

# 7 Quadrilaterals and Other Polygons

- 7.1 Angles of Polygons
- 7.2 Properties of Parallelograms
- 7.3 Proving That a Quadrilateral Is a Parallelogram
- 7.4 Properties of Special Parallelograms
- 7.5 Properties of Trapezoids and Kites
- 7.6 Tessellations



Diamond (p. 368)



Window (p. 357)



Amusement Park Ride (p. 341)



Arrow (p. 337)



Gazebo (p. 329)

# Maintaining Mathematical Proficiency

## Using Structure to Solve a Multi-Step Equation

**Example 1** Solve  $3(2 + x) = -9$  by interpreting the expression  $2 + x$  as a single quantity.

$$3(2 + x) = -9$$

Write the equation.

$$\frac{3(2 + x)}{3} = \frac{-9}{3}$$

Divide each side by 3.

$$2 + x = -3$$

Simplify.

$$\underline{-2} \quad \underline{-2}$$

Subtract 2 from each side.

$$x = -5$$

Simplify.

Solve the equation by interpreting the expression in parentheses as a single quantity.

1.  $4(7 - x) = 16$

2.  $7(1 - x) + 2 = -19$

3.  $3(x - 5) + 8(x - 5) = 22$

## Identifying Parallel and Perpendicular Lines

**Example 2** Determine which of the lines are parallel and which are perpendicular.

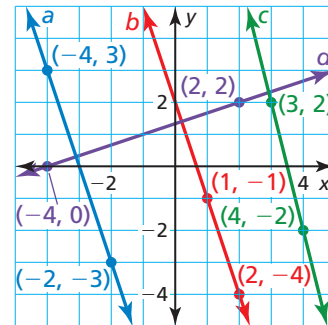
Find the slope of each line.

Line *a*:  $m = \frac{3 - (-3)}{-4 - (-2)} = -3$

Line *b*:  $m = \frac{-1 - (-4)}{1 - 2} = -3$

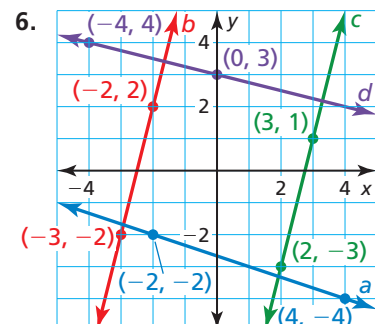
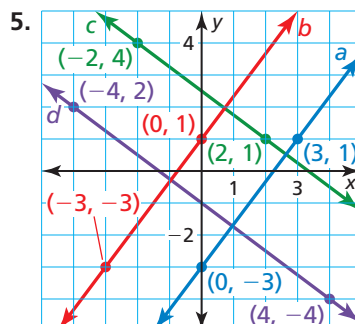
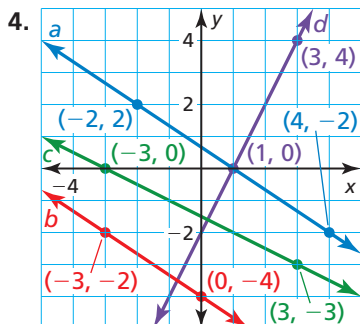
Line *c*:  $m = \frac{2 - (-2)}{3 - 4} = -4$

Line *d*:  $m = \frac{2 - 0}{2 - (-4)} = \frac{1}{3}$



Because lines *a* and *b* have the same slope, lines *a* and *b* are parallel. Because  $\frac{1}{3}(-3) = -1$ , lines *a* and *d* are perpendicular and lines *b* and *d* are perpendicular.

Determine which lines are parallel and which are perpendicular.

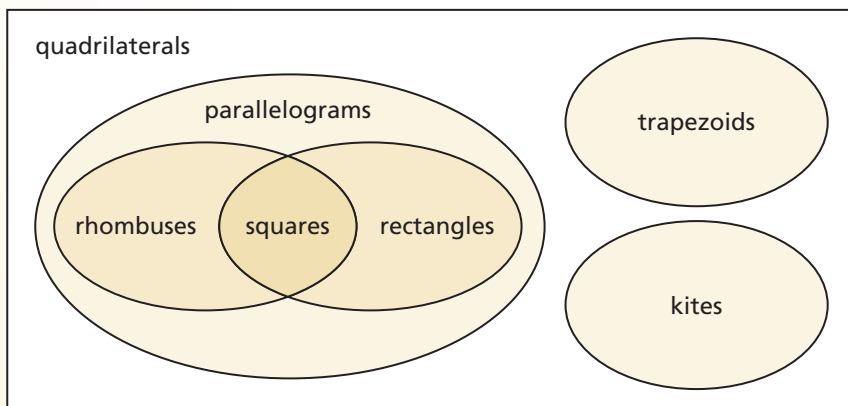


7. **ABSTRACT REASONING** Explain why interpreting an expression as a single quantity does not contradict the order of operations.

## Mapping Relationships

### Core Concept

#### Classifications of Quadrilaterals



#### EXAMPLE 1 Writing Statements about Quadrilaterals

Use the Venn diagram above to write three true statements about different types of quadrilaterals.

#### SOLUTION

Here are three true statements that can be made about the relationships shown in the Venn diagram.

- All rhombuses are parallelograms.
- Some rhombuses are rectangles.
- No trapezoids are parallelograms.

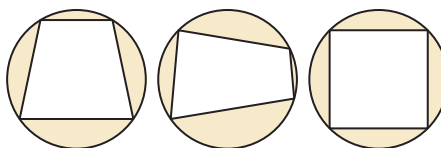
## Monitoring Progress

Use the Venn diagram above to decide whether each statement is true or false.

Explain your reasoning.

1. Some trapezoids are kites.
2. No kites are parallelograms.
3. All parallelograms are rectangles.
4. Some quadrilaterals are squares.
5. Example 1 lists three true statements based on the Venn diagram above. Write six more true statements based on the Venn diagram.

6. A cyclic quadrilateral is a quadrilateral that can be circumscribed by a circle so that the circle touches each vertex. Redraw the Venn diagram so that it includes cyclic quadrilaterals.



# 7.1 Angles of Polygons

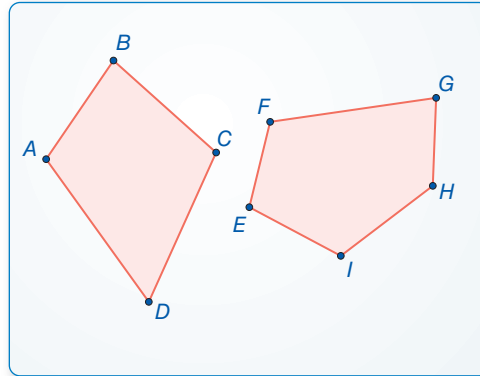
**Essential Question** What is the sum of the measures of the interior angles of a polygon?

## EXPLORATION 1 The Sum of the Angle Measures of a Polygon

**Work with a partner.** Use dynamic geometry software.

- a. Draw a quadrilateral and a pentagon. Find the sum of the measures of the interior angles of each polygon.

Sample



- b. Draw other polygons and find the sums of the measures of their interior angles. Record your results in the table below.

Number of sides, $n$	3	4	5	6	7	8	9
Sum of angle measures, $S$							

- c. Plot the data from your table in a coordinate plane.  
d. Write a function that fits the data. Explain what the function represents.

### CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to reason inductively about data.

## EXPLORATION 2 Measure of One Angle in a Regular Polygon

**Work with a partner.**

- a. Use the function you found in Exploration 1 to write a new function that gives the measure of one interior angle in a regular polygon with  $n$  sides.  
b. Use the function in part (a) to find the measure of one interior angle of a regular pentagon. Use dynamic geometry software to check your result by constructing a regular pentagon and finding the measure of one of its interior angles.  
c. Copy your table from Exploration 1 and add a row for the measure of one interior angle in a regular polygon with  $n$  sides. Complete the table. Use dynamic geometry software to check your results.

### Communicate Your Answer

3. What is the sum of the measures of the interior angles of a polygon?  
4. Find the measure of one interior angle in a regular dodecagon (a polygon with 12 sides).

# 7.1 Lesson

## Core Vocabulary

diagonal, p. 324  
 equilateral polygon, p. 325  
 equiangular polygon, p. 325  
 regular polygon, p. 325

### Previous

polygon  
 convex  
 interior angles  
 exterior angles

## What You Will Learn

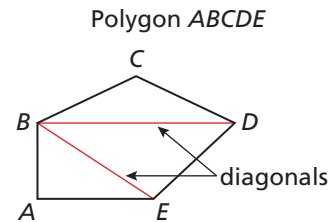
- ▶ Use the interior angle measures of polygons.
- ▶ Use the exterior angle measures of polygons.

## Using Interior Angle Measures of Polygons

In a polygon, two vertices that are endpoints of the same side are called *consecutive vertices*.

A **diagonal** of a polygon is a segment that joins two nonconsecutive vertices.

As you can see, the diagonals from one vertex divide a polygon into triangles. Dividing a polygon with  $n$  sides into  $(n - 2)$  triangles shows that the sum of the measures of the interior angles of a polygon is a multiple of  $180^\circ$ .



$A$  and  $B$  are consecutive vertices.

Vertex  $B$  has two diagonals,  $\overline{BD}$  and  $\overline{BE}$ .

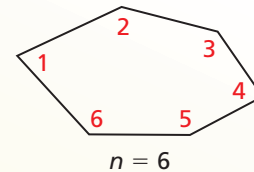
## Theorem

### Theorem 7.1 Polygon Interior Angles Theorem

The sum of the measures of the interior angles of a convex  $n$ -gon is  $(n - 2) \cdot 180^\circ$ .

$$m\angle 1 + m\angle 2 + \cdots + m\angle n = (n - 2) \cdot 180^\circ$$

*Proof* Ex. 42 (for pentagons), p. 329



## REMEMBER

A polygon is *convex* when no line that contains a side of the polygon contains a point in the interior of the polygon.

### EXAMPLE 1 Finding the Sum of Angle Measures in a Polygon

Find the sum of the measures of the interior angles of the figure.



### SOLUTION

The figure is a convex octagon. It has 8 sides. Use the Polygon Interior Angles Theorem.

$$\begin{aligned} (n - 2) \cdot 180^\circ &= (8 - 2) \cdot 180^\circ && \text{Substitute 8 for } n. \\ &= 6 \cdot 180^\circ && \text{Subtract.} \\ &= 1080^\circ && \text{Multiply.} \end{aligned}$$

- ▶ The sum of the measures of the interior angles of the figure is  $1080^\circ$ .

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1. The coin shown is in the shape of an 11-gon. Find the sum of the measures of the interior angles.



## EXAMPLE 2 Finding the Number of Sides of a Polygon

The sum of the measures of the interior angles of a convex polygon is  $900^\circ$ . Classify the polygon by the number of sides.

### SOLUTION

Use the Polygon Interior Angles Theorem to write an equation involving the number of sides  $n$ . Then solve the equation to find the number of sides.

$$\begin{aligned}(n - 2) \cdot 180^\circ &= 900^\circ && \text{Polygon Interior Angles Theorem} \\ n - 2 &= 5 && \text{Divide each side by } 180^\circ. \\ n &= 7 && \text{Add 2 to each side.}\end{aligned}$$

► The polygon has 7 sides. It is a heptagon.

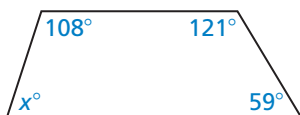
## Corollary

### Corollary 7.1 Corollary to the Polygon Interior Angles Theorem

The sum of the measures of the interior angles of a quadrilateral is  $360^\circ$ .

*Proof* Ex. 43, p. 330

## EXAMPLE 3 Finding an Unknown Interior Angle Measure



Find the value of  $x$  in the diagram.

### SOLUTION

The polygon is a quadrilateral. Use the Corollary to the Polygon Interior Angles Theorem to write an equation involving  $x$ . Then solve the equation.

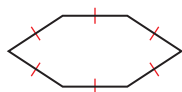
$$\begin{aligned}x^\circ + 108^\circ + 121^\circ + 59^\circ &= 360^\circ && \text{Corollary to the Polygon Interior Angles Theorem} \\ x + 288 &= 360 && \text{Combine like terms.} \\ x &= 72 && \text{Subtract 288 from each side.}\end{aligned}$$

► The value of  $x$  is 72.

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- The sum of the measures of the interior angles of a convex polygon is  $1440^\circ$ . Classify the polygon by the number of sides.
- The measures of the interior angles of a quadrilateral are  $x^\circ$ ,  $3x^\circ$ ,  $5x^\circ$ , and  $7x^\circ$ . Find the measures of all the interior angles.

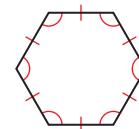
In an **equilateral polygon**, all sides are congruent.



In an **equiangular polygon**, all angles in the interior of the polygon are congruent.



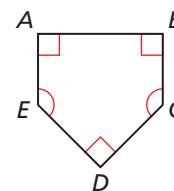
A **regular polygon** is a convex polygon that is both equilateral and equiangular.



### EXAMPLE 4 Finding Angle Measures in Polygons

A home plate for a baseball field is shown.

- Is the polygon regular? Explain your reasoning.
- Find the measures of  $\angle C$  and  $\angle E$ .



#### SOLUTION

- The polygon is not equilateral or equiangular. So, the polygon is not regular.
- Find the sum of the measures of the interior angles.

$$(n - 2) \cdot 180^\circ = (5 - 2) \cdot 180^\circ = 540^\circ \quad \text{Polygon Interior Angles Theorem}$$

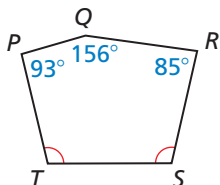
Then write an equation involving  $x$  and solve the equation.

$$x^\circ + x^\circ + 90^\circ + 90^\circ + 90^\circ = 540^\circ \quad \text{Write an equation.}$$

$$2x + 270 = 540 \quad \text{Combine like terms.}$$

$$x = 135 \quad \text{Solve for } x.$$

► So,  $m\angle C = m\angle E = 135^\circ$ .

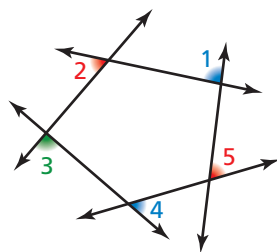


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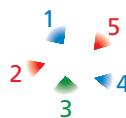
- Find  $m\angle S$  and  $m\angle T$  in the diagram.
- Explain how to find the number of sides of a regular polygon when given an interior angle measure.

## Using Exterior Angle Measures of Polygons

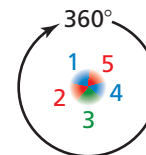
Unlike the sum of the interior angle measures of a convex polygon, the sum of the exterior angle measures does *not* depend on the number of sides of the polygon. The diagrams suggest that the sum of the measures of the exterior angles, one angle at each vertex, of a pentagon is  $360^\circ$ . In general, this sum is  $360^\circ$  for any convex polygon.



**Step 1** Shade one exterior angle at each vertex.



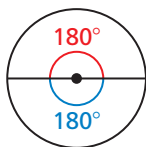
**Step 2** Cut out the exterior angles.



**Step 3** Arrange the exterior angles to form  $360^\circ$ .

### JUSTIFYING STEPS

To help justify this conclusion, you can visualize a circle containing two straight angles. So, there are  $180^\circ + 180^\circ$ , or  $360^\circ$ , in a circle.



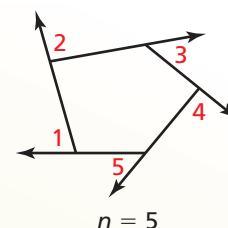
## Theorem

### Theorem 7.2 Polygon Exterior Angles Theorem

The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is  $360^\circ$ .

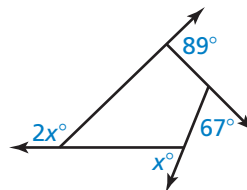
$$m\angle 1 + m\angle 2 + \cdots + m\angle n = 360^\circ$$

*Proof* Ex. 51, p. 330



### EXAMPLE 5 Finding an Unknown Exterior Angle Measure

Find the value of  $x$  in the diagram.



#### SOLUTION

Use the Polygon Exterior Angles Theorem to write and solve an equation.

$$x^\circ + 2x^\circ + 89^\circ + 67^\circ = 360^\circ \quad \text{Polygon Exterior Angles Theorem}$$

$$3x + 156 = 360 \quad \text{Combine like terms.}$$

$$x = 68 \quad \text{Solve for } x.$$

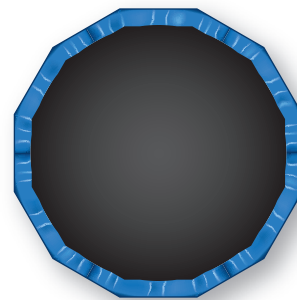
► The value of  $x$  is 68.

### REMEMBER

A *dodecagon* is a polygon with 12 sides and 12 vertices.

### EXAMPLE 6 Finding Angle Measures in Regular Polygons

The trampoline shown is shaped like a regular dodecagon.



- Find the measure of each interior angle.
- Find the measure of each exterior angle.

#### SOLUTION

- Use the Polygon Interior Angles Theorem to find the sum of the measures of the interior angles.

$$\begin{aligned} (n - 2) \cdot 180^\circ &= (12 - 2) \cdot 180^\circ \\ &= 1800^\circ \end{aligned}$$

Then find the measure of one interior angle. A regular dodecagon has 12 congruent interior angles. Divide  $1800^\circ$  by 12.

$$\frac{1800^\circ}{12} = 150^\circ$$

► The measure of each interior angle in the dodecagon is  $150^\circ$ .

- By the Polygon Exterior Angles Theorem, the sum of the measures of the exterior angles, one angle at each vertex, is  $360^\circ$ . Divide  $360^\circ$  by 12 to find the measure of one of the 12 congruent exterior angles.

$$\frac{360^\circ}{12} = 30^\circ$$

► The measure of each exterior angle in the dodecagon is  $30^\circ$ .

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- An interior angle and an adjacent exterior angle of a polygon form a linear pair. How can you use this fact as another method to find the measure of each exterior angle in Example 6?
- Explain how to find the number of sides of a regular polygon when given an exterior angle measure.



## Vocabulary and Core Concept Check

- VOCABULARY** Why do vertices connected by a diagonal of a polygon have to be nonconsecutive?
- WHICH ONE DOESN'T BELONG?** Which sum does *not* belong with the other three? Explain your reasoning.

the sum of the measures of the interior angles of a quadrilateral

the sum of the measures of the exterior angles of a quadrilateral

the sum of the measures of the interior angles of a pentagon

the sum of the measures of the exterior angles of a pentagon

## Monitoring Progress and Modeling with Mathematics

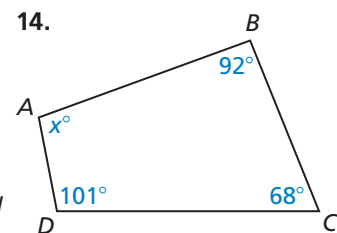
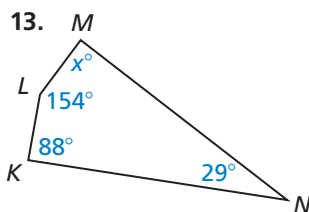
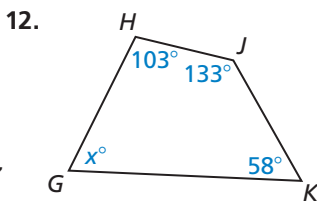
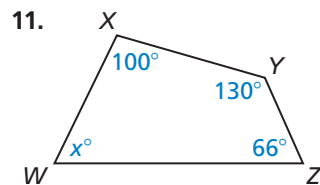
In Exercises 3–6, find the sum of the measures of the interior angles of the indicated convex polygon. (See Example 1.)

- nonagon
- 14-gon
- 16-gon
- 20-gon

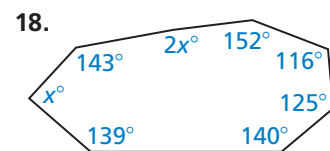
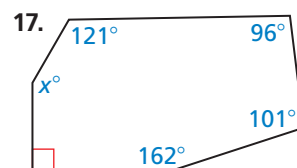
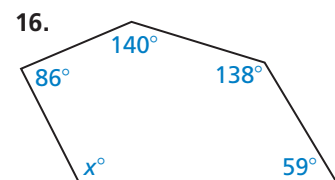
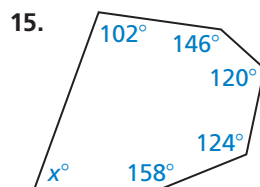
In Exercises 7–10, the sum of the measures of the interior angles of a convex polygon is given. Classify the polygon by the number of sides. (See Example 2.)

- $720^\circ$
- $1080^\circ$
- $2520^\circ$
- $3240^\circ$

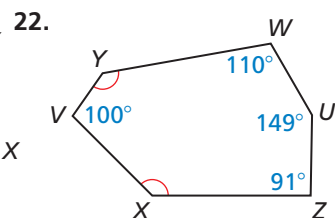
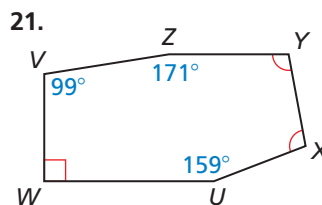
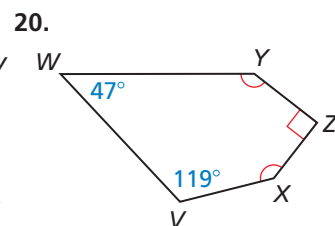
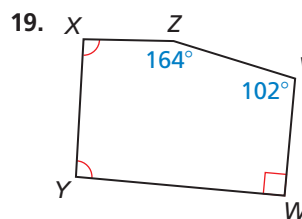
In Exercises 11–14, find the value of  $x$ . (See Example 3.)



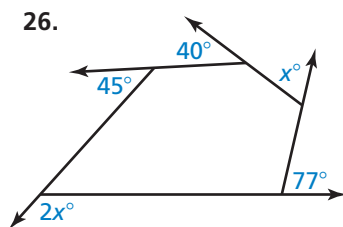
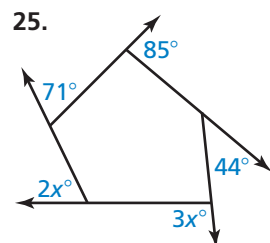
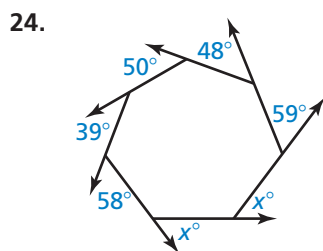
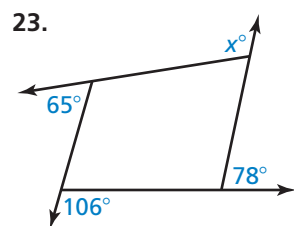
In Exercises 15–18, find the value of  $x$ .



In Exercises 19–22, find the measures of  $\angle X$  and  $\angle Y$ . (See Example 4.)




In Exercises 23–26, find the value of  $x$ . (See Example 5.)



