4 Inequalities

4.1 Writing and Graphing Inequalities
4.2 Solving Inequalities Using Addition or Subtraction
4.3 Solving Inequalities Using Multiplication or Division
4.4 Solving Two-Step Inequalities
What You Learned Before

**Graphing Inequalities**

**Example 1** Graph \( x \geq 2 \).

- Use a closed circle because 2 is a solution.
- Shade the number line on the side where you found the solution.
- Test a number to the left of 2, \( x = 0 \) is not a solution.
- Test a number to the right of 2, \( x = 6 \) is a solution.

**Try It Yourself**

Graph the inequality.

1. \( x \geq 1 \)
2. \( x < 5 \)
3. \( x \leq 20 \)
4. \( x > 13 \)

**Comparing Numbers**

**Example 2** Compare \( -\frac{1}{3} \) and \( -\frac{5}{6} \).

- \( -\frac{5}{6} \) is to the left of \( -\frac{1}{3} \).
- \( \therefore \) So, \( -\frac{5}{6} < -\frac{1}{3} \).

**Try It Yourself**

Copy and complete the statement using < or >.

5. \( -\frac{2}{3} \quad 3 \quad 8 \)
6. \( -\frac{1}{2} \quad 7 \quad 8 \)
7. \( -\frac{1}{5} \quad 1 \quad 10 \)
8. \( -1.4 \quad 1.2 \)
9. \( -2.2 \quad -4.6 \)
10. \( -1.9 \quad -1.1 \)
Essential Question How can you use a number line to represent solutions of an inequality?

ACTIVITY: Understanding Inequality Statements

Work with a partner. Read the statement. Circle each number that makes the statement true, and then answer the questions.

a. “You are in at least 5 of the photos.”

−3 −2 −1 0 1 2 3 4 5 6

● What do you notice about the numbers that you circled?
● Is the number 5 included? Why or why not?
● Write four other numbers that make the statement true.

b. “The temperature is less than −4 degrees Fahrenheit.”

−7 −6 −5 −4 −3 −2 −1 0 1 2

● What do you notice about the numbers that you circled?
● Can the temperature be exactly −4 degrees Fahrenheit? Explain.
● Write four other numbers that make the statement true.

c. “More than 3 students from our school are in the chess tournament.”

−3 −2 −1 0 1 2 3 4 5 6

● What do you notice about the numbers that you circled?
● Is the number 3 included? Why or why not?
● Write four other numbers that make the statement true.

d. “The balance in a yearbook fund is no more than −$5.”

−7 −6 −5 −4 −3 −2 −1 0 1 2

● What do you notice about the numbers that you circled?
● Is the number −5 included? Why or why not?
● Write four other numbers that make the statement true.
Work with a partner.

a. Consider the statement “$x$ is a number such that $x > -1.5$.”
   - Can the number be exactly $-1.5$? Explain.
   - Make a number line. Shade the part of the number line that shows the numbers that make the statement true.
   - Write four other numbers that are not integers that make the statement true.

b. Consider the statement “$x$ is a number such that $x \leq \frac{5}{2}$.”
   - Can the number be exactly $\frac{5}{2}$? Explain.
   - Make a number line. Shade the part of the number line that shows the numbers that make the statement true.
   - Write four other numbers that are not integers that make the statement true.

Work with a partner. Write an inequality for each graph. Then, in words, describe all the values of $x$ that make the inequality true.

a. 

b. 

c. 

d. 

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you use a number line to represent solutions of an inequality?

5. **STRUCTURE** Is $x \geq -1.4$ the same as $-1.4 \leq x$? Explain.

Use what you learned about writing and graphing inequalities to complete Exercises 4 and 5 on page 128.
An **inequality** is a mathematical sentence that compares expressions. It contains the symbols $<$, $>$, $\leq$, or $\geq$. To write an inequality, look for the following phrases to determine where to place the inequality symbol.

