure of a major arc** is the difference of 360° and the measure of the related minor arc. The measure of a semicircle is 180°.



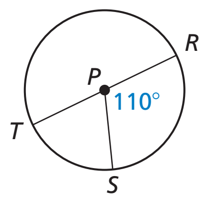
EXAMPLE 1 Finding Measures of Arcs

Find the measure of each arc of $\odot P$, where $\overline{RT}$ is a diameter.

a. $\overset{\frown}{RS}$

b. $\overset{\frown}{RTS}$

c. $\overset{\frown}{RST}$



SOLUTION

a. $\overset{\frown}{RS}$ is a minor arc, so $m\overset{\frown}{RS} = m\angle RPS = 110°$.

b. $\overset{\frown}{RTS}$ is a major arc, so $m\overset{\frown}{RTS} = 360° - 110° = 250°$.

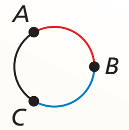
c. $\overline{RT}$ is a diameter, so $\overset{\frown}{RST}$ is a semicircle, and $m\overset{\frown}{RST} = 180°$.

Two arcs of the same circle are **adjacent arcs** when they intersect at exactly one point. You can add the measures of two adjacent arcs.

Postulate

Postulate 10.1 Arc Addition Postulate

The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

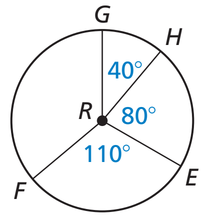


$m\widehat{ABC} = m\widehat{AB} + m\widehat{BC}$

EXAMPLE 2 Using the Arc Addition Postulate

Find the measure of each arc.

a. $\widehat{GE}$ **b.** $\widehat{GEF}$ **c.** $\widehat{GF}$



SOLUTION

a. $m\widehat{GE} = m\widehat{GH} + m\widehat{HE} = 40° + 80° = 120°$

b. $m\widehat{GEF} = m\widehat{GE} + m\widehat{EF} = 120° + 110° = 230°$

c. $m\widehat{GF} = 360° - m\widehat{GEF} = 360° - 230° = 130°$

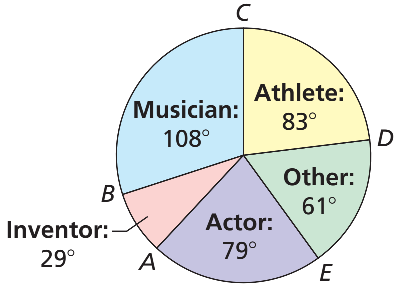
EXAMPLE 3 Finding Measures of Arcs

A recent survey asked teenagers whether they would rather meet a famous musician, athlete, actor, inventor, or other person. The circle graph shows the results. Find the indicated arc measures.

a. $m\widehat{AC}$ **b.** $m\widehat{ACD}$

c. $m\widehat{ADC}$ **d.** $m\widehat{EBD}$

Whom Would You Rather Meet?



SOLUTION

a. $m\widehat{AC} = m\widehat{AB} + m\widehat{BC}$

$= 29° + 108°$

$= 137°$

b. $m\widehat{ACD} = m\widehat{AC} + m\widehat{CD}$

$= 137° + 83°$

$= 220°$

c. $m\widehat{ADC} = 360° - m\widehat{AC}$

$= 360° - 137°$

$= 223°$

d. $m\widehat{EBD} = 360° - m\widehat{ED}$

$= 360° - 61°$

$= 299°$

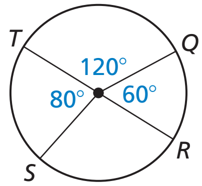
Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Identify the given arc as a *major arc*, *minor arc*, or *semicircle*. Then find the measure of the arc.

1. $\widehat{TQ}$ **2.** $\widehat{QRT}$ **3.** $\widehat{TQR}$

4. $\widehat{QS}$ **5.** $\widehat{TS}$ **6.** $\widehat{RST}$



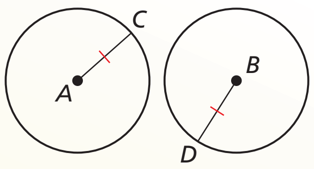
Identifying Congruent Arcs

Two circles are **congruent circles** if and only if a rigid motion or a composition of rigid motions maps one circle onto the other. This statement is equivalent to the Congruent Circles Theorem below.

Theorem

Theorem 10.3 Congruent Circles Theorem

Two circles are congruent circles if and only if they have the same radius.



Proof Ex. 35, p. 488

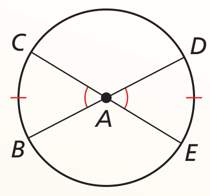
$\odot A \cong \odot B$ if and only if $\overline{AC} \cong \overline{BD}$.

Two arcs are **congruent arcs** if and only if they have the same measure and they are arcs of the same circle or of congruent circles.

Theorem

Theorem 10.4 Congruent Central Angles Theorem

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding central angles are congruent.



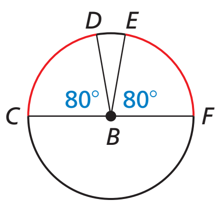
Proof Ex. 37, p. 488

$\overset{\frown}{BC} \cong \overset{\frown}{DE}$ if and only if $\angle BAC \cong \angle DAE$.

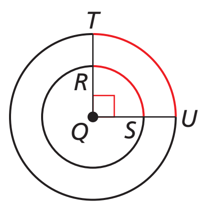
EXAMPLE 4 **Identifying Congruent Arcs**

Tell whether the red arcs are congruent. Explain why or why not.

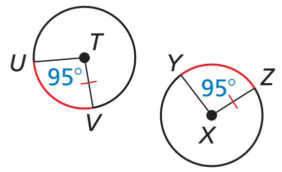
a.



b.



c.



SOLUTION

a. $\overset{\frown}{CD} \cong \overset{\frown}{EF}$ by the Congruent Central Angles Theorem because they are arcs of the same circle and they have congruent central angles, $\angle CBD \cong \angle FBE$.

b. $\overset{\frown}{RS}$ and $\overset{\frown}{TU}$ have the same measure, but are not congruent because they are arcs of circles that are not congruent.

c. $\overset{\frown}{UV} \cong \overset{\frown}{YZ}$ by the Congruent Central Angles Theorem because they are arcs of congruent circles and they have congruent central angles, $\angle UTV \cong \angle YXZ$.

STUDY TIP

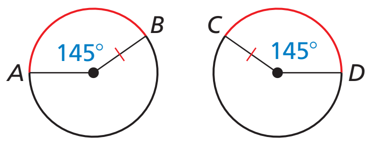
The two circles in part (c) are congruent by the Congruent Circles Theorem because they have the same radius.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Tell whether the red arcs are congruent. Explain why or why not.

7.



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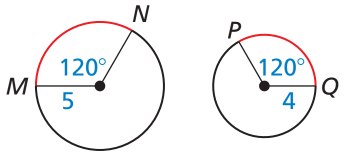


Proving Circles Are Similar

Theorem

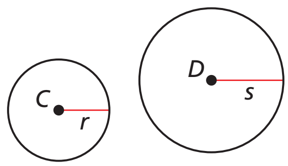
Theorem 10.5 Similar Circles Theorem

All circles are similar.

Proof p. 485; Ex. 33, p. 488

PROOF Similar Circles Theorem

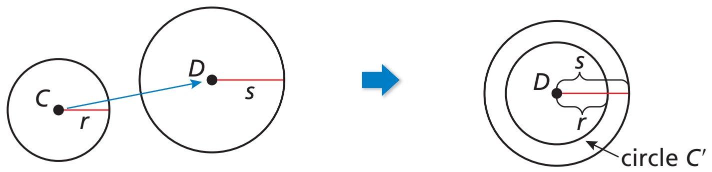
All circles are similar.



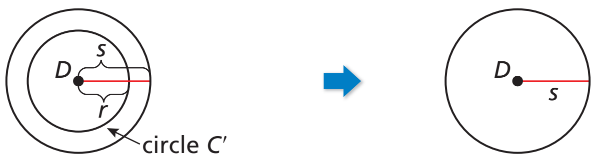
Given $\odot C$ with center C and radius r, $\odot D$ with center D and radius s

Prove $\odot C \sim \odot D$

First, translate $\odot C$ so that point C maps to point D. The image of $\odot C$ is $\odot C'$ with center D. So, $\odot C'$ and $\odot D$ are concentric circles.



$\odot C'$ is the set of all points that are r units from point D. Dilate $\odot C'$ using center of dilation D and scale factor $\frac{s}{r}$.



This dilation maps the set of all the points that are r units from point D to the set of all points that are $\frac{s}{r}(r) = s$ units from point D. $\odot D$ is the set of all points that are s units from point D. So, this dilation maps $\odot C'$ to $\odot D$.

Because a similarity transformation maps $\odot C$ to $\odot D$, $\odot C \sim \odot D$.

Two arcs are **similar arcs** if and only if they have the same measure. All congruent arcs are similar, but not all similar arcs are congruent. For instance, in Example 4, the pairs of arcs in parts (a), (b), and (c) are similar but only the pairs of arcs in parts (a) and (c) are congruent.

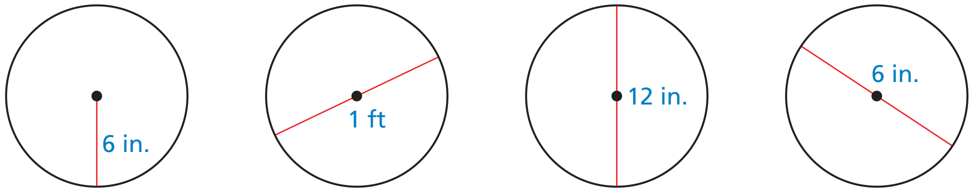
10.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** Copy and complete: If $\angle ACB$ and $\angle DCE$ are congruent central angles of $\odot C$, then $\overset{\frown}{AB}$ and $\overset{\frown}{DE}$ are ______________.

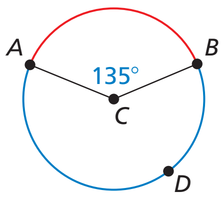
2. **WHICH ONE DOESN'T BELONG?** Which circle does *not* belong with the other three? Explain your reasoning.



Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, name the red minor arc and find its measure. Then name the blue major arc and find its measure.

3.



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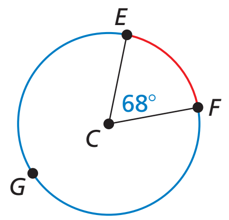

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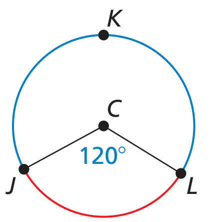

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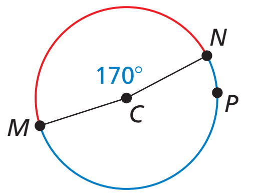
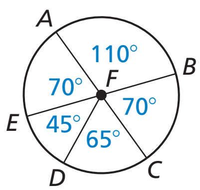


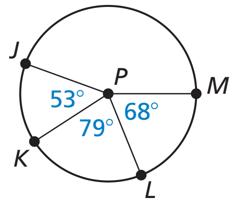
In Exercises 7–14, identify the given arc as a *major arc*, *minor arc*, or *semicircle*. Then find the measure of the arc. *(See Example 1.)*

7. $\overset{\frown}{BC}$
8. $\overset{\frown}{DC}$
9. $\overset{\frown}{ED}$
10. $\overset{\frown}{AE}$
11. $\overset{\frown}{EAB}$
12. $\overset{\frown}{ABC}$
13. $\overset{\frown}{BAC}$
14. $\overset{\frown}{EBD}$

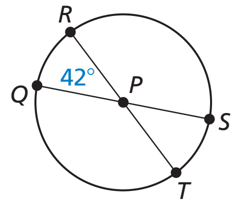


In Exercises 15 and 16, find the measure of each arc. *(See Example 2.)*

15. a. $\overset{\frown}{JL}$
    b. $\overset{\frown}{KM}$
    c. $\overset{\frown}{JLM}$
    d. $\overset{\frown}{JM}$

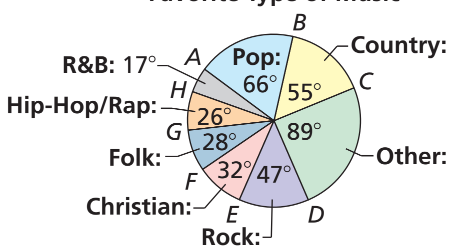


16. a. $\overset{\frown}{RS}$
    b. $\overset{\frown}{QRS}$
    c. $\overset{\frown}{QST}$
    d. $\overset{\frown}{QT}$



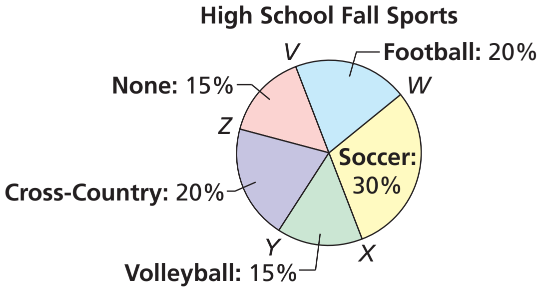
17. **MODELING WITH MATHEMATICS** A recent survey asked high school students their favorite type of music. The results are shown in the circle graph. Find each indicated arc measure. *(See Example 3.)*

Favorite Type of Music



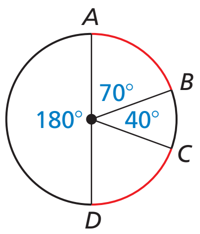
a. $m\overset{\frown}{AE}$ b. $m\overset{\frown}{ACE}$ c. $m\overset{\frown}{GDC}$
d. $m\overset{\frown}{BHC}$ e. $m\overset{\frown}{FD}$ f. $m\overset{\frown}{FBD}$

18. **ABSTRACT REASONING** The circle graph shows the percentages of students enrolled in fall sports at a high school. Is it possible to find the measure of each minor arc? If so, find the measure of the arc for each category shown. If not, explain why it is not possible.



In Exercises 19–22, tell whether the red arcs are congruent. Explain why or why not. *(See Example 4.)*

19.



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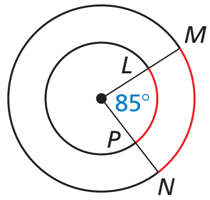

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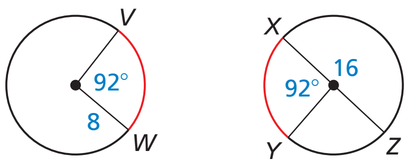

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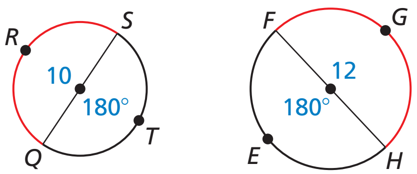
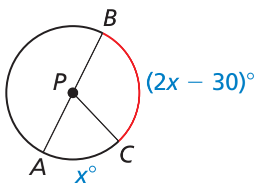
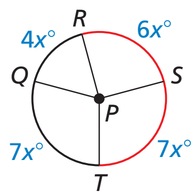


MATHEMATICAL CONNECTIONS **In Exercises 23 and 24, find the value of x. Then find the measure of the red arc.**

23.

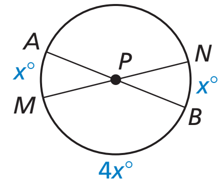


24.

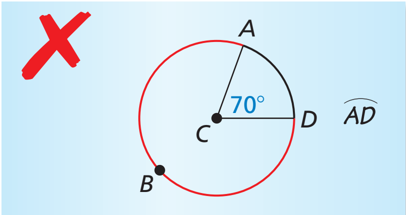


25. **MAKING AN ARGUMENT** Your friend claims that any two arcs with the same measure are similar. Your cousin claims that any two arcs with the same measure are congruent. Who is correct? Explain.

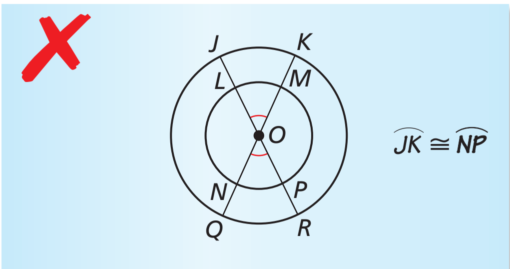
26. **MAKING AN ARGUMENT** Your friend claims that there is not enough information given to find the value of x. Is your friend correct? Explain your reasoning.



27. **ERROR ANALYSIS** Describe and correct the error in naming the red arc.



28. **ERROR ANALYSIS** Describe and correct the error in naming congruent arcs.



29. **ATTENDING TO PRECISION** Two diameters of $\odot P$ are $\overline{AB}$ and $\overline{CD}$. Find $m\overset{\frown}{ACD}$ and $m\overset{\frown}{AC}$ when $m\overset{\frown}{AD} = 20°$.

30. **REASONING** In $\odot R$, $m\overset{\frown}{AB} = 60°$, $m\overset{\frown}{BC} = 25°$, $m\overset{\frown}{CD} = 70°$, and $m\overset{\frown}{DE} = 20°$. Find two possible measures of $\overset{\frown}{AE}$.