In Exercises 27–30, find the measure of each interior angle and each exterior angle of the indicated regular polygon. (See Example 6.)

27. pentagon                      28. 18-gon  
29. 45-gon                        30. 90-gon


**ERROR ANALYSIS** In Exercises 31 and 32, describe and correct the error in finding the measure of one exterior angle of a regular polygon.

31.  
$$(n - 2) \cdot 180^\circ = (5 - 2) \cdot 180^\circ$$

$$= 3 \cdot 180^\circ$$

$$= 540^\circ$$

The sum of the measures of the angles is  $540^\circ$ . There are five angles, so the measure of one exterior angle is  $\frac{540^\circ}{5} = 108^\circ$ .

32.  There are a total of 10 exterior angles, two at each vertex, so the measure of one exterior angle is  $\frac{360^\circ}{10} = 36^\circ$ .

33. **MODELING WITH MATHEMATICS** The base of a jewelry box is shaped like a regular hexagon. What is the measure of each interior angle of the jewelry box base?

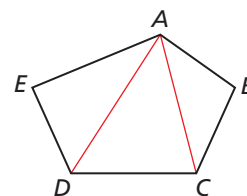
34. **MODELING WITH MATHEMATICS** The floor of the gazebo shown is shaped like a regular decagon. Find the measure of each interior angle of the regular decagon. Then find the measure of each exterior angle.



35. **WRITING A FORMULA** Write a formula to find the number of sides  $n$  in a regular polygon given that the measure of one interior angle is  $x^\circ$ .
36. **WRITING A FORMULA** Write a formula to find the number of sides  $n$  in a regular polygon given that the measure of one exterior angle is  $x^\circ$ .

**REASONING** In Exercises 37–40, find the number of sides for the regular polygon described.

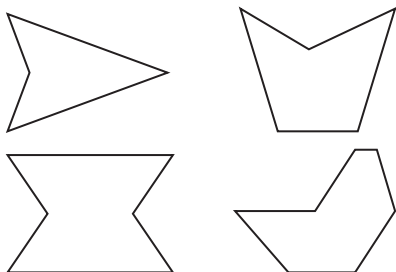
37. Each interior angle has a measure of  $156^\circ$ .
38. Each interior angle has a measure of  $165^\circ$ .
39. Each exterior angle has a measure of  $9^\circ$ .
40. Each exterior angle has a measure of  $6^\circ$ .
41. **DRAWING CONCLUSIONS** Which of the following angle measures are possible interior angle measures of a regular polygon? Explain your reasoning. Select all that apply.
- (A)  $162^\circ$     (B)  $171^\circ$     (C)  $75^\circ$     (D)  $40^\circ$
42. **PROVING A THEOREM** The Polygon Interior Angles Theorem (Theorem 7.1) states that the sum of the measures of the interior angles of a convex  $n$ -gon is  $(n - 2) \cdot 180^\circ$ . Write a paragraph proof of this theorem for the case when  $n = 5$ .



43. **PROVING A COROLLARY** Write a paragraph proof of the Corollary to the Polygon Interior Angles Theorem (Corollary 7.1).
44. **MAKING AN ARGUMENT** Your friend claims that to find the interior angle measures of a regular polygon, you do not have to use the Polygon Interior Angles Theorem (Theorem 7.1). You instead can use the Polygon Exterior Angles Theorem (Theorem 7.2) and then the Linear Pair Postulate (Postulate 2.8). Is your friend correct? Explain your reasoning.
45. **MATHEMATICAL CONNECTIONS** In an equilateral hexagon, four of the exterior angles each have a measure of  $x^\circ$ . The other two exterior angles each have a measure of twice the sum of  $x$  and 48. Find the measure of each exterior angle.

46. **THOUGHT PROVOKING** For a concave polygon, is it true that at least one of the interior angle measures must be greater than  $180^\circ$ ? If not, give an example. If so, explain your reasoning.

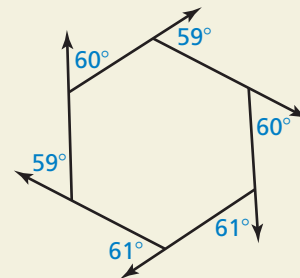
47. **WRITING EXPRESSIONS** Write an expression to find the sum of the measures of the interior angles for a concave polygon. Explain your reasoning.



48. **ANALYZING RELATIONSHIPS** Polygon  $ABCDEFGH$  is a regular octagon. Suppose sides  $\overline{AB}$  and  $\overline{CD}$  are extended to meet at a point  $P$ . Find  $m\angle BPC$ . Explain your reasoning. Include a diagram with your answer.

49. **MULTIPLE REPRESENTATIONS** The formula for the measure of each interior angle in a regular polygon can be written in function notation.
- Write a function  $h(n)$ , where  $n$  is the number of sides in a regular polygon and  $h(n)$  is the measure of any interior angle in the regular polygon.
  - Use the function to find  $h(9)$ .
  - Use the function to find  $n$  when  $h(n) = 150^\circ$ .
  - Plot the points for  $n = 3, 4, 5, 6, 7$ , and 8. What happens to the value of  $h(n)$  as  $n$  gets larger?

50. **HOW DO YOU SEE IT?** Is the hexagon a regular hexagon? Explain your reasoning.



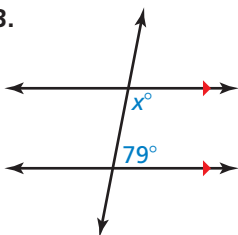
51. **PROVING A THEOREM** Write a paragraph proof of the Polygon Exterior Angles Theorem (Theorem 7.2). (*Hint:* In a convex  $n$ -gon, the sum of the measures of an interior angle and an adjacent exterior angle at any vertex is  $180^\circ$ .)
52. **ABSTRACT REASONING** You are given a convex polygon. You are asked to draw a new polygon by increasing the sum of the interior angle measures by  $540^\circ$ . How many more sides does your new polygon have? Explain your reasoning.

## Maintaining Mathematical Proficiency

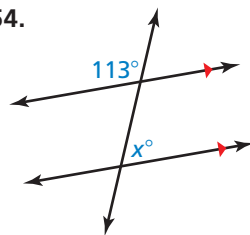
Reviewing what you learned in previous grades and lessons

Find the value of  $x$ .

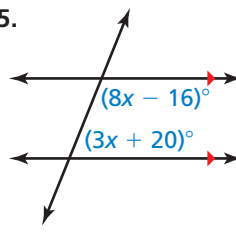
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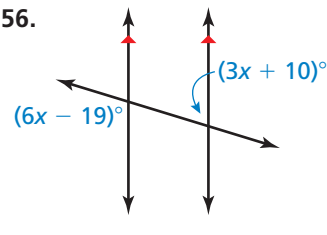
54.



55.



56.



## 7.2 Properties of Parallelograms

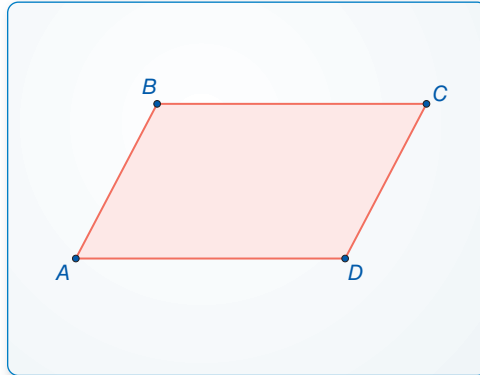
**Essential Question** What are the properties of parallelograms?

### EXPLORATION 1 Discovering Properties of Parallelograms

**Work with a partner.** Use dynamic geometry software.

- a. Construct any parallelogram and label it  $ABCD$ . Explain your process.

Sample



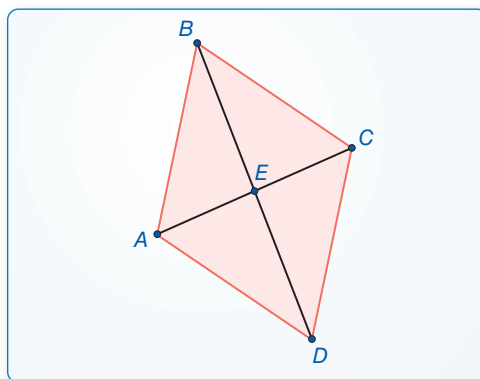
- b. Find the angle measures of the parallelogram. What do you observe?  
c. Find the side lengths of the parallelogram. What do you observe?  
d. Repeat parts (a)–(c) for several other parallelograms. Use your results to write conjectures about the angle measures and side lengths of a parallelogram.

### EXPLORATION 2 Discovering a Property of Parallelograms

**Work with a partner.** Use dynamic geometry software.

- a. Construct any parallelogram and label it  $ABCD$ .  
b. Draw the two diagonals of the parallelogram. Label the point of intersection  $E$ .

Sample



- c. Find the segment lengths  $AE$ ,  $BE$ ,  $CE$ , and  $DE$ . What do you observe?  
d. Repeat parts (a)–(c) for several other parallelograms. Use your results to write a conjecture about the diagonals of a parallelogram.

### MAKING SENSE OF PROBLEMS

To be proficient in math, you need to analyze givens, constraints, relationships, and goals.

### Communicate Your Answer

3. What are the properties of parallelograms?

# 7.2 Lesson

## Core Vocabulary

parallelogram, p. 332

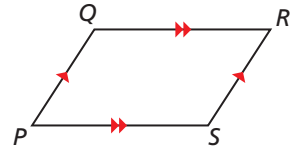
**Previous**  
 quadrilateral  
 diagonal  
 interior angles  
 segment bisector

## What You Will Learn

- ▶ Use properties to find side lengths and angles of parallelograms.
- ▶ Use parallelograms in the coordinate plane.

## Using Properties of Parallelograms

A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel. In  $\square PQRS$ ,  $\overline{PQ} \parallel \overline{RS}$  and  $\overline{QR} \parallel \overline{PS}$  by definition. The theorems below describe other properties of parallelograms.



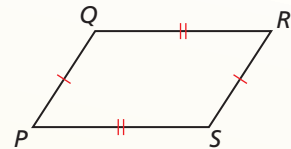
## Theorems

### Theorem 7.3 Parallelogram Opposite Sides Theorem

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

If  $PQRS$  is a parallelogram, then  $\overline{PQ} \cong \overline{RS}$  and  $\overline{QR} \cong \overline{SP}$ .

*Proof* p. 332

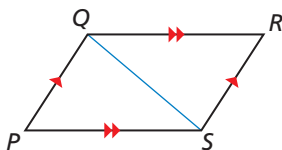
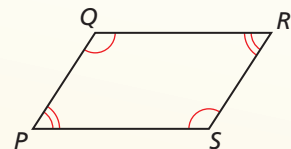


### Theorem 7.4 Parallelogram Opposite Angles Theorem

If a quadrilateral is a parallelogram, then its opposite angles are congruent.

If  $PQRS$  is a parallelogram, then  $\angle P \cong \angle R$  and  $\angle Q \cong \angle S$ .

*Proof* Ex. 37, p. 337



## PROOF Parallelogram Opposite Sides Theorem

**Given**  $PQRS$  is a parallelogram.

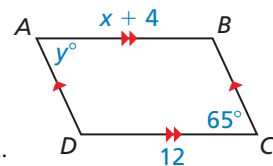
**Prove**  $\overline{PQ} \cong \overline{RS}$ ,  $\overline{QR} \cong \overline{SP}$

- Plan for Proof**
- a. Draw diagonal  $\overline{QS}$  to form  $\triangle PQS$  and  $\triangle RSQ$ .
  - b. Use the ASA Congruence Theorem (Thm. 5.10) to show that  $\triangle PQS \cong \triangle RSQ$ .
  - c. Use congruent triangles to show that  $\overline{PQ} \cong \overline{RS}$  and  $\overline{QR} \cong \overline{SP}$ .

Plan in Action	STATEMENTS	REASONS
	1. $PQRS$ is a parallelogram.	1. Given
	a. 2. Draw $\overline{QS}$ .	2. Through any two points, there exists exactly one line.
	3. $\overline{PQ} \parallel \overline{RS}$ , $\overline{QR} \parallel \overline{PS}$	3. Definition of parallelogram
	b. 4. $\angle PQS \cong \angle RSQ$ , $\angle PSQ \cong \angle RQS$	4. Alternate Interior Angles Theorem (Thm. 3.2)
	5. $\overline{QS} \cong \overline{SQ}$	5. Reflexive Property of Congruence (Thm. 2.1)
	6. $\triangle PQS \cong \triangle RSQ$	6. ASA Congruence Theorem (Thm. 5.10)
	c. 7. $\overline{PQ} \cong \overline{RS}$ , $\overline{QR} \cong \overline{SP}$	7. Corresponding parts of congruent triangles are congruent.

### EXAMPLE 1 Using Properties of Parallelograms

Find the values of  $x$  and  $y$ .



#### SOLUTION

$ABCD$  is a parallelogram by the definition of a parallelogram. Use the Parallelogram Opposite Sides Theorem to find the value of  $x$ .

$$AB = CD \quad \text{Opposite sides of a parallelogram are congruent.}$$

$$x + 4 = 12 \quad \text{Substitute } x + 4 \text{ for } AB \text{ and } 12 \text{ for } CD.$$

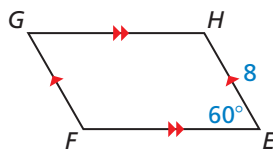
$$x = 8 \quad \text{Subtract 4 from each side.}$$

By the Parallelogram Opposite Angles Theorem,  $\angle A \cong \angle C$ , or  $m\angle A = m\angle C$ . So,  $y^\circ = 65^\circ$ .

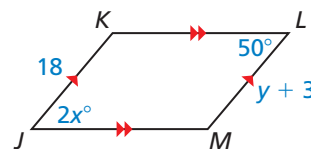
► In  $\square ABCD$ ,  $x = 8$  and  $y = 65$ .

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1. Find  $FG$  and  $m\angle G$ .

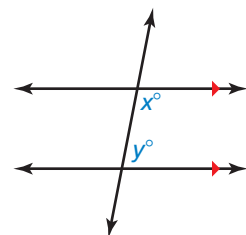


2. Find the values of  $x$  and  $y$ .



The Consecutive Interior Angles Theorem (Theorem 3.4) states that if two parallel lines are cut by a transversal, then the pairs of consecutive interior angles formed are supplementary.

A pair of consecutive angles in a parallelogram is like a pair of consecutive interior angles between parallel lines. This similarity suggests the Parallelogram Consecutive Angles Theorem.



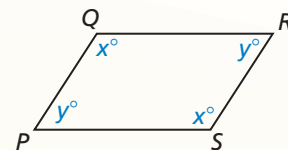
## Theorems

### Theorem 7.5 Parallelogram Consecutive Angles Theorem

If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

If  $PQRS$  is a parallelogram, then  $x^\circ + y^\circ = 180^\circ$ .

*Proof* Ex. 38, p. 337

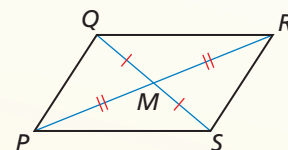


### Theorem 7.6 Parallelogram Diagonals Theorem

If a quadrilateral is a parallelogram, then its diagonals bisect each other.

If  $PQRS$  is a parallelogram, then  $\overline{QM} \cong \overline{SM}$  and  $\overline{PM} \cong \overline{RM}$ .

*Proof* p. 334

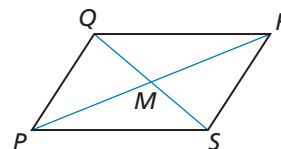


## PROOF

 Parallelogram Diagonals Theorem

**Given**  $PQRS$  is a parallelogram. Diagonals  $\overline{PR}$  and  $\overline{QS}$  intersect at point  $M$ .

**Prove**  $M$  bisects  $\overline{QS}$  and  $\overline{PR}$ .



STATEMENTS	REASONS
1. $PQRS$ is a parallelogram.	1. Given
2. $\overline{PQ} \parallel \overline{RS}$	2. Definition of a parallelogram
3. $\angle QPR \cong \angle SRP$ , $\angle PQS \cong \angle RSQ$	3. Alternate Interior Angles Theorem (Thm. 3.2)
4. $\overline{PQ} \cong \overline{RS}$	4. Parallelogram Opposite Sides Theorem
5. $\triangle PMQ \cong \triangle RMS$	5. ASA Congruence Theorem (Thm. 5.10)
6. $\overline{QM} \cong \overline{SM}$ , $\overline{PM} \cong \overline{RM}$	6. Corresponding parts of congruent triangles are congruent.
7. $M$ bisects $\overline{QS}$ and $\overline{PR}$ .	7. Definition of segment bisector

## EXAMPLE 2

 Using Properties of a Parallelogram

As shown, part of the extending arm of a desk lamp is a parallelogram. The angles of the parallelogram change as the lamp is raised and lowered. Find  $m\angle BCD$  when  $m\angle ADC = 110^\circ$ .



### SOLUTION

By the Parallelogram Consecutive Angles Theorem, the consecutive angle pairs in  $\square ABCD$  are supplementary. So,  $m\angle ADC + m\angle BCD = 180^\circ$ . Because  $m\angle ADC = 110^\circ$ ,  $m\angle BCD = 180^\circ - 110^\circ = 70^\circ$ .

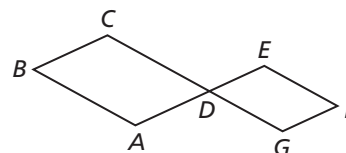
## EXAMPLE 3

 Writing a Two-Column Proof

Write a two-column proof.

**Given**  $ABCD$  and  $GDEF$  are parallelograms.

**Prove**  $\angle B \cong \angle F$



STATEMENTS	REASONS
1. $ABCD$ and $GDEF$ are parallelograms.	1. Given
2. $\angle CDA \cong \angle B$ , $\angle EDG \cong \angle F$	2. If a quadrilateral is a parallelogram, then its opposite angles are congruent.
3. $\angle CDA \cong \angle EDG$	3. Vertical Angles Congruence Theorem (Thm. 2.6)
4. $\angle B \cong \angle F$	4. Transitive Property of Congruence (Thm. 2.2)

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- WHAT IF?** In Example 2, find  $m\angle BCD$  when  $m\angle ADC$  is twice the measure of  $\angle BCD$ .
- Using the figure and the given statement in Example 3, prove that  $\angle C$  and  $\angle F$  are supplementary angles.

## Using Parallelograms in the Coordinate Plane

### JUSTIFYING STEPS

In Example 4, you can use either diagonal to find the coordinates of the intersection. Using diagonal  $\overline{OM}$  helps simplify the calculation because one endpoint is  $(0, 0)$ .

### EXAMPLE 4 Using Parallelograms in the Coordinate Plane

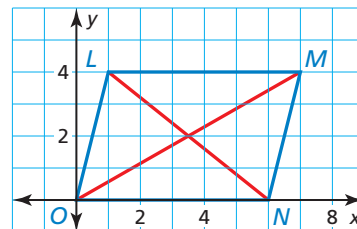
Find the coordinates of the intersection of the diagonals of  $\square LMNO$  with vertices  $L(1, 4)$ ,  $M(7, 4)$ ,  $N(6, 0)$ , and  $O(0, 0)$ .

#### SOLUTION

By the Parallelogram Diagonals Theorem, the diagonals of a parallelogram bisect each other. So, the coordinates of the intersection are the midpoints of diagonals  $\overline{LN}$  and  $\overline{OM}$ .

$$\text{coordinates of midpoint of } \overline{OM} = \left( \frac{7+0}{2}, \frac{4+0}{2} \right) = \left( \frac{7}{2}, 2 \right) \quad \text{Midpoint Formula}$$

► The coordinates of the intersection of the diagonals are  $\left( \frac{7}{2}, 2 \right)$ . You can check your answer by graphing  $\square LMNO$  and drawing the diagonals. The point of intersection appears to be correct.



### REMEMBER

When graphing a polygon in the coordinate plane, the name of the polygon gives the order of the vertices.

### EXAMPLE 5 Using Parallelograms in the Coordinate Plane

Three vertices of  $\square WXYZ$  are  $W(-1, -3)$ ,  $X(-3, 2)$ , and  $Z(4, -4)$ . Find the coordinates of vertex  $Y$ .

#### SOLUTION

**Step 1** Graph the vertices  $W$ ,  $X$ , and  $Z$ .

**Step 2** Find the slope of  $\overline{WX}$ .

$$\text{slope of } \overline{WX} = \frac{2 - (-3)}{-3 - (-1)} = \frac{5}{-2} = -\frac{5}{2}$$

**Step 3** Start at  $Z(4, -4)$ . Use the rise and run from Step 2 to find vertex  $Y$ .

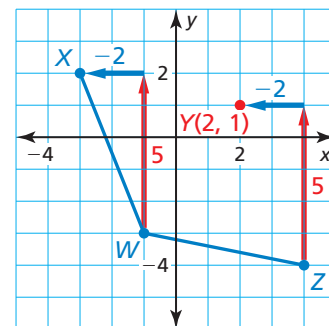
A rise of 5 represents a change of 5 units up. A run of  $-2$  represents a change of 2 units left.

So, plot the point that is 5 units up and 2 units left from  $Z(4, -4)$ . The point is  $(2, 1)$ . Label it as vertex  $Y$ .

**Step 4** Find the slopes of  $\overline{XY}$  and  $\overline{WZ}$  to verify that they are parallel.

$$\text{slope of } \overline{XY} = \frac{1 - 2}{2 - (-3)} = \frac{-1}{5} = -\frac{1}{5} \quad \text{slope of } \overline{WZ} = \frac{-4 - (-3)}{4 - (-1)} = \frac{-1}{5} = -\frac{1}{5}$$

► So, the coordinates of vertex  $Y$  are  $(2, 1)$ .



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- Find the coordinates of the intersection of the diagonals of  $\square STUV$  with vertices  $S(-2, 3)$ ,  $T(1, 5)$ ,  $U(6, 3)$ , and  $V(3, 1)$ .
- Three vertices of  $\square ABCD$  are  $A(2, 4)$ ,  $B(5, 2)$ , and  $C(3, -1)$ . Find the coordinates of vertex  $D$ .



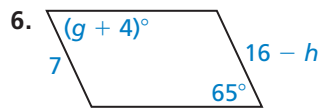
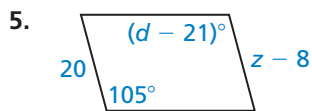
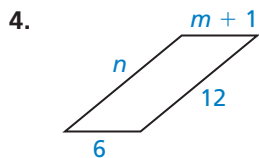
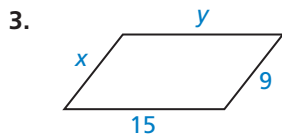
# 7.2 Exercises

## Vocabulary and Core Concept Check

- VOCABULARY** Why is a parallelogram always a quadrilateral, but a quadrilateral is only sometimes a parallelogram?
- WRITING** You are given one angle measure of a parallelogram. Explain how you can find the other angle measures of the parallelogram.