### Key Vocabulary
- **inequality**, p. 126
- **solution of an inequality**, p. 126
- **solution set**, p. 126
- **graph of an inequality**, p. 127

### Inequality Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>$&lt;$</th>
<th>$&gt;$</th>
<th>$\leq$</th>
<th>$\geq$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Key Phrases</td>
<td>• is less than</td>
<td>• is greater than</td>
<td>• is less than or equal to</td>
<td>• is greater than or equal to</td>
</tr>
<tr>
<td></td>
<td>• is fewer than</td>
<td>• is more than</td>
<td>• is at most</td>
<td>• is at least</td>
</tr>
<tr>
<td></td>
<td>• is no more than</td>
<td></td>
<td>• is no less than</td>
<td></td>
</tr>
</tbody>
</table>

### Example 1 Writing an Inequality

A number $q$ plus 5 is greater than or equal to $-7.9$. Write this word sentence as an inequality.

\[
q + 5 \geq -7.9
\]

An inequality is $q + 5 \geq -7.9$.

### On Your Own

Write the word sentence as an inequality.

1. A number $x$ is at most $-10$.
2. Twice a number $y$ is more than $-\frac{5}{2}$.

### Reading

The symbol $\leq$ means *is not less than or equal to*.

### Solution of an Inequality

A **solution of an inequality** is a value that makes the inequality true. An inequality can have more than one solution. The set of all solutions of an inequality is called the **solution set**.

<table>
<thead>
<tr>
<th>Value of $x$</th>
<th>$x + 2 \leq -1$</th>
<th>Is the inequality true?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2$</td>
<td>$-2 + 2 \leq -1$</td>
<td>$-2 \not\leq -1$ no</td>
</tr>
<tr>
<td></td>
<td>$0 \not\leq -1$</td>
<td></td>
</tr>
<tr>
<td>$-3$</td>
<td>$-3 + 2 \leq -1$</td>
<td>$-3 \leq -1$ yes</td>
</tr>
<tr>
<td></td>
<td>$-1 \leq -1$</td>
<td></td>
</tr>
<tr>
<td>$-4$</td>
<td>$-4 + 2 \leq -1$</td>
<td>$-4 \leq -1$ yes</td>
</tr>
<tr>
<td></td>
<td>$-2 \leq -1$</td>
<td></td>
</tr>
</tbody>
</table>
**Example 2 Checking Solutions**

Tell whether \(-2\) is a solution of each inequality.

a. \(y - 5 \geq -6\)

\[
\begin{align*}
  y - 5 & \geq -6 \\
  -2 - 5 & \geq -6 \\
  -7 & \geq -6
\end{align*}
\]

\(-7 \geq -6 \quad \times \quad \text{Simplify.}\)

\(-7\) is not greater than or equal to \(-6\).

\[
\therefore \text{So, } -2 \text{ is not a solution of the inequality.}
\]

b. \(-5.5y < 14\)

\[
\begin{align*}
  -5.5y & < 14 \\
  -5.5(-2) & < 14 \\
  11 & < 14 \quad \checkmark
\end{align*}
\]

11 is less than 14.

\[
\therefore \text{So, } -2 \text{ is a solution of the inequality.}
\]

**On Your Own**

Tell whether \(-5\) is a solution of the inequality.

3. \(x + 12 > 7\) 
4. \(1 - 2p \leq -9\) 
5. \(n \div 2.5 \geq -3\)

The graph of an inequality shows all the solutions of the inequality on a number line. An open circle \(\circ\) is used when a number is not a solution. A closed circle \(\bullet\) is used when a number is a solution. An arrow to the left or right shows that the graph continues in that direction.

**Example 3 Graphing an Inequality**

Graph \(y > -8\).

Use an open circle because \(-8\) is not a solution.

Test a number to the left of \(-8\). \(y = -12\) is not a solution.

Test a number to the right of \(-8\). \(y = 0\) is a solution.

Shade the number line on the side where you found the solution.

**On Your Own**

Graph the inequality on a number line.

6. \(x < -1\) 
7. \(z \geq 4\) 
8. \(s \leq 1.4\) 
9. \(-\frac{1}{2} < t\)

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Section 4.1 Writing and Graphing Inequalities 127
1. **PRECISION** Should you use an open circle or a closed circle in the graph of the inequality \( b \geq -42 \)? Explain.

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Write “both” inequalities.
   - \( k \) is less than or equal to \(-3\).
   - \( k \) is no more than \(-3\).
   - \( k \) is at most \(-3\).
   - \( k \) is at least \(-3\).

3. **REASONING** Do \( x < 5 \) and \( 5 < x \) represent the same inequality? Explain.

---

**Write the word sentence as an inequality.**

6. A number \( y \) is no more than \(-8\).
7. A number \( w \) added to 2.3 is more than 18.
8. A number \( t \) multiplied by \(-4\) is at least \(-\frac{2}{5}\).
9. A number \( b \) minus 4.2 is less than \(-7.5\).