31. **MODELING WITH MATHEMATICS** On a regulation dartboard, the outermost circle is divided into twenty congruent sections. What is the measure of each arc in this circle?

32. MODELING WITH MATHEMATICS You can use the time zone wheel to find the time in different locations across the world. For example, to find the time in Tokyo when it is 4 P.M. in San Francisco, rotate the small wheel until 4 P.M. and San Francisco line up, as shown. Then look at Tokyo to see that it is 9 A.M. there.

a. What is the arc measure between each time zone on the wheel?

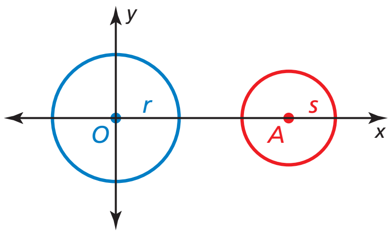
b. What is the measure of the minor arc from the Tokyo zone to the Anchorage zone?

c. If two locations differ by 180° on the wheel, then it is 3 P.M. at one location when it is _____ at the other location.

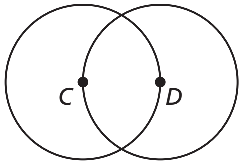
33. PROVING A THEOREM Write a coordinate proof of the Similar Circles Theorem (Theorem 10.5).

Given $\odot O$ with center $O(0, 0)$ and radius r, $\odot A$ with center $A(a, 0)$ and radius s

Prove $\odot O \sim \odot A$



34. ABSTRACT REASONING Is there enough information to tell whether $\odot C \cong \odot D$? Explain your reasoning.

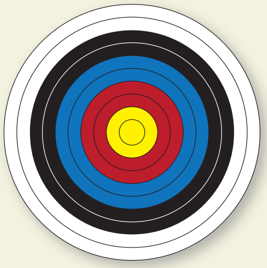


35. PROVING A THEOREM Use the diagram on page 484 to prove each part of the biconditional in the Congruent Circles Theorem (Theorem 10.3).

a. **Given** $\overline{AC} \cong \overline{BD}$
Prove $\odot A \cong \odot B$

b. **Given** $\odot A \cong \odot B$
Prove $\overline{AC} \cong \overline{BD}$

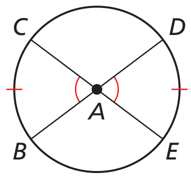
36. HOW DO YOU SEE IT? Are the circles on the target *similar* or *congruent*? Explain your reasoning.



37. PROVING A THEOREM Use the diagram to prove each part of the biconditional in the Congruent Central Angles Theorem (Theorem 10.4).

a. **Given** $\angle BAC \cong \angle DAE$
Prove $\overset{\frown}{BC} \cong \overset{\frown}{DE}$

b. **Given** $\overset{\frown}{BC} \cong \overset{\frown}{DE}$
Prove $\angle BAC \cong \angle DAE$



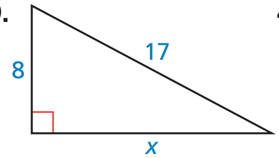
38. THOUGHT PROVOKING Write a formula for the length of a circular arc. Justify your answer.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the value of x. Tell whether the side lengths form a Pythagorean triple.

39.



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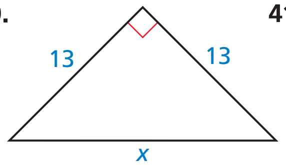

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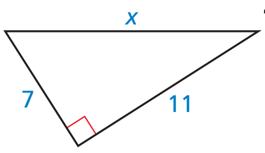

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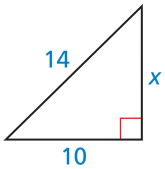


10.3 Using Chords

Essential Question What are two ways to determine when a chord is a diameter of a circle?

EXPLORATION 1 Drawing Diameters

Work with a partner. Use dynamic geometry software to construct a circle of radius 5 with center at the origin. Draw a diameter that has the given point as an endpoint. Explain how you know that the chord you drew is a diameter.

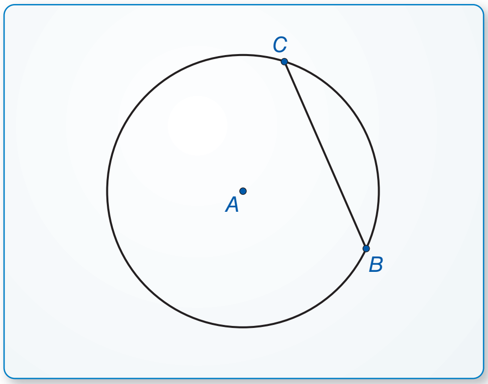
a. (4, 3) **b.** (0, 5) **c.** (−3, 4) **d.** (−5, 0)

LOOKING FOR STRUCTURE

To be proficient in math, you need to look closely to discern a pattern or structure.

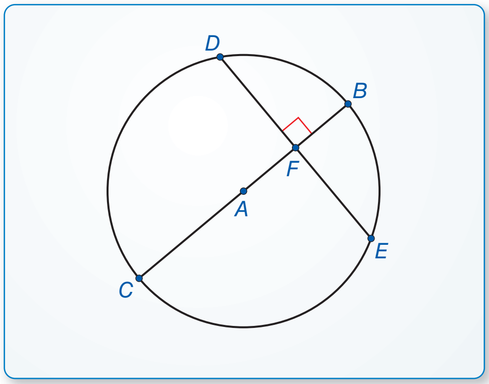
EXPLORATION 2 Writing a Conjecture about Chords

Work with a partner. Use dynamic geometry software to construct a chord $\overline{BC}$ of a circle A. Construct a chord on the perpendicular bisector of $\overline{BC}$. What do you notice? Change the original chord and the circle several times. Are your results always the same? Use your results to write a conjecture.



EXPLORATION 3 A Chord Perpendicular to a Diameter

Work with a partner. Use dynamic geometry software to construct a diameter $\overline{BC}$ of a circle A. Then construct a chord $\overline{DE}$ perpendicular to $\overline{BC}$ at point F. Find the lengths DF and EF. What do you notice? Change the chord perpendicular to $\overline{BC}$ and the circle several times. Do you always get the same results? Write a conjecture about a chord that is perpendicular to a diameter of a circle.



Communicate Your Answer

4. What are two ways to determine when a chord is a diameter of a circle?

10.3 Lesson

What You Will Learn

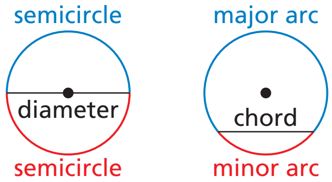
- Use chords of circles to find lengths and arc measures.

Core Vocabulary

Previous
chord
arc
diameter

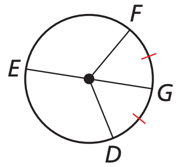
Using Chords of Circles

Recall that a *chord* is a segment with endpoints on a circle. Because its endpoints lie on the circle, any chord divides the circle into two arcs. A diameter divides a circle into two semicircles. Any other chord divides a circle into a minor arc and a major arc.



READING

If $\widehat{GD} \cong \widehat{GF}$, then the point G, and any line, segment, or ray that contains G, bisects $\widehat{FD}$.

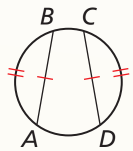


$\overline{EG}$ bisects $\widehat{FD}$.

Theorems

Theorem 10.6 Congruent Corresponding Chords Theorem

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

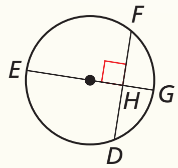


Proof Ex. 19, p. 494

$\widehat{AB} \cong \widehat{CD}$ if and only if $\overline{AB} \cong \overline{CD}$.

Theorem 10.7 Perpendicular Chord Bisector Theorem

If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

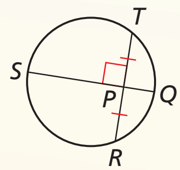


If $\overline{EG}$ is a diameter and $\overline{EG} \perp \overline{DF}$, then $\overline{HD} \cong \overline{HF}$ and $\widehat{GD} \cong \widehat{GF}$.

Proof Ex. 22, p. 494

Theorem 10.8 Perpendicular Chord Bisector Converse

If one chord of a circle is a perpendicular bisector of another chord, then the first chord is a diameter.

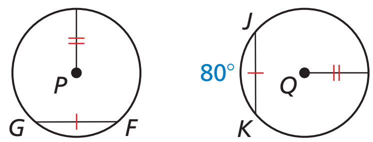


If $\overline{QS}$ is a perpendicular bisector of $\overline{TR}$, then $\overline{QS}$ is a diameter of the circle.

Proof Ex. 23, p. 494

EXAMPLE 1 Using Congruent Chords to Find an Arc Measure

In the diagram, $\odot P \cong \odot Q$, $\overline{FG} \cong \overline{JK}$, and $m\widehat{JK} = 80°$. Find $m\widehat{FG}$.

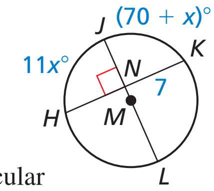


SOLUTION

Because $\overline{FG}$ and $\overline{JK}$ are congruent chords in congruent circles, the corresponding minor arcs $\widehat{FG}$ and $\widehat{JK}$ are congruent by the Congruent Corresponding Chords Theorem.

- So, $m\widehat{FG} = m\widehat{JK} = 80°$.

EXAMPLE 2 Using a Diameter



a. Find HK. **b.** Find $m\widehat{HK}$.

SOLUTION

a. Diameter $\overline{JL}$ is perpendicular to $\overline{HK}$. So, by the Perpendicular Chord Bisector Theorem, $\overline{JL}$ bisects $\overline{HK}$, and $HN = NK$.

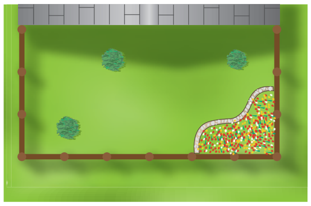
▶ So, $HK = 2(NK) = 2(7) = 14$.

b. Diameter $\overline{JL}$ is perpendicular to $\overline{HK}$. So, by the Perpendicular Chord Bisector Theorem, $\overline{JL}$ bisects $\widehat{HK}$, and $m\widehat{HJ} = m\widehat{JK}$.

$m\widehat{HJ} = m\widehat{JK}$	Perpendicular Chord Bisector Theorem
$11x° = (70 + x)°$	Substitute.
$10x = 70$	Subtract x from each side.
$x = 7$	Divide each side by 10.

▶ So, $m\widehat{HJ} = m\widehat{JK} = (70 + x)° = (70 + 7)° = 77°$, and $m\widehat{HK} = 2(m\widehat{HJ}) = 2(77°) = 154°$.

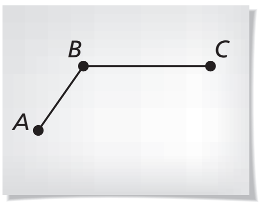
EXAMPLE 3 Using Perpendicular Bisectors



Three bushes are arranged in a garden, as shown. Where should you place a sprinkler so that it is the same distance from each bush?

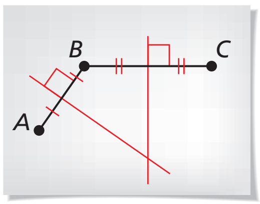
SOLUTION

Step 1



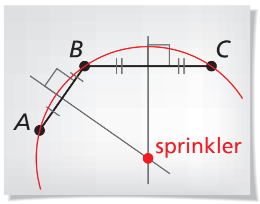
Label the bushes A, B, and C, as shown. Draw segments $\overline{AB}$ and $\overline{BC}$.

Step 2



Draw the perpendicular bisectors of $\overline{AB}$ and $\overline{BC}$. By the Perpendicular Chord Bisector Converse, these lie on diameters of the circle containing A, B, and C.

Step 3



Find the point where the perpendicular bisectors intersect. This is the center of the circle, which is equidistant from points A, B, and C.

Monitoring Progress

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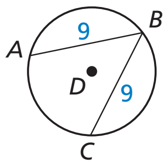
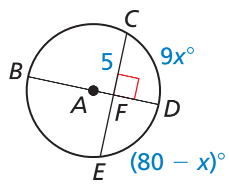
In Exercises 1 and 2, use the diagram of ⊙D.



1. If $m\widehat{AB} = 110°$, find $m\widehat{BC}$.

2. If $m\widehat{AC} = 150°$, find $m\widehat{AB}$.

In Exercises 3 and 4, find the indicated length or arc measure.



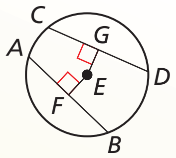
3. CE

4. $m\widehat{CE}$

Theorem

Theorem 10.9 Equidistant Chords Theorem

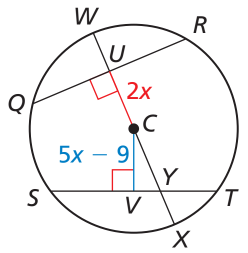
In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.



Proof Ex. 25, p. 494

$\overline{AB} \cong \overline{CD}$ if and only if $EF = EG$.

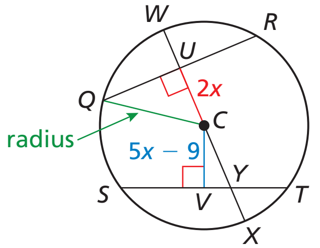
EXAMPLE 4 Using Congruent Chords to Find a Circle's Radius



In the diagram, $QR = ST = 16$, $CU = 2x$, and $CV = 5x - 9$. Find the radius of $\odot C$.

SOLUTION

Because $\overline{CQ}$ is a segment whose endpoints are the center and a point on the circle, it is a radius of $\odot C$. Because $\overline{CU} \perp \overline{QR}$, $\triangle QUC$ is a right triangle. Apply properties of chords to find the lengths of the legs of $\triangle QUC$.



Step 1 Find CU.

Because $\overline{QR}$ and $\overline{ST}$ are congruent chords, $\overline{QR}$ and $\overline{ST}$ are equidistant from C by the Equidistant Chords Theorem. So, $CU = CV$.

$CU = CV$	Equidistant Chords Theorem
$2x = 5x - 9$	Substitute.
$x = 3$	Solve for x.

So, $CU = 2x = 2(3) = 6$.

Step 2 Find QU.

Because diameter $\overline{WX} \perp \overline{QR}$, $\overline{WX}$ bisects $\overline{QR}$ by the Perpendicular Chord Bisector Theorem.

So, $QU = \frac{1}{2}(16) = 8$.

Step 3 Find CQ.

Because the lengths of the legs are $CU = 6$ and $QU = 8$, $\triangle QUC$ is a right triangle with the Pythagorean triple 6, 8, 10. So, $CQ = 10$.

▶ So, the radius of $\odot C$ is 10 units.

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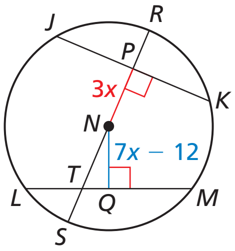


5. In the diagram, $JK = LM = 24$, $NP = 3x$, and $NQ = 7x - 12$. Find the radius of $\odot N$.

10.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

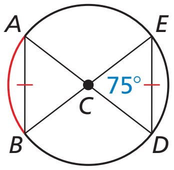
Vocabulary and Core Concept Check

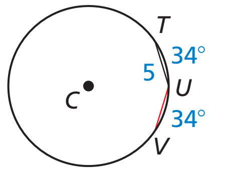
1. **WRITING** Describe what it means to bisect a chord.

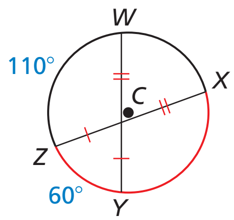
2. **WRITING** Two chords of a circle are perpendicular and congruent. Does one of them have to be a diameter? Explain your reasoning.

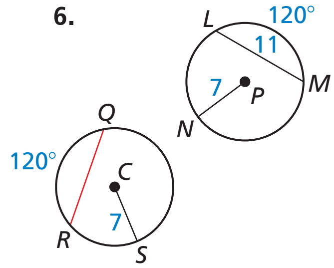
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the measure of the red arc or chord in ⊙*C*. *(See Example 1.)*

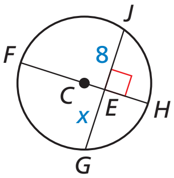
3. 

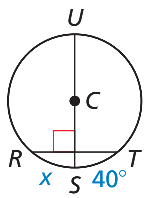
4. 

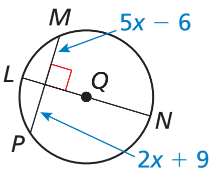
5. 

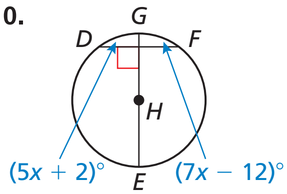
6. 

In Exercises 7–10, find the value of *x*. *(See Example 2.)*

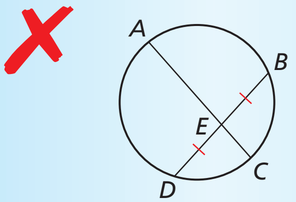
7. 

8. 

9. 

10. 

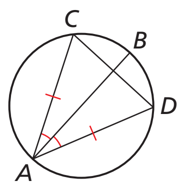
11. **ERROR ANALYSIS** Describe and correct the error in reasoning.