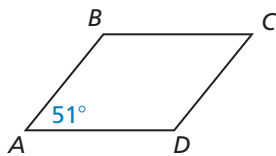
## Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the value of each variable in the parallelogram. (See Example 1.)

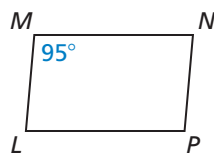


In Exercises 7 and 8, find the measure of the indicated angle in the parallelogram. (See Example 2.)

7. Find  $m\angle B$ .

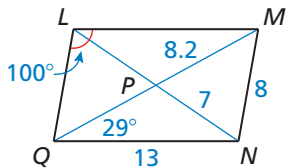


8. Find  $m\angle N$ .

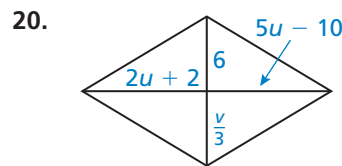
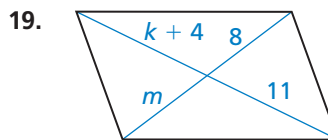
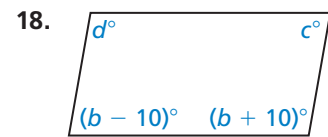
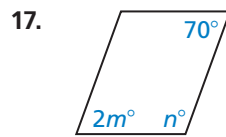


In Exercises 9–16, find the indicated measure in  $\square LMNQ$ . Explain your reasoning.

- $LM$
- $LP$
- $LQ$
- $MQ$
- $m\angle LMN$
- $m\angle NQL$
- $m\angle MNQ$
- $m\angle LMQ$



In Exercises 17–20, find the value of each variable in the parallelogram.



**ERROR ANALYSIS** In Exercises 21 and 22, describe and correct the error in using properties of parallelograms.

21. A parallelogram  $STUV$  with angle  $S = 50^\circ$ . A red  $X$  is next to it.

Because quadrilateral  $STUV$  is a parallelogram,  $\angle S \cong \angle V$ . So,  $m\angle V = 50^\circ$ .

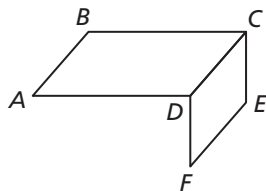
22. A parallelogram  $GHJK$  with diagonals intersecting at  $F$ . A red  $X$  is next to it.

Because quadrilateral  $GHJK$  is a parallelogram,  $\overline{GF} \cong \overline{FH}$ .

**PROOF** In Exercises 23 and 24, write a two-column proof. (See Example 3.)

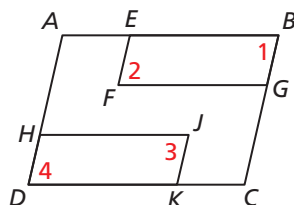
23. **Given**  $ABCD$  and  $CEFD$  are parallelograms.

**Prove**  $\overline{AB} \cong \overline{FE}$



24. **Given**  $ABCD$ ,  $EBGF$ , and  $HJKD$  are parallelograms.

**Prove**  $\angle 2 \cong \angle 3$



In Exercises 25 and 26, find the coordinates of the intersection of the diagonals of the parallelogram with the given vertices. (See Example 4.)

25.  $W(-2, 5)$ ,  $X(2, 5)$ ,  $Y(4, 0)$ ,  $Z(0, 0)$   
 26.  $Q(-1, 3)$ ,  $R(5, 2)$ ,  $S(1, -2)$ ,  $T(-5, -1)$

In Exercises 27–30, three vertices of  $\square DEFG$  are given. Find the coordinates of the remaining vertex. (See Example 5.)

27.  $D(0, 2)$ ,  $E(-1, 5)$ ,  $G(4, 0)$   
 28.  $D(-2, -4)$ ,  $F(0, 7)$ ,  $G(1, 0)$   
 29.  $D(-4, -2)$ ,  $E(-3, 1)$ ,  $F(3, 3)$   
 30.  $E(-1, 4)$ ,  $F(5, 6)$ ,  $G(8, 0)$

**MATHEMATICAL CONNECTIONS** In Exercises 31 and 32, find the measure of each angle.

31. The measure of one interior angle of a parallelogram is 0.25 times the measure of another angle.  
 32. The measure of one interior angle of a parallelogram is 50 degrees more than 4 times the measure of another angle.  
 33. **MAKING AN ARGUMENT** In quadrilateral  $ABCD$ ,  $m\angle B = 124^\circ$ ,  $m\angle A = 56^\circ$ , and  $m\angle C = 124^\circ$ . Your friend claims quadrilateral  $ABCD$  could be a parallelogram. Is your friend correct? Explain your reasoning.

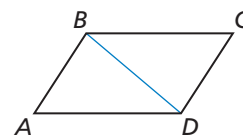
34. **ATTENDING TO PRECISION**  $\angle J$  and  $\angle K$  are consecutive angles in a parallelogram,  $m\angle J = (3x + 7)^\circ$ , and  $m\angle K = (5x - 11)^\circ$ . Find the measure of each angle.

35. **CONSTRUCTION** Construct any parallelogram and label it  $ABCD$ . Draw diagonals  $\overline{AC}$  and  $\overline{BD}$ . Explain how to use paper folding to verify the Parallelogram Diagonals Theorem (Theorem 7.6) for  $\square ABCD$ .

36. **MODELING WITH MATHEMATICS** The feathers on an arrow form two congruent parallelograms. The parallelograms are reflections of each other over the line that contains their shared side. Show that  $m\angle 2 = 2m\angle 1$ .



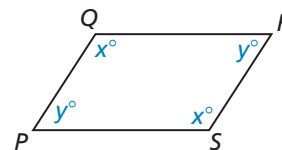
37. **PROVING A THEOREM** Use the diagram to write a two-column proof of the Parallelogram Opposite Angles Theorem (Theorem 7.4).



**Given**  $ABCD$  is a parallelogram.

**Prove**  $\angle A \cong \angle C$ ,  $\angle B \cong \angle D$

38. **PROVING A THEOREM** Use the diagram to write a two-column proof of the Parallelogram Consecutive Angles Theorem (Theorem 7.5).



**Given**  $PQRS$  is a parallelogram.

**Prove**  $x^\circ + y^\circ = 180^\circ$

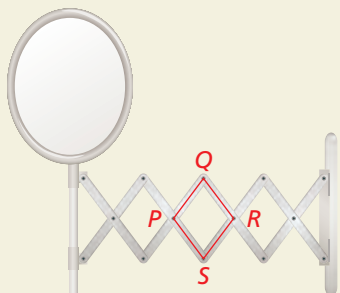
39. **PROBLEM SOLVING** The sides of  $\square MNPQ$  are represented by the expressions below. Sketch  $\square MNPQ$  and find its perimeter.

$$\begin{aligned} MQ &= -2x + 37 & QP &= y + 14 \\ NP &= x - 5 & MN &= 4y + 5 \end{aligned}$$

40. **PROBLEM SOLVING** In  $\square LMNP$ , the ratio of  $LM$  to  $MN$  is 4:3. Find  $LM$  when the perimeter of  $\square LMNP$  is 28.

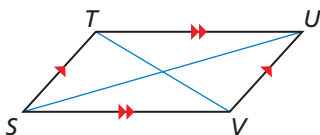
41. **ABSTRACT REASONING** Can you prove that two parallelograms are congruent by proving that all their corresponding sides are congruent? Explain your reasoning.

42. **HOW DO YOU SEE IT?** The mirror shown is attached to the wall by an arm that can extend away from the wall. In the figure, points  $P$ ,  $Q$ ,  $R$ , and  $S$  are the vertices of a parallelogram. This parallelogram is one of several that change shape as the mirror is extended.



- What happens to  $m\angle P$  as  $m\angle Q$  increases? Explain.
- What happens to  $QS$  as  $m\angle Q$  decreases? Explain.
- What happens to the overall distance between the mirror and the wall when  $m\angle Q$  decreases? Explain.

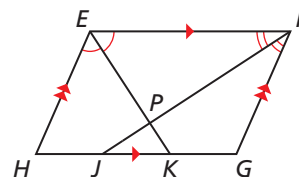
43. **MATHEMATICAL CONNECTIONS** In  $\square STUV$ ,  $m\angle TSU = 32^\circ$ ,  $m\angle USV = (x^2)^\circ$ ,  $m\angle TUV = 12x^\circ$ , and  $\angle TUV$  is an acute angle. Find  $m\angle USV$ .



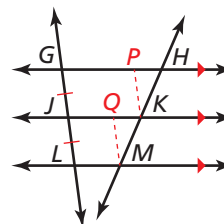
44. **THOUGHT PROVOKING** Is it possible that any triangle can be partitioned into four congruent triangles that can be rearranged to form a parallelogram? Explain your reasoning.

45. **CRITICAL THINKING** Points  $W(1, 2)$ ,  $X(3, 6)$ , and  $Y(6, 4)$  are three vertices of a parallelogram. How many parallelograms can be created using these three vertices? Find the coordinates of each point that could be the fourth vertex.

46. **PROOF** In the diagram,  $\overline{EK}$  bisects  $\angle FEH$ , and  $\overline{FJ}$  bisects  $\angle EFG$ . Prove that  $\overline{EK} \perp \overline{FJ}$ . (Hint: Write equations using the angle measures of the triangles and quadrilaterals formed.)



47. **PROOF** Prove the *Congruent Parts of Parallel Lines Corollary*: If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.



**Given**  $\overrightarrow{GH} \parallel \overrightarrow{JK} \parallel \overrightarrow{LM}$ ,  $\overline{GJ} \cong \overline{JL}$

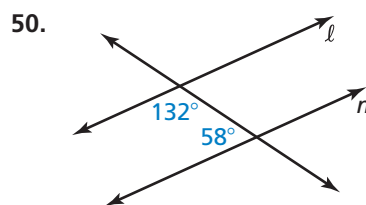
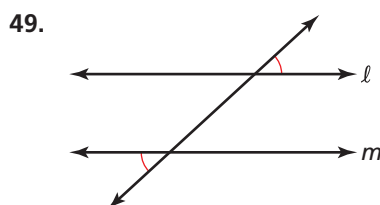
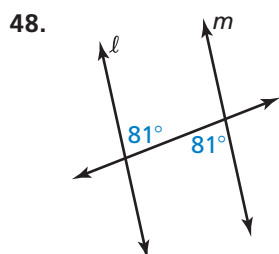
**Prove**  $\overline{HK} \cong \overline{KM}$

(Hint: Draw  $\overline{KP}$  and  $\overline{MQ}$  such that quadrilateral  $GPKJ$  and quadrilateral  $JQML$  are parallelograms.)

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Determine whether lines  $\ell$  and  $m$  are parallel. Explain your reasoning.

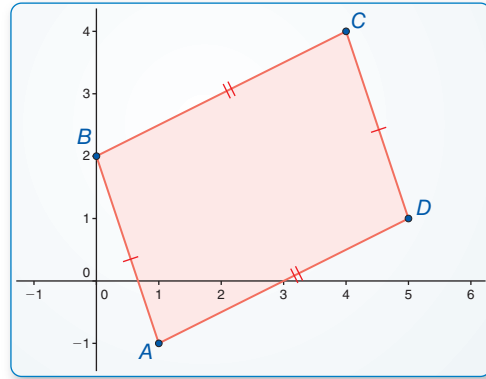


# 7.3 Proving That a Quadrilateral Is a Parallelogram

**Essential Question** How can you prove that a quadrilateral is a parallelogram?

## EXPLORATION 1 Proving That a Quadrilateral Is a Parallelogram

**Work with a partner.**  
Use dynamic geometry software.



**Sample**  
Points  
 $A(1, -1)$   
 $B(0, 2)$   
 $C(4, 4)$   
 $D(5, 1)$   
Segments  
 $AB = 3.16$   
 $BC = 4.47$   
 $CD = 3.16$   
 $DA = 4.47$

### REASONING ABSTRACTLY

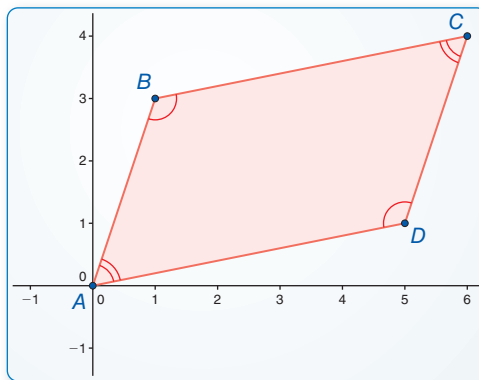
To be proficient in math, you need to know and flexibly use different properties of objects.

- Construct any quadrilateral  $ABCD$  whose opposite sides are congruent.
- Is the quadrilateral a parallelogram? Justify your answer.
- Repeat parts (a) and (b) for several other quadrilaterals. Then write a conjecture based on your results.
- Write the converse of your conjecture. Is the converse true? Explain.

## EXPLORATION 2 Proving That a Quadrilateral Is a Parallelogram

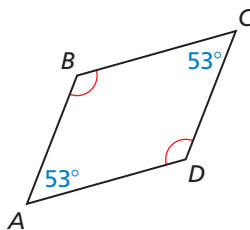
**Work with a partner.** Use dynamic geometry software.

- Construct any quadrilateral  $ABCD$  whose opposite angles are congruent.
- Is the quadrilateral a parallelogram? Justify your answer.



**Sample**  
Points  
 $A(0, 0)$   
 $B(1, 3)$   
 $C(6, 4)$   
 $D(5, 1)$   
Angles  
 $\angle A = 60.26^\circ$   
 $\angle B = 119.74^\circ$   
 $\angle C = 60.26^\circ$   
 $\angle D = 119.74^\circ$

- Repeat parts (a) and (b) for several other quadrilaterals. Then write a conjecture based on your results.
- Write the converse of your conjecture. Is the converse true? Explain.



### Communicate Your Answer

- How can you prove that a quadrilateral is a parallelogram?
- Is the quadrilateral at the left a parallelogram? Explain your reasoning.

# 7.3 Lesson

## Core Vocabulary

Previous  
diagonal  
parallelogram

## What You Will Learn

- ▶ Identify and verify parallelograms.
- ▶ Show that a quadrilateral is a parallelogram in the coordinate plane.

## Identifying and Verifying Parallelograms

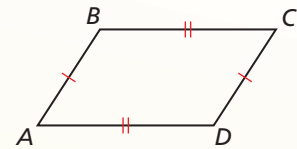
Given a parallelogram, you can use the Parallelogram Opposite Sides Theorem (Theorem 7.3) and the Parallelogram Opposite Angles Theorem (Theorem 7.4) to prove statements about the sides and angles of the parallelogram. The converses of the theorems are stated below. You can use these and other theorems in this lesson to prove that a quadrilateral with certain properties is a parallelogram.

## Theorems

### Theorem 7.7 Parallelogram Opposite Sides Converse

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

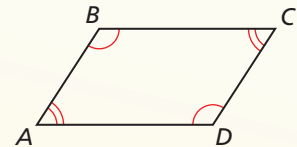
If  $\overline{AB} \cong \overline{CD}$  and  $\overline{BC} \cong \overline{DA}$ , then  $ABCD$  is a parallelogram.



### Theorem 7.8 Parallelogram Opposite Angles Converse

If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If  $\angle A \cong \angle C$  and  $\angle B \cong \angle D$ , then  $ABCD$  is a parallelogram.



*Proof* Ex. 39, p. 347

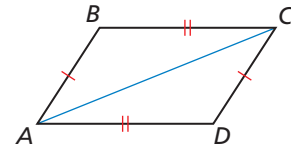
## PROOF

### Parallelogram Opposite Sides Converse

**Given**  $\overline{AB} \cong \overline{CD}, \overline{BC} \cong \overline{DA}$

**Prove**  $ABCD$  is a parallelogram.

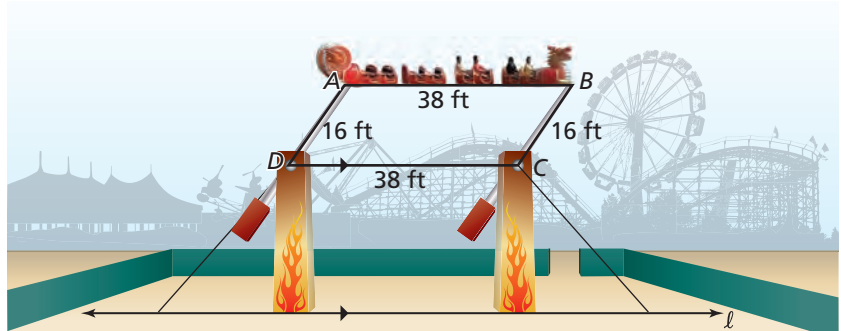
- Plan for Proof**
- a. Draw diagonal  $\overline{AC}$  to form  $\triangle ABC$  and  $\triangle CDA$ .
  - b. Use the SSS Congruence Theorem (Thm. 5.8) to show that  $\triangle ABC \cong \triangle CDA$ .
  - c. Use the Alternate Interior Angles Converse (Thm. 3.6) to show that opposite sides are parallel.



Plan in Action	STATEMENTS	REASONS
	a. 1. $\overline{AB} \cong \overline{CD}, \overline{BC} \cong \overline{DA}$	1. Given
	2. Draw $\overline{AC}$ .	2. Through any two points, there exists exactly one line.
	3. $\overline{AC} \cong \overline{CA}$	3. Reflexive Property of Congruence (Thm. 2.1)
	b. 4. $\triangle ABC \cong \triangle CDA$	4. SSS Congruence Theorem (Thm. 5.8)
	c. 5. $\angle BAC \cong \angle DCA,$ $\angle BCA \cong \angle DAC$	5. Corresponding parts of congruent triangles are congruent.
	6. $\overline{AB} \parallel \overline{CD}, \overline{BC} \parallel \overline{DA}$	6. Alternate Interior Angles Converse (Thm. 3.6)
	7. $ABCD$ is a parallelogram.	7. Definition of parallelogram

### EXAMPLE 1 Identifying a Parallelogram

An amusement park ride has a moving platform attached to four swinging arms. The platform swings back and forth, higher and higher, until it goes over the top and around in a circular motion. In the diagram below,  $\overline{AD}$  and  $\overline{BC}$  represent two of the swinging arms, and  $\overline{DC}$  is parallel to the ground (line  $\ell$ ). Explain why the moving platform  $\overline{AB}$  is always parallel to the ground.



#### SOLUTION

The shape of quadrilateral  $ABCD$  changes as the moving platform swings around, but its side lengths do not change. Both pairs of opposite sides are congruent, so  $ABCD$  is a parallelogram by the Parallelogram Opposite Sides Converse.

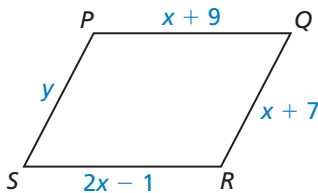
By the definition of a parallelogram,  $\overline{AB} \parallel \overline{DC}$ . Because  $\overline{DC}$  is parallel to line  $\ell$ ,  $\overline{AB}$  is also parallel to line  $\ell$  by the Transitive Property of Parallel Lines (Theorem 3.9). So, the moving platform is parallel to the ground.

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- In quadrilateral  $WXYZ$ ,  $m\angle W = 42^\circ$ ,  $m\angle X = 138^\circ$ , and  $m\angle Y = 42^\circ$ . Find  $m\angle Z$ . Is  $WXYZ$  a parallelogram? Explain your reasoning.

### EXAMPLE 2 Finding Side Lengths of a Parallelogram

For what values of  $x$  and  $y$  is quadrilateral  $PQRS$  a parallelogram?



#### SOLUTION

By the Parallelogram Opposite Sides Converse, if both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. Find  $x$  so that  $\overline{PQ} \cong \overline{SR}$ .

$$PQ = SR \quad \text{Set the segment lengths equal.}$$

$$x + 9 = 2x - 1 \quad \text{Substitute } x + 9 \text{ for } PQ \text{ and } 2x - 1 \text{ for } SR.$$

$$10 = x \quad \text{Solve for } x.$$

When  $x = 10$ ,  $PQ = 10 + 9 = 19$  and  $SR = 2(10) - 1 = 19$ . Find  $y$  so that  $\overline{PS} \cong \overline{QR}$ .

$$PS = QR \quad \text{Set the segment lengths equal.}$$

$$y = x + 7 \quad \text{Substitute } y \text{ for } PS \text{ and } x + 7 \text{ for } QR.$$

$$y = 10 + 7 \quad \text{Substitute } 10 \text{ for } x.$$

$$y = 17 \quad \text{Add.}$$

When  $x = 10$  and  $y = 17$ ,  $PS = 17$  and  $QR = 10 + 7 = 17$ .

- Quadrilateral  $PQRS$  is a parallelogram when  $x = 10$  and  $y = 17$ .

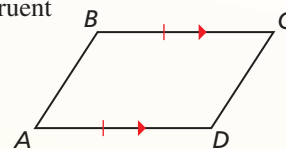
## Theorems

### Theorem 7.9 Opposite Sides Parallel and Congruent Theorem

If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.

If  $\overline{BC} \parallel \overline{AD}$  and  $\overline{BC} \cong \overline{AD}$ , then  $ABCD$  is a parallelogram.

*Proof* Ex. 40, p. 347

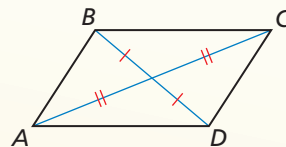


### Theorem 7.10 Parallelogram Diagonals Converse

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

If  $\overline{BD}$  and  $\overline{AC}$  bisect each other, then  $ABCD$  is a parallelogram.

*Proof* Ex. 41, p. 347



### EXAMPLE 3 Identifying a Parallelogram

The doorway shown is part of a building in England. Over time, the building has leaned sideways. Explain how you know that  $SV = TU$ .

#### SOLUTION

In the photograph,  $\overline{ST} \parallel \overline{UV}$  and  $\overline{ST} \cong \overline{UV}$ . By the Opposite Sides Parallel and Congruent Theorem, quadrilateral  $STUV$  is a parallelogram. By the Parallelogram Opposite Sides Theorem (Theorem 7.3), you know that opposite sides of a parallelogram are congruent. So,  $SV = TU$ .



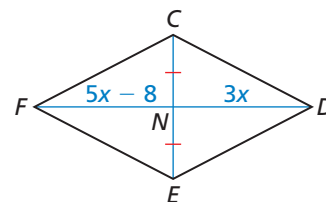
### EXAMPLE 4 Finding Diagonal Lengths of a Parallelogram

For what value of  $x$  is quadrilateral  $CDEF$  a parallelogram?

#### SOLUTION

By the Parallelogram Diagonals Converse, if the diagonals of  $CDEF$  bisect each other, then it is a parallelogram. You are given that  $\overline{CN} \cong \overline{EN}$ .

Find  $x$  so that  $\overline{FN} \cong \overline{DN}$ .



$$FN = DN$$

Set the segment lengths equal.

$$5x - 8 = 3x$$

Substitute  $5x - 8$  for  $FN$  and  $3x$  for  $DN$ .

$$2x - 8 = 0$$

Subtract  $3x$  from each side.

$$2x = 8$$

Add 8 to each side.