10. **ERROR ANALYSIS** Describe and correct the error in writing the word sentence as an inequality.
    
    Twice a number \( x \) is at most \(-24\).
    \[ 2x \geq -24 \]

---

**Tell whether the given value is a solution of the inequality.**

11. \( n + 8 \leq 13; n = 4 \)
12. \( 5h > -15; h = -5 \)
13. \( p + 1.4 \leq 0.5; p = 0.1 \)
14. \( \frac{a}{6} > -4; a = -18 \)
15. \( -\frac{2}{3}s \geq 6; s = -9 \)
16. \( \frac{7}{8} - 3k < -\frac{1}{2}; k = \frac{1}{4} \)

---

**Graph the inequality on a number line.**

17. \( r \leq -9 \)
18. \( g > 2.75 \)
19. \( x \geq -3\frac{1}{2} \)
20. \( z < 1\frac{1}{4} \)

21. **FOOD TRUCK** Each day at lunchtime, at least 53 people buy food from a food truck. Write an inequality that represents this situation.
Tell whether the given value is a solution of the inequality.

22. \(4k < k + 8; k = 3\)

23. \(\frac{w}{3} \geq w - 12; w = 15\)

24. \(7 - 2y > 3y + 13; y = -1\)

25. \(\frac{3}{4}b - 2 \leq 2b + 8; b = -4\)

26. **MODELING** A subway ride for a student costs $1.25. A monthly pass costs $35.
   
   a. Write an inequality that represents the number of times you must ride the subway for the monthly pass to be a better deal.
   
   b. You ride the subway about 45 times per month. Should you buy the monthly pass? Explain.

27. **LOGIC** Consider the inequality \(b > -2\).
   
   a. Describe the values of \(b\) that are solutions of the inequality.
   
   b. Describe the values of \(b\) that are not solutions of the inequality. Write an inequality for these values.
   
   c. What do all the values in parts (a) and (b) represent? Is this true for any inequality?

28. **Critical Thinking** A postal service says that a rectangular package can have a maximum combined length and girth of 108 inches. The girth of a package is the distance around the perimeter of a face that does not include the length.
   
   a. Write an inequality that represents the allowable dimensions for the package.
   
   b. Find three different sets of allowable dimensions that are reasonable for the package. Find the volume of each package.

---

**Fair Game Review** What you learned in previous grades & lessons

**Solve the equation. Check your solution. (Section 3.3)**

29. \(p - 8 = 3\)

30. \(8.7 + w = 5.1\)

31. \(x - 2 = -9\)

**32. MULTIPLE CHOICE** Which expression has a value less than \(-5\)? (Section 1.2)

- A. \(5 + 8\)
- B. \(-9 + 5\)
- C. \(1 + (-8)\)
- D. \(7 + (-2)\)
Essential Question: How can you use addition or subtraction to solve an inequality?

1. **ACTIVITY: Writing an Inequality**

   Work with a partner. Members of the Boy Scouts must be less than 18 years old. In 4 years, your friend will still be eligible to be a scout.

   a. Which of the following represents your friend’s situation? What does $x$ represent? Explain your reasoning.

   $x + 4 > 18$  $x + 4 < 18$

   $x + 4 \geq 18$  $x + 4 \leq 18$

   b. Graph the possible ages of your friend on a number line. Explain how you decided what to graph.

2. **ACTIVITY: Writing an Inequality**

   Work with a partner. Supercooling is the process of lowering the temperature of a liquid or a gas below its freezing point without it becoming a solid. Water can be supercooled to 86°F below its normal freezing point (32°F) and still not freeze.

   a. Let $x$ represent the temperature of water. Which inequality represents the temperature at which water can be a liquid or a gas? Explain your reasoning.

   $x - 32 > -86$  $x - 32 < -86$

   $x - 32 \geq -86$  $x - 32 \leq -86$

   b. On a number line, graph the possible temperatures at which water can be a liquid or a gas. Explain how you decided what to graph.
Work with a partner. Complete the following steps for Activity 1. Then repeat the steps for Activity 2.

- Use your inequality from part (a). Replace the inequality symbol with an equal sign.
- Solve the equation.
- Replace the equal sign with the original inequality symbol.
- Graph this new inequality.
- Compare the graph with your graph in part (b). What do you notice?