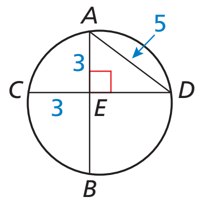


Because $\overline{AC}$ bisects $\overline{DB}$, $\widehat{BC} \cong \widehat{CD}$.

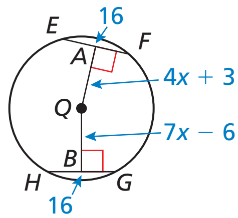
12. **PROBLEM SOLVING** In the cross section of the submarine shown, the control panels are parallel and the same length. Describe a method you can use to find the center of the cross section. Justify your method. *(See Example 3.)*

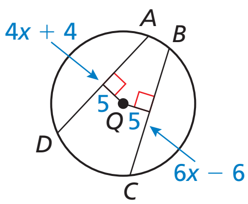
In Exercises 13 and 14, determine whether $\overline{AB}$ is a diameter of the circle. Explain your reasoning.

13. 

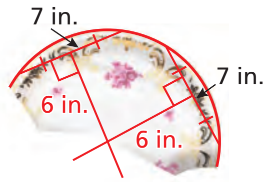
14. 

In Exercises 15 and 16, find the radius of ⊙*Q*. *(See Example 4.)*

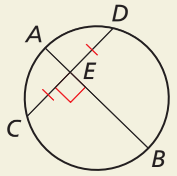
15. 

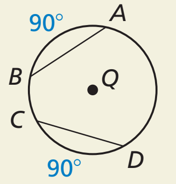
16. 

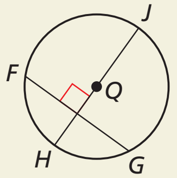
17. **PROBLEM SOLVING** An archaeologist finds part of a circular plate. What was the diameter of the plate to the nearest tenth of an inch? Justify your answer.

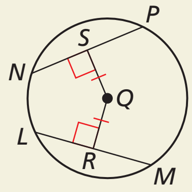


18. HOW DO YOU SEE IT? What can you conclude from each diagram? Name a theorem that justifies your answer.

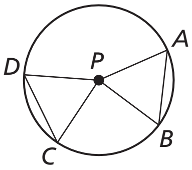
a.


b.


c.


d.


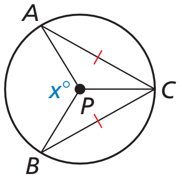
19. PROVING A THEOREM Use the diagram to prove each part of the biconditional in the Congruent Corresponding Chords Theorem (Theorem 10.6).



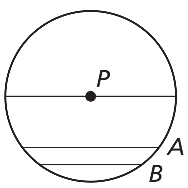
a. Given $\overline{AB}$ and $\overline{CD}$ are congruent chords.
Prove $\widehat{AB} \cong \widehat{CD}$

b. Given $\widehat{AB} \cong \widehat{CD}$
Prove $\overline{AB} \cong \overline{CD}$

20. MATHEMATICAL CONNECTIONS In $\odot P$, all the arcs shown have integer measures. Show that x must be even.



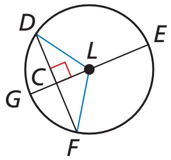
21. REASONING In $\odot P$, the lengths of the parallel chords are 20, 16, and 12. Find $m\widehat{AB}$. Explain your reasoning.



22. PROVING A THEOREM Use congruent triangles to prove the Perpendicular Chord Bisector Theorem (Theorem 10.7).

Given $\overline{EG}$ is a diameter of $\odot L$.
$\overline{EG} \perp \overline{DF}$

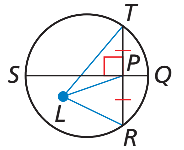
Prove $\overline{DC} \cong \overline{FC}$, $\widehat{DG} \cong \widehat{FG}$



23. PROVING A THEOREM Write a proof of the Perpendicular Chord Bisector Converse (Theorem 10.8).

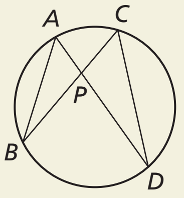
Given $\overline{QS}$ is a perpendicular bisector of $\overline{RT}$.

Prove $\overline{QS}$ is a diameter of the circle L.



(*Hint*: Plot the center L and draw $\triangle LPT$ and $\triangle LPR$.)

24. THOUGHT PROVOKING Consider two chords that intersect at point P. Do you think that $\frac{AP}{BP} = \frac{CP}{DP}$? Justify your answer.



25. PROVING A THEOREM Use the diagram with the Equidistant Chords Theorem (Theorem 10.9) on page 492 to prove both parts of the biconditional of this theorem.

26. MAKING AN ARGUMENT A car is designed so that the rear wheel is only partially visible below the body of the car. The bottom edge of the panel is parallel to the ground. Your friend claims that the point where the tire touches the ground bisects $\widehat{AB}$. Is your friend correct? Explain your reasoning.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the missing interior angle measure.

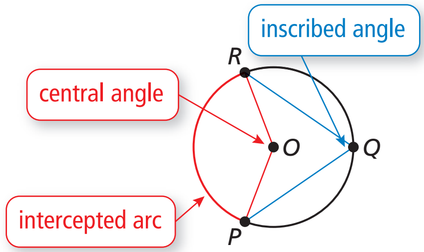
27. Quadrilateral $JKLM$ has angle measures $m\angle J = 32°$, $m\angle K = 25°$, and $m\angle L = 44°$. Find $m\angle M$.

28. Pentagon $PQRST$ has angle measures $m\angle P = 85°$, $m\angle Q = 134°$, $m\angle R = 97°$, and $m\angle S = 102°$. Find $m\angle T$.

10.4 Inscribed Angles and Polygons

Essential Question How are inscribed angles related to their intercepted arcs? How are the angles of an inscribed quadrilateral related to each other?

An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. An arc that lies between two lines, rays, or segments is called an **intercepted arc**. A polygon is an **inscribed polygon** when all its vertices lie on a circle.



EXPLORATION 1 Inscribed Angles and Central Angles

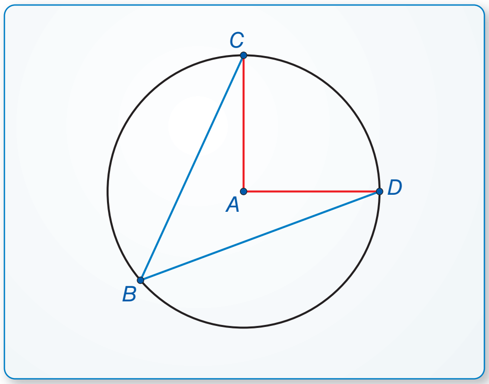
Work with a partner. Use dynamic geometry software.

a. Construct an inscribed angle in a circle. Then construct the corresponding central angle.

b. Measure both angles. How is the inscribed angle related to its intercepted arc?

c. Repeat parts (a) and (b) several times. Record your results in a table. Write a conjecture about how an inscribed angle is related to its intercepted arc.

Sample



ATTENDING TO PRECISION

To be proficient in math, you need to communicate precisely with others.

EXPLORATION 2 A Quadrilateral with Inscribed Angles

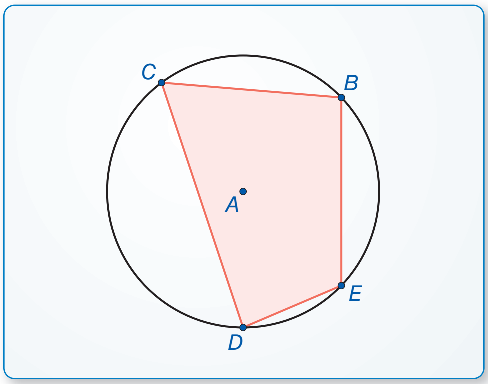
Work with a partner. Use dynamic geometry software.

a. Construct a quadrilateral with each vertex on a circle.

b. Measure all four angles. What relationships do you notice?

c. Repeat parts (a) and (b) several times. Record your results in a table. Then write a conjecture that summarizes the data.

Sample



Communicate Your Answer

3. How are inscribed angles related to their intercepted arcs? How are the angles of an inscribed quadrilateral related to each other?

4. Quadrilateral $EFGH$ is inscribed in $\odot C$, and $m\angle E = 80°$. What is $m\angle G$? Explain.

10.4 Lesson

Core Vocabulary

inscribed angle, *p. 496*
intercepted arc, *p. 496*
subtend, *p. 496*
inscribed polygon, *p. 498*
circumscribed circle, *p. 498*

What You Will Learn

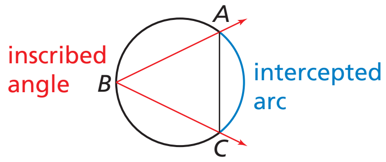
- Use inscribed angles.
- Use inscribed polygons.

Using Inscribed Angles

Core Concept

Inscribed Angle and Intercepted Arc

An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. An arc that lies between two lines, rays, or segments is called an **intercepted arc**. If the endpoints of a chord or arc lie on the sides of an inscribed angle, then the chord or arc is said to **subtend** the angle.

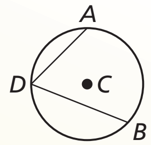


$\angle B$ intercepts $\overset{\frown}{AC}$.
$\overset{\frown}{AC}$ subtends $\angle B$.
$\overline{AC}$ subtends $\angle B$.

Theorem

Theorem 10.10 Measure of an Inscribed Angle Theorem

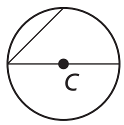
The measure of an inscribed angle is one-half the measure of its intercepted arc.



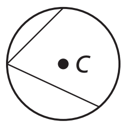
$m\angle ADB = \frac{1}{2} m\overset{\frown}{AB}$

Proof Ex. 37, p. 502

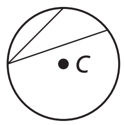
The proof of the Measure of an Inscribed Angle Theorem involves three cases.



Case 1 Center C is on a side of the inscribed angle.



Case 2 Center C is inside the inscribed angle.



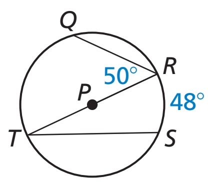
Case 3 Center C is outside the inscribed angle.

EXAMPLE 1 Using Inscribed Angles

Find the indicated measure.

a. $m\angle T$

b. $m\overset{\frown}{QR}$



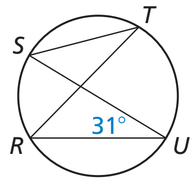
SOLUTION

a. $m\angle T = \frac{1}{2} m\overset{\frown}{RS} = \frac{1}{2}(48^\circ) = 24^\circ$

b. $m\overset{\frown}{TQ} = 2m\angle R = 2 \cdot 50^\circ = 100^\circ$
Because $\overset{\frown}{TQR}$ is a semicircle, $m\overset{\frown}{QR} = 180^\circ - m\overset{\frown}{TQ} = 180^\circ - 100^\circ = 80^\circ$.

EXAMPLE 2 Finding the Measure of an Intercepted Arc

Find $m\overset{\frown}{RS}$ and $m\angle STR$. What do you notice about $\angle STR$ and $\angle RUS$?



SOLUTION

From the Measure of an Inscribed Angle Theorem, you know that $m\overset{\frown}{RS} = 2m\angle RUS = 2(31°) = 62°$.

Also, $m\angle STR = \frac{1}{2}m\overset{\frown}{RS} = \frac{1}{2}(62°) = 31°$.

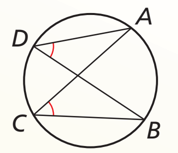
▶ So, $\angle STR \cong \angle RUS$.

Example 2 suggests the Inscribed Angles of a Circle Theorem.

Theorem

Theorem 10.11 Inscribed Angles of a Circle Theorem

If two inscribed angles of a circle intercept the same arc, then the angles are congruent.

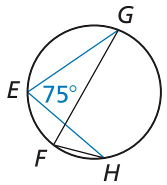


$\angle ADB \cong \angle ACB$

Proof Ex. 38, p. 502

EXAMPLE 3 Finding the Measure of an Angle

Given $m\angle E = 75°$, find $m\angle F$.



SOLUTION

Both $\angle E$ and $\angle F$ intercept $\overset{\frown}{GH}$. So, $\angle E \cong \angle F$ by the Inscribed Angles of a Circle Theorem.

▶ So, $m\angle F = m\angle E = 75°$.

Monitoring Progress

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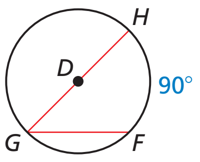
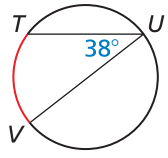
Find the measure of the red arc or angle.

1.



2.



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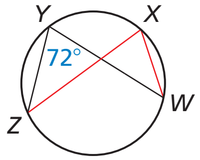
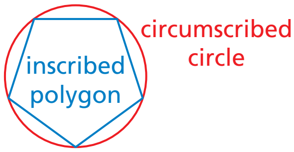


Using Inscribed Polygons

Core Concept

Inscribed Polygon

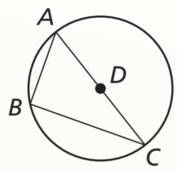
A polygon is an **inscribed polygon** when all its vertices lie on a circle. The circle that contains the vertices is a **circumscribed circle**.



Theorems

Theorem 10.12 Inscribed Right Triangle Theorem

If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.

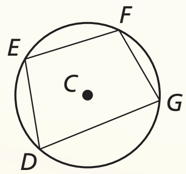


$m\angle ABC = 90°$ if and only if $\overline{AC}$ is a diameter of the circle.

Proof Ex. 39, p. 502

Theorem 10.13 Inscribed Quadrilateral Theorem

A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.



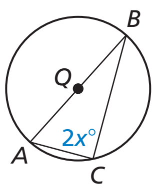
D, E, F, and G lie on $\odot C$ if and only if $m\angle D + m\angle F = m\angle E + m\angle G = 180°$.

Proof Ex. 40, p. 502; *BigIdeasMath.com*

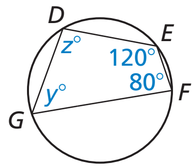
EXAMPLE 4 Using Inscribed Polygons

Find the value of each variable.

a.



b.



SOLUTION

a. $\overline{AB}$ is a diameter. So, $\angle C$ is a right angle, and $m\angle C = 90°$ by the Inscribed Right Triangle Theorem.

$$2x° = 90°$$

$$x = 45$$

▶ The value of x is 45.

b. *DEFG* is inscribed in a circle, so opposite angles are supplementary by the Inscribed Quadrilateral Theorem.

$$m\angle D + m\angle F = 180° \qquad m\angle E + m\angle G = 180°$$

$$z + 80 = 180 \qquad 120 + y = 180$$

$$z = 100 \qquad y = 60$$

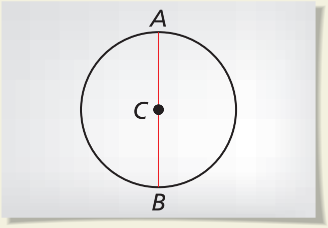
▶ The value of z is 100 and the value of y is 60.

CONSTRUCTION Constructing a Square Inscribed in a Circle

Given $\odot C$, construct a square inscribed in a circle.

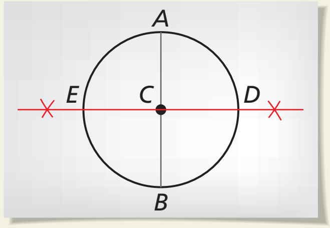
SOLUTION

Step 1



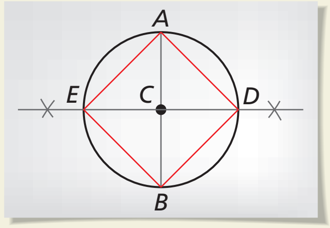
Draw a diameter
Draw any diameter. Label the endpoints *A* and *B*.

Step 2



Construct a perpendicular bisector
Construct the perpendicular bisector of the diameter. Label the points where it intersects $\odot C$ as points *D* and *E*.

Step 3



Form a square
Connect points *A*, *D*, *B*, and *E* to form a square.

EXAMPLE 5 Using a Circumscribed Circle

Your camera has a 90° field of vision, and you want to photograph the front of a statue. You stand at a location in which the front of the statue is all that appears in your camera's field of vision, as shown. You want to change your location. Where else can you stand so that the front of the statue is all that appears in your camera's field of vision?

SOLUTION

From the Inscribed Right Triangle Theorem, you know that if a right triangle is inscribed in a circle, then the hypotenuse of the triangle is a diameter of the circle. So, draw the circle that has the front of the statue as a diameter.

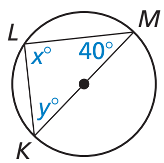
▶ The statue fits perfectly within your camera's 90° field of vision from any point on the semicircle in front of the statue.

Monitoring Progress

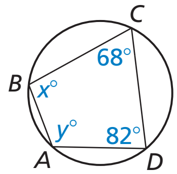
Help in English and Spanish at *BigIdeasMath.com*

Find the value of each variable.

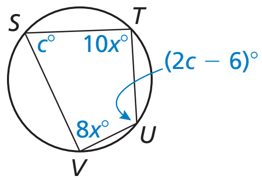
4.



5.



6.