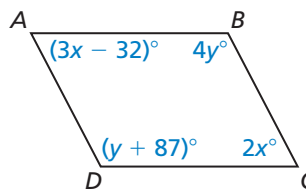
$$x = 4$$

Divide each side by 2.

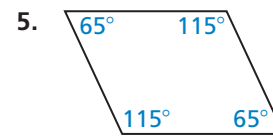
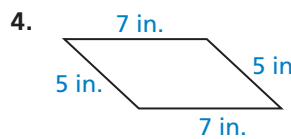
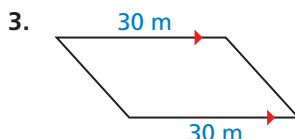
When  $x = 4$ ,  $FN = 5(4) - 8 = 12$  and  $DN = 3(4) = 12$ .

► Quadrilateral  $CDEF$  is a parallelogram when  $x = 4$ .

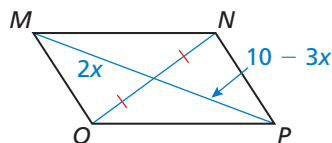
2. For what values of  $x$  and  $y$  is quadrilateral  $ABCD$  a parallelogram? Explain your reasoning.



State the theorem you can use to show that the quadrilateral is a parallelogram.



6. For what value of  $x$  is quadrilateral  $MNPQ$  a parallelogram? Explain your reasoning.



## Concept Summary

### Ways to Prove a Quadrilateral Is a Parallelogram

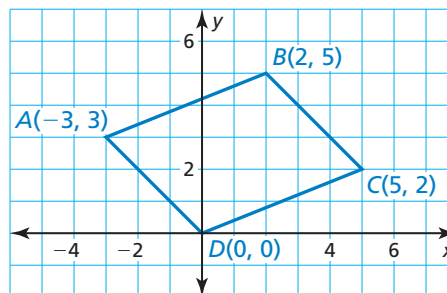
1. Show that both pairs of opposite sides are parallel. ( <i>Definition</i> )	
2. Show that both pairs of opposite sides are congruent. ( <i>Parallelogram Opposite Sides Converse</i> )	
3. Show that both pairs of opposite angles are congruent. ( <i>Parallelogram Opposite Angles Converse</i> )	
4. Show that one pair of opposite sides are congruent and parallel. ( <i>Opposite Sides Parallel and Congruent Theorem</i> )	
5. Show that the diagonals bisect each other. ( <i>Parallelogram Diagonals Converse</i> )	



## Using Coordinate Geometry

### EXAMPLE 5 Identifying a Parallelogram in the Coordinate Plane

Show that quadrilateral  $ABCD$  is a parallelogram.



### SOLUTION

**Method 1** Show that a pair of sides are congruent and parallel. Then apply the Opposite Sides Parallel and Congruent Theorem.

First, use the Distance Formula to show that  $\overline{AB}$  and  $\overline{CD}$  are congruent.

$$AB = \sqrt{[2 - (-3)]^2 + (5 - 3)^2} = \sqrt{29}$$

$$CD = \sqrt{(5 - 0)^2 + (2 - 0)^2} = \sqrt{29}$$

Because  $AB = CD = \sqrt{29}$ ,  $\overline{AB} \cong \overline{CD}$ .

Then, use the slope formula to show that  $\overline{AB} \parallel \overline{CD}$ .

$$\text{slope of } \overline{AB} = \frac{5 - 3}{2 - (-3)} = \frac{2}{5}$$

$$\text{slope of } \overline{CD} = \frac{2 - 0}{5 - 0} = \frac{2}{5}$$

Because  $\overline{AB}$  and  $\overline{CD}$  have the same slope, they are parallel.

►  $\overline{AB}$  and  $\overline{CD}$  are congruent and parallel. So,  $ABCD$  is a parallelogram by the Opposite Sides Parallel and Congruent Theorem.

**Method 2** Show that opposite sides are congruent. Then apply the Parallelogram Opposite Sides Converse. In Method 1, you already have shown that because  $AB = CD = \sqrt{29}$ ,  $\overline{AB} \cong \overline{CD}$ . Now find  $AD$  and  $BC$ .

$$AD = \sqrt{(-3 - 0)^2 + (3 - 0)^2} = 3\sqrt{2}$$

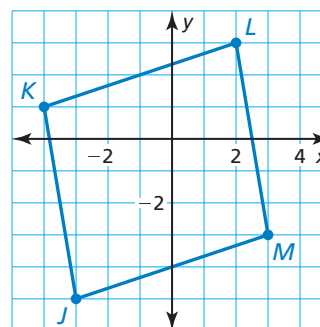
$$BC = \sqrt{(2 - 5)^2 + (5 - 2)^2} = 3\sqrt{2}$$

Because  $AD = BC = 3\sqrt{2}$ ,  $\overline{AD} \cong \overline{BC}$ .

►  $\overline{AB} \cong \overline{CD}$  and  $\overline{AD} \cong \overline{BC}$ . So,  $ABCD$  is a parallelogram by the Parallelogram Opposite Sides Converse.

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- Show that quadrilateral  $JKLM$  is a parallelogram.
- Refer to the Concept Summary on the previous page. Explain two other methods you can use to show that quadrilateral  $ABCD$  in Example 5 is a parallelogram.



# 7.3 Exercises

## Vocabulary and Core Concept Check

- WRITING** A quadrilateral has four congruent sides. Is the quadrilateral a parallelogram? Justify your answer.
- DIFFERENT WORDS, SAME QUESTION** Which is different? Find “both” answers.

Construct a quadrilateral with opposite sides congruent.

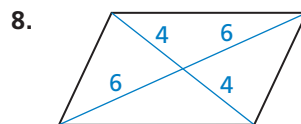
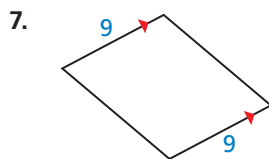
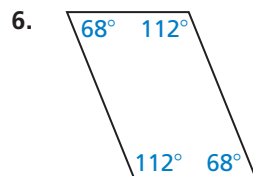
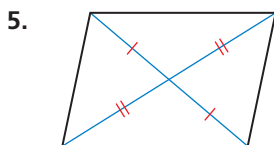
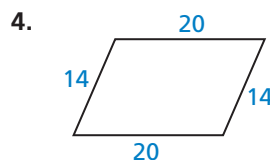
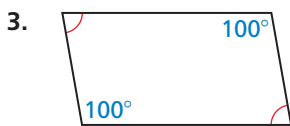
Construct a quadrilateral with one pair of parallel sides.

Construct a quadrilateral with opposite angles congruent.

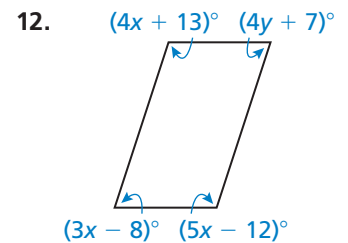
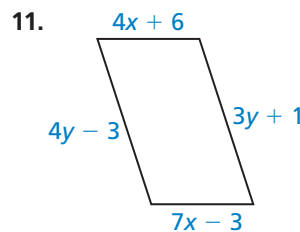
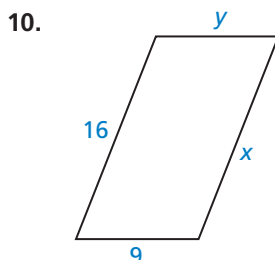
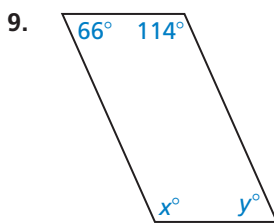
Construct a quadrilateral with one pair of opposite sides congruent and parallel.

## Monitoring Progress and Modeling with Mathematics

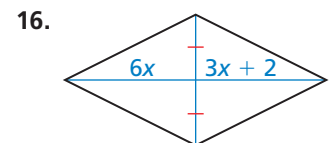
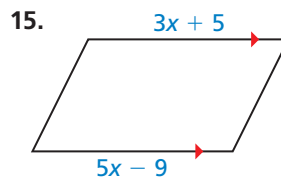
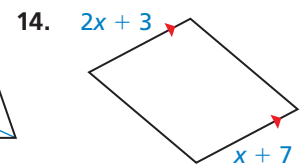
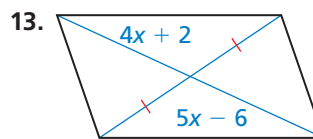
In Exercises 3–8, state which theorem you can use to show that the quadrilateral is a parallelogram. (See Examples 1 and 3.)



In Exercises 9–12, find the values of  $x$  and  $y$  that make the quadrilateral a parallelogram. (See Example 2.)



In Exercises 13–16, find the value of  $x$  that makes the quadrilateral a parallelogram. (See Example 4.)



In Exercises 17–20, graph the quadrilateral with the given vertices in a coordinate plane. Then show that the quadrilateral is a parallelogram. (See Example 5.)

17.  $A(0, 1), B(4, 4), C(12, 4), D(8, 1)$

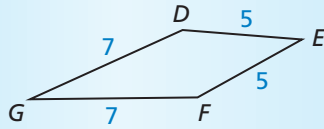
18.  $E(-3, 0), F(-3, 4), G(3, -1), H(3, -5)$

19.  $J(-2, 3), K(-5, 7), L(3, 6), M(6, 2)$

20.  $N(-5, 0), P(0, 4), Q(3, 0), R(-2, -4)$

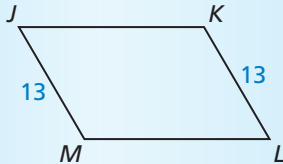
**ERROR ANALYSIS** In Exercises 21 and 22, describe and correct the error in identifying a parallelogram.

21.



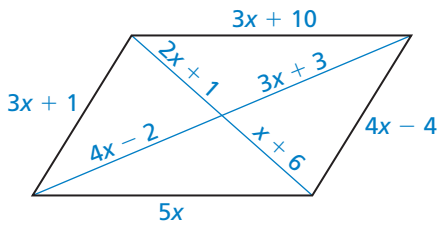
*DEFG is a parallelogram by the Parallelogram Opposite Sides Converse (Theorem 7.7).*

22.

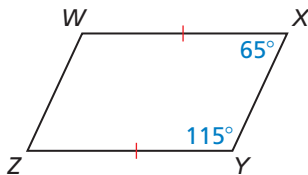


*JKLM is a parallelogram by the Opposite Sides Parallel and Congruent Theorem (Theorem 7.9).*

23. **MATHEMATICAL CONNECTIONS** What value of  $x$  makes the quadrilateral a parallelogram? Explain how you found your answer.



24. **MAKING AN ARGUMENT** Your friend says you can show that quadrilateral  $WXYZ$  is a parallelogram by using the Consecutive Interior Angles Converse (Theorem 3.8) and the Opposite Sides Parallel and Congruent Theorem (Theorem 7.9). Is your friend correct? Explain your reasoning.



**ANALYZING RELATIONSHIPS** In Exercises 25–27, write the indicated theorems as a biconditional statement.

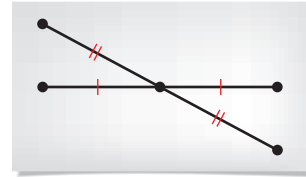
25. Parallelogram Opposite Sides Theorem (Theorem 7.3) and Parallelogram Opposite Sides Converse (Theorem 7.7)
26. Parallelogram Opposite Angles Theorem (Theorem 7.4) and Parallelogram Opposite Angles Converse (Theorem 7.8)

27. Parallelogram Diagonals Theorem (Theorem 7.6) and Parallelogram Diagonals Converse (Theorem 7.10)

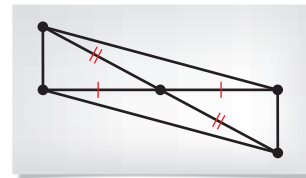
28. **CONSTRUCTION** Describe a method that uses the Opposite Sides Parallel and Congruent Theorem (Theorem 7.9) to construct a parallelogram. Then construct a parallelogram using your method.

29. **REASONING** Follow the steps below to construct a parallelogram. Explain why this method works. State a theorem to support your answer.

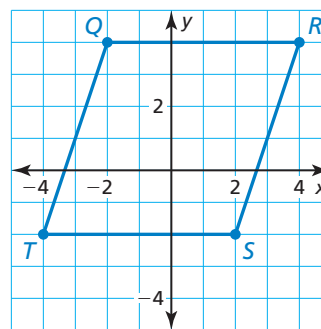
**Step 1** Use a ruler to draw two segments that intersect at their midpoints.



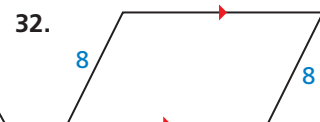
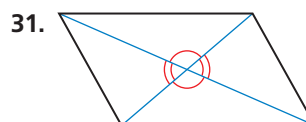
**Step 2** Connect the endpoints of the segments to form a parallelogram.



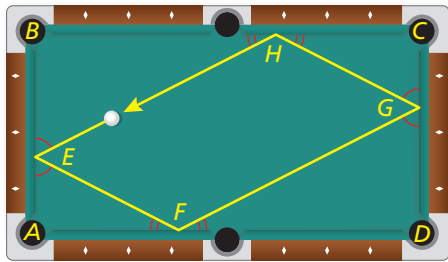
30. **MAKING AN ARGUMENT** Your brother says to show that quadrilateral  $QRST$  is a parallelogram, you must show that  $\overline{QR} \parallel \overline{TS}$  and  $\overline{QT} \parallel \overline{RS}$ . Your sister says that you must show that  $\overline{QR} \cong \overline{TS}$  and  $\overline{QT} \cong \overline{RS}$ . Who is correct? Explain your reasoning.



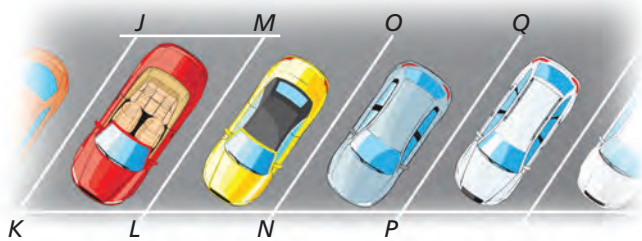
**REASONING** In Exercises 31 and 32, your classmate incorrectly claims that the marked information can be used to show that the figure is a parallelogram. Draw a quadrilateral with the same marked properties that is clearly *not* a parallelogram.



33. **MODELING WITH MATHEMATICS** You shoot a pool ball, and it rolls back to where it started, as shown in the diagram. The ball bounces off each wall at the same angle at which it hits the wall.

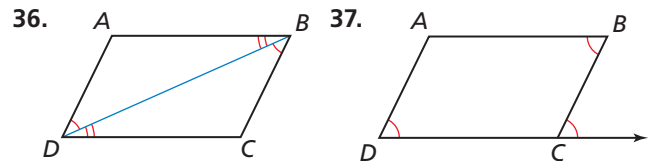
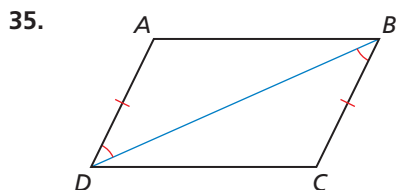


- The ball hits the first wall at an angle of  $63^\circ$ . So  $m\angle AEF = m\angle BEH = 63^\circ$ . What is  $m\angle AFE$ ? Explain your reasoning.
  - Explain why  $m\angle FGD = 63^\circ$ .
  - What is  $m\angle GHC$ ?  $m\angle EHB$ ?
  - Is quadrilateral  $EFGH$  a parallelogram? Explain your reasoning.
34. **MODELING WITH MATHEMATICS** In the diagram of the parking lot shown,  $m\angle JKL = 60^\circ$ ,  $JK = LM = 21$  feet, and  $KL = JM = 9$  feet.

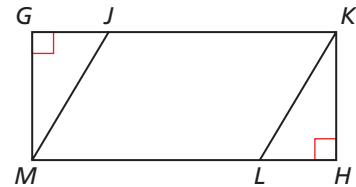


- Explain how to show that parking space  $JKLM$  is a parallelogram.
- Find  $m\angle JML$ ,  $m\angle KJM$ , and  $m\angle KLM$ .
- $\overline{LM} \parallel \overline{NO}$  and  $\overline{NO} \parallel \overline{PQ}$ . Which theorem could you use to show that  $\overline{JK} \parallel \overline{PQ}$ ?

**REASONING** In Exercises 35–37, describe how to prove that  $ABCD$  is a parallelogram.



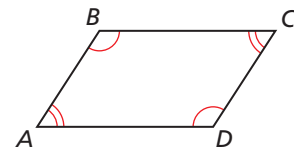
38. **REASONING** Quadrilateral  $JKLM$  is a parallelogram. Describe how to prove that  $\triangle MGJ \cong \triangle KHL$ .



39. **PROVING A THEOREM** Prove the Parallelogram Opposite Angles Converse (Theorem 7.8). (*Hint*: Let  $x^\circ$  represent  $m\angle A$  and  $m\angle C$ . Let  $y^\circ$  represent  $m\angle B$  and  $m\angle D$ . Write and simplify an equation involving  $x$  and  $y$ .)

**Given**  $\angle A \cong \angle C$ ,  $\angle B \cong \angle D$

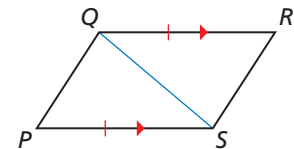
**Prove**  $ABCD$  is a parallelogram.



40. **PROVING A THEOREM** Use the diagram of  $PQRS$  with the auxiliary line segment drawn to prove the Opposite Sides Parallel and Congruent Theorem (Theorem 7.9).

**Given**  $\overline{QR} \parallel \overline{PS}$ ,  $\overline{QR} \cong \overline{PS}$

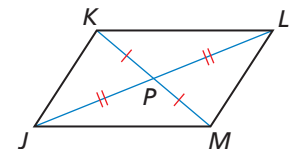
**Prove**  $PQRS$  is a parallelogram.



41. **PROVING A THEOREM** Prove the Parallelogram Diagonals Converse (Theorem 7.10).

**Given** Diagonals  $\overline{JL}$  and  $\overline{KM}$  bisect each other.

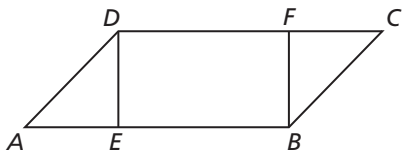
**Prove**  $JKLM$  is a parallelogram.



42. **PROOF** Write a proof.

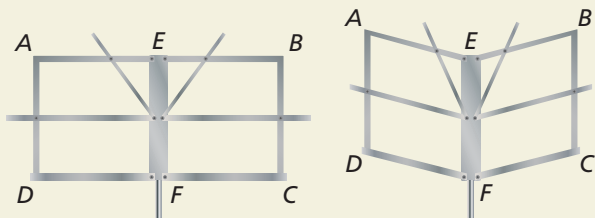
**Given**  $DEBF$  is a parallelogram.  
 $AE = CF$

**Prove**  $ABCD$  is a parallelogram.

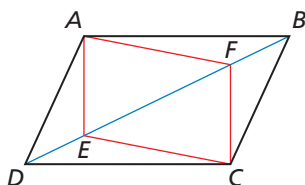


43. **REASONING** Three interior angle measures of a quadrilateral are  $67^\circ$ ,  $67^\circ$ , and  $113^\circ$ . Is this enough information to conclude that the quadrilateral is a parallelogram? Explain your reasoning.

44. **HOW DO YOU SEE IT?** A music stand can be folded up, as shown. In the diagrams,  $AEFD$  and  $EBCF$  are parallelograms. Which labeled segments remain parallel as the stand is folded?



45. **CRITICAL THINKING** In the diagram,  $ABCD$  is a parallelogram,  $BF = DE = 12$ , and  $CF = 8$ . Find  $AE$ . Explain your reasoning.



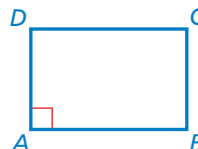
46. **THOUGHT PROVOKING** Create a regular hexagon using congruent parallelograms.

47. **WRITING** The Parallelogram Consecutive Angles Theorem (Theorem 7.5) says that if a quadrilateral is a parallelogram, then its consecutive angles are supplementary. Write the converse of this theorem. Then write a plan for proving the converse. Include a diagram.

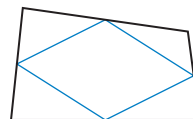
48. **PROOF** Write a proof.

**Given**  $ABCD$  is a parallelogram.  
 $\angle A$  is a right angle.

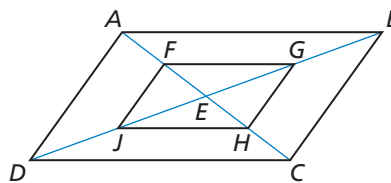
**Prove**  $\angle B$ ,  $\angle C$ , and  $\angle D$  are right angles.



49. **ABSTRACT REASONING** The midpoints of the sides of a quadrilateral have been joined to form what looks like a parallelogram. Show that a quadrilateral formed by connecting the midpoints of the sides of any quadrilateral is always a parallelogram. (*Hint*: Draw a diagonal. Include a diagonal of the larger quadrilateral. Show how two sides of the smaller quadrilateral relate to the diagonal.)



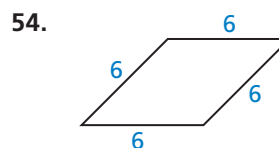
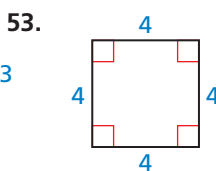
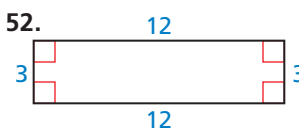
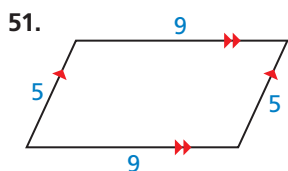
50. **CRITICAL THINKING** Show that if  $ABCD$  is a parallelogram with its diagonals intersecting at  $E$ , then you can connect the midpoints  $F$ ,  $G$ ,  $H$ , and  $J$  of  $\overline{AE}$ ,  $\overline{BE}$ ,  $\overline{CE}$ , and  $\overline{DE}$ , respectively, to form another parallelogram,  $FGHJ$ .



## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

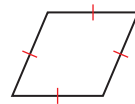
Classify the quadrilateral.



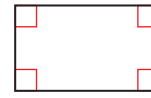
# 7.4 Properties of Special Parallelograms

**Essential Question** What are the properties of the diagonals of rectangles, rhombuses, and squares?

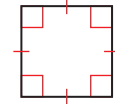
Recall the three types of parallelograms shown below.