### Activity: Temperatures of Continents

Work with a partner. The table shows the lowest recorded temperature on each continent. Write an inequality that represents each statement. Then solve and graph the inequality.

<table>
<thead>
<tr>
<th>Continent</th>
<th>Lowest Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Africa</td>
<td>−11°F</td>
</tr>
<tr>
<td>Antarctica</td>
<td>−129°F</td>
</tr>
<tr>
<td>Asia</td>
<td>−90°F</td>
</tr>
<tr>
<td>Australia</td>
<td>−9.4°F</td>
</tr>
<tr>
<td>Europe</td>
<td>−67°F</td>
</tr>
<tr>
<td>North America</td>
<td>−81.4°F</td>
</tr>
<tr>
<td>South America</td>
<td>−27°F</td>
</tr>
</tbody>
</table>

**a.** The temperature at a weather station in Asia is more than 150°F greater than the record low in Asia.

**b.** The temperature at a research station in Antarctica is at least 80°F greater than the record low in Antarctica.

### What Is Your Answer?

5. **IN YOUR OWN WORDS** How can you use addition or subtraction to solve an inequality?

6. Describe a real-life situation that you can represent with an inequality. Write the inequality. Graph the solution on a number line.

**Practice** Use what you learned about solving inequalities to complete Exercises 3–5 on page 134.
**Key Ideas**

**Addition Property of Inequality**

**Words** When you add the same number to each side of an inequality, the inequality remains true.

**Numbers** $-4 < 3$

**Algebra** If $a < b$, then $a + c < b + c$.

\[
\begin{align*}
+2 & \quad +2 \\
-2 & \quad < \\
5 & \\
\end{align*}
\]

If $a > b$, then $a + c > b + c$.

**Subtraction Property of Inequality**

**Words** When you subtract the same number from each side of an inequality, the inequality remains true.

**Numbers** $-2 < 2$

**Algebra** If $a < b$, then $a - c < b - c$.

\[
\begin{align*}
-3 & \quad -3 \\
-5 & \quad < \\
-1 & \\
\end{align*}
\]

If $a > b$, then $a - c > b - c$.

These properties are also true for $\leq$ and $\geq$.

**Example 1**

**Solving an Inequality Using Addition**

Solve $x - 5 < -3$. Graph the solution.

\[
x - 5 < -3 \quad \text{Write the inequality.}
\]

\[
\begin{align*}
+5 & \quad +5 \\
x & \quad < \\
2 & \quad \text{Simplify.}
\end{align*}
\]

The solution is $x < 2$.

**On Your Own**

Solve the inequality. Graph the solution.

1. $y - 6 > -7$
2. $b - 3.8 \leq 1.7$
3. $\frac{1}{2} > z - \frac{1}{4}$
EXAMPLE 2 Solving an Inequality Using Subtraction

Solve $13 \leq x + 14$. Graph the solution.

Write the inequality.

$\begin{align*}
13 & \leq x + 14 \\
-14 & \quad -14 \\
\hline
-1 & \leq x
\end{align*}$

Subtraction Property of Inequality

Simplify.

The solution is $x \geq -1$.

On Your Own

Solve the inequality. Graph the solution.

4. $w + 7 \leq 4$

5. $12.5 \geq d + 10$

6. $x + \frac{3}{4} > 1 \frac{1}{2}$

EXAMPLE 3 Real-Life Application

A person can be no taller than 6.25 feet to become an astronaut pilot for NASA. Your friend is 5 feet 9 inches tall. Write and solve an inequality that represents how much your friend can grow and still meet the requirement.

Words

Current height plus amount your friend can grow is no more than the height limit.

Variable

Let $h$ be the possible amounts your friend can grow.

Inequality

$5.75 + h \leq 6.25$

Write the inequality.

$\begin{align*}
5.75 + h & \leq 6.25 \\
-5.75 & \quad -5.75 \\
\hline
h & \leq 0.5
\end{align*}$

Subtraction Property of Inequality

Simplify.

So, your friend can grow no more than 0.5 foot, or 6 inches.

On Your Own

7. Your cousin is 5 feet 3 inches tall. Write and solve an inequality that represents how much your cousin can grow and still meet the requirement.
1. **REASONING** Is the inequality \( c + 3 > 5 \) the same as \( c > 5 - 3 \)? Explain.