7. Construct an equilateral triangle. Then, construct the circumscribed circle of that triangle. Repeat this process for a regular hexagon.

10.4 Exercises

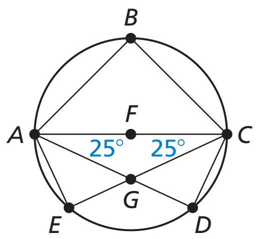
Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** If a circle is circumscribed about a polygon, then the polygon is an ________________.

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

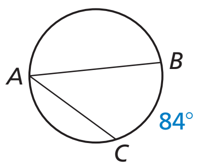
Find $m\angle ABC$.	Find $m\angle AGC$.
Find $m\angle AEC$.	Find $m\angle ADC$.



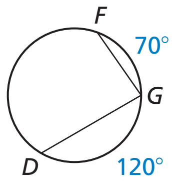
Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, find the indicated measure. *(See Examples 1 and 2.)*

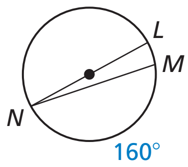
3. $m\angle A$



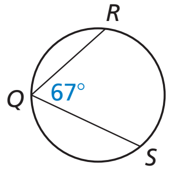
4. $m\angle G$



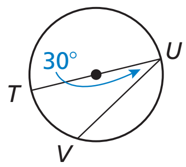
5. $m\angle N$



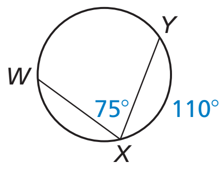
6. $m\widehat{RS}$



7. $m\widehat{VU}$

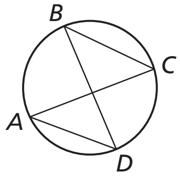


8. $m\widehat{WX}$

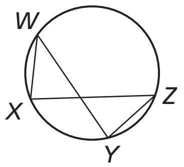


In Exercises 9 and 10, name two pairs of congruent angles.

9.

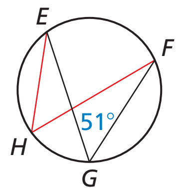


10.



In Exercises 11 and 12, find the measure of the red arc or angle. *(See Example 3.)*

11.



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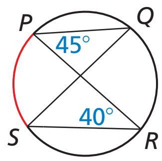
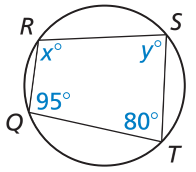


In Exercises 13–16, find the value of each variable. *(See Example 4.)*

13.



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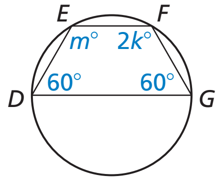
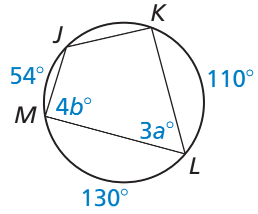
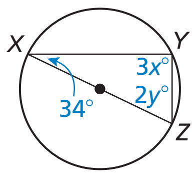


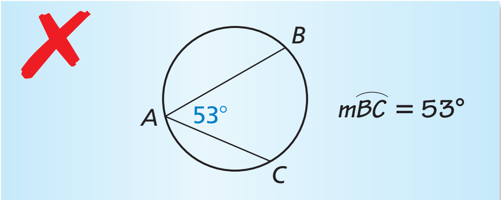
15.



16.



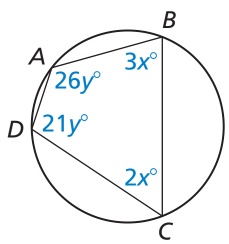
17. **ERROR ANALYSIS** Describe and correct the error in finding $m\widehat{BC}$.



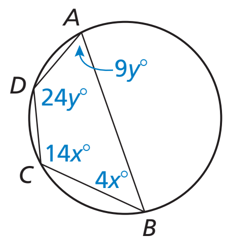
18. **MODELING WITH MATHEMATICS** A *carpenter's square* is an L-shaped tool used to draw right angles. You need to cut a circular piece of wood into two semicircles. How can you use the carpenter's square to draw a diameter on the circular piece of wood? *(See Example 5.)*

MATHEMATICAL CONNECTIONS **In Exercises 19–21, find the values of x and y. Then find the measures of the interior angles of the polygon.**

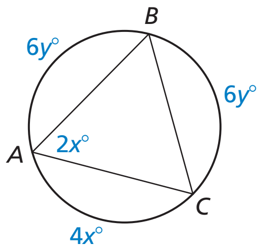
19.



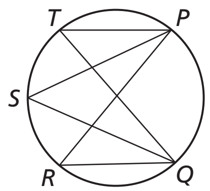
20.



21.



22. **MAKING AN ARGUMENT** Your friend claims that $\angle PTQ \cong \angle PSQ \cong \angle PRQ$. Is your friend correct? Explain your reasoning.



23. **CONSTRUCTION** The side length of an inscribed regular hexagon is equal to the radius of the circumscribed circle. Construct a circle. Then, construct the inscribed regular hexagon.

24. Explain how you can construct an inscribed equilateral triangle in a given circle using your answer to Exercise 23.

REASONING **In Exercises 25–30, determine whether a quadrilateral of the given type can always be inscribed inside a circle. Explain your reasoning.**

25. square

26. rectangle

27. parallelogram

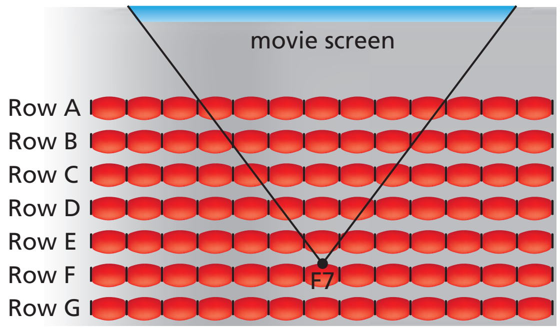
28. kite

29. rhombus

30. isosceles trapezoid

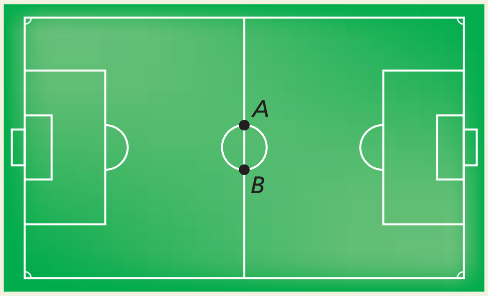
31. **MODELING WITH MATHEMATICS** Three moons, A, B, and C, are in the same circular orbit 100,000 kilometers above the surface of a planet. The planet is 20,000 kilometers in diameter and $m\angle ABC = 90°$. Draw a diagram of the situation. How far is moon A from moon C?

32. **MODELING WITH MATHEMATICS** At the movie theater, you want to choose a seat that has the best *viewing angle*, so that you can be close to the screen and still see the whole screen without moving your eyes. You previously decided that seat F7 has the best viewing angle, but this time someone else is already sitting there. Where else can you sit so that your seat has the same viewing angle as seat F7? Explain.



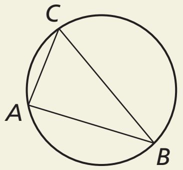
33. **WRITING** A right triangle is inscribed in a circle, and the radius of the circle is given. Explain how to find the length of the hypotenuse.

34. **HOW DO YOU SEE IT?** Let point Y represent your location on the soccer field below. What type of angle is $\angle AYB$ if you stand anywhere on the circle except at point A or point B?



35. WRITING Explain why the diagonals of a rectangle inscribed in a circle are diameters of the circle.

36. THOUGHT PROVOKING The figure shows a circle that is circumscribed about $\triangle ABC$. Is it possible to circumscribe a circle about any triangle? Justify your answer.

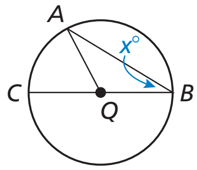


37. PROVING A THEOREM If an angle is inscribed in $\odot Q$, the center Q can be on a side of the inscribed angle, inside the inscribed angle, or outside the inscribed angle. Prove each case of the Measure of an Inscribed Angle Theorem (Theorem 10.10).

a. Case 1

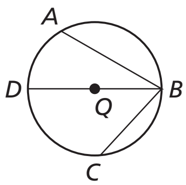
Given $\angle ABC$ is inscribed in $\odot Q$.
Let $m\angle B = x°$.
Center Q lies on $\overline{BC}$.

Prove $m\angle ABC = \frac{1}{2} m\overset{\frown}{AC}$

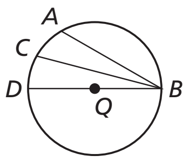


(*Hint*: Show that $\triangle AQB$ is isosceles. Then write $m\overset{\frown}{AC}$ in terms of x.)

b. Case 2 Use the diagram and auxiliary line to write **Given** and **Prove** statements for Case 2. Then write a proof.



c. Case 3 Use the diagram and auxiliary line to write **Given** and **Prove** statements for Case 3. Then write a proof.



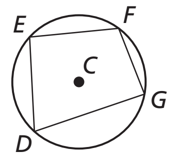
38. PROVING A THEOREM Write a paragraph proof of the Inscribed Angles of a Circle Theorem (Theorem 10.11). First, draw a diagram and write **Given** and **Prove** statements.

39. PROVING A THEOREM The Inscribed Right Triangle Theorem (Theorem 10.12) is written as a conditional statement and its converse. Write a plan for proof for each statement.

40. PROVING A THEOREM Copy and complete the paragraph proof for one part of the Inscribed Quadrilateral Theorem (Theorem 10.13).

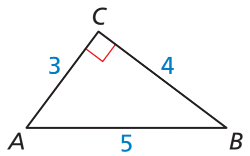
Given $\odot C$ with inscribed quadrilateral $DEFG$

Prove $m\angle D + m\angle F = 180°$,
$m\angle E + m\angle G = 180°$

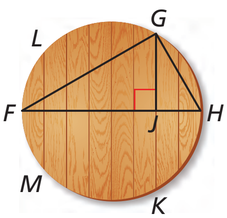


By the Arc Addition Postulate (Postulate 10.1), $m\overset{\frown}{EFG}$ + ____ = 360° and $m\overset{\frown}{FGD} + m\overset{\frown}{DEF} = 360°$. Using the ________ Theorem, $m\overset{\frown}{EDG} = 2m\angle F$, $m\overset{\frown}{EFG} = 2m\angle D$, $m\overset{\frown}{DEF} = 2m\angle G$, and $m\overset{\frown}{FGD} = 2m\angle E$. By the Substitution Property of Equality, $2m\angle D$ + ___ = 360°, so ___. Similarly, ___.

41. CRITICAL THINKING In the diagram, $\angle C$ is a right angle. If you draw the smallest possible circle through C tangent to $\overline{AB}$, the circle will intersect $\overline{AC}$ at J and $\overline{BC}$ at K. Find the exact length of $\overline{JK}$.



42. CRITICAL THINKING You are making a circular cutting board. To begin, you glue eight 1-inch boards together, as shown. Then you draw and cut a circle with an 8-inch diameter from the boards.



a. $\overline{FH}$ is a diameter of the circular cutting board. Write a proportion relating GJ and JH. State a theorem to justify your answer.

b. Find FJ, JH, and GJ. What is the length of the cutting board seam labeled $\overline{GK}$?

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. Check your solution.

43. $3x = 145$

44. $\frac{1}{2}x = 63$

45. $240 = 2x$

46. $75 = \frac{1}{2}(x - 30)$

10.5 Angle Relationships in Circles

Essential Question When a chord intersects a tangent line or another chord, what relationships exist among the angles and arcs formed?

EXPLORATION 1 Angles Formed by a Chord and Tangent Line

Work with a partner. Use dynamic geometry software.

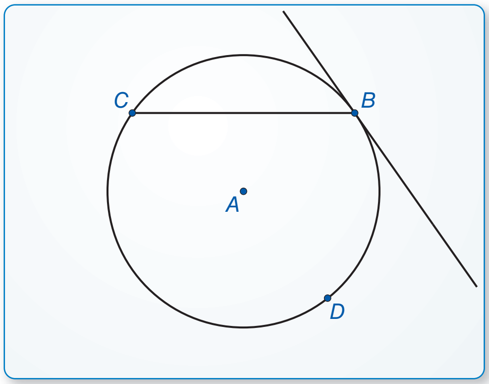
a. Construct a chord in a circle. At one of the endpoints of the chord, construct a tangent line to the circle.

b. Find the measures of the two angles formed by the chord and the tangent line.

c. Find the measures of the two circular arcs determined by the chord.

d. Repeat parts (a)–(c) several times. Record your results in a table. Then write a conjecture that summarizes the data.

Sample



EXPLORATION 2 Angles Formed by Intersecting Chords

Work with a partner. Use dynamic geometry software.

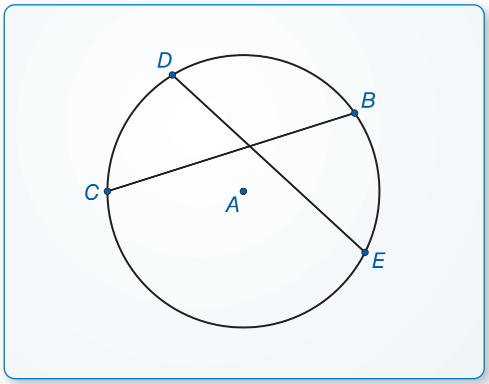
a. Construct two chords that intersect inside a circle.

b. Find the measure of one of the angles formed by the intersecting chords.

c. Find the measures of the arcs intercepted by the angle in part (b) and its vertical angle. What do you observe?

d. Repeat parts (a)–(c) several times. Record your results in a table. Then write a conjecture that summarizes the data.

Sample



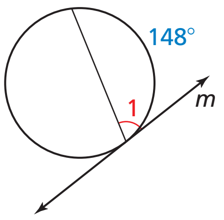
CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to understand and use stated assumptions, definitions, and previously established results.

Communicate Your Answer

3. When a chord intersects a tangent line or another chord, what relationships exist among the angles and arcs formed?

4. Line m is tangent to the circle in the figure at the left. Find the measure of $\angle 1$.



5. Two chords intersect inside a circle to form a pair of vertical angles with measures of 55°. Find the sum of the measures of the arcs intercepted by the two angles.

10.5 Lesson

Core Vocabulary

circumscribed angle, *p. 506*

Previous
tangent
chord
secant

What You Will Learn

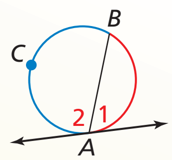
- Find angle and arc measures.
- Use circumscribed angles.

Finding Angle and Arc Measures

Theorem

Theorem 10.14 Tangent and Intersected Chord Theorem

If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one-half the measure of its intercepted arc.



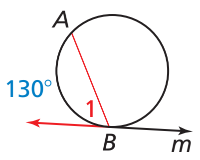
Proof Ex. 33, p. 510

$m\angle 1 = \frac{1}{2}m\overset{\frown}{AB}$ $\quad m\angle 2 = \frac{1}{2}m\overset{\frown}{BCA}$

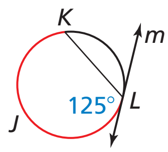
EXAMPLE 1 Finding Angle and Arc Measures

Line *m* is tangent to the circle. Find the measure of the red angle or arc.

a.



b.



SOLUTION

a. $m\angle 1 = \frac{1}{2}(130°) = 65°$

b. $m\overset{\frown}{KJL} = 2(125°) = 250°$

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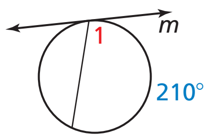
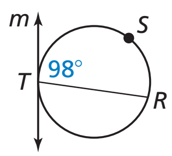
Line *m* is tangent to the circle. Find the indicated measure.

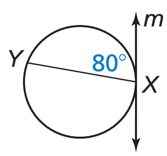
1. $m\angle 1$



2. $m\overset{\frown}{RST}$



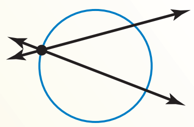
3. $m\overset{\frown}{XY}$



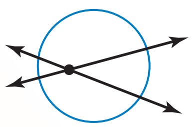
Core Concept

Intersecting Lines and Circles

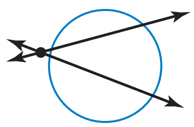
If two nonparallel lines intersect a circle, there are three places where the lines can intersect.



on the circle



inside the circle

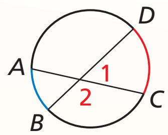


outside the circle

Theorems

Theorem 10.15 Angles Inside the Circle Theorem

If two chords intersect *inside* a circle, then the measure of each angle is one-half the *sum* of the measures of the arcs intercepted by the angle and its vertical angle.



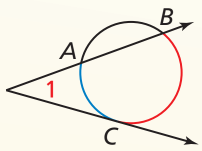
$m\angle 1 = \frac{1}{2}(m\widehat{DC} + m\widehat{AB})$,

$m\angle 2 = \frac{1}{2}(m\widehat{AD} + m\widehat{BC})$

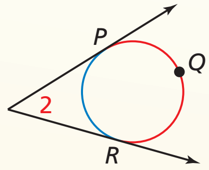
Proof Ex. 35, p. 510

Theorem 10.16 Angles Outside the Circle Theorem

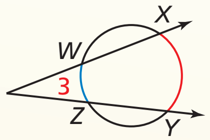
If a tangent and a secant, two tangents, or two secants intersect *outside* a circle, then the measure of the angle formed is one-half the *difference* of the measures of the intercepted arcs.