Rhombus



Rectangle



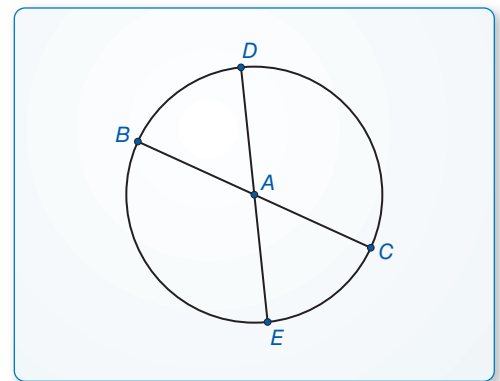
Square

## EXPLORATION 1 Identifying Special Quadrilaterals

**Work with a partner.** Use dynamic geometry software.

- Draw a circle with center  $A$ .
- Draw two diameters of the circle. Label the endpoints  $B$ ,  $C$ ,  $D$ , and  $E$ .
- Draw quadrilateral  $BDCE$ .
- Is  $BDCE$  a parallelogram? rectangle? rhombus? square? Explain your reasoning.
- Repeat parts (a)–(d) for several other circles. Write a conjecture based on your results.

**Sample**

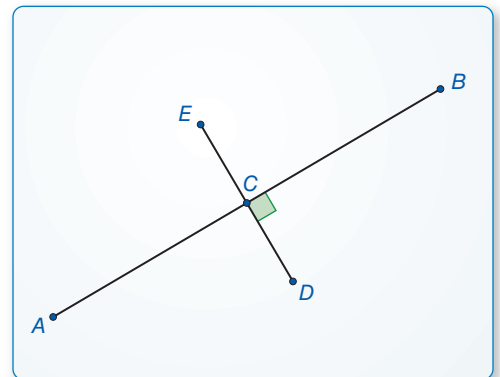


## EXPLORATION 2 Identifying Special Quadrilaterals

**Work with a partner.** Use dynamic geometry software.

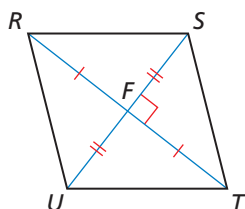
- Construct two segments that are perpendicular bisectors of each other. Label the endpoints  $A$ ,  $B$ ,  $D$ , and  $E$ . Label the intersection  $C$ .
- Draw quadrilateral  $AEBD$ .
- Is  $AEBD$  a parallelogram? rectangle? rhombus? square? Explain your reasoning.
- Repeat parts (a)–(c) for several other segments. Write a conjecture based on your results.

**Sample**



## CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to make conjectures and build a logical progression of statements to explore the truth of your conjectures.



## Communicate Your Answer

- What are the properties of the diagonals of rectangles, rhombuses, and squares?
- Is  $RSTU$  a parallelogram? rectangle? rhombus? square? Explain your reasoning.
- What type of quadrilateral has congruent diagonals that bisect each other?

# 7.4 Lesson

## Core Vocabulary

rhombus, p. 350  
rectangle, p. 350  
square, p. 350

**Previous**  
quadrilateral  
parallelogram  
diagonal

## What You Will Learn

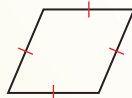
- ▶ Use properties of special parallelograms.
- ▶ Use properties of diagonals of special parallelograms.
- ▶ Use coordinate geometry to identify special types of parallelograms.

## Using Properties of Special Parallelograms

In this lesson, you will learn about three special types of parallelograms: *rhombuses*, *rectangles*, and *squares*.

## Core Concept

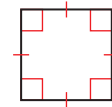
### Rhombuses, Rectangles, and Squares



A **rhombus** is a parallelogram with four congruent sides.



A **rectangle** is a parallelogram with four right angles.



A **square** is a parallelogram with four congruent sides and four right angles.

You can use the corollaries below to prove that a quadrilateral is a rhombus, rectangle, or square, without first proving that the quadrilateral is a parallelogram.

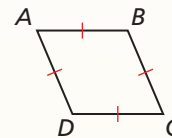
## Corollaries

### Corollary 7.2 Rhombus Corollary

A quadrilateral is a rhombus if and only if it has four congruent sides.

$ABCD$  is a rhombus if and only if  $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ .

*Proof* Ex. 79, p. 358

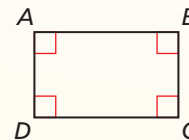


### Corollary 7.3 Rectangle Corollary

A quadrilateral is a rectangle if and only if it has four right angles.

$ABCD$  is a rectangle if and only if  $\angle A$ ,  $\angle B$ ,  $\angle C$ , and  $\angle D$  are right angles.

*Proof* Ex. 80, p. 358

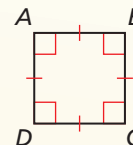


### Corollary 7.4 Square Corollary

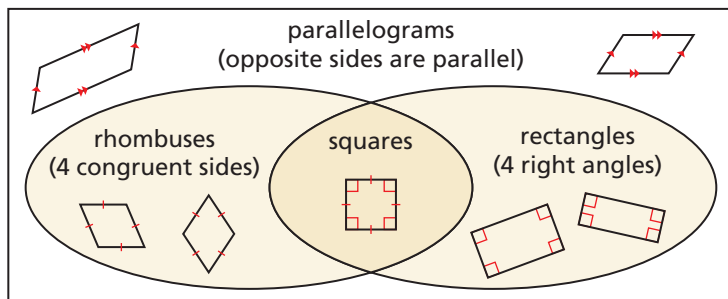
A quadrilateral is a square if and only if it is a rhombus and a rectangle.

$ABCD$  is a square if and only if  $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$  and  $\angle A$ ,  $\angle B$ ,  $\angle C$ , and  $\angle D$  are right angles.

*Proof* Ex. 81, p. 358



The Venn diagram below illustrates some important relationships among parallelograms, rhombuses, rectangles, and squares. For example, you can see that a square is a rhombus because it is a parallelogram with four congruent sides. Because it has four right angles, a square is also a rectangle.



### EXAMPLE 1 Using Properties of Special Quadrilaterals

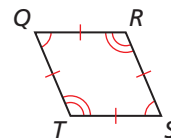
For any rhombus  $QRST$ , decide whether the statement is *always* or *sometimes* true. Draw a diagram and explain your reasoning.

a.  $\angle Q \cong \angle S$

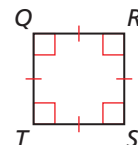
b.  $\angle Q \cong \angle R$

#### SOLUTION

a. By definition, a rhombus is a parallelogram with four congruent sides. By the Parallelogram Opposite Angles Theorem (Theorem 7.4), opposite angles of a parallelogram are congruent. So,  $\angle Q \cong \angle S$ . The statement is *always* true.

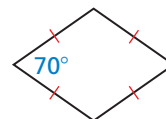


b. If rhombus  $QRST$  is a square, then all four angles are congruent right angles. So,  $\angle Q \cong \angle R$  when  $QRST$  is a square. Because not all rhombuses are also squares, the statement is *sometimes* true.



### EXAMPLE 2 Classifying Special Quadrilaterals

Classify the special quadrilateral. Explain your reasoning.



#### SOLUTION

The quadrilateral has four congruent sides. By the Rhombus Corollary, the quadrilateral is a rhombus. Because one of the angles is not a right angle, the rhombus cannot be a square.

### Monitoring Progress Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

- For any square  $JKLM$ , is it *always* or *sometimes* true that  $\overline{JK} \perp \overline{KL}$ ? Explain your reasoning.
- For any rectangle  $EFGH$ , is it *always* or *sometimes* true that  $\overline{FG} \cong \overline{GH}$ ? Explain your reasoning.
- A quadrilateral has four congruent sides and four congruent angles. Sketch the quadrilateral and classify it.



## Using Properties of Diagonals

### Theorems

#### STUDY TIP

The diagonals of a rhombus divide the rhombus into four congruent right triangles.

#### READING

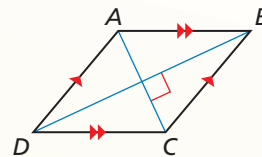
Recall that biconditionals, such as the Rhombus Opposite Angles Theorem, can be rewritten as two parts. To prove a biconditional, you must prove both parts.

#### Theorem 7.11 Rhombus Diagonals Theorem

A parallelogram is a rhombus if and only if its diagonals are perpendicular.

$\square ABCD$  is a rhombus if and only if  $\overline{AC} \perp \overline{BD}$ .

*Proof* p. 352; Ex. 72, p. 357

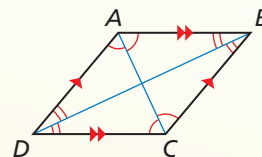


#### Theorem 7.12 Rhombus Opposite Angles Theorem

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

$\square ABCD$  is a rhombus if and only if  $\overline{AC}$  bisects  $\angle BCD$  and  $\angle BAD$ , and  $\overline{BD}$  bisects  $\angle ABC$  and  $\angle ADC$ .

*Proof* Exs. 73 and 74, p. 357



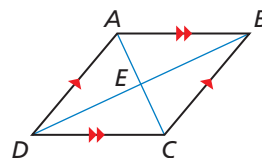
#### PROOF

#### Part of Rhombus Diagonals Theorem

**Given**  $ABCD$  is a rhombus.

**Prove**  $\overline{AC} \perp \overline{BD}$

$ABCD$  is a rhombus. By the definition of a rhombus,  $\overline{AB} \cong \overline{BC}$ . Because a rhombus is a parallelogram and the diagonals of a parallelogram bisect each other,  $\overline{BD}$  bisects  $\overline{AC}$  at  $E$ . So,  $\overline{AE} \cong \overline{EC}$ .  $\overline{BE} \cong \overline{BE}$  by the Reflexive Property of Congruence (Theorem 2.1). So,  $\triangle AEB \cong \triangle CEB$  by the SSS Congruence Theorem (Theorem 5.8).  $\angle AEB \cong \angle CEB$  because corresponding parts of congruent triangles are congruent. Then by the Linear Pair Postulate (Postulate 2.8),  $\angle AEB$  and  $\angle CEB$  are supplementary. Two congruent angles that form a linear pair are right angles, so  $m\angle AEB = m\angle CEB = 90^\circ$  by the definition of a right angle. So,  $\overline{AC} \perp \overline{BD}$  by the definition of perpendicular lines.



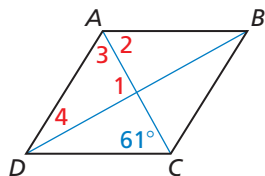
#### EXAMPLE 3

#### Finding Angle Measures in a Rhombus

Find the measures of the numbered angles in rhombus  $ABCD$ .

#### SOLUTION

Use the Rhombus Diagonals Theorem and the Rhombus Opposite Angles Theorem to find the angle measures.



$$m\angle 1 = 90^\circ$$

The diagonals of a rhombus are perpendicular.

$$m\angle 2 = 61^\circ$$

Alternate Interior Angles Theorem (Theorem 3.2)

$$m\angle 3 = 61^\circ$$

Each diagonal of a rhombus bisects a pair of opposite angles, and  $m\angle 2 = 61^\circ$ .

$$m\angle 1 + m\angle 3 + m\angle 4 = 180^\circ$$

Triangle Sum Theorem (Theorem 5.1)

$$90^\circ + 61^\circ + m\angle 4 = 180^\circ$$

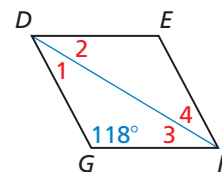
Substitute  $90^\circ$  for  $m\angle 1$  and  $61^\circ$  for  $m\angle 3$ .

$$m\angle 4 = 29^\circ$$

Solve for  $m\angle 4$ .

► So,  $m\angle 1 = 90^\circ$ ,  $m\angle 2 = 61^\circ$ ,  $m\angle 3 = 61^\circ$ , and  $m\angle 4 = 29^\circ$ .

- In Example 3, what is  $m\angle ADC$  and  $m\angle BCD$ ?
- Find the measures of the numbered angles in rhombus  $DEFG$ .



## STUDY TIP

The diagonals of a square divide the square into four congruent triangles with angle measures of  $45^\circ$ ,  $45^\circ$ , and  $90^\circ$ .

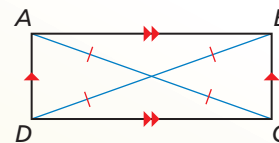
## Theorem

### Theorem 7.13 Rectangle Diagonals Theorem

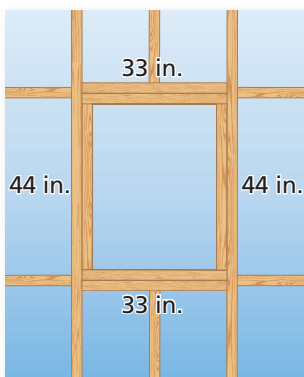
A parallelogram is a rectangle if and only if its diagonals are congruent.

$\square ABCD$  is a rectangle if and only if  $\overline{AC} \cong \overline{BD}$ .

*Proof* Exs. 85 and 86, p. 358



### EXAMPLE 4 Identifying a Rectangle



You are building a frame for a window. The window will be installed in the opening shown in the diagram.

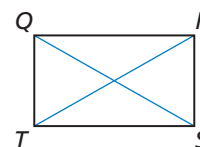
- The opening must be a rectangle. Given the measurements in the diagram, can you assume that it is? Explain.
- You measure the diagonals of the opening. The diagonals are 54.8 inches and 55.3 inches. What can you conclude about the shape of the opening?

### SOLUTION

- No, you cannot. The boards on opposite sides are the same length, so they form a parallelogram. But you do not know whether the angles are right angles.
- By the Rectangle Diagonals Theorem, the diagonals of a rectangle are congruent. The diagonals of the quadrilateral formed by the boards are not congruent, so the boards do not form a rectangle.

### EXAMPLE 5 Finding Diagonal Lengths in a Rectangle

In rectangle  $QRST$ ,  $QS = 5x - 31$  and  $RT = 2x + 11$ . Find the lengths of the diagonals of  $QRST$ .



### SOLUTION

By the Rectangle Diagonals Theorem, the diagonals of a rectangle are congruent. Find  $x$  so that  $\overline{QS} \cong \overline{RT}$ .

$$QS = RT$$

Set the diagonal lengths equal.

$$5x - 31 = 2x + 11$$

Substitute  $5x - 31$  for  $QS$  and  $2x + 11$  for  $RT$ .

$$3x - 31 = 11$$

Subtract  $2x$  from each side.

$$3x = 42$$

Add 31 to each side.

$$x = 14$$

Divide each side by 3.

When  $x = 14$ ,  $QS = 5(14) - 31 = 39$  and  $RT = 2(14) + 11 = 39$ .

- Each diagonal has a length of 39 units.

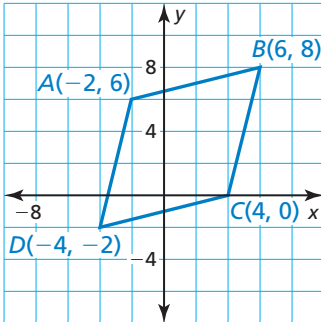
- Suppose you measure only the diagonals of the window opening in Example 4 and they have the same measure. Can you conclude that the opening is a rectangle? Explain.
- WHAT IF?** In Example 5,  $QS = 4x - 15$  and  $RT = 3x + 8$ . Find the lengths of the diagonals of  $QRST$ .

## Using Coordinate Geometry

### EXAMPLE 6 Identifying a Parallelogram in the Coordinate Plane

Decide whether  $\square ABCD$  with vertices  $A(-2, 6)$ ,  $B(6, 8)$ ,  $C(4, 0)$ , and  $D(-4, -2)$  is a *rectangle*, a *rhombus*, or a *square*. Give all names that apply.

#### SOLUTION



- Understand the Problem** You know the vertices of  $\square ABCD$ . You need to identify the type of parallelogram.
- Make a Plan** Begin by graphing the vertices. From the graph, it appears that all four sides are congruent and there are no right angles.

Check the lengths and slopes of the diagonals of  $\square ABCD$ . If the diagonals are congruent, then  $\square ABCD$  is a rectangle. If the diagonals are perpendicular, then  $\square ABCD$  is a rhombus. If they are both congruent and perpendicular, then  $\square ABCD$  is a rectangle, a rhombus, and a square.

- Solve the Problem** Use the Distance Formula to find  $AC$  and  $BD$ .

$$AC = \sqrt{(-2 - 4)^2 + (6 - 0)^2} = \sqrt{72} = 6\sqrt{2}$$

$$BD = \sqrt{[6 - (-4)]^2 + [8 - (-2)]^2} = \sqrt{200} = 10\sqrt{2}$$

Because  $6\sqrt{2} \neq 10\sqrt{2}$ , the diagonals are not congruent. So,  $\square ABCD$  is not a rectangle. Because it is not a rectangle, it also cannot be a square.

Use the slope formula to find the slopes of the diagonals  $\overline{AC}$  and  $\overline{BD}$ .

$$\text{slope of } \overline{AC} = \frac{6 - 0}{-2 - 4} = \frac{6}{-6} = -1 \quad \text{slope of } \overline{BD} = \frac{8 - (-2)}{6 - (-4)} = \frac{10}{10} = 1$$

Because the product of the slopes of the diagonals is  $-1$ , the diagonals are perpendicular.

► So,  $\square ABCD$  is a rhombus.

- Look Back** Check the side lengths of  $\square ABCD$ . Each side has a length of  $2\sqrt{17}$  units, so  $\square ABCD$  is a rhombus. Check the slopes of two consecutive sides.

$$\text{slope of } \overline{AB} = \frac{8 - 6}{6 - (-2)} = \frac{2}{8} = \frac{1}{4} \quad \text{slope of } \overline{BC} = \frac{8 - 0}{6 - 4} = \frac{8}{2} = 4$$

Because the product of these slopes is not  $-1$ ,  $\overline{AB}$  is not perpendicular to  $\overline{BC}$ .

So,  $\angle ABC$  is not a right angle, and  $\square ABCD$  cannot be a rectangle or a square. ✓

- Decide whether  $\square PQRS$  with vertices  $P(-5, 2)$ ,  $Q(0, 4)$ ,  $R(2, -1)$ , and  $S(-3, -3)$  is a *rectangle*, a *rhombus*, or a *square*. Give all names that apply.

# 7.4 Exercises

## Vocabulary and Core Concept Check

- VOCABULARY** What is another name for an equilateral rectangle?
- WRITING** What should you look for in a parallelogram to know if the parallelogram is also a rhombus?

## Monitoring Progress and Modeling with Mathematics

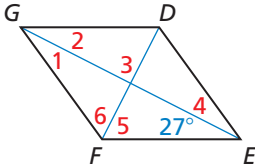
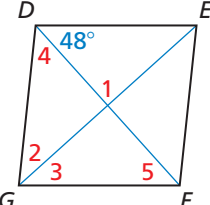
In Exercises 3–8, for any rhombus  $JKLM$ , decide whether the statement is *always* or *sometimes* true. Draw a diagram and explain your reasoning. (See Example 1.)

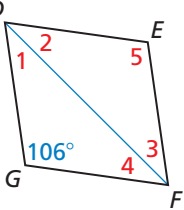
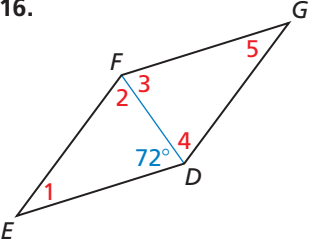
- $\angle L \cong \angle M$
- $\overline{JM} \cong \overline{KL}$
- $\overline{JL} \cong \overline{KM}$
- $\angle K \cong \angle M$
- $\overline{JK} \cong \overline{KL}$
- $\angle JKM \cong \angle LKM$

In Exercises 9–12, classify the quadrilateral. Explain your reasoning. (See Example 2.)

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In Exercises 13–16, find the measures of the numbered angles in rhombus  $DEFG$ . (See Example 3.)

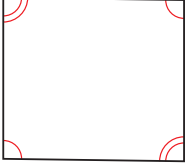
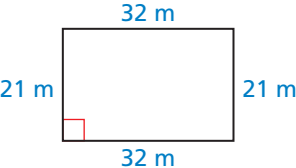
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In Exercises 17–22, for any rectangle  $WXYZ$ , decide whether the statement is *always* or *sometimes* true. Draw a diagram and explain your reasoning.

- $\angle W \cong \angle X$
- $\overline{WX} \cong \overline{XY}$
- $\overline{WY} \perp \overline{XZ}$
- $\overline{WX} \cong \overline{YZ}$
- $\overline{WY} \cong \overline{XZ}$
- $\angle WXZ \cong \angle YXZ$

In Exercises 23 and 24, determine whether the quadrilateral is a rectangle. (See Example 4.)

- 
- 

In Exercises 25–28, find the lengths of the diagonals of rectangle  $WXYZ$ . (See Example 5.)

- $WY = 6x - 7$   
 $XZ = 3x + 2$
- $WY = 24x - 8$   
 $XZ = -18x + 13$
- $WY = 14x + 10$   
 $XZ = 11x + 22$
- $WY = 16x + 2$   
 $XZ = 36x - 6$

In Exercises 29–34, name each quadrilateral—*parallelogram, rectangle, rhombus, or square*—for which the statement is always true.

29. It is equiangular.
30. It is equiangular and equilateral.
31. The diagonals are perpendicular.
32. Opposite sides are congruent.
33. The diagonals bisect each other.
34. The diagonals bisect opposite angles.
35. **ERROR ANALYSIS** Quadrilateral  $PQRS$  is a rectangle. Describe and correct the error in finding the value of  $x$ .

**X**

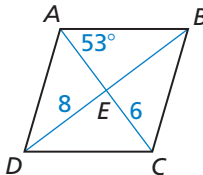
$m\angle QSR = m\angle QSP$   
 $x^\circ = 58^\circ$   
 $x = 58$

36. **ERROR ANALYSIS** Quadrilateral  $PQRS$  is a rhombus. Describe and correct the error in finding the value of  $x$ .

**X**

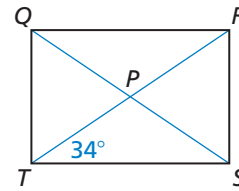
$m\angle QRP = m\angle SQR$   
 $x^\circ = 37^\circ$   
 $x = 37$

In Exercises 37–42, the diagonals of rhombus  $ABCD$  intersect at  $E$ . Given that  $m\angle BAC = 53^\circ$ ,  $DE = 8$ , and  $EC = 6$ , find the indicated measure.



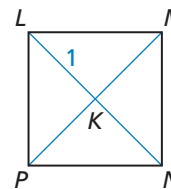
- |                   |                   |
|-------------------|-------------------|
| 37. $m\angle DAC$ | 38. $m\angle AED$ |
| 39. $m\angle ADC$ | 40. $DB$          |
| 41. $AE$          | 42. $AC$          |

In Exercises 43–48, the diagonals of rectangle  $QRST$  intersect at  $P$ . Given that  $m\angle PTS = 34^\circ$  and  $QS = 10$ , find the indicated measure.



- |                   |                   |
|-------------------|-------------------|
| 43. $m\angle QTR$ | 44. $m\angle QRT$ |
| 45. $m\angle SRT$ | 46. $QP$          |
| 47. $RT$          | 48. $RP$          |

In Exercises 49–54, the diagonals of square  $LMNP$  intersect at  $K$ . Given that  $LK = 1$ , find the indicated measure.