2. **WHICH ONE DOESN'T BELONG?** Which inequality does not belong with the other three? Explain your reasoning.

\[
\begin{align*}
w + \frac{7}{4} &> \frac{3}{4} \\
\frac{w}{4} - \frac{3}{4} &> -\frac{7}{4} \\
\frac{w}{4} + \frac{7}{4} &< \frac{3}{4} \\
\frac{3}{4} &< w + \frac{7}{4}
\end{align*}
\]

**Practice and Problem Solving**

Solve the inequality. Graph the solution.

1. \( x + 7 \geq 18 \)
2. \( a - 2 > 4 \)
3. \( 8 + k \leq -3 \)
4. \( -12 < y - 6 \)
5. \( 3 \leq 7 + g \)
6. \( t - 5 \leq -7 \)
7. \( n - 4 < 5 \)
8. \( \frac{2}{7} > b + \frac{5}{7} \)
9. \( z - 4.7 \geq -1.6 \)
10. \( p + \frac{1}{4} \geq 2 \)
11. \( \frac{8}{5} > s + \frac{12}{5} \)
12. \( \frac{-7}{8} \geq m - \frac{13}{8} \)
13. \( -9.1 < d - 6.3 \)
14. \( h - 6 \leq -8.4 \)
15. \( r + 0.2 < -0.7 \)
16. \( \frac{-3}{5} \leq x \)
17. \( \frac{-3}{5} \leq x \)

**ERROR ANALYSIS** Describe and correct the error in solving the inequality or graphing the solution of the inequality.

18. \( x - 7 > -2 \)

\[
\begin{align*}
x - 7 &> -2 \\
+7 &+7 \\
x &> 9
\end{align*}
\]

19. \( 8 \leq x + 3 \)

\[
\begin{align*}
\frac{8}{3} &\leq x + 3 \\
-3 &-3 \\
5 &\leq x
\end{align*}
\]

20. **AIRPLANE** A small airplane can hold 44 passengers. Fifteen passengers board the plane.

a. Write and solve an inequality that represents the additional number of passengers that can board the plane.

b. Can 30 more passengers board the plane? Explain.
Write and solve an inequality that represents \( x \).

21. The perimeter is less than 28 feet.

22. The base is greater than the height.

23. The perimeter is less than or equal to 51 meters.

24. **REASONING** The solution of \( d + s > -3 \) is \( d > -7 \). What is the value of \( s \)?

25. **BIRDFEEDER** The hole for a birdfeeder post is 3 feet deep. The top of the post needs to be at least 5 feet above the ground. Write and solve an inequality that represents the required length of the post.

26. **SHOPPING** You can spend up to $35 on a shopping trip.
   
   a. You want to buy a shirt that costs $14. Write and solve an inequality that represents the amount of money you will have left if you buy the shirt.
   
   b. You notice that the shirt is on sale for 30% off. How does this change the inequality?
   
   c. Do you have enough money to buy the shirt that is on sale and a pair of pants that costs $23? Explain.

27. **POWER** A circuit overloads at 2400 watts of electricity. A portable heater that uses 1050 watts of electricity is plugged into the circuit.
   
   a. Write and solve an inequality that represents the additional number of watts you can plug in without overloading the circuit.
   
   b. In addition to the portable heater, what two other items in the table can you plug in at the same time without overloading the circuit? Is there more than one possibility? Explain.

28. **Number Sense** The possible values of \( x \) are given by \( x + 8 \leq 6 \). What is the greatest possible value of \( 7x \)?

### Fair Game Review

**What you learned in previous grades & lessons**

Solve the equation. Check your solution. **(Section 3.4)**

29. \( 4x = 36 \) 
30. \( \frac{w}{3} = -9 \) 
31. \( -2b = 44 \) 
32. \( 60 = \frac{3}{4}h \)

33. **MULTIPLE CHOICE** Which fraction is equivalent to \(-2.4\)? **(Section 2.1)**

   - A. \( \frac{-12}{5} \)
   - B. \( \frac{-51}{25} \)
   - C. \( \frac{-8}{5} \)
   - D. \( \frac{-6}{25} \)
You can use a **Y chart** to compare two topics. List differences in the branches and similarities in the base of the Y. Here is an example of a Y chart that compares solving equations and solving inequalities.

### Solving Equations
- The sign between two expressions is an equal sign, =.
- One number is the solution.
- Use inverse operations to group numbers on one side.
- Use inverse operations to group variables on one side.
- Solve for the variable.