$m\angle 1 = \frac{1}{2}(m\widehat{BC} - m\widehat{AC})$



$m\angle 2 = \frac{1}{2}(m\widehat{PQR} - m\widehat{PR})$



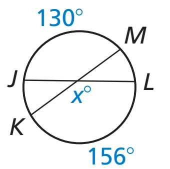
$m\angle 3 = \frac{1}{2}(m\widehat{XY} - m\widehat{WZ})$

Proof Ex. 37, p. 510

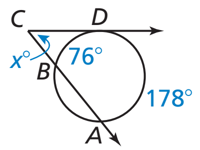
EXAMPLE 2 Finding an Angle Measure

Find the value of x.

a.



b.



SOLUTION

a. The chords $\overline{JL}$ and $\overline{KM}$ intersect inside the circle. Use the Angles Inside the Circle Theorem.

$$x° = \frac{1}{2}(m\widehat{JM} + m\widehat{LK})$$

$$x° = \frac{1}{2}(130° + 156°)$$

$$x = 143$$

▶ So, the value of x is 143.

b. The tangent $\overrightarrow{CD}$ and the secant $\overrightarrow{CB}$ intersect outside the circle. Use the Angles Outside the Circle Theorem.

$$m\angle BCD = \frac{1}{2}(m\widehat{AD} - m\widehat{BD})$$

$$x° = \frac{1}{2}(178° - 76°)$$

$$x = 51$$

▶ So, the value of x is 51.

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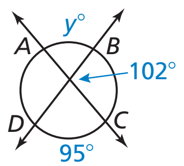
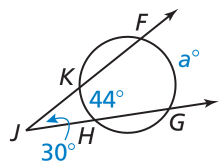
Find the value of the variable.

4.



5.

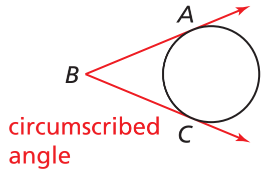


Using Circumscribed Angles

Core Concept

Circumscribed Angle

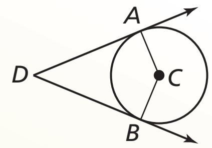
A **circumscribed angle** is an angle whose sides are tangent to a circle.



Theorem

Theorem 10.17 Circumscribed Angle Theorem

The measure of a circumscribed angle is equal to 180° minus the measure of the central angle that intercepts the same arc.



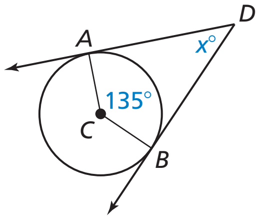
Proof Ex. 38, p. 510

$m\angle ADB = 180° - m\angle ACB$

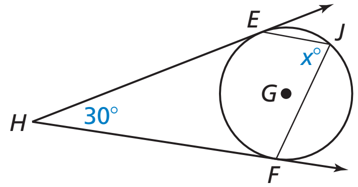
EXAMPLE 3 **Finding Angle Measures**

Find the value of x.

a.



b.



SOLUTION

a. Use the Circumscribed Angle Theorem to find $m\angle ADB$.

$m\angle ADB = 180° - m\angle ACB$	Circumscribed Angle Theorem
$x° = 180° - 135°$	Substitute.
$x = 45$	Subtract.

▶ So, the value of x is 45.

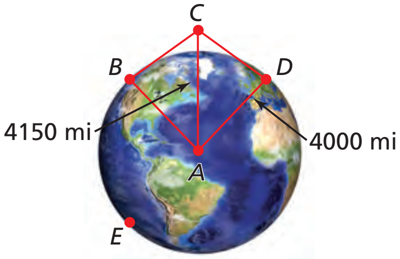
b. Use the Measure of an Inscribed Angle Theorem (Theorem 10.10) and the Circumscribed Angle Theorem to find $m\angle EJF$.

$m\angle EJF = \frac{1}{2}m\overparen{EF}$	Measure of an Inscribed Angle Theorem
$m\angle EJF = \frac{1}{2}m\angle EGF$	Definition of minor arc
$m\angle EJF = \frac{1}{2}(180° - m\angle EHF)$	Circumscribed Angle Theorem
$m\angle EJF = \frac{1}{2}(180° - 30°)$	Substitute.
$x = \frac{1}{2}(180 - 30)$	Substitute.
$x = 75$	Simplify.

▶ So, the value of x is 75.

EXAMPLE 4 Modeling with Mathematics

The northern lights are bright flashes of colored light between 50 and 200 miles above Earth. A flash occurs 150 miles above Earth at point C. What is the measure of $\overset{\frown}{BD}$, the portion of Earth from which the flash is visible? (Earth's radius is approximately 4000 miles.)



Not drawn to scale

SOLUTION

1. **Understand the Problem** You are given the approximate radius of Earth and the distance above Earth that the flash occurs. You need to find the measure of the arc that represents the portion of Earth from which the flash is visible.

2. **Make a Plan** Use properties of tangents, triangle congruence, and angles outside a circle to find the arc measure.

3. **Solve the Problem** Because $\overline{CB}$ and $\overline{CD}$ are tangents, $\overline{CB} \perp \overline{AB}$ and $\overline{CD} \perp \overline{AD}$ by the Tangent Line to Circle Theorem (Theorem 10.1). Also, $\overline{BC} \cong \overline{DC}$ by the External Tangent Congruence Theorem (Theorem 10.2), and $\overline{CA} \cong \overline{CA}$ by the Reflexive Property of Congruence (Theorem 2.1). So, $\triangle ABC \cong \triangle ADC$ by the Hypotenuse-Leg Congruence Theorem (Theorem 5.9). Because corresponding parts of congruent triangles are congruent, $\angle BCA \cong \angle DCA$. Solve right $\triangle CBA$ to find that $m\angle BCA \approx 74.5°$. So, $m\angle BCD \approx 2(74.5°) = 149°$.

COMMON ERROR

Because the value for $m\angle BCD$ is an approximation, use the symbol $\approx$ instead of $=$.

$m\angle BCD = 180° - m\angle BAD$	Circumscribed Angle Theorem
$m\angle BCD = 180° - m\overset{\frown}{BD}$	Definition of minor arc
$149° \approx 180° - m\overset{\frown}{BD}$	Substitute.
$31° \approx m\overset{\frown}{BD}$	Solve for $m\overset{\frown}{BD}$.

▶ The measure of the arc from which the flash is visible is about 31°.

4. **Look Back** You can use inverse trigonometric ratios to find $m\angle BAC$ and $m\angle DAC$.

$$m\angle BAC = \cos^{-1}\left(\frac{4000}{4150}\right) \approx 15.5°$$

$$m\angle DAC = \cos^{-1}\left(\frac{4000}{4150}\right) \approx 15.5°$$

So, $m\angle BAD \approx 15.5° + 15.5° = 31°$, and therefore $m\overset{\frown}{BD} \approx 31°$.

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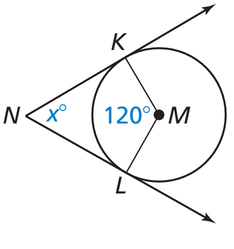
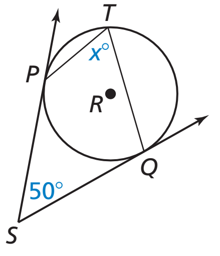
Find the value of *x*.

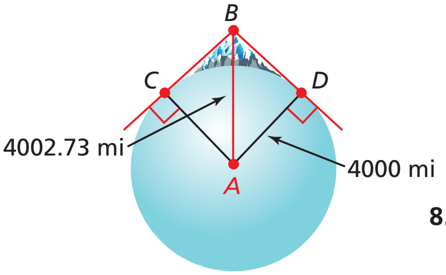
6.



7.



8. You are on top of Mount Rainier on a clear day. You are about 2.73 miles above sea level at point B. Find $m\overset{\frown}{CD}$, which represents the part of Earth that you can see.



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10.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

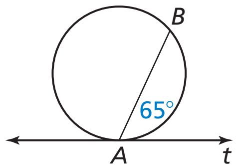
1. **COMPLETE THE SENTENCE** Points A, B, C, and D are on a circle, and $\overleftrightarrow{AB}$ intersects $\overleftrightarrow{CD}$ at point P. If $m\angle APC = \frac{1}{2}(m\widehat{BD} - m\widehat{AC})$, then point P is ______ the circle.

2. **WRITING** Explain how to find the measure of a circumscribed angle.

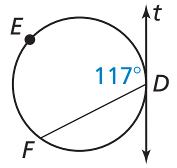
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, line t is tangent to the circle. Find the indicated measure. *(See Example 1.)*

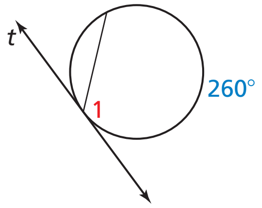
3. $m\widehat{AB}$



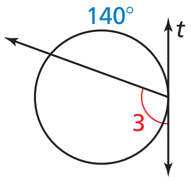
4. $m\widehat{DEF}$



5. $m\angle 1$

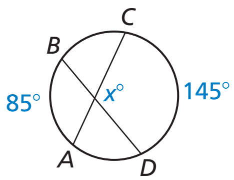


6. $m\angle 3$



In Exercises 7–14, find the value of x. *(See Examples 2 and 3.)*

7.



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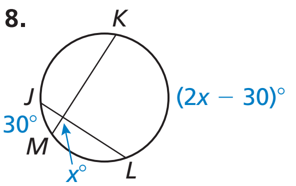

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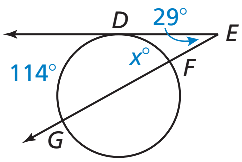

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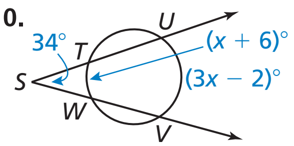

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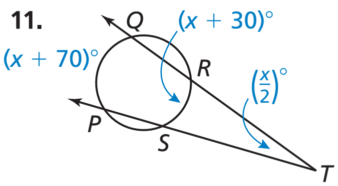

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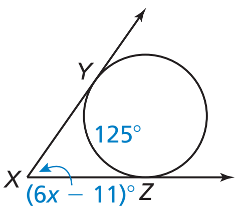
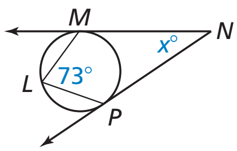
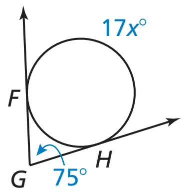


13.

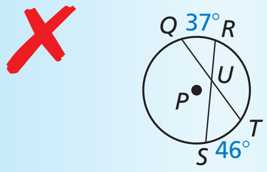


14.



ERROR ANALYSIS **In Exercises 15 and 16, describe and correct the error in finding the angle measure.**

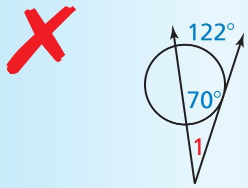
15.



$m\angle SUT = m\widehat{ST} = 46°$

So, $m\angle SUT = 46°$.

16.

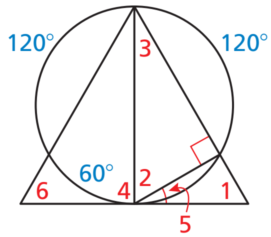


$m\angle 1 = 122° - 70°$

$= 52°$

So, $m\angle 1 = 52°$.

In Exercises 17–22, find the indicated angle measure. Justify your answer.



17. $m\angle 1$

18. $m\angle 2$

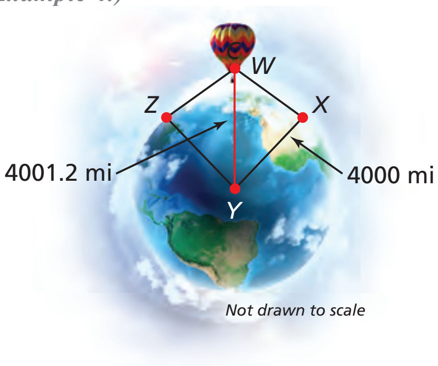
19. $m\angle 3$

20. $m\angle 4$

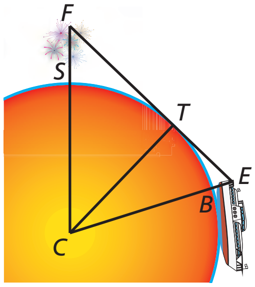
21. $m\angle 5$

22. $m\angle 6$

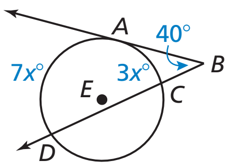
23. PROBLEM SOLVING You are flying in a hot air balloon about 1.2 miles above the ground. Find the measure of the arc that represents the part of Earth you can see. The radius of Earth is about 4000 miles. *(See Example 4.)*



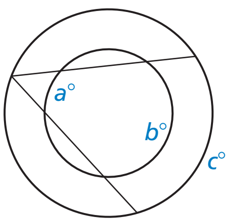
24. PROBLEM SOLVING You are watching fireworks over San Diego Bay S as you sail away in a boat. The highest point the fireworks reach F is about 0.2 mile above the bay. Your eyes E are about 0.01 mile above the water. At point B you can no longer see the fireworks because of the curvature of Earth. The radius of Earth is about 4000 miles, and $\overline{FE}$ is tangent to Earth at point T. Find $m\overset{\frown}{SB}$. Round your answer to the nearest tenth.



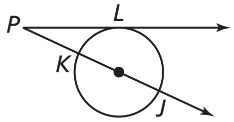
25. MATHEMATICAL CONNECTIONS In the diagram, $\overrightarrow{BA}$ is tangent to $\odot E$. Write an algebraic expression for $m\overset{\frown}{CD}$ in terms of x. Then find $m\overset{\frown}{CD}$.



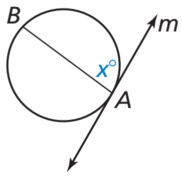
26. MATHEMATICAL CONNECTIONS The circles in the diagram are concentric. Write an algebraic expression for c in terms of a and b.



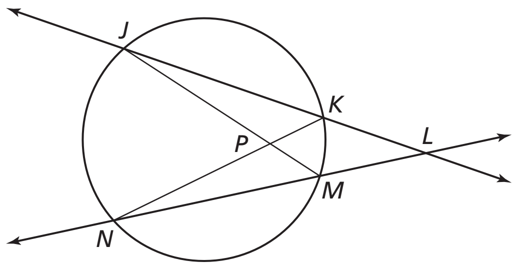
27. ABSTRACT REASONING In the diagram, $\overrightarrow{PL}$ is tangent to the circle, and $\overline{KJ}$ is a diameter. What is the range of possible angle measures of $\angle LPJ$? Explain your reasoning.



28. ABSTRACT REASONING In the diagram, $\overline{AB}$ is any chord that is not a diameter of the circle. Line m is tangent to the circle at point A. What is the range of possible values of x? Explain your reasoning. (The diagram is not drawn to scale.)



29. PROOF In the diagram, $\overleftrightarrow{JL}$ and $\overleftrightarrow{NL}$ are secant lines that intersect at point L. Prove that $m\angle JPN > m\angle JLN$.



30. MAKING AN ARGUMENT Your friend claims that it is possible for a circumscribed angle to have the same measure as its intercepted arc. Is your friend correct? Explain your reasoning.

31. REASONING Points A and B are on a circle, and t is a tangent line containing A and another point C.

a. Draw two diagrams that illustrate this situation.

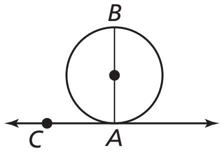
b. Write an equation for $m\overset{\frown}{AB}$ in terms of $m\angle BAC$ for each diagram.

c. For what measure of $\angle BAC$ can you use either equation to find $m\overset{\frown}{AB}$? Explain.

32. REASONING $\triangle XYZ$ is an equilateral triangle inscribed in $\odot P$. $\overline{AB}$ is tangent to $\odot P$ at point X, $\overline{BC}$ is tangent to $\odot P$ at point Y, and $\overline{AC}$ is tangent to $\odot P$ at point Z. Draw a diagram that illustrates this situation. Then classify $\triangle ABC$ by its angles and sides. Justify your answer.

33. PROVING A THEOREM To prove the Tangent and Intersected Chord Theorem (Theorem 10.14), you must prove three cases.

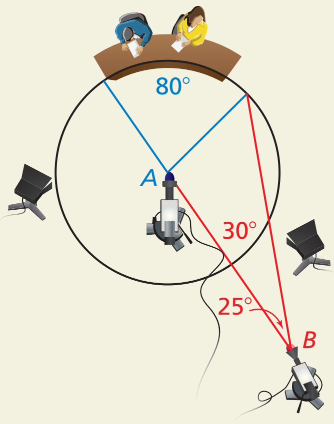
a. The diagram shows the case where $\overline{AB}$ contains the center of the circle. Use the Tangent Line to Circle Theorem (Theorem 10.1) to write a paragraph proof for this case.



b. Draw a diagram and write a proof for the case where the center of the circle is in the interior of $\angle CAB$.

c. Draw a diagram and write a proof for the case where the center of the circle is in the exterior of $\angle CAB$.