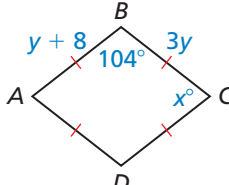
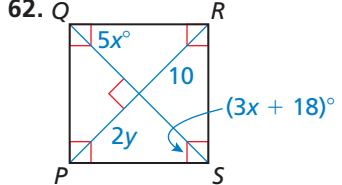


- |                   |                   |
|-------------------|-------------------|
| 49. $m\angle MKN$ | 50. $m\angle LMK$ |
| 51. $m\angle LPK$ | 52. $KN$          |
| 53. $LN$          | 54. $MP$          |

In Exercises 55–60, decide whether  $\square JKLM$  is a rectangle, a rhombus, or a square. Give all names that apply. Explain your reasoning. (See Example 6.)

55.  $J(-4, 2)$ ,  $K(0, 3)$ ,  $L(1, -1)$ ,  $M(-3, -2)$
56.  $J(-2, 7)$ ,  $K(7, 2)$ ,  $L(-2, -3)$ ,  $M(-11, 2)$
57.  $J(3, 1)$ ,  $K(3, -3)$ ,  $L(-2, -3)$ ,  $M(-2, 1)$
58.  $J(-1, 4)$ ,  $K(-3, 2)$ ,  $L(2, -3)$ ,  $M(4, -1)$
59.  $J(5, 2)$ ,  $K(1, 9)$ ,  $L(-3, 2)$ ,  $M(1, -5)$
60.  $J(5, 2)$ ,  $K(2, 5)$ ,  $L(-1, 2)$ ,  $M(2, -1)$

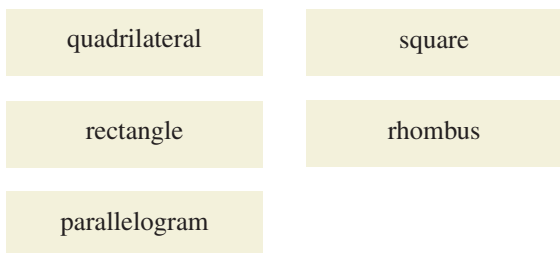
**MATHEMATICAL CONNECTIONS** In Exercises 61 and 62, classify the quadrilateral. Explain your reasoning. Then find the values of  $x$  and  $y$ .

- |  |   |
|--|---|
| 61.  | 62.  |
|--|---|

63. **DRAWING CONCLUSIONS** In the window,  $\overline{BD} \cong \overline{DF} \cong \overline{BH} \cong \overline{HF}$ . Also,  $\angle HAB$ ,  $\angle BCD$ ,  $\angle DEF$ , and  $\angle FGH$  are right angles.



- a. Classify  $HBDF$  and  $ACEG$ . Explain your reasoning.
- b. What can you conclude about the lengths of the diagonals  $\overline{AE}$  and  $\overline{GC}$ ? Given that these diagonals intersect at  $J$ , what can you conclude about the lengths of  $\overline{AJ}$ ,  $\overline{JE}$ ,  $\overline{CJ}$ , and  $\overline{JG}$ ? Explain.
64. **ABSTRACT REASONING** Order the terms in a diagram so that each term builds off the previous term(s). Explain why each figure is in the location you chose.

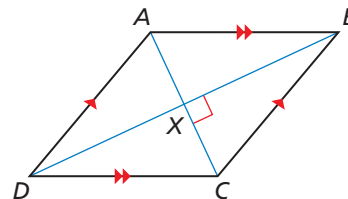


**CRITICAL THINKING** In Exercises 65–70, complete each statement with *always*, *sometimes*, or *never*. Explain your reasoning.

65. A square is \_\_\_\_\_ a rhombus.
66. A rectangle is \_\_\_\_\_ a square.
67. A rectangle \_\_\_\_\_ has congruent diagonals.
68. The diagonals of a square \_\_\_\_\_ bisect its angles.
69. A rhombus \_\_\_\_\_ has four congruent angles.
70. A rectangle \_\_\_\_\_ has perpendicular diagonals.

71. **USING TOOLS** You want to mark off a square region for a garden at school. You use a tape measure to mark off a quadrilateral on the ground. Each side of the quadrilateral is 2.5 meters long. Explain how you can use the tape measure to make sure that the quadrilateral is a square.

72. **PROVING A THEOREM** Use the plan for proof below to write a paragraph proof for one part of the Rhombus Diagonals Theorem (Theorem 7.11).



**Given**  $ABCD$  is a parallelogram.  
 $\overline{AC} \perp \overline{BD}$

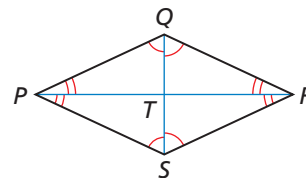
**Prove**  $ABCD$  is a rhombus.

**Plan for Proof** Because  $ABCD$  is a parallelogram, its diagonals bisect each other at  $X$ . Use  $\overline{AC} \perp \overline{BD}$  to show that  $\triangle BXC \cong \triangle DXC$ . Then show that  $\overline{BC} \cong \overline{DC}$ . Use the properties of a parallelogram to show that  $ABCD$  is a rhombus.

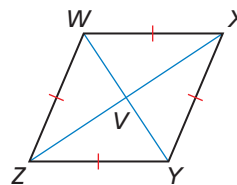
**PROVING A THEOREM** In Exercises 73 and 74, write a proof for part of the Rhombus Opposite Angles Theorem (Theorem 7.12).

73. **Given**  $PQRS$  is a parallelogram.  
 $\overline{PR}$  bisects  $\angle SPQ$  and  $\angle QRS$ .  
 $\overline{SQ}$  bisects  $\angle PSR$  and  $\angle RQP$ .

**Prove**  $PQRS$  is a rhombus.

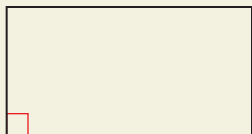


74. **Given**  $WXYZ$  is a rhombus.  
**Prove**  $\overline{WY}$  bisects  $\angle ZWX$  and  $\angle XYZ$ .  
 $\overline{ZX}$  bisects  $\angle WZY$  and  $\angle YXW$ .



75. **CRITICAL THINKING** Which quadrilateral could be called a regular quadrilateral? Explain your reasoning.

76. **HOW DO YOU SEE IT?** What other information do you need to determine whether the figure is a rectangle?



77. **REASONING** Are all rhombuses similar? Are all squares similar? Explain your reasoning.

78. **THOUGHT PROVOKING** Use the Rhombus Diagonals Theorem (Theorem 7.11) to explain why every rhombus has at least two lines of symmetry.

**PROVING A COROLLARY** In Exercises 79–81, write the corollary as a conditional statement and its converse. Then explain why each statement is true.

79. Rhombus Corollary (Corollary 7.2)

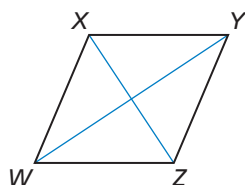
80. Rectangle Corollary (Corollary 7.3)

81. Square Corollary (Corollary 7.4)

82. **PROOF** Write a proof in the style of your choice.

**Given**  $\triangle XYZ \cong \triangle XWZ$ ,  $\angle XYW \cong \angle ZWY$

**Prove**  $WXYZ$  is a rhombus.

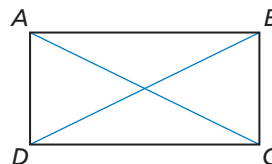


83. **MAKING AN ARGUMENT** Your friend claims a rhombus will never have congruent diagonals because it would have to be a rectangle. Is your friend correct? Explain your reasoning.

84. **PROOF** Write a proof in the style of your choice.

**Given**  $\overline{BC} \cong \overline{AD}$ ,  $\overline{BC} \perp \overline{DC}$ ,  $\overline{AD} \perp \overline{DC}$

**Prove**  $ABCD$  is a rectangle.



**PROVING A THEOREM** In Exercises 85 and 86, write a proof for part of the Rectangle Diagonals Theorem (Theorem 7.13).

85. **Given**  $PQRS$  is a rectangle.

**Prove**  $\overline{PR} \cong \overline{SQ}$

86. **Given**  $PQRS$  is a parallelogram.

$\overline{PR} \cong \overline{SQ}$

**Prove**  $PQRS$  is a rectangle.

87. **PROOF** Write a proof in the style of your choice.

**Given**  $ABCD$  is a rhombus.

**Prove**  $\overline{AC}$  and  $\overline{BD}$  divide the rhombus into four congruent right triangles.

88. **PROOF** Write a proof in the style of your choice.

**Given**  $PQRS$  is a square.

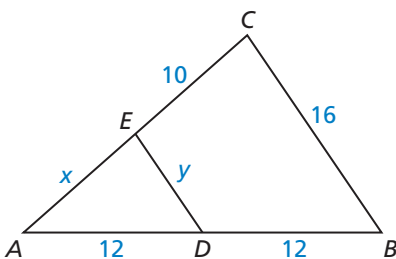
**Prove**  $\overline{PR}$  and  $\overline{QS}$  divide the square into four congruent triangles with angle measures of  $45^\circ$ ,  $45^\circ$ , and  $90^\circ$ .

## Maintaining Mathematical Proficiency

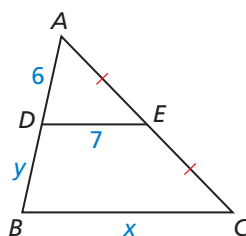
Reviewing what you learned in previous grades and lessons

$\overline{DE}$  is a midsegment of  $\triangle ABC$ . Find the values of  $x$  and  $y$ .

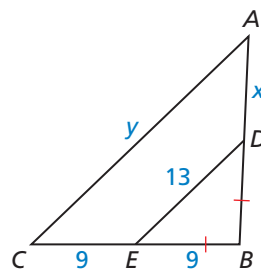
89.



90.



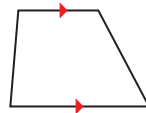
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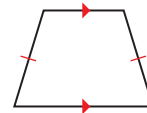
# 7.5 Properties of Trapezoids and Kites

**Essential Question** What are some properties of trapezoids and kites?

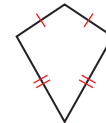
Recall the types of quadrilaterals shown below.



Trapezoid



Isosceles Trapezoid



Kite

## PERSEVERE IN SOLVING PROBLEMS

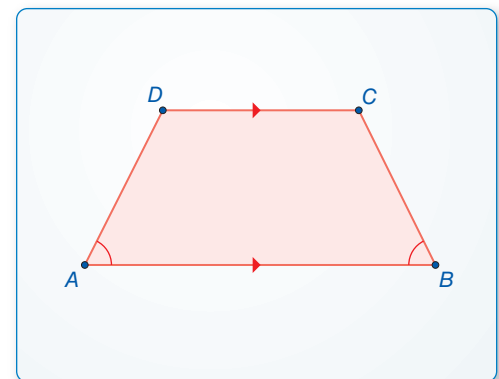
To be proficient in math, you need to draw diagrams of important features and relationships, and search for regularity or trends.

### EXPLORATION 1 Making a Conjecture about Trapezoids

**Work with a partner.** Use dynamic geometry software.

- Construct a trapezoid whose base angles are congruent. Explain your process.
- Is the trapezoid isosceles? Justify your answer.
- Repeat parts (a) and (b) for several other trapezoids. Write a conjecture based on your results.

#### Sample

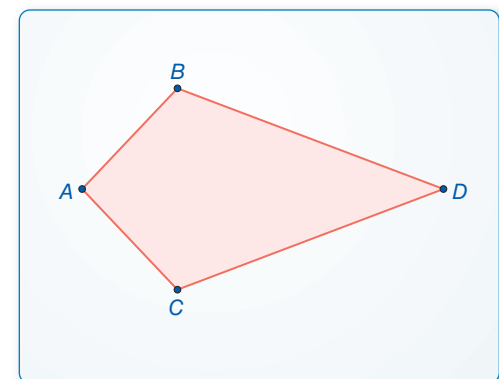


### EXPLORATION 2 Discovering a Property of Kites

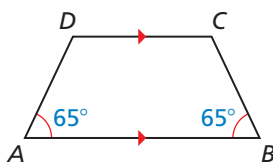
**Work with a partner.** Use dynamic geometry software.

- Construct a kite. Explain your process.
- Measure the angles of the kite. What do you observe?
- Repeat parts (a) and (b) for several other kites. Write a conjecture based on your results.

#### Sample



## Communicate Your Answer



- What are some properties of trapezoids and kites?
- Is the trapezoid at the left isosceles? Explain.
- A quadrilateral has angle measures of  $70^\circ$ ,  $70^\circ$ ,  $110^\circ$ , and  $110^\circ$ . Is the quadrilateral a kite? Explain.



# 7.5 Lesson

## Core Vocabulary

trapezoid, p. 360  
 bases, p. 360  
 base angles, p. 360  
 legs, p. 360  
 isosceles trapezoid, p. 360  
 midsegment of a trapezoid,  
 p. 362  
 kite, p. 363

### Previous

diagonal  
 parallelogram

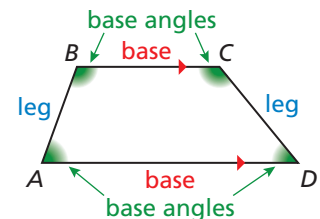
## What You Will Learn

- ▶ Use properties of trapezoids.
- ▶ Use the Trapezoid Midsegment Theorem to find distances.
- ▶ Use properties of kites.
- ▶ Identify quadrilaterals.

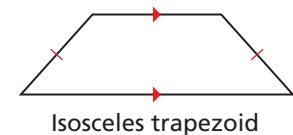
## Using Properties of Trapezoids

A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are the **bases**.

**Base angles** of a trapezoid are two consecutive angles whose common side is a base. A trapezoid has two pairs of base angles. For example, in trapezoid  $ABCD$ ,  $\angle A$  and  $\angle D$  are one pair of base angles, and  $\angle B$  and  $\angle C$  are the second pair. The nonparallel sides are the **legs** of the trapezoid.



If the legs of a trapezoid are congruent, then the trapezoid is an **isosceles trapezoid**.



### EXAMPLE 1 Identifying a Trapezoid in the Coordinate Plane

Show that  $ORST$  is a trapezoid. Then decide whether it is isosceles.

#### SOLUTION

**Step 1** Compare the slopes of opposite sides.

$$\text{slope of } \overline{RS} = \frac{4 - 3}{2 - 0} = \frac{1}{2}$$

$$\text{slope of } \overline{OT} = \frac{2 - 0}{4 - 0} = \frac{2}{4} = \frac{1}{2}$$

The slopes of  $\overline{RS}$  and  $\overline{OT}$  are the same, so  $\overline{RS} \parallel \overline{OT}$ .

$$\text{slope of } \overline{ST} = \frac{2 - 4}{4 - 2} = \frac{-2}{2} = -1 \quad \text{slope of } \overline{RO} = \frac{3 - 0}{0 - 0} = \frac{3}{0} \text{ Undefined}$$

The slopes of  $\overline{ST}$  and  $\overline{RO}$  are not the same, so  $\overline{ST}$  is not parallel to  $\overline{RO}$ .

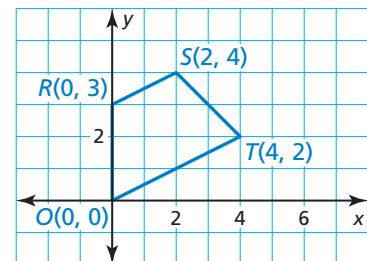
▶ Because  $ORST$  has exactly one pair of parallel sides, it is a trapezoid.

**Step 2** Compare the lengths of legs  $\overline{RO}$  and  $\overline{ST}$ .

$$RO = |3 - 0| = 3 \quad ST = \sqrt{(2 - 4)^2 + (4 - 2)^2} = \sqrt{8} = 2\sqrt{2}$$

Because  $RO \neq ST$ , legs  $\overline{RO}$  and  $\overline{ST}$  are *not* congruent.

▶ So,  $ORST$  is not an isosceles trapezoid.



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1. The points  $A(-5, 6)$ ,  $B(4, 9)$ ,  $C(4, 4)$ , and  $D(-2, 2)$  form the vertices of a quadrilateral. Show that  $ABCD$  is a trapezoid. Then decide whether it is isosceles.

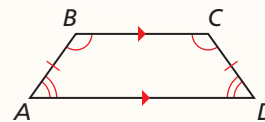
## Theorems

### Theorem 7.14 Isosceles Trapezoid Base Angles Theorem

If a trapezoid is isosceles, then each pair of base angles is congruent.

If trapezoid  $ABCD$  is isosceles, then  $\angle A \cong \angle D$  and  $\angle B \cong \angle C$ .

*Proof* Ex. 39, p. 367

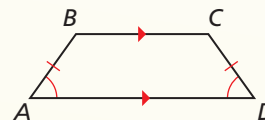


### Theorem 7.15 Isosceles Trapezoid Base Angles Converse

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

If  $\angle A \cong \angle D$  (or if  $\angle B \cong \angle C$ ), then trapezoid  $ABCD$  is isosceles.

*Proof* Ex. 40, p. 367

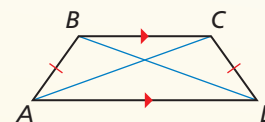


### Theorem 7.16 Isosceles Trapezoid Diagonals Theorem

A trapezoid is isosceles if and only if its diagonals are congruent.

Trapezoid  $ABCD$  is isosceles if and only if  $\overline{AC} \cong \overline{BD}$ .

*Proof* Ex. 51, p. 368



### EXAMPLE 2 Using Properties of Isosceles Trapezoids

The stone above the arch in the diagram is an isosceles trapezoid. Find  $m\angle K$ ,  $m\angle M$ , and  $m\angle J$ .

#### SOLUTION

**Step 1** Find  $m\angle K$ .  $JKLM$  is an isosceles trapezoid.

So,  $\angle K$  and  $\angle L$  are congruent base angles, and  $m\angle K = m\angle L = 85^\circ$ .

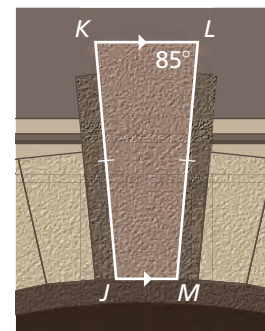
**Step 2** Find  $m\angle M$ . Because  $\angle L$  and  $\angle M$  are

consecutive interior angles formed by  $\overline{LM}$  intersecting two parallel lines, they are supplementary. So,  $m\angle M = 180^\circ - 85^\circ = 95^\circ$ .

**Step 3** Find  $m\angle J$ . Because  $\angle J$  and  $\angle M$  are

a pair of base angles, they are congruent, and  $m\angle J = m\angle M = 95^\circ$ .

► So,  $m\angle K = 85^\circ$ ,  $m\angle M = 95^\circ$ , and  $m\angle J = 95^\circ$ .

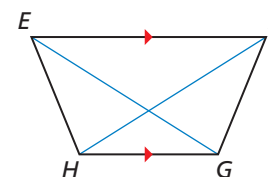


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In Exercises 2 and 3, use trapezoid  $EFGH$ .

2. If  $EG = FH$ , is trapezoid  $EFGH$  isosceles? Explain.

3. If  $m\angle HEF = 70^\circ$  and  $m\angle FGH = 110^\circ$ , is trapezoid  $EFGH$  isosceles? Explain.

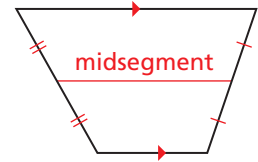


## READING

The midsegment of a trapezoid is sometimes called the *median* of the trapezoid.

## Using the Trapezoid Midsegment Theorem

Recall that a midsegment of a triangle is a segment that connects the midpoints of two sides of the triangle. The **midsegment of a trapezoid** is the segment that connects the midpoints of its legs. The theorem below is similar to the Triangle Midsegment Theorem (Thm. 6.8).



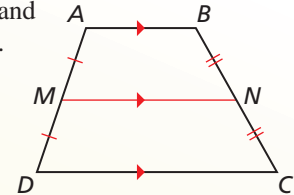
## Theorem

### Theorem 7.17 Trapezoid Midsegment Theorem

The midsegment of a trapezoid is parallel to each base, and its length is one-half the sum of the lengths of the bases.

If  $\overline{MN}$  is the midsegment of trapezoid  $ABCD$ , then  $\overline{MN} \parallel \overline{AB}$ ,  $\overline{MN} \parallel \overline{DC}$ , and  $MN = \frac{1}{2}(AB + CD)$ .

*Proof* Ex. 49, p. 368

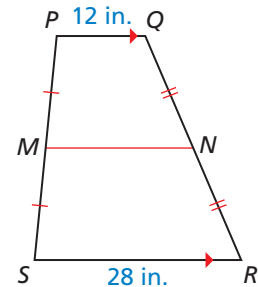


### EXAMPLE 3 Using the Midsegment of a Trapezoid

In the diagram,  $\overline{MN}$  is the midsegment of trapezoid  $PQRS$ . Find  $MN$ .

#### SOLUTION

$$\begin{aligned} MN &= \frac{1}{2}(PQ + SR) && \text{Trapezoid Midsegment Theorem} \\ &= \frac{1}{2}(12 + 28) && \text{Substitute 12 for PQ and 28 for SR.} \\ &= 20 && \text{Simplify.} \end{aligned}$$



► The length of  $\overline{MN}$  is 20 inches.

### EXAMPLE 4 Using a Midsegment in the Coordinate Plane

Find the length of midsegment  $\overline{YZ}$  in trapezoid  $STUV$ .

#### SOLUTION

**Step 1** Find the lengths of  $\overline{SV}$  and  $\overline{TU}$ .

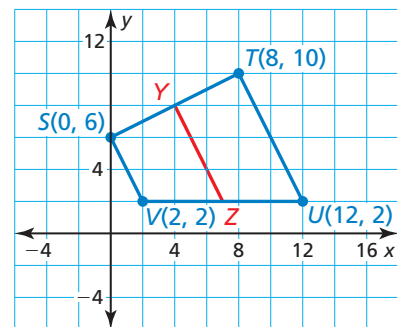
$$SV = \sqrt{(0 - 2)^2 + (6 - 2)^2} = \sqrt{20} = 2\sqrt{5}$$

$$TU = \sqrt{(8 - 12)^2 + (10 - 2)^2} = \sqrt{80} = 4\sqrt{5}$$

**Step 2** Multiply the sum of  $SV$  and  $TU$  by  $\frac{1}{2}$ .

$$YZ = \frac{1}{2}(2\sqrt{5} + 4\sqrt{5}) = \frac{1}{2}(6\sqrt{5}) = 3\sqrt{5}$$

► So, the length of  $\overline{YZ}$  is  $3\sqrt{5}$  units.

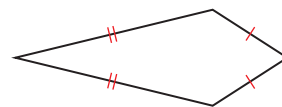


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- In trapezoid  $JKLM$ ,  $\angle J$  and  $\angle M$  are right angles, and  $JK = 9$  centimeters. The length of midsegment  $\overline{NP}$  of trapezoid  $JKLM$  is 12 centimeters. Sketch trapezoid  $JKLM$  and its midsegment. Find  $ML$ . Explain your reasoning.
- Explain another method you can use to find the length of  $\overline{YZ}$  in Example 4.