### Solving Inequalities
- The sign between two expressions is an inequality symbol: <, >, ≤, or ≥.
- More than one number can be a solution.
- Use inverse operations to group numbers on one side.

**On Your Own**

Make Y charts to help you study and compare these topics.

1. writing equations and writing inequalities
2. graphing the solution of an equation and graphing the solution of an inequality
3. graphing inequalities that use > and graphing inequalities that use <
4. graphing inequalities that use > or < and graphing inequalities that use ≥ or ≤
5. solving inequalities using addition and solving inequalities using subtraction

After you complete this chapter, make Y charts for the following topics.

6. solving inequalities using multiplication and solving inequalities using division
7. solving two-step equations and solving two-step inequalities
8. Pick two other topics that you studied earlier in this course and make a Y chart to compare them.
Write the word sentence as an inequality.  
1. A number $y$ plus 2 is greater than $-5$.  
2. A number $s$ minus 2.4 is at least 8.

Tell whether the given value is a solution of the inequality.  
3. $8p < -3; p = -2$  
4. $z + 2 > -4; z = -8$

Graph the inequality on a number line.  
5. $x < -12$  
6. $v > \frac{5}{4}$  
7. $b \geq \frac{1}{3}$  
8. $q \leq 4.2$

Solve the inequality. Graph the solution.  
9. $n + 2 \leq -6$  
10. $t - \frac{3}{7} > \frac{6}{7}$  
11. $-\frac{3}{4} \geq w + 1$  
12. $y - 2.6 < -3.4$

13. **STUDYING** You plan to study at least 1.5 hours for a geography test. Write an inequality that represents this situation.  

14. **FITNESS TEST** The three requirements to pass a fitness test are shown.  

   a. Write and graph three inequalities that represent the requirements.
   
   b. You can jog 2500 meters, perform 30 push-ups, and perform 20 pull-ups. Do you satisfy the requirements of the test? Explain.

15. **NUMBER LINE** Use tape on the floor to make the number line shown. All units are in feet. You are standing at $-\frac{7}{2}$. You want to move to a number greater than $-\frac{3}{2}$. Write and solve an inequality that represents the distance you must move.  

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**Sections 4.1–4.2 Quiz**

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4.3 Solving Inequalities Using Multiplication or Division

**Essential Question** How can you use multiplication or division to solve an inequality?

1. **ACTIVITY: Using a Table to Solve an Inequality**

Work with a partner.
- Copy and complete the table.
- Decide which graph represents the solution of the inequality.
- Write the solution of the inequality.

a. \(4x > 12\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4x)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4x &gt; 12)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. \(-3x \leq 9\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-3x)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-3x \leq 9)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. **ACTIVITY: Solving an Inequality**

Work with a partner.

a. Solve \(-3x \leq 9\) by adding \(3x\) to each side of the inequality first. Then solve the resulting inequality.

b. Compare the solution in part (a) with the solution in Activity 1(b).
Work with a partner.

- Copy and complete the table.
- Decide which graph represents the solution of the inequality.
- Write the solution of the inequality.

### Activity: Using a Table to Solve an Inequality

**a.** \( \frac{x}{3} < 1 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{x}{3} )</td>
<td>0</td>
<td>1/3</td>
<td>2/3</td>
<td>1</td>
<td>4/3</td>
<td>5/3</td>
</tr>
<tr>
<td>( \frac{x}{3} &lt; 1 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Graph 1](image1)

**b.** \( \frac{x}{-4} \geq \frac{3}{4} \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{x}{-4} )</td>
<td>5/4</td>
<td>1</td>
<td>3/4</td>
<td>1/2</td>
<td>1/4</td>
<td>0</td>
<td>1/4</td>
</tr>
<tr>
<td>( \frac{x}{-4} \geq \frac{3}{4} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Graph 2](image2)

### Activity: Writing Rules

Work with a partner. Use a table to solve each inequality.

a. \(-2x \leq 10\)  
b. \(-6x > 0\)  
c. \(\frac{x}{-4} < 1\)  
d. \(\frac{x}{-8} \geq \frac{1}{8}\)

Write a set of rules that describes how to solve inequalities like those in Activities 1 and 3. Then use your rules to solve each of the four inequalities above.

### What Is Your Answer?

5. **In Your Own Words**  How can you use multiplication or division to solve an inequality?