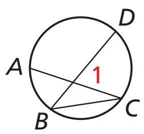
34. HOW DO YOU SEE IT? In the diagram, television cameras are positioned at A and B to record what happens on stage. The stage is an arc of $\odot A$. You would like the camera at B to have a 30° view of the stage. Should you move the camera closer or farther away? Explain your reasoning.



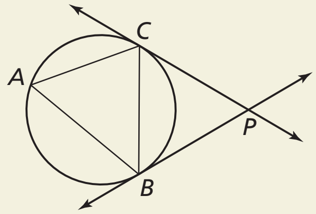
35. PROVING A THEOREM Write a proof of the Angles Inside the Circle Theorem (Theorem 10.15).

Given Chords $\overline{AC}$ and $\overline{BD}$ intersect inside a circle.

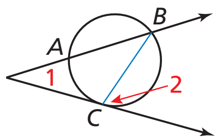
Prove $m\angle 1 = \frac{1}{2}(m\overparen{DC} + m\overparen{AB})$



36. THOUGHT PROVOKING In the figure, $\overleftrightarrow{BP}$ and $\overleftrightarrow{CP}$ are tangent to the circle. Point A is any point on the major arc formed by the endpoints of the chord $\overline{BC}$. Label all congruent angles in the figure. Justify your reasoning.



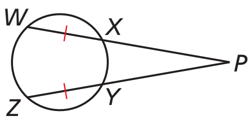
37. PROVING A THEOREM Use the diagram below to prove the Angles Outside the Circle Theorem (Theorem 10.16) for the case of a tangent and a secant. Then copy the diagrams for the other two cases on page 505 and draw appropriate auxiliary segments. Use your diagrams to prove each case.



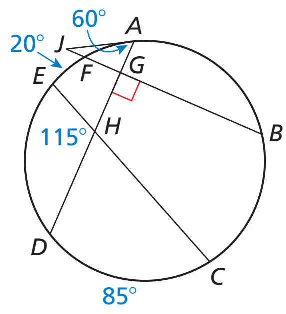
38. PROVING A THEOREM Prove that the Circumscribed Angle Theorem (Theorem 10.17) follows from the Angles Outside the Circle Theorem (Theorem 10.16).

In Exercises 39 and 40, find the indicated measure(s). Justify your answer.

39. Find $m\angle P$ when $m\overparen{WZY} = 200°$.



40. Find $m\overparen{AB}$ and $m\overparen{ED}$.



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation.

41. $x^2 + x = 12$

42. $x^2 = 12x + 35$

43. $-3 = x^2 + 4x$

10.6 Segment Relationships in Circles

Essential Question What relationships exist among the segments formed by two intersecting chords or among segments of two secants that intersect outside a circle?

EXPLORATION 1 Segments Formed by Two Intersecting Chords

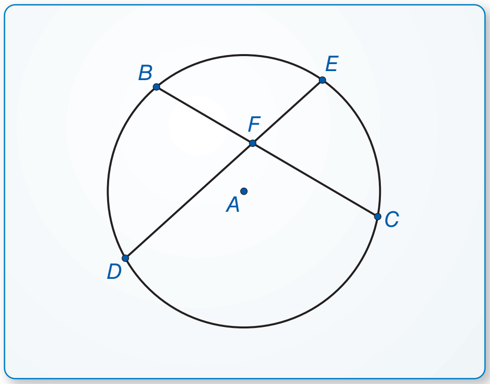
Work with a partner. Use dynamic geometry software.

a. Construct two chords $\overline{BC}$ and $\overline{DE}$ that intersect in the interior of a circle at a point F.

b. Find the segment lengths BF, CF, DF, and EF and complete the table. What do you observe?

BF	CF	$BF \cdot CF$
DF	EF	$DF \cdot EF$

Sample



c. Repeat parts (a) and (b) several times. Write a conjecture about your results.

REASONING ABSTRACTLY

To be proficient in math, you need to make sense of quantities and their relationships in problem situations.

EXPLORATION 2 Secants Intersecting Outside a Circle

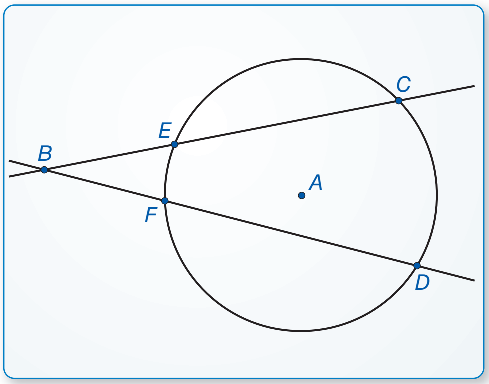
Work with a partner. Use dynamic geometry software.

a. Construct two secants $\overleftrightarrow{BC}$ and $\overleftrightarrow{BD}$ that intersect at a point B outside a circle, as shown.

b. Find the segment lengths BE, BC, BF, and BD, and complete the table. What do you observe?

BE	BC	$BE \cdot BC$
BF	BD	$BF \cdot BD$

Sample

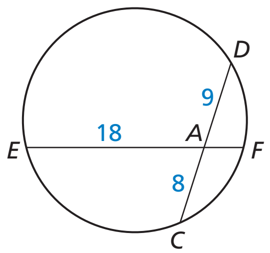


c. Repeat parts (a) and (b) several times. Write a conjecture about your results.

Communicate Your Answer

3. What relationships exist among the segments formed by two intersecting chords or among segments of two secants that intersect outside a circle?

4. Find the segment length AF in the figure at the left.



10.6 Lesson

What You Will Learn

- Use segments of chords, tangents, and secants.

Core Vocabulary

segments of a chord, *p. 512*
tangent segment, *p. 513*
secant segment, *p. 513*
external segment, *p. 513*

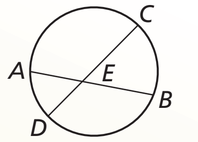
Using Segments of Chords, Tangents, and Secants

When two chords intersect in the interior of a circle, each chord is divided into two segments that are called **segments of the chord**.

Theorem

Theorem 10.18 Segments of Chords Theorem

If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

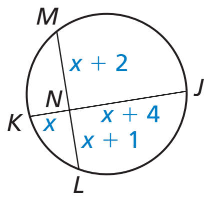


$EA \cdot EB = EC \cdot ED$

Proof Ex. 19, p. 516

EXAMPLE 1 Using Segments of Chords

Find ML and JK.



SOLUTION

$NK \cdot NJ = NL \cdot NM$ — Segments of Chords Theorem

$x \cdot (x + 4) = (x + 1) \cdot (x + 2)$ — Substitute.

$x^2 + 4x = x^2 + 3x + 2$ — Simplify.

$4x = 3x + 2$ — Subtract x^2 from each side.

$x = 2$ — Subtract $3x$ from each side.

Find ML and JK by substitution.

$ML = (x + 2) + (x + 1)$
$= 2 + 2 + 2 + 1$
$= 7$

$JK = x + (x + 4)$
$= 2 + 2 + 4$
$= 8$

- So, $ML = 7$ and $JK = 8$.

Monitoring Progress

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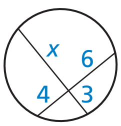
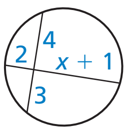
Find the value of x.

1.



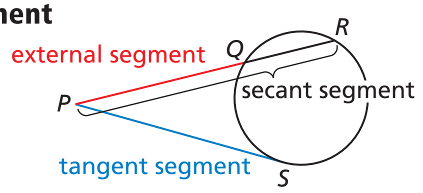
2.



Core Concept

Tangent Segment and Secant Segment

A **tangent segment** is a segment that is tangent to a circle at an endpoint. A **secant segment** is a segment that contains a chord of a circle and has exactly one endpoint outside the circle. The part of a secant segment that is outside the circle is called an **external segment**.

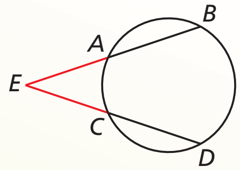


$\overline{PS}$ is a tangent segment.
$\overline{PR}$ is a secant segment.
$\overline{PQ}$ is the external segment of $\overline{PR}$.

Theorem

Theorem 10.19 Segments of Secants Theorem

If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.

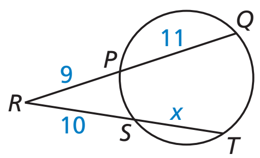


$EA \cdot EB = EC \cdot ED$

Proof Ex. 20, p. 516

EXAMPLE 2 Using Segments of Secants

Find the value of x.



SOLUTION

$RP \cdot RQ = RS \cdot RT$	Segments of Secants Theorem
$9 \cdot (11 + 9) = 10 \cdot (x + 10)$	Substitute.
$180 = 10x + 100$	Simplify.
$80 = 10x$	Subtract 100 from each side.
$8 = x$	Divide each side by 10.

▶ The value of x is 8.

Monitoring Progress

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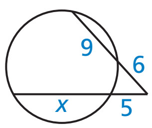
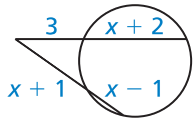
Find the value of x.

3.



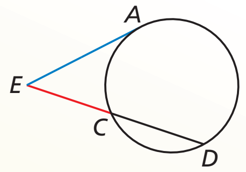
4.



Theorem

Theorem 10.20 Segments of Secants and Tangents Theorem

If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.

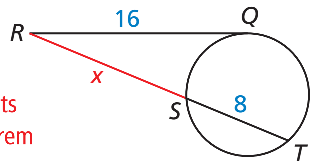


$EA^2 = EC \cdot ED$

Proof Exs. 21 and 22, p. 516

EXAMPLE 3 Using Segments of Secants and Tangents

Find RS.



SOLUTION

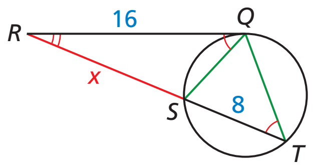
$RQ^2 = RS \cdot RT$	Segments of Secants and Tangents Theorem
$16^2 = x \cdot (x + 8)$	Substitute.
$256 = x^2 + 8x$	Simplify.
$0 = x^2 + 8x - 256$	Write in standard form.
$x = \dfrac{-8 \pm \sqrt{8^2 - 4(1)(-256)}}{2(1)}$	Use Quadratic Formula.
$x = -4 \pm 4\sqrt{17}$	Simplify.

Use the positive solution because lengths cannot be negative.

▶ So, $x = -4 + 4\sqrt{17} \approx 12.49$, and $RS \approx 12.49$.

ANOTHER WAY

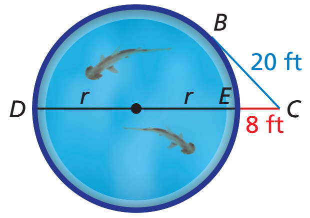
In Example 3, you can draw segments $\overline{QS}$ and $\overline{QT}$.



Because $\angle RQS$ and $\angle RTQ$ intercept the same arc, they are congruent. By the Reflexive Property of Congruence (Theorem 2.2), $\angle QRS \cong \angle TRQ$. So, $\triangle RSQ \sim \triangle RQT$ by the AA Similarity Theorem (Theorem 8.3). You can use this fact to write and solve a proportion to find x.

EXAMPLE 4 Finding the Radius of a Circle

Find the radius of the aquarium tank.



SOLUTION

$CB^2 = CE \cdot CD$	Segments of Secants and Tangents Theorem
$20^2 = 8 \cdot (2r + 8)$	Substitute.
$400 = 16r + 64$	Simplify.
$336 = 16r$	Subtract 64 from each side.
$21 = r$	Divide each side by 16.

▶ So, the radius of the tank is 21 feet.

Monitoring Progress

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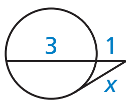
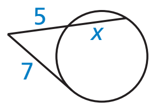
Find the value of x.

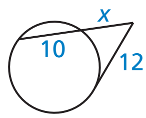
5.



6.



7.



8. WHAT IF? In Example 4, $CB = 35$ feet and $CE = 14$ feet. Find the radius of the tank.

10.6 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

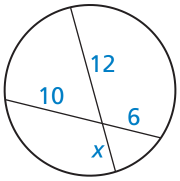
Vocabulary and Core Concept Check

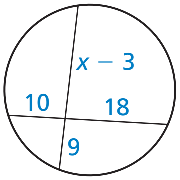
1. **VOCABULARY** The part of the secant segment that is outside the circle is called a(n) ____________.

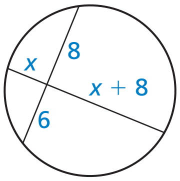
2. **WRITING** Explain the difference between a tangent segment and a secant segment.

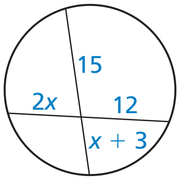
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the value of x. *(See Example 1.)*

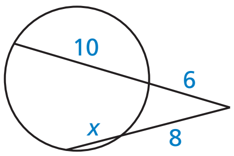
3. 

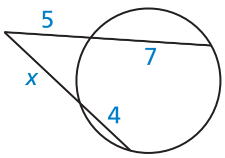
4. 

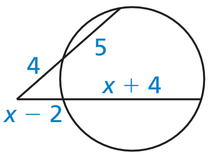
5. 

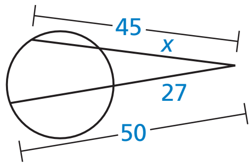
6. 

In Exercises 7–10, find the value of x. *(See Example 2.)*

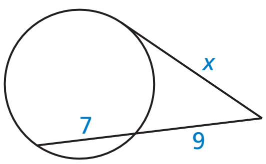
7. 

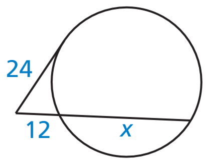
8. 

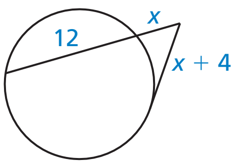
9. 

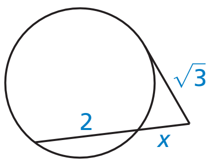
10. 

In Exercises 11–14, find the value of x. *(See Example 3.)*

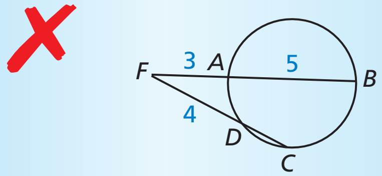
11. 

12. 

13. 

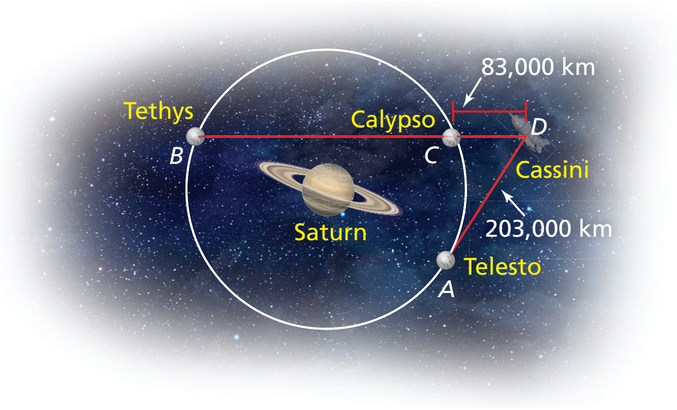
14. 

15. **ERROR ANALYSIS** Describe and correct the error in finding CD.

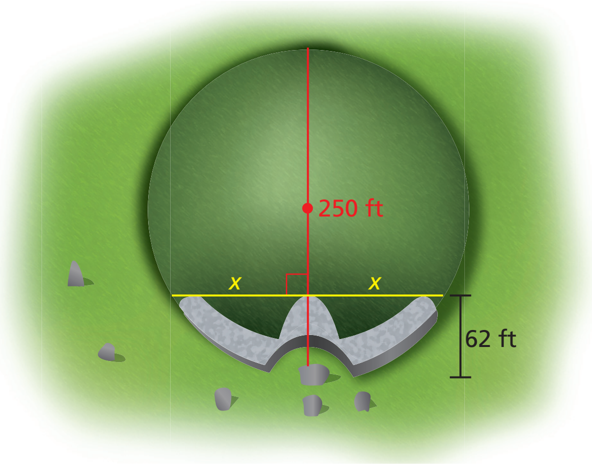


$$CD \cdot DF = AB \cdot AF$$
$$CD \cdot 4 = 5 \cdot 3$$
$$CD \cdot 4 = 15$$
$$CD = 3.75$$

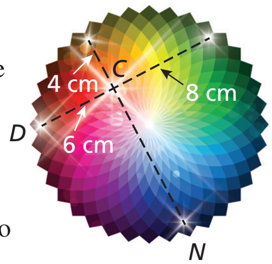
16. **MODELING WITH MATHEMATICS** The Cassini spacecraft is on a mission in orbit around Saturn until September 2017. Three of Saturn's moons, Tethys, Calypso, and Telesto, have nearly circular orbits of radius 295,000 kilometers. The diagram shows the positions of the moons and the spacecraft on one of Cassini's missions. Find the distance DB from Cassini to Tethys when $\overline{AD}$ is tangent to the circular orbit. *(See Example 4.)*



17. MODELING WITH MATHEMATICS The circular stone mound in Ireland called Newgrange has a diameter of 250 feet. A passage 62 feet long leads toward the center of the mound. Find the perpendicular distance x from the end of the passage to either side of the mound.