## Using Properties of Kites

A **kite** is a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent.



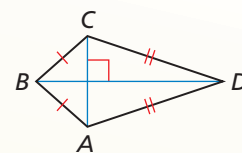
## Theorems

### Theorem 7.18 Kite Diagonals Theorem

If a quadrilateral is a kite, then its diagonals are perpendicular.

If quadrilateral  $ABCD$  is a kite, then  $\overline{AC} \perp \overline{BD}$ .

*Proof* p. 363



### STUDY TIP

The congruent angles of a kite are formed by the noncongruent adjacent sides.

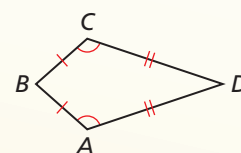


### Theorem 7.19 Kite Opposite Angles Theorem

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

If quadrilateral  $ABCD$  is a kite and  $\overline{BC} \cong \overline{BA}$ , then  $\angle A \cong \angle C$  and  $\angle B \cong \angle D$ .

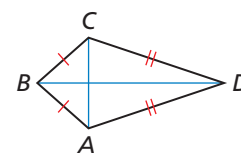
*Proof* Ex. 47, p. 368



### PROOF Kite Diagonals Theorem

**Given**  $ABCD$  is a kite,  $\overline{BC} \cong \overline{BA}$ , and  $\overline{DC} \cong \overline{DA}$ .

**Prove**  $\overline{AC} \perp \overline{BD}$



#### STATEMENTS

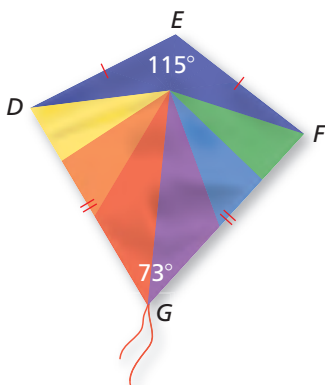
- $ABCD$  is a kite with  $\overline{BC} \cong \overline{BA}$  and  $\overline{DC} \cong \overline{DA}$ .
- $B$  and  $D$  lie on the  $\perp$  bisector of  $\overline{AC}$ .
- $\overline{BD}$  is the  $\perp$  bisector of  $\overline{AC}$ .
- $\overline{AC} \perp \overline{BD}$

#### REASONS

- Given
- Converse of the  $\perp$  Bisector Theorem (Theorem 6.2)
- Through any two points, there exists exactly one line.
- Definition of  $\perp$  bisector

### EXAMPLE 5 Finding Angle Measures in a Kite

Find  $m\angle D$  in the kite shown.



#### SOLUTION

By the Kite Opposite Angles Theorem,  $DEFG$  has exactly one pair of congruent opposite angles. Because  $\angle E \cong \angle G$ ,  $\angle D$  and  $\angle F$  must be congruent. So,  $m\angle D = m\angle F$ . Write and solve an equation to find  $m\angle D$ .

$$m\angle D + m\angle F + 115^\circ + 73^\circ = 360^\circ$$

$$m\angle D + m\angle D + 115^\circ + 73^\circ = 360^\circ$$

$$2m\angle D + 188^\circ = 360^\circ$$

$$m\angle D = 86^\circ$$

Corollary to the Polygon Interior Angles Theorem (Corollary 7.1)

Substitute  $m\angle D$  for  $m\angle F$ .

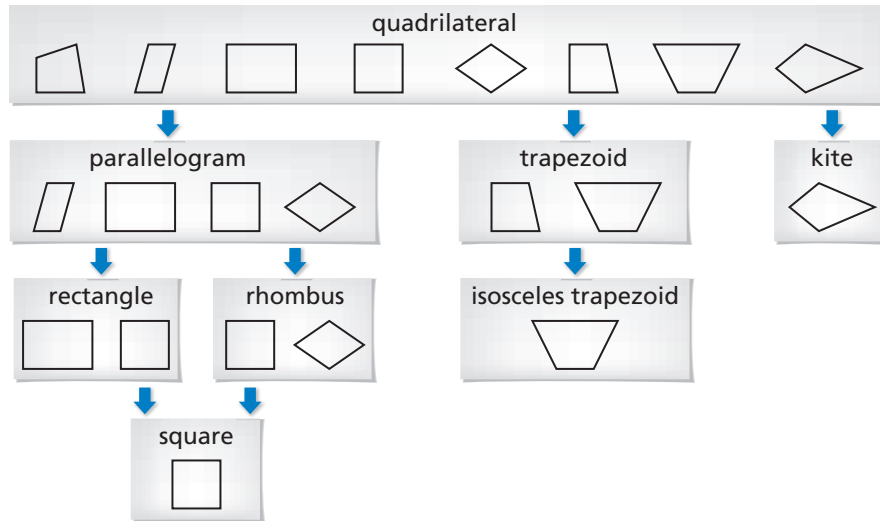
Combine like terms.

Solve for  $m\angle D$ .

6. In a kite, the measures of the angles are  $3x^\circ$ ,  $75^\circ$ ,  $90^\circ$ , and  $120^\circ$ . Find the value of  $x$ . What are the measures of the angles that are congruent?

## Identifying Special Quadrilaterals

The diagram shows relationships among the special quadrilaterals you have studied in this chapter. Each shape in the diagram has the properties of the shapes linked above it. For example, a rhombus has the properties of a parallelogram and a quadrilateral.



### EXAMPLE 6 Identifying a Quadrilateral

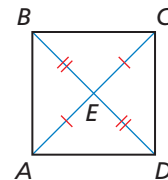
#### READING DIAGRAMS

In Example 6,  $ABCD$  looks like a square. But you must rely only on marked information when you interpret a diagram.

What is the most specific name for quadrilateral  $ABCD$ ?

#### SOLUTION

The diagram shows  $\overline{AE} \cong \overline{CE}$  and  $\overline{BE} \cong \overline{DE}$ . So, the diagonals bisect each other. By the Parallelogram Diagonals Converse (Theorem 7.10),  $ABCD$  is a parallelogram.



Rectangles, rhombuses, and squares are also parallelograms. However, there is no information given about the side lengths or angle measures of  $ABCD$ . So, you cannot determine whether it is a rectangle, a rhombus, or a square.

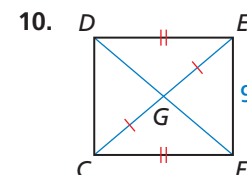
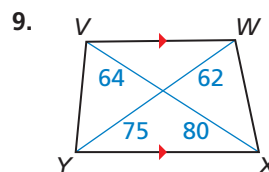
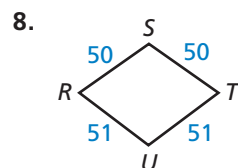
► So, the most specific name for  $ABCD$  is a parallelogram.

## Monitoring Progress

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7. Quadrilateral  $DEFG$  has at least one pair of opposite sides congruent. What types of quadrilaterals meet this condition?

Give the most specific name for the quadrilateral. Explain your reasoning.



# 7.5 Exercises

## Vocabulary and Core Concept Check

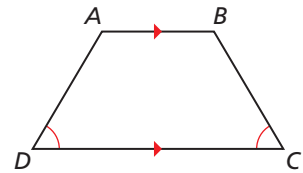
- WRITING** Describe the differences between a trapezoid and a kite.
- DIFFERENT WORDS, SAME QUESTION** Which is different? Find “both” answers.

Is there enough information to prove that trapezoid  $ABCD$  is isosceles?

Is there enough information to prove that  $\overline{AB} \cong \overline{DC}$ ?

Is there enough information to prove that the non-parallel sides of trapezoid  $ABCD$  are congruent?

Is there enough information to prove that the legs of trapezoid  $ABCD$  are congruent?

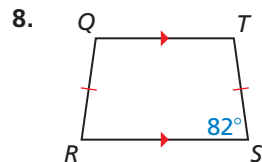
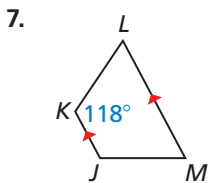


## Monitoring Progress and Modeling with Mathematics

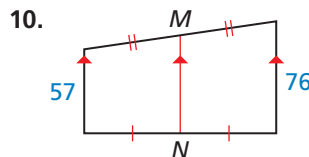
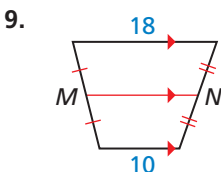
In Exercises 3–6, show that the quadrilateral with the given vertices is a trapezoid. Then decide whether it is isosceles. (See Example 1.)

- $W(1, 4), X(1, 8), Y(-3, 9), Z(-3, 3)$
- $D(-3, 3), E(-1, 1), F(1, -4), G(-3, 0)$
- $M(-2, 0), N(0, 4), P(5, 4), Q(8, 0)$
- $H(1, 9), J(4, 2), K(5, 2), L(8, 9)$

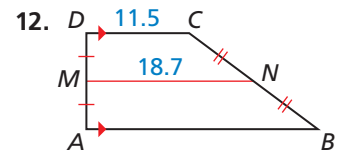
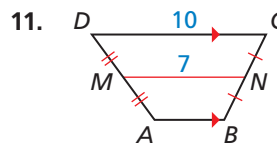
In Exercises 7 and 8, find the measure of each angle in the isosceles trapezoid. (See Example 2.)



In Exercises 9 and 10, find the length of the midsegment of the trapezoid. (See Example 3.)



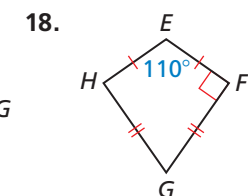
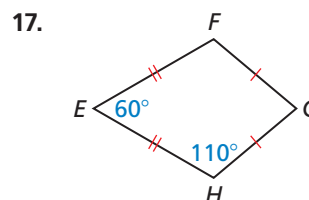
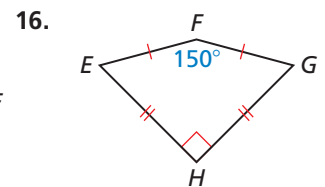
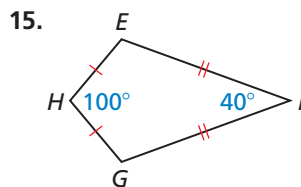
In Exercises 11 and 12, find  $AB$ .



In Exercises 13 and 14, find the length of the midsegment of the trapezoid with the given vertices. (See Example 4.)

- $A(2, 0), B(8, -4), C(12, 2), D(0, 10)$
- $S(-2, 4), T(-2, -4), U(3, -2), V(13, 10)$

In Exercises 15–18, find  $m\angle G$ . (See Example 5.)



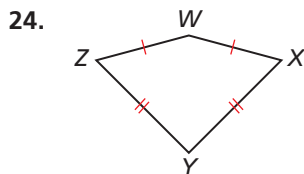
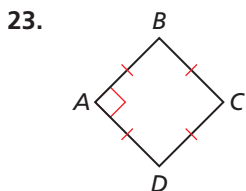
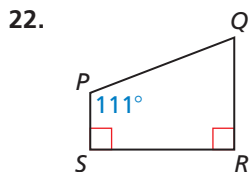
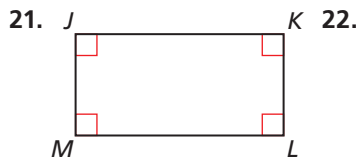
19. **ERROR ANALYSIS** Describe and correct the error in finding  $DC$ .

$DC = AB - MN$   
 $DC = 14 - 8$   
 $DC = 6$

20. **ERROR ANALYSIS** Describe and correct the error in finding  $m\angle A$ .

Opposite angles of a kite are congruent, so  $m\angle A = 50^\circ$ .

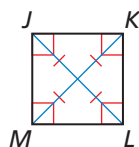
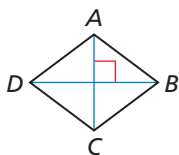
In Exercises 21–24, give the most specific name for the quadrilateral. Explain your reasoning. (See Example 6.)



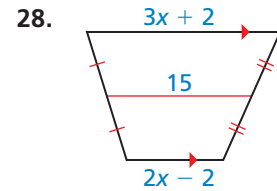
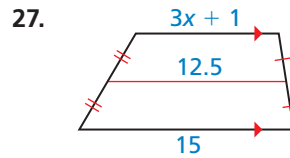
**REASONING** In Exercises 25 and 26, tell whether enough information is given in the diagram to classify the quadrilateral by the indicated name. Explain.

25. rhombus

26. square

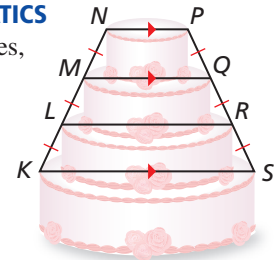


**MATHEMATICAL CONNECTIONS** In Exercises 27 and 28, find the value of  $x$ .

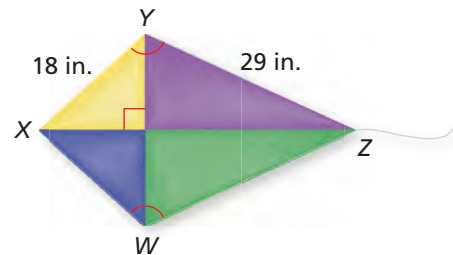


29. **MODELING WITH MATHEMATICS**

In the diagram,  $NP = 8$  inches, and  $LR = 20$  inches. What is the diameter of the bottom layer of the cake?



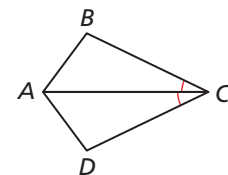
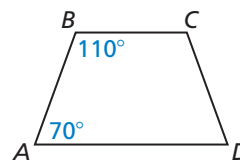
30. **PROBLEM SOLVING** You and a friend are building a kite. You need a stick to place from  $X$  to  $W$  and a stick to place from  $W$  to  $Z$  to finish constructing the frame. You want the kite to have the geometric shape of a kite. How long does each stick need to be? Explain your reasoning.



**REASONING** In Exercises 31–34, determine which pairs of segments or angles must be congruent so that you can prove that  $ABCD$  is the indicated quadrilateral. Explain your reasoning. (There may be more than one right answer.)

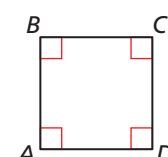
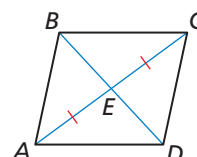
31. isosceles trapezoid

32. kite



33. parallelogram

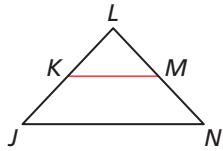
34. square



35. **PROOF** Write a proof.

**Given**  $\overline{JL} \cong \overline{LN}$ ,  $\overline{KM}$  is a midsegment of  $\triangle JLN$ .

**Prove** Quadrilateral  $JKMN$  is an isosceles trapezoid.

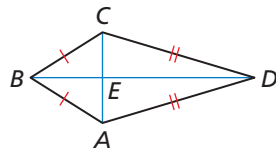


36. **PROOF** Write a proof.

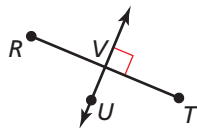
**Given**  $ABCD$  is a kite.

$$\overline{AB} \cong \overline{CB}, \overline{AD} \cong \overline{CD}$$

**Prove**  $\overline{CE} \cong \overline{AE}$

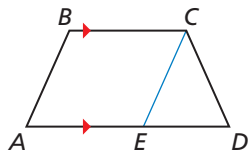


37. **ABSTRACT REASONING** Point  $U$  lies on the perpendicular bisector of  $\overline{RT}$ . Describe the set of points  $S$  for which  $RSTU$  is a kite.



38. **REASONING** Determine whether the points  $A(4, 5)$ ,  $B(-3, 3)$ ,  $C(-6, -13)$ , and  $D(6, -2)$  are the vertices of a kite. Explain your reasoning.

**PROVING A THEOREM** In Exercises 39 and 40, use the diagram to prove the given theorem. In the diagram,  $\overline{EC}$  is drawn parallel to  $\overline{AB}$ .



39. Isosceles Trapezoid Base Angles Theorem (Theorem 7.14)

**Given**  $ABCD$  is an isosceles trapezoid.

$$\overline{BC} \parallel \overline{AD}$$

**Prove**  $\angle A \cong \angle D$ ,  $\angle B \cong \angle C$

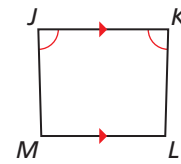
40. Isosceles Trapezoid Base Angles Converse (Theorem 7.15)

**Given**  $ABCD$  is a trapezoid.

$$\angle A \cong \angle D, \overline{BC} \parallel \overline{AD}$$

**Prove**  $ABCD$  is an isosceles trapezoid.

41. **MAKING AN ARGUMENT** Your cousin claims there is enough information to prove that  $JKLM$  is an isosceles trapezoid. Is your cousin correct? Explain.



42. **MATHEMATICAL CONNECTIONS** The bases of a trapezoid lie on the lines  $y = 2x + 7$  and  $y = 2x - 5$ . Write the equation of the line that contains the midsegment of the trapezoid.

43. **CONSTRUCTION**  $\overline{AC}$  and  $\overline{BD}$  bisect each other.

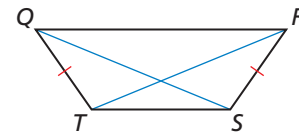
a. Construct quadrilateral  $ABCD$  so that  $\overline{AC}$  and  $\overline{BD}$  are congruent, but not perpendicular. Classify the quadrilateral. Justify your answer.

b. Construct quadrilateral  $ABCD$  so that  $\overline{AC}$  and  $\overline{BD}$  are perpendicular, but not congruent. Classify the quadrilateral. Justify your answer.

44. **PROOF** Write a proof.

**Given**  $QRST$  is an isosceles trapezoid.

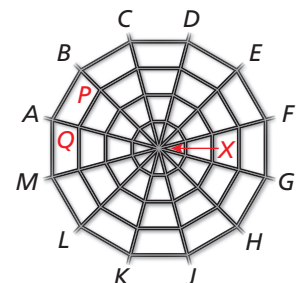
**Prove**  $\angle TQS \cong \angle SRT$



45. **MODELING WITH MATHEMATICS** A plastic spiderweb is made in the shape of a regular dodecagon (12-sided polygon).  $\overline{AB} \parallel \overline{PQ}$ , and  $X$  is equidistant from the vertices of the dodecagon.

a. Are you given enough information to prove that  $ABPQ$  is an isosceles trapezoid?

b. What is the measure of each interior angle of  $ABPQ$ ?



46. **ATTENDING TO PRECISION** In trapezoid  $PQRS$ ,  $\overline{PQ} \parallel \overline{RS}$  and  $\overline{MN}$  is the midsegment of  $PQRS$ . If  $RS = 5 \cdot PQ$ , what is the ratio of  $MN$  to  $RS$ ?

(A) 3 : 5

(B) 5 : 3

(C) 1 : 2

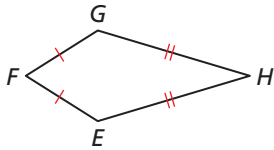
(D) 3 : 1



47. **PROVING A THEOREM** Use the plan for proof below to write a paragraph proof of the Kite Opposite Angles Theorem (Theorem 7.19).

**Given**  $EFGH$  is a kite.  
 $\overline{EF} \cong \overline{FG}$ ,  $\overline{EH} \cong \overline{GH}$

**Prove**  $\angle E \cong \angle G$ ,  $\angle F \cong \angle H$



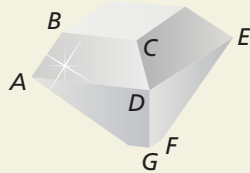
**Plan for Proof** First show that  $\angle E \cong \angle G$ . Then use an indirect argument to show that  $\angle F \cong \angle H$ .

48. **HOW DO YOU SEE IT?** One of the earliest shapes used for cut diamonds is called the *table cut*, as shown in the figure. Each face of a cut gem is called a *facet*.



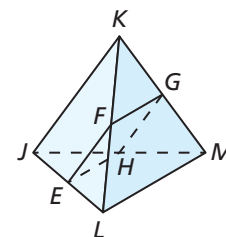
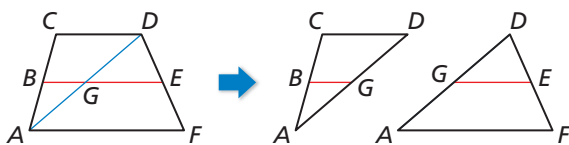
STEM  
VIDEO

- a.  $\overline{BC} \parallel \overline{AD}$ , and  $\overline{AB}$  and  $\overline{DC}$  are not parallel. What shape is the facet labeled  $ABCD$ ?



- b.  $\overline{DE} \parallel \overline{GF}$ , and  $\overline{DG}$  and  $\overline{EF}$  are congruent but not parallel. What shape is the facet labeled  $DEFG$ ?

49. **PROVING A THEOREM** In the diagram below,  $\overline{BG}$  is the midsegment of  $\triangle ACD$ , and  $\overline{GE}$  is the midsegment of  $\triangle ADF$ . Use the diagram to prove the Trapezoid Midsegment Theorem (Theorem 7.17).



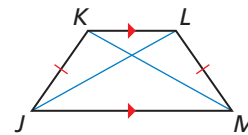
50. **THOUGHT PROVOKING** Is SSASS a valid congruence theorem for kites? Justify your answer.

51. **PROVING A THEOREM** To prove the biconditional statement in the Isosceles Trapezoid Diagonals Theorem (Theorem 7.16), you must prove both parts separately.

- a. Prove part of the Isosceles Trapezoid Diagonals Theorem (Theorem 7.16).

**Given**  $JKLM$  is an isosceles trapezoid.  
 $\overline{KL} \parallel \overline{JM}$ ,  $\overline{JK} \cong \overline{LM}$

**Prove**  $\overline{JL} \cong \overline{KM}$



- b. Write the other part of the Isosceles Trapezoid Diagonals Theorem (Theorem 7.16) as a conditional. Then prove the statement is true.

52. **PROOF** What special type of quadrilateral is  $EFGH$ ? Write a proof to show that your answer is correct.

**Given** In the three-dimensional figure,  $\overline{JK} \cong \overline{LM}$ .  
 $E, F, G,$  and  $H$  are the midpoints of  $\overline{JL}$ ,  $\overline{KL}$ ,  $\overline{KM}$ , and  $\overline{JM}$ , respectively.

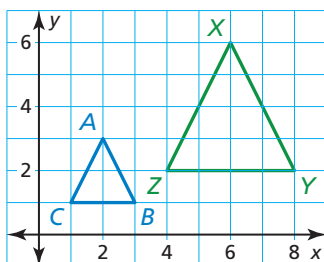
**Prove**  $EFGH$  is a \_\_\_\_\_.

## Maintaining Mathematical Proficiency

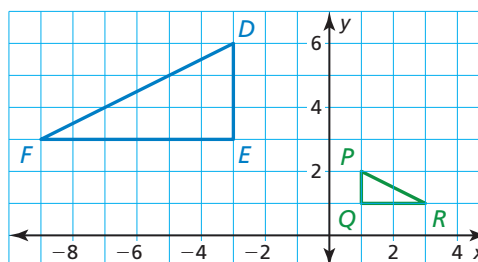
Reviewing what you learned in previous grades and lessons

Describe a similarity transformation that maps the blue preimage to the green image.