Use what you learned about solving inequalities using multiplication or division to complete Exercises 4–9 on page 143.

**Section 4.3**  Solving Inequalities Using Multiplication or Division
**Key Idea**

**Multiplication and Division Properties of Inequality (Case 1)**

**Words**  
When you multiply or divide each side of an inequality by the same positive number, the inequality remains true.

**Numbers**  

\[
\begin{align*}
-4 &< 6 \\
4 &> -6 \\
2 \cdot (-4) &< 2 \cdot 6 \\
\frac{4}{2} &> \frac{-6}{2} \\
-8 &< 12 \\
2 &> -3
\end{align*}
\]

**Algebra**  
If \(a < b\) and \(c\) is positive, then  
\[
\frac{a \cdot c}{c} < \frac{b \cdot c}{c}
\]
If \(a > b\) and \(c\) is positive, then  
\[
\frac{a \cdot c}{c} > \frac{b \cdot c}{c}
\]

These properties are also true for \(\leq\) and \(\geq\).

---

**EXAMPLE 1**

**Solving an Inequality Using Multiplication**

Solve \(\frac{x}{5} \leq -3\). Graph the solution.

\[
\frac{x}{5} \leq -3  \\
5 \cdot \frac{x}{5} \leq 5 \cdot (-3)  \\
x \leq -15
\]

\[
\text{The solution is } x \leq -15.
\]

\[
\text{On Your Own}
\]

Solve the inequality. Graph the solution.

1. \(n \div 3 < 1\)
2. \(-0.5 \leq \frac{m}{10}\)
3. \(-3 > \frac{2}{3}p\)
EXAMPLE 2 Solving an Inequality Using Division

Solve \( 6x > -18 \). Graph the solution.

\[
\begin{align*}
6x & > -18 & \text{Write the inequality.} \\
\frac{6x}{6} & > \frac{-18}{6} & \text{Division Property of Inequality} \\
x & > -3 & \text{Simplify.}
\end{align*}
\]

\( x > -3 \)

The solution is \( x > -3 \).

\( x = -6 \) is not a solution. \( x = 0 \) is a solution.

On Your Own

Solve the inequality. Graph the solution.

4. \( 4b \geq 2 \) \hspace{1cm} 5. \( 12k \leq -24 \) \hspace{1cm} 6. \( -15 < 2.5q \)

Key Idea

Multiplication and Division Properties of Inequality (Case 2)

**Words** When you multiply or divide each side of an inequality by the same negative number, the direction of the inequality symbol must be reversed for the inequality to remain true.

**Numbers**

\[
\begin{align*}
-4 < 6 & \quad 4 > -6 \\
-2 \cdot (-4) & \geq -2 \cdot 6 & \frac{4}{-2} & \leq \frac{-6}{-2} \\
8 & > -12 & -2 & < 3
\end{align*}
\]

**Algebra** If \( a < b \) and \( c \) is negative, then

\[
\frac{a}{c} \leq \frac{b}{c}
\]

If \( a > b \) and \( c \) is negative, then

\[
\frac{a}{c} \leq \frac{b}{c}
\]

These properties are also true for \( \leq \) and \( \geq \).
EXAMPLE 3 Solving an Inequality Using Multiplication

Solve $-\frac{3}{2} n \leq 6$. Graph the solution.

$$-\frac{3}{2} n \leq 6$$

Write the inequality.

$$-\frac{2}{3} \cdot \left(-\frac{3}{2} n\right) \geq -\frac{2}{3} \cdot 6$$

Use the Multiplication Property of Inequality.
Reverse the inequality symbol.

$$n \geq -4$$

Simplify.

 declaración: The solution is $n \geq -4$.

On Your Own

Solve the inequality. Graph the solution.

7. $\frac{x}{-3} > -4$

8. $0.5 \leq -\frac{y}{2}$

9. $-12 \geq \frac{6}{5} m$

10. $-\frac{2}{3} h \leq -8$

EXAMPLE 4 Solving an Inequality Using Division

Solve $-3z > -4.5$. Graph the solution.

$$-3z > -4.5$$

Write the inequality.

$$-3z \leq -4.5$$

Use the Division Property of Inequality.
Reverse the inequality symbol.

$$z < 1.5$$

Simplify.

declaración: The solution is $z < 1.5$.

On Your Own

Solve the inequality. Graph the solution.