18. MODELING WITH MATHEMATICS You are designing an animated logo for your website. Sparkles leave point C and move to the outer circle along the segments shown so that all of the sparkles reach the outer circle at the same time. Sparkles travel from point C to point D at 2 centimeters per second. How fast should sparkles move from point C to point N? Explain.



19. PROVING A THEOREM Write a two-column proof of the Segments of Chords Theorem (Theorem 10.18).

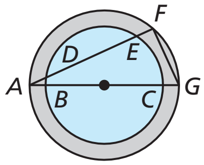
Plan for Proof Use the diagram from page 512. Draw $\overline{AC}$ and $\overline{DB}$. Show that $\triangle EAC$ and $\triangle EDB$ are similar. Use the fact that corresponding side lengths in similar triangles are proportional.

20. PROVING A THEOREM Prove the Segments of Secants Theorem (Theorem 10.19). (*Hint*: Draw a diagram and add auxiliary line segments to form similar triangles.)

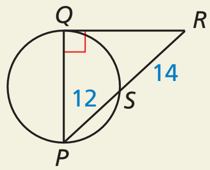
21. PROVING A THEOREM Use the Tangent Line to Circle Theorem (Theorem 10.1) to prove the Segments of Secants and Tangents Theorem (Theorem 10.20) for the special case when the secant segment contains the center of the circle.

22. PROVING A THEOREM Prove the Segments of Secants and Tangents Theorem (Theorem 10.20). (*Hint*: Draw a diagram and add auxiliary line segments to form similar triangles.)

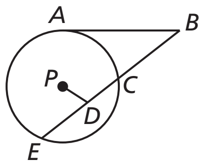
23. WRITING EQUATIONS In the diagram of the water well, AB, AD, and DE are known. Write an equation for BC using these three measurements.



24. HOW DO YOU SEE IT? Which two theorems would you need to use to find PQ? Explain your reasoning.



25. CRITICAL THINKING In the figure, $AB = 12$, $BC = 8$, $DE = 6$, $PD = 4$, and A is a point of tangency. Find the radius of $\odot P$.



26. THOUGHT PROVOKING Circumscribe a triangle about a circle. Then, using the points of tangency, inscribe a triangle in the circle. Must it be true that the two triangles are similar? Explain your reasoning.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation by completing the square.

27. $x^2 + 4x = 45$

28. $x^2 - 2x - 1 = 8$

29. $2x^2 + 12x + 20 = 34$

30. $-4x^2 + 8x + 44 = 16$

10.7 Circles in the Coordinate Plane

Essential Question What is the equation of a circle with center (h, k) and radius r in the coordinate plane?

EXPLORATION 1 The Equation of a Circle with Center at the Origin

Work with a partner. Use dynamic geometry software to construct and determine the equations of circles centered at (0, 0) in the coordinate plane, as described below.

a. Complete the first two rows of the table for circles with the given radii. Complete the other rows for circles with radii of your choice.

b. Write an equation of a circle with center (0, 0) and radius r.

Radius	Equation of circle
1	
2	

EXPLORATION 2 The Equation of a Circle with Center (*h*, *k*)

Work with a partner. Use dynamic geometry software to construct and determine the equations of circles of radius 2 in the coordinate plane, as described below.

a. Complete the first two rows of the table for circles with the given centers. Complete the other rows for circles with centers of your choice.

b. Write an equation of a circle with center (h, k) and radius 2.

Center	Equation of circle
(0, 0)	
(2, 0)	

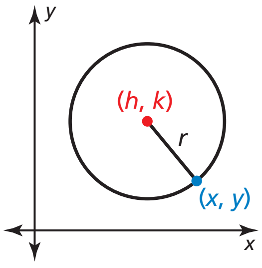
c. Write an equation of a circle with center (h, k) and radius r.

EXPLORATION 3 Deriving the Standard Equation of a Circle

Work with a partner. Consider a circle with radius r and center (h, k).

Write the Distance Formula to represent the distance d between a point (x, y) on the circle and the center (h, k) of the circle. Then square each side of the Distance Formula equation.

How does your result compare with the equation you wrote in part (c) of Exploration 2?



MAKING SENSE OF PROBLEMS

To be proficient in math, you need to explain correspondences between equations and graphs.

Communicate Your Answer

4. What is the equation of a circle with center (h, k) and radius r in the coordinate plane?

5. Write an equation of the circle with center $(4, -1)$ and radius 3.

10.7 Lesson

Core Vocabulary

standard equation of a circle, *p. 518*

Previous
Pythagorean Theorem
Distance Formula
Midpoint Formula

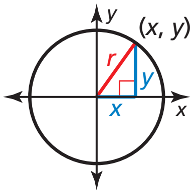
What You Will Learn

- Write and graph equations of circles.
- Write coordinate proofs involving circles.
- Solve real-life problems using graphs of circles.

Writing and Graphing Equations of Circles

Let (x, y) represent any point on a circle with center at the origin and radius r. By the Pythagorean Theorem (Theorem 9.1),

$$x^2 + y^2 = r^2.$$



This is the equation of a circle with center at the origin and radius r.

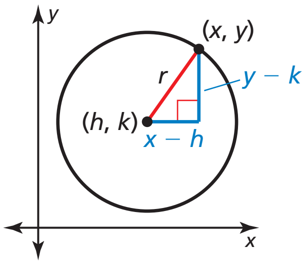
Core Concept

Standard Equation of a Circle

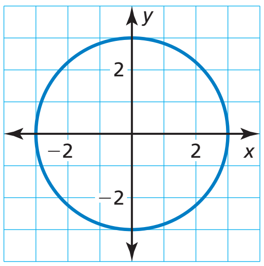
Let (x, y) represent any point on a circle with center (h, k) and radius r. By the Pythagorean Theorem (Theorem 9.1),

$$(x - h)^2 + (y - k)^2 = r^2.$$

This is the **standard equation of a circle** with center (h, k) and radius r.



EXAMPLE 1 Writing the Standard Equation of a Circle



Write the standard equation of each circle.

a. the circle shown at the left

b. a circle with center $(0, -9)$ and radius 4.2

SOLUTION

a. The radius is 3, and the center is at the origin.

$(x - h)^2 + (y - k)^2 = r^2$ — Standard equation of a circle

$(x - 0)^2 + (y - 0)^2 = 3^2$ — Substitute.

$x^2 + y^2 = 9$ — Simplify.

The standard equation of the circle is $x^2 + y^2 = 9$.

b. The radius is 4.2, and the center is at $(0, -9)$.

$(x - h)^2 + (y - k)^2 = r^2$ — Standard equation of a circle

$(x - 0)^2 + [y - (-9)]^2 = 4.2^2$ — Substitute.

$x^2 + (y + 9)^2 = 17.64$ — Simplify.

The standard equation of the circle is $x^2 + (y + 9)^2 = 17.64$.

Monitoring Progress

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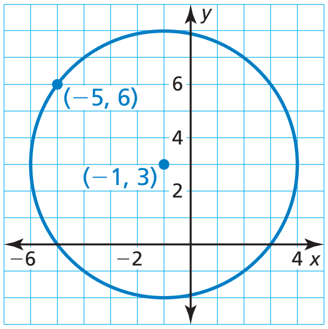
Write the standard equation of the circle with the given center and radius.

1. center: $(0, 0)$, radius: 2.5

2. center: $(-2, 5)$, radius: 7

EXAMPLE 2 Writing the Standard Equation of a Circle



The point $(-5, 6)$ is on a circle with center $(-1, 3)$. Write the standard equation of the circle.

SOLUTION

To write the standard equation, you need to know the values of h, k, and r. To find r, find the distance between the center and the point $(-5, 6)$ on the circle.

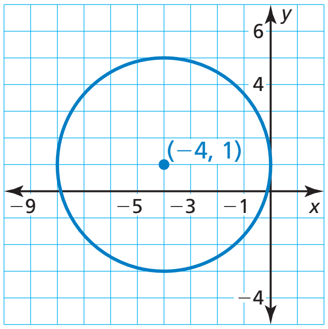
$$r = \sqrt{[-5 - (-1)]^2 + (6 - 3)^2} \quad \text{Distance Formula}$$
$$= \sqrt{(-4)^2 + 3^2} \quad \text{Simplify.}$$
$$= 5 \quad \text{Simplify.}$$

Substitute the values for the center and the radius into the standard equation of a circle.

$$(x - h)^2 + (y - k)^2 = r^2 \quad \text{Standard equation of a circle}$$
$$[x - (-1)]^2 + (y - 3)^2 = 5^2 \quad \text{Substitute } (h, k) = (-1, 3) \text{ and } r = 5.$$
$$(x + 1)^2 + (y - 3)^2 = 25 \quad \text{Simplify.}$$

▶ The standard equation of the circle is $(x + 1)^2 + (y - 3)^2 = 25$.

EXAMPLE 3 Graphing a Circle



Find the center and radius of each circle. Then graph each circle.

a. the circle whose standard equation is $(x + 4)^2 + (y - 1)^2 = 16$

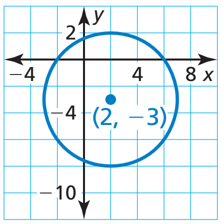
b. the circle that has $(-1, 1)$ and $(5, -7)$ as endpoints of a diameter

SOLUTION

a. $[x - (-4)]^2 + (y - 1)^2 = 4^2$ Rewrite the equation to find the center and the radius.

▶ The center is $(-4, 1)$, and the radius is 4. Use a compass to graph the circle.

b. To find h and k, use the Midpoint Formula. To find r, find the distance between the center and one of the endpoints of a diameter.



$$(h, k) = \left(\frac{-1 + 5}{2}, \frac{1 + (-7)}{2}\right) \quad \text{Midpoint Formula}$$
$$= (2, -3) \quad \text{Simplify.}$$
$$r = \sqrt{(-1 - 2)^2 + [1 - (-3)]^2} \quad \text{Distance Formula}$$
$$= \sqrt{(-3)^2 + 4^2} \quad \text{Simplify.}$$
$$= 5 \quad \text{Simplify.}$$

▶ The center is $(2, -3)$, and the radius is 5. Use a compass to graph the circle.

Monitoring Progress

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3. The point $(3, 4)$ is on a circle with center $(1, 4)$. Write the standard equation of the circle.

4. The endpoints of the diameter of a circle are $(6, -8)$ and $(-4, 16)$. Find the center and the radius of the circle. Then graph the circle.

Writing Coordinate Proofs Involving Circles

EXAMPLE 4 Writing a Coordinate Proof Involving a Circle

Prove or disprove that the point $(\sqrt{2}, \sqrt{2})$ lies on the circle centered at the origin and containing the point (2, 0).

SOLUTION

The circle centered at the origin and containing the point (2, 0) has the following radius.

$$r = \sqrt{(x-h)^2 + (y-k)^2} = \sqrt{(2-0)^2 + (0-0)^2} = 2$$

So, a point lies on the circle if and only if the distance from that point to the origin is 2. The distance from $(\sqrt{2}, \sqrt{2})$ to (0, 0) is

$$d = \sqrt{(\sqrt{2}-0)^2 + (\sqrt{2}-0)^2} = 2.$$

▶ So, the point $(\sqrt{2}, \sqrt{2})$ lies on the circle centered at the origin and containing the point (2, 0).

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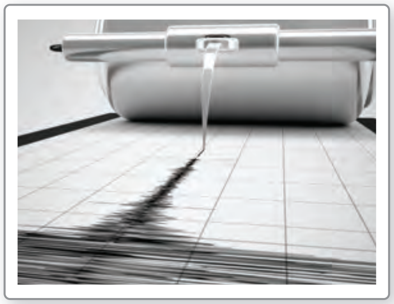
5. Prove or disprove that the point $(1, \sqrt{5})$ lies on the circle centered at the origin and containing the point (0, 1).

Solving Real-Life Problems

STEM VIDEO

EXAMPLE 5 Using Graphs of Circles

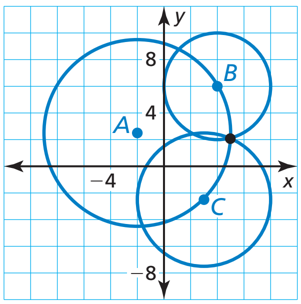


The epicenter of an earthquake is the point on Earth's surface directly above the earthquake's origin. A seismograph can be used to determine the distance to the epicenter of an earthquake. Seismographs are needed in three different places to locate an earthquake's epicenter.

Use the seismograph readings from locations A, B, and C to find the epicenter of an earthquake.

- The epicenter is 7 miles away from $A(-2, 2.5)$.
- The epicenter is 4 miles away from $B(4, 6)$.
- The epicenter is 5 miles away from $C(3, -2.5)$.

SOLUTION



The set of all points equidistant from a given point is a circle, so the epicenter is located on each of the following circles.

$\odot A$ with center $(-2, 2.5)$ and radius 7

$\odot B$ with center $(4, 6)$ and radius 4

$\odot C$ with center $(3, -2.5)$ and radius 5

To find the epicenter, graph the circles on a coordinate plane where each unit corresponds to one mile. Find the point of intersection of the three circles.

▶ The epicenter is at about (5, 2).

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6. Why are three seismographs needed to locate an earthquake's epicenter?

10.7 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

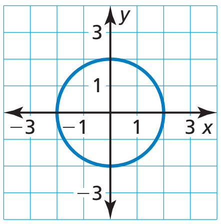
1. **VOCABULARY** What is the standard equation of a circle?

2. **WRITING** Explain why knowing the location of the center and one point on a circle is enough to graph the circle.

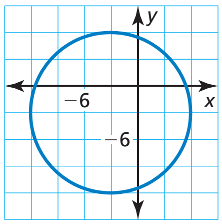
Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, write the standard equation of the circle. *(See Example 1.)*

3.



4.



5. a circle with center (0, 0) and radius 7

6. a circle with center (4, 1) and radius 5

7. a circle with center (−3, 4) and radius 1

8. a circle with center (3, −5) and radius 7

In Exercises 9–11, use the given information to write the standard equation of the circle. *(See Example 2.)*

9. The center is (0, 0), and a point on the circle is (0, 6).

10. The center is (1, 2), and a point on the circle is (4, 2).

11. The center is (0, 0), and a point on the circle is (3, −7).

12. **ERROR ANALYSIS** Describe and correct the error in writing the standard equation of a circle.

> ✗ The standard equation of a circle with center (−3, −5) and radius 3 is $(x - 3)^2 + (y - 5)^2 = 9$.

In Exercises 13–18, find the center and radius of the circle. Then graph the circle. *(See Example 3.)*

13. $x^2 + y^2 = 49$

14. $(x + 5)^2 + (y - 3)^2 = 9$

15. $(x - 12)^2 + (y + 6)^2 = 72$

16. endpoints of a diameter: (−1, −1), (−25, −11)

17. endpoints of a diameter: (4, 0), (−2, 8)

18. endpoints of a diameter: (−3, −5), (5, 5)

In Exercises 19–22, prove or disprove the statement. *(See Example 4.)*

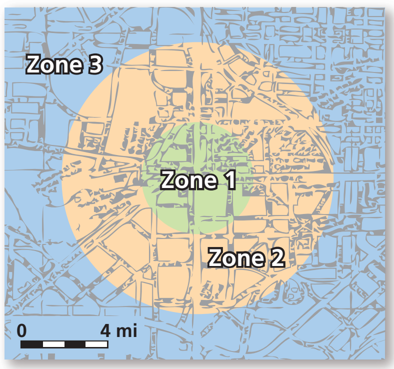
19. The point (2, 3) lies on the circle centered at the origin with radius 8.

20. The point $(4, \sqrt{5})$ lies on the circle centered at the origin with radius 3.

21. The point $(\sqrt{6}, 2)$ lies on the circle centered at the origin and containing the point (3, −1).

22. The point $(\sqrt{7}, 5)$ lies on the circle centered at the origin and containing the point (5, 2).

23. **MODELING WITH MATHEMATICS** A city's commuter system has three zones. Zone 1 serves people living within 3 miles of the city's center. Zone 2 serves those between 3 and 7 miles from the center. Zone 3 serves those over 7 miles from the center. *(See Example 5.)*



a. Graph this situation on a coordinate plane where each unit corresponds to 1 mile. Locate the city's center at the origin.

b. Determine which zone serves people whose homes are represented by the points (3, 4), (6, 5), (1, 2), (0, 3), and (1, 6).

24. MODELING WITH MATHEMATICS Telecommunication towers can be used to transmit cellular phone calls. A graph with units measured in kilometers shows towers at points (0, 0), (0, 5), and (6, 3). These towers have a range of about 3 kilometers.

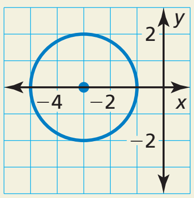
a. Sketch a graph and locate the towers. Are there any locations that may receive calls from more than one tower? Explain your reasoning.

b. The center of City A is located at (−2, 2.5), and the center of City B is located at (5, 4). Each city has a radius of 1.5 kilometers. Which city seems to have better cell phone coverage? Explain your reasoning.