53.



54.



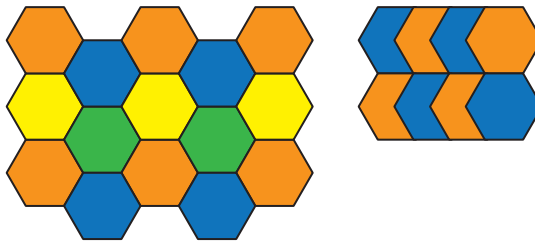
# 7.6 Tessellations

**Essential Question** How can you use patterns to completely cover a plane?

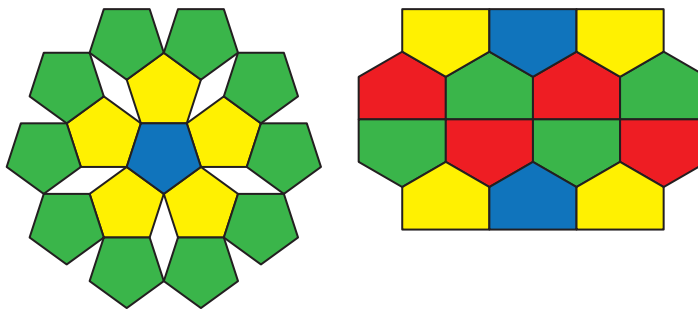
## EXPLORATION 1 Comparing Images

**Work with a partner.** The images are made using the indicated polygon. Describe any similarities and any differences in the images.

a. hexagon



b. pentagon



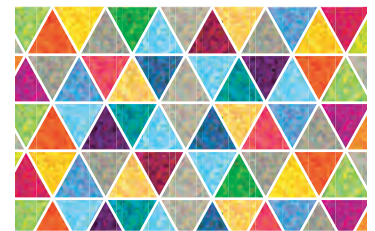
### ATTENDING TO PRECISION

Mathematically proficient students communicate ideas clearly to others.

## EXPLORATION 2 Drawing a Tessellation

**Work with a partner.**

- The image at the right shows a *tessellation*. Which of the images in Exploration 1 are also tessellations? Explain your reasoning.
- Use what you learned in part (a) to draw your own tessellation.



### Communicate Your Answer

- How can you use patterns to completely cover a plane?
- Describe a tessellation that is found in real life.

# 7.6 Lesson

## Core Vocabulary

tessellation, p. 370  
regular tessellation, p. 372

### Previous

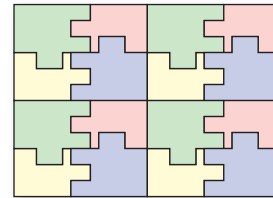
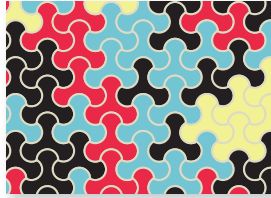
regular polygon  
interior angles

## What You Will Learn

- ▶ Determine whether a shape tessellates.
- ▶ Find angle measures in tessellations of polygons.
- ▶ Determine whether a regular polygon tessellates a plane.

## Tessellations

A **tessellation** is the covering of a plane with figures so that there are no gaps or overlaps. A figure that forms a tessellation is said to tessellate the plane.



### EXAMPLE 1 Determining Whether Shapes Tessellate

Determine whether each shape tessellates.

a. parallelogram

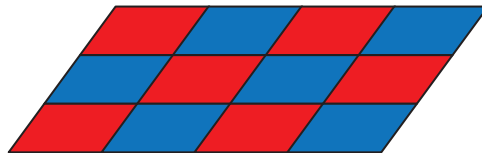


b. circle



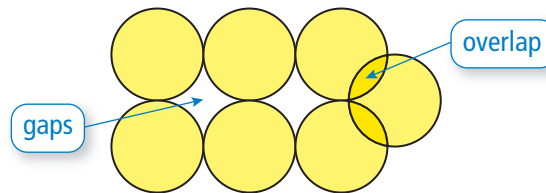
### SOLUTION

a.



- ▶ There are no gaps or overlaps. So, the parallelogram tessellates.

b.



- ▶ The circle does not tessellate.

## Monitoring Progress



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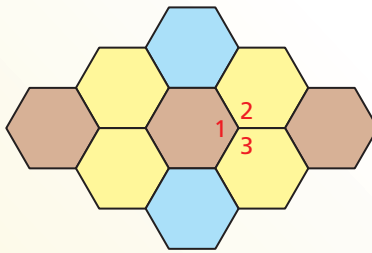
1. Determine whether the trapezoid tessellates.



## Core Concept

### Angle Measures in a Tessellation of Polygons

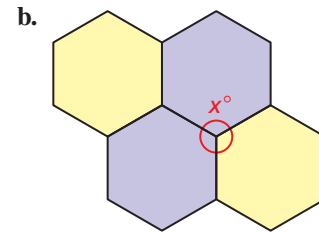
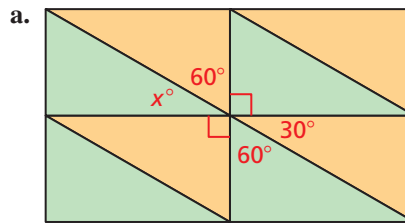
The sum of the angle measures around a point of intersection in a tessellation of polygons is  $360^\circ$ .



$$m\angle 1 + m\angle 2 + m\angle 3 = 360^\circ$$

### EXAMPLE 2 Finding Angle Measures in Tessellations

Find  $x$  in each tessellation.



### SOLUTION

$$\begin{aligned} \text{a. } 60^\circ + 90^\circ + 30^\circ + 60^\circ + 90^\circ + x^\circ &= 360^\circ \\ 330 + x &= 360 \\ x &= 30 \end{aligned}$$

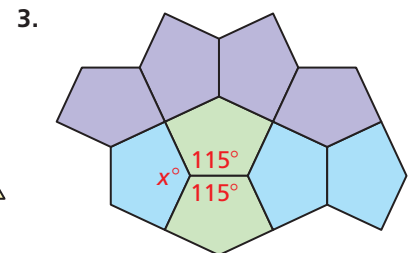
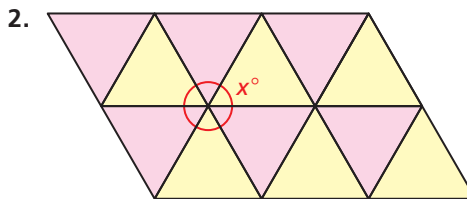
► So, the value of  $x$  is 30.

b. There are 3 congruent angles around the point of intersection in the tessellation. So, the measure of each angle is  $360^\circ \div 3 = 120^\circ$ .

► So, the value of  $x$  is 120.

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Find  $x$  in the tessellation.



A **regular tessellation** is a tessellation of congruent regular polygons. In a regular tessellation, each angle around a point of intersection is congruent. This means that the measure of each angle around the point of intersection is a factor of 360.

## Core Concept

### Regular Tessellations

A regular polygon tessellates a plane if the measure of an interior angle of the polygon is a factor of 360.

### EXAMPLE 3 Determining Whether Regular Polygons Tessellate

Determine whether each polygon tessellates.

- a. regular pentagon                      b. regular hexagon                      c. regular octagon

### SOLUTION

- a. A pentagon has 5 sides. By the Polygon Interior Angles Theorem (Thm. 7.1), the sum of the measures of the interior angles is  $(5 - 2) \cdot 180^\circ = 3 \cdot 180^\circ = 540^\circ$ . Therefore, the measure of each interior angle of a regular pentagon is  $540^\circ \div 5 = 108^\circ$ .

108 is not a factor of 360.

- ▶ A regular pentagon does not tessellate.

- b. A hexagon has 6 sides. By the Polygon Interior Angles Theorem (Thm 7.1), the sum of the measures of the interior angles is  $(6 - 2) \cdot 180^\circ = 4 \cdot 180^\circ = 720^\circ$ . Therefore, the measure of each interior angle of a regular hexagon is  $720^\circ \div 6 = 120^\circ$ .

120 is a factor of 360.

- ▶ A regular hexagon tessellates.

- c. An octagon has 8 sides. By the Polygon Interior Angles Theorem (Thm 7.1), the sum of the measures of the interior angles is  $(8 - 2) \cdot 180^\circ = 6 \cdot 180^\circ = 1080^\circ$ . Therefore, the measure of each interior angle of a regular octagon is  $1080^\circ \div 8 = 135^\circ$ .

135 is not a factor of 360.

- ▶ A regular octagon does not tessellate.

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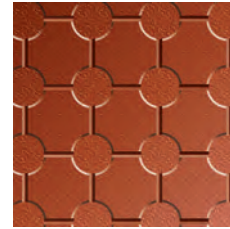
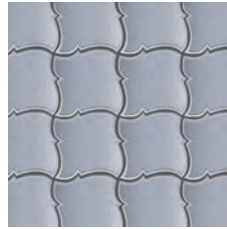
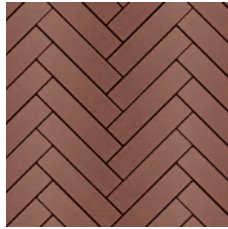
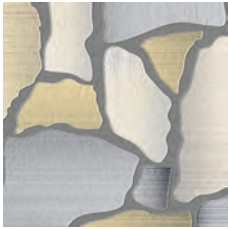
Determine whether the polygon tessellates.

4. regular quadrilateral  
5. regular heptagon

# 7.6 Exercises

## Vocabulary and Core Concept Check

- WRITING** In your own words, describe how to tell if a tessellation is a regular tessellation.
- WHICH ONE DOESN'T BELONG** Which image does *not* belong with the other three? Explain your reasoning.



## Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, determine whether the shape tessellates. (See Example 1.)

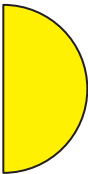
3. rectangle



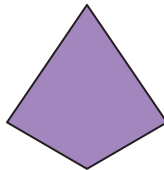
4. rhombus



5. semicircle

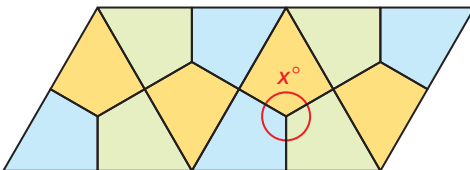


6. kite

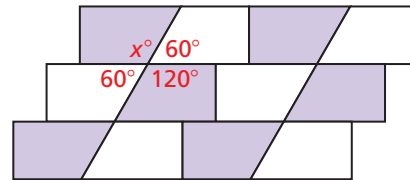


In Exercises 7–10, find  $x$  in the tessellation. (See Example 2.)

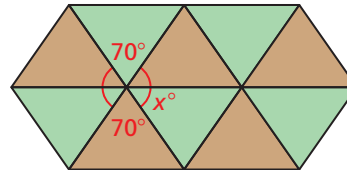
7.



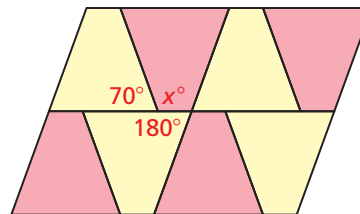
8.



9.



10.



In Exercises 11–14, determine whether the polygon tessellates. (See Example 3.)

- regular triangle
- square
- regular nonagon
- regular decagon

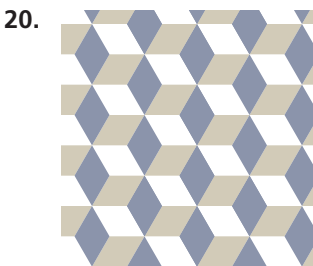
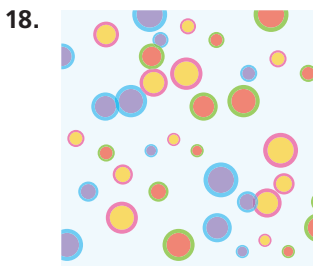
15. **ERROR ANALYSIS** Describe and correct the error in determining whether a regular dodecagon tessellates.



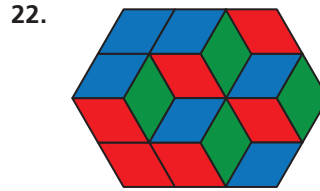
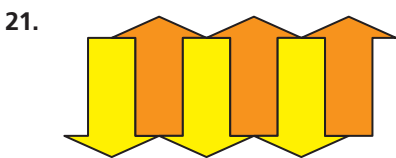
A dodecagon has 12 sides. So, the measure of each interior angle is  $360^\circ \div 12 = 30^\circ$ . Because 30 is a factor of 360, a regular dodecagon tessellates.

16. **MAKING AN ARGUMENT** Your friend claims it is possible to draw a tessellation that has exactly 6 angles around a point of intersection, each with a measure of  $30^\circ$ . Is your friend correct? Explain your reasoning.

In Exercises 17–20, determine whether the image shows a tessellation.

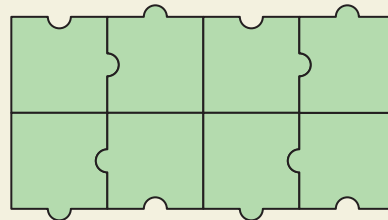


In Exercises 21 and 22, describe the transformation(s) used to make the tessellation.



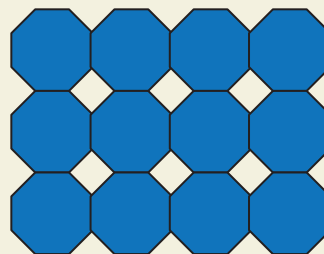
23. **WRITING** Determine whether each shape will *always*, *sometimes*, or *never* tessellate. Explain your reasoning.
- circle
  - triangle
  - quadrilateral
  - pentagon

24. **HOW DO YOU SEE IT?** Consider the tessellation shown below. How can you use a triangle to make a similar tessellation? Justify your answer.



25. **REASONING** Is it possible to list every regular polygon that tessellates? Explain your reasoning.

26. **THOUGHT PROVOKING** A pattern is made using octagons as shown. Explain how you can create a tessellating composite figure using one of the octagons.



## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the coordinates of the image after a dilation centered at the origin with scale factor  $k$ .

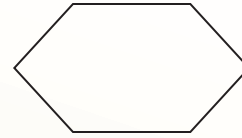
27.  $A(-2, 1), B(-2, 5), C(2, 1), D(2, 5); k = 2$       28.  $J(-8, -4), K(0, 6), L(2, 2); k = \frac{1}{2}$   
 29.  $P(1, 1), Q(2, -1), R(-1, -2); k = -3$       30.  $T(-8, 8), U(0, 0), V(-4, 2); k = -\frac{1}{4}$

### 7.1 Angles of Polygons (pp. 323–330)

Find the sum of the measures of the interior angles of the figure.

The figure is a convex hexagon. It has 6 sides. Use the Polygon Interior Angles Theorem (Theorem 7.1).

$$\begin{aligned}(n - 2) \cdot 180^\circ &= (6 - 2) \cdot 180^\circ && \text{Substitute 6 for } n. \\ &= 4 \cdot 180^\circ && \text{Subtract.} \\ &= 720^\circ && \text{Multiply.}\end{aligned}$$

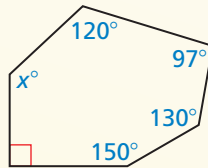


► The sum of the measures of the interior angles of the figure is  $720^\circ$ .

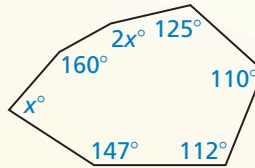
- Find the sum of the measures of the interior angles of a regular 30-gon. Then find the measure of each interior angle and each exterior angle.

Find the value of  $x$ .

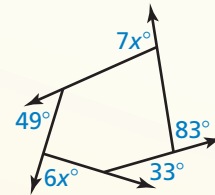
2.



3.



4.

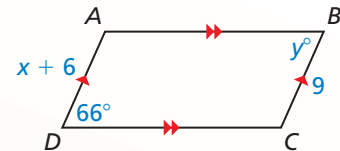


### 7.2 Properties of Parallelograms (pp. 331–338)

Find the values of  $x$  and  $y$ .

$ABCD$  is a parallelogram by the definition of a parallelogram. Use the Parallelogram Opposite Sides Theorem (Thm. 7.3) to find the value of  $x$ .

$$\begin{aligned}AD &= BC && \text{Opposite sides of a parallelogram are congruent.} \\ x + 6 &= 9 && \text{Substitute } x + 6 \text{ for } AD \text{ and } 9 \text{ for } BC. \\ x &= 3 && \text{Subtract 6 from each side.}\end{aligned}$$

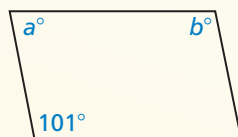


By the Parallelogram Opposite Angles Theorem (Thm. 7.4),  $\angle D \cong \angle B$ , or  $m\angle D = m\angle B$ . So,  $y^\circ = 66^\circ$ .

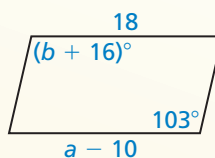
► In  $\square ABCD$ ,  $x = 3$  and  $y = 66$ .

Find the value of each variable in the parallelogram.

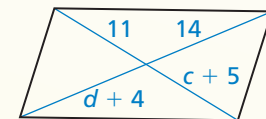
5.



6.



7.



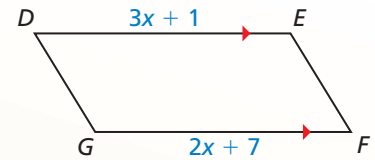
- Find the coordinates of the intersection of the diagonals of  $\square QRST$  with vertices  $Q(-8, 1)$ ,  $R(2, 1)$ ,  $S(4, -3)$ , and  $T(-6, -3)$ .
- Three vertices of  $\square JKLM$  are  $J(1, 4)$ ,  $K(5, 3)$ , and  $L(6, -3)$ . Find the coordinates of vertex  $M$ .



### 7.3 Proving That a Quadrilateral Is a Parallelogram (pp. 339–348)

For what value of  $x$  is quadrilateral  $DEFG$  a parallelogram?

By the Opposite Sides Parallel and Congruent Theorem (Thm. 7.9), if one pair of opposite sides are congruent and parallel, then  $DEFG$  is a parallelogram. You are given that  $\overline{DE} \parallel \overline{FG}$ . Find  $x$  so that  $\overline{DE} \cong \overline{FG}$ .



$$DE = FG \quad \text{Set the segment lengths equal.}$$

$$3x + 1 = 2x + 7 \quad \text{Substitute } 3x + 1 \text{ for } DE \text{ and } 2x + 7 \text{ for } FG.$$

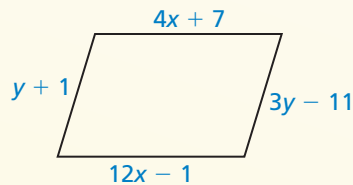
$$x + 1 = 7 \quad \text{Subtract } 2x \text{ from each side.}$$

$$x = 6 \quad \text{Subtract 1 from each side.}$$

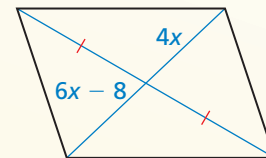
When  $x = 6$ ,  $DE = 3(6) + 1 = 19$  and  $FG = 2(6) + 7 = 19$ .

► Quadrilateral  $DEFG$  is a parallelogram when  $x = 6$ .

10. Find the values of  $x$  and  $y$  that make the quadrilateral a parallelogram.



11. Find the value of  $x$  that makes the quadrilateral a parallelogram.



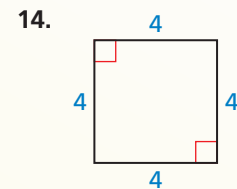
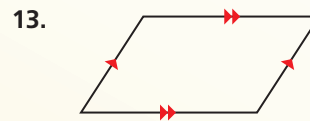
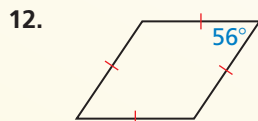
### 7.4 Properties of Special Parallelograms (pp. 349–358)

Classify the special quadrilateral. Explain your reasoning.

The quadrilateral has four right angles. By the Rectangle Corollary (Corollary 7.3), the quadrilateral is a rectangle. Because the four sides are not marked as congruent, you cannot conclude that the rectangle is a square.



Classify the special quadrilateral. Explain your reasoning.



15. Find the lengths of the diagonals of rectangle  $WXYZ$  where  $WY = -2x + 34$  and  $XZ = 3x - 26$ .

## 7.5 Properties of Trapezoids and Kites (pp. 359–368)

Find the length of midsegment  $\overline{EF}$  in trapezoid  $ABCD$ .

**Step 1** Find the lengths of  $\overline{AD}$  and  $\overline{BC}$ .

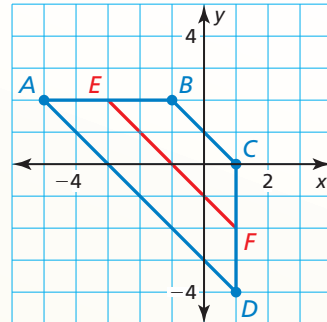
$$\begin{aligned} AD &= \sqrt{[1 - (-5)]^2 + (-4 - 2)^2} \\ &= \sqrt{72} \\ &= 6\sqrt{2} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{[1 - (-1)]^2 + (0 - 2)^2} \\ &= \sqrt{8} \\ &= 2\sqrt{2} \end{aligned}$$

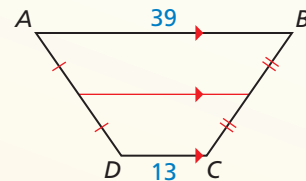
**Step 2** Multiply the sum of  $AD$  and  $BC$  by  $\frac{1}{2}$ .

$$EF = \frac{1}{2}(6\sqrt{2} + 2\sqrt{2}) = \frac{1}{2}(8\sqrt{2}) = 4\sqrt{2}$$

▶ So, the length of  $\overline{EF}$  is  $4\sqrt{2}$  units.



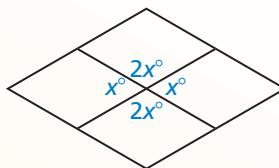
16. Find the length of the midsegment of trapezoid  $ABCD$ .
17. Find the length of the midsegment of trapezoid  $JKLM$  with vertices  $J(6, 10)$ ,  $K(10, 6)$ ,  $L(8, 2)$ , and  $M(2, 2)$ .
18. A kite has angle measures of  $7x^\circ$ ,  $65^\circ$ ,  $85^\circ$ , and  $105^\circ$ . Find the value of  $x$ . What are the measures of the angles that are congruent?



## 7.6 Tessellations (pp. 369–374)

Find  $x$  in the tessellation.

$$\begin{aligned} x^\circ + x^\circ + 2x^\circ + 2x^\circ &= 360^\circ \\ 6x &= 360 \\ x &= 60 \end{aligned}$$



▶ So, the value of  $x$  is 60.

Find  $x$  in the tessellation.