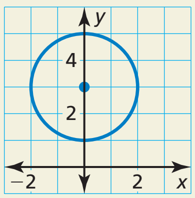
25. REASONING Sketch the graph of the circle whose equation is $x^2 + y^2 = 16$. Then sketch the graph of the circle after the translation $(x, y) \rightarrow (x - 2, y - 4)$. What is the equation of the image? Make a conjecture about the equation of the image of a circle centered at the origin after a translation m units to the left and n units down.

26. HOW DO YOU SEE IT? Match each graph with its equation.

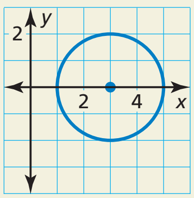
a.



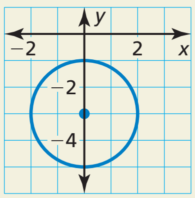
b.



c.



d.



A. $x^2 + (y + 3)^2 = 4$ **B.** $(x - 3)^2 + y^2 = 4$

C. $(x + 3)^2 + y^2 = 4$ **D.** $x^2 + (y - 3)^2 = 4$

27. USING STRUCTURE The vertices of $\triangle XYZ$ are $X(4, 5)$, $Y(4, 13)$, and $Z(8, 9)$. Find the equation of the circle circumscribed about $\triangle XYZ$. Justify your answer.

28. THOUGHT PROVOKING A circle has center (h, k) and contains point (a, b). Write the equation of the line tangent to the circle at point (a, b).

MATHEMATICAL CONNECTIONS In Exercises 29–31, use the equations to determine whether the line is *a tangent*, *a secant*, *a secant that contains the diameter*, or *none of these*. Explain your reasoning.

29. Circle: $(x - 4)^2 + (y - 3)^2 = 9$
Line: $y = 6$

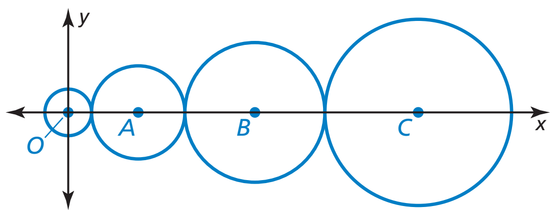
30. Circle: $(x + 2)^2 + (y - 2)^2 = 16$
Line: $y = 2x - 4$

31. Circle: $(x - 5)^2 + (y + 1)^2 = 4$
Line: $y = \frac{1}{5}x - 3$

32. Find the center and radius of the circle whose equation is $x^2 + y^2 - 6x = 7$. (*Hint:* use completing the square.)

33. MAKING AN ARGUMENT Your friend claims that the equation of a circle passing through the points (−1, 0) and (1, 0) is $x^2 - 2yk + y^2 = 1$ with center $(0, k)$. Is your friend correct? Explain your reasoning.

34. REASONING Four tangent circles are centered on the x-axis. The radius of $\odot A$ is twice the radius of $\odot O$. The radius of $\odot B$ is three times the radius of $\odot O$. The radius of $\odot C$ is four times the radius of $\odot O$. All circles have integer radii, and the point (63, 16) is on $\odot C$. What is the equation of $\odot A$? Explain your reasoning.



Maintaining Mathematical Proficiency

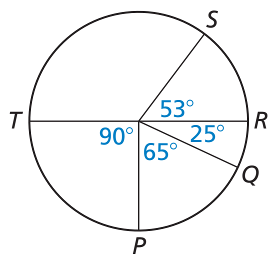
Reviewing what you learned in previous grades and lessons

Identify the arc as a *major arc*, *minor arc*, or *semicircle*. Then find the measure of the arc.

35. $\overset{\frown}{RS}$ **36.** $\overset{\frown}{PR}$

37. $\overset{\frown}{PRT}$ **38.** $\overset{\frown}{ST}$

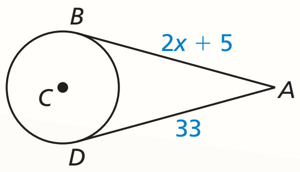
39. $\overset{\frown}{RST}$ **40.** $\overset{\frown}{QS}$



Dynamic Solutions available at *BigIdeasMath.com*

10.1 Lines and Segments That Intersect Circles *(pp. 473–480)*

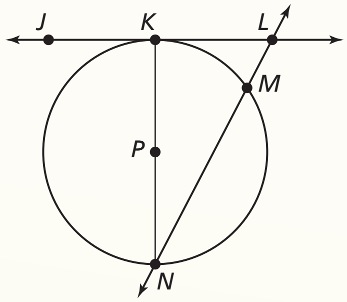
In the diagram, $\overline{AB}$ is tangent to $\odot C$ at B and $\overline{AD}$ is tangent to $\odot C$ at D. Find the value of x.



$AB = AD$	External Tangent Congruence Theorem (Theorem 10.2)
$2x + 5 = 33$	Substitute.
$x = 14$	Solve for x.

▶ The value of x is 14.

Tell whether the line, ray, or segment is best described as a *radius*, *chord*, *diameter*, *secant*, or *tangent* of $\odot P$.



1. $\overline{PK}$

2. $\overline{NM}$

3. $\overrightarrow{JL}$

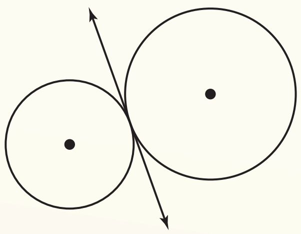
4. $\overline{KN}$

5. $\overleftrightarrow{NL}$

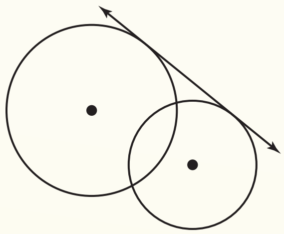
6. $\overline{PN}$

Tell whether the common tangent is *internal* or *external*.

7.

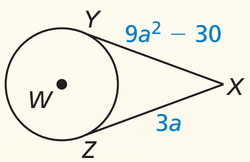


8.

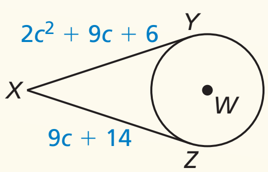


Points Y and Z are points of tangency. Find the value of the variable.

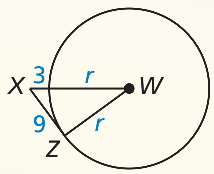
9.



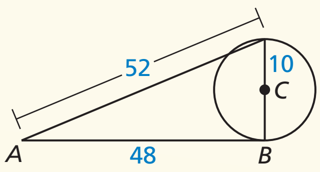
10.



11.

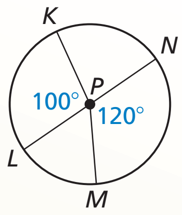


12. Tell whether $\overline{AB}$ is tangent to $\odot C$. Explain.



10.2 Finding Arc Measures *(pp. 481–488)*

Find the measure of each arc of $\odot P$, where $\overline{LN}$ is a diameter.



a. $\widehat{MN}$

▶ $\widehat{MN}$ is a minor arc, so $m\widehat{MN} = m\angle MPN = 120°$.

b. $\widehat{NLM}$

▶ $\widehat{NLM}$ is a major arc, so $m\widehat{NLM} = 360° - 120° = 240°$.

c. $\widehat{NML}$

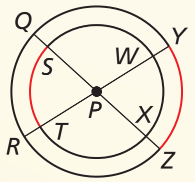
▶ $\overline{NL}$ is a diameter, so $\widehat{NML}$ is a semicircle, and $m\widehat{NML} = 180°$.

Use the diagram above to find the measure of the indicated arc.

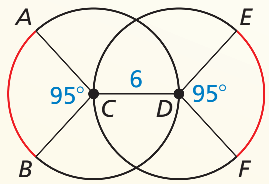
13. $\widehat{KL}$ **14.** $\widehat{LM}$ **15.** $\widehat{KM}$ **16.** $\widehat{KN}$

Tell whether the red arcs are congruent. Explain why or why not.

17.



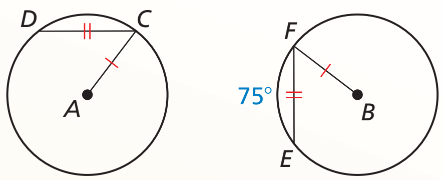
18.



10.3 Using Chords *(pp. 489–494)*

In the diagram, $\odot A \cong \odot B$, $\overline{CD} \cong \overline{FE}$, and $m\widehat{FE} = 75°$. Find $m\widehat{CD}$.

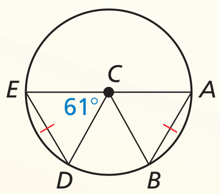
Because $\overline{CD}$ and $\overline{FE}$ are congruent chords in congruent circles, the corresponding minor arcs $\widehat{CD}$ and $\widehat{FE}$ are congruent by the Congruent Corresponding Chords Theorem (Theorem 10.6).



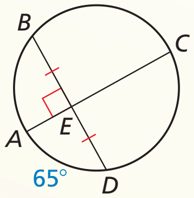
▶ So, $m\widehat{CD} = m\widehat{FE} = 75°$.

Find the measure of $\widehat{AB}$.

19.



20.



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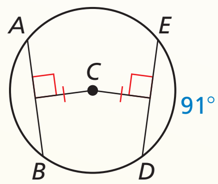
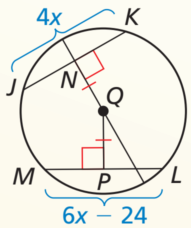


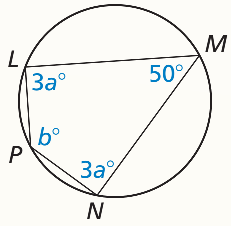
22. In the diagram, $QN = QP = 10$, $JK = 4x$, and $LM = 6x - 24$. Find the radius of $\odot Q$.



10.4 Inscribed Angles and Polygons (pp. 495–502)

Find the value of each variable.

LMNP is inscribed in a circle, so opposite angles are supplementary by the Inscribed Quadrilateral Theorem (Theorem 10.13).

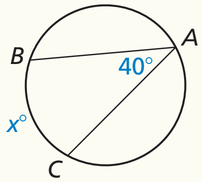


$$m\angle L + m\angle N = 180^\circ \qquad m\angle P + m\angle M = 180^\circ$$
$$3a^\circ + 3a^\circ = 180^\circ \qquad b^\circ + 50^\circ = 180^\circ$$
$$6a = 180 \qquad b = 130$$
$$a = 30$$

▶ The value of a is 30, and the value of b is 130.

Find the value(s) of the variable(s).

23.



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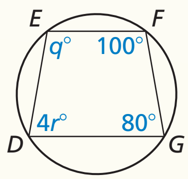

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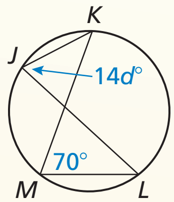

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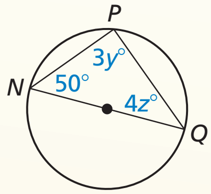

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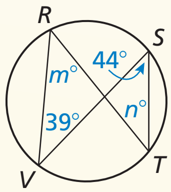
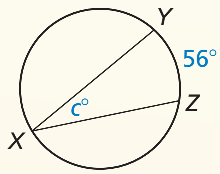


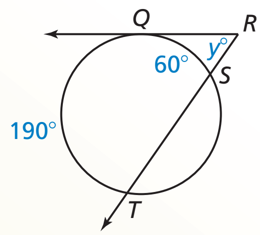
28.



10.5 Angle Relationships in Circles (pp. 503–510)

Find the value of *y*.

The tangent $\overrightarrow{RQ}$ and secant $\overrightarrow{RT}$ intersect outside the circle, so you can use the Angles Outside the Circle Theorem (Theorem 10.16).

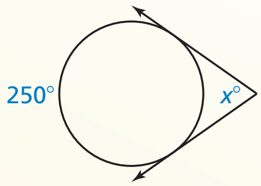


$y^\circ = \frac{1}{2}(m\overset{\frown}{QT} - m\overset{\frown}{SQ})$	Angles Outside the Circle Theorem
$y^\circ = \frac{1}{2}(190^\circ - 60^\circ)$	Substitute.
$y = 65$	Simplify.

▶ The value of y is 65.

Find the value of x.

29.



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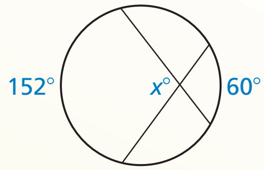
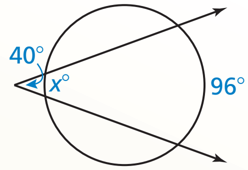
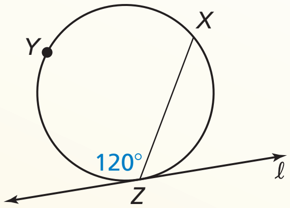


31.



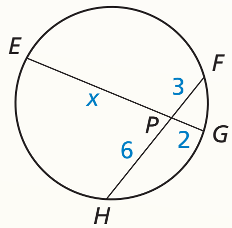
32. Line ℓ is tangent to the circle. Find $m\overparen{XYZ}$.



10.6 Segment Relationships in Circles (pp. 511–516)

Find the value of x.

The chords $\overline{EG}$ and $\overline{FH}$ intersect inside the circle, so you can use the Segments of Chords Theorem (Theorem 10.18).

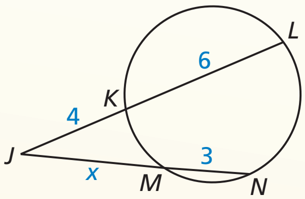


$EP \cdot PG = FP \cdot PH$	Segments of Chords Theorem
$x \cdot 2 = 3 \cdot 6$	Substitute.
$x = 9$	Simplify.

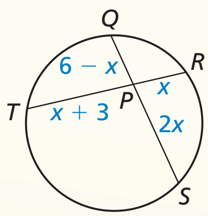
▶ The value of x is 9.

Find the value of x.

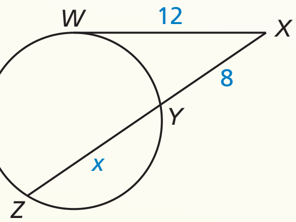
33.



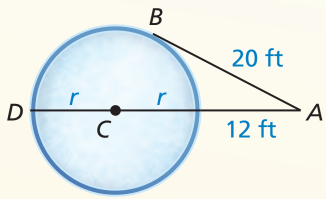
34.



35.

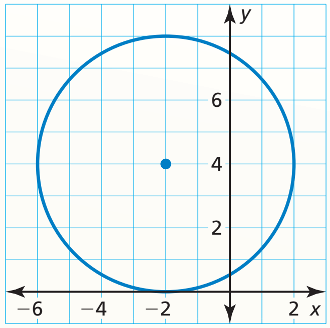


36. A local park has a circular ice skating rink. You are standing at point A, about 12 feet from the edge of the rink. The distance from you to a point of tangency on the rink is about 20 feet. Estimate the radius of the rink.



10.7 Circles in the Coordinate Plane (pp. 517–522)

Write the standard equation of the circle shown.



The radius is 4, and the center is $(-2, 4)$.

$(x - h)^2 + (y - k)^2 = r^2$ Standard equation of a circle

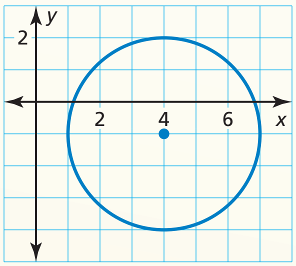
$[x - (-2)]^2 + (y - 4)^2 = 4^2$ Substitute.

$(x + 2)^2 + (y - 4)^2 = 16$ Simplify.

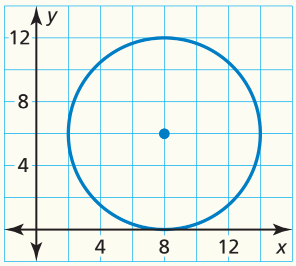
▶ The standard equation of the circle is $(x + 2)^2 + (y - 4)^2 = 16$.

Write the standard equation of the circle shown.

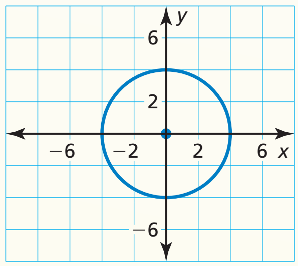
37.



38.



39.



Write the standard equation of the circle with the given center and radius.

40. center: $(0, 0)$, radius: 9

41. center: $(-5, 2)$, radius: 1.3

42. center: $(6, 21)$, radius: 4

43. center: $(-3, 2)$, radius: 16

44. center: $(10, 7)$, radius: 3.5

45. center: $(0, 0)$, radius: 5.2

46. The point $(-7, 1)$ is on a circle with center $(-7, 6)$. Write the standard equation of the circle.

47. The equation of a circle is $(x - 6)^2 + (y + 4)^2 = 4$. Find the center and the radius of the circle. Then graph the circle.

48. Prove or disprove that the point $(4, -3)$ lies on the circle centered at the origin and containing the point $(-5, 0)$.