11. $-5z < 35$

12. $-2a > -9$

13. $-1.5 < 3n$

14. $-4.2 \geq -0.7w$
4.3 Exercises

Vocabulary and Concept Check

1. **WRITING** Explain how to solve \( \frac{x}{3} < -2 \).
2. **PRECISION** Explain how solving \( 4x < -16 \) is different from solving \(-4x < 16\).
3. **OPEN-ENDED** Write an inequality that you can solve using the Division Property of Inequality where the direction of the inequality symbol must be reversed.

Practice and Problem Solving

Use a table to solve the inequality.

4. \( 2x < 2 \)  
5. \( -3x \leq 3 \)  
6. \( -6x > 18 \)

7. \( \frac{x}{-5} \geq 7 \)  
8. \( \frac{x}{-1} > \frac{2}{5} \)  
9. \( \frac{x}{3} \leq \frac{1}{2} \)

Solve the inequality. Graph the solution.

10. \( 2n > 20 \)  
11. \( \frac{c}{9} \leq -4 \)  
12. \( 2.2m < 11 \)

13. \( -16 > x \div 2 \)  
14. \( \frac{1}{6}w \geq 2.5 \)  
15. \( 7 < 3.5k \)

16. \( 3x \leq -\frac{5}{4} \)  
17. \( 4.2y \leq -12.6 \)  
18. \( 11.3 > \frac{b}{4.3} \)

19. **ERROR ANALYSIS** Describe and correct the error in solving the inequality.

Write the word sentence as an inequality. Then solve the inequality.

20. The quotient of a number and 4 is at most 5.
21. A number divided by 7 is less than -3.
22. Six times a number is at least -24.
23. The product of -2 and a number is greater than 30.

24. **SMART PHONE** You earn $9.20 per hour at your summer job. Write and solve an inequality that represents the number of hours you need to work in order to buy a smart phone that costs $299.

Section 4.3 Solving Inequalities Using Multiplication or Division 143
25. **AVOCADOS** You have $9.60 to buy avocados for a guacamole recipe. Avocados cost $2.40 each.
   a. Write and solve an inequality that represents the number of avocados you can buy.
   b. Are there infinitely many solutions in this context? Explain.

26. **SCIENCE PROJECT** Students in a science class are divided into 6 equal groups with at least 4 students in each group for a project. Write and solve an inequality that represents the number of students in the class.

Solve the inequality. Graph the solution.

27. \(-5n \leq 15\) \hspace{1cm} 28. \(-7w > 49\) \hspace{1cm} 29. \(-\frac{1}{3}h \geq 8\)

30. \(-9 < -\frac{1}{5}x\) \hspace{1cm} 31. \(-3y < -14\) \hspace{1cm} 32. \(-2d \geq 26\)

33. \(4.5 > -\frac{m}{6}\) \hspace{1cm} 34. \(k - \frac{0.25}{3} \leq 36\)

36. **ERROR ANALYSIS** Describe and correct the error in solving the inequality.

37. **TEMPERATURE** It is currently 0°C outside. The temperature is dropping 2.5°C every hour. Write and solve an inequality that represents the number of hours that must pass for the temperature to drop below \(-20°C\).

38. **STORAGE** You are moving some of your belongings into a storage facility.
   a. Write and solve an inequality that represents the number of boxes that you can stack vertically in the storage unit.
   b. Can you stack 6 boxes vertically in the storage unit? Explain.

Write and solve an inequality that represents \(x\).

39. Area \(\geq 120 \text{ cm}^2\) \hspace{1cm} 40. Area \(< 20 \text{ ft}^2\)
41. **AMUSEMENT PARK** You and four friends are planning a visit to an amusement park. You want to keep the cost below $100 per person. Write and solve an inequality that represents the total cost of visiting the amusement park.

42. **LOGIC** When you multiply or divide each side of an inequality by the same negative number, you must reverse the direction of the inequality symbol. Explain why.

43. **PROJECT** Choose two novels to research.
   a. Use the Internet or a magazine to complete the table.
   b. Use the table to find and compare the average number of copies sold per month for each novel. Which novel do you consider to be the most successful? Explain.
   c. Assume each novel continues to sell at the average rate. Write and solve an inequality that represents the number of months it will take for the total number of copies sold to exceed twice the current number sold.

<table>
<thead>
<tr>
<th>Author</th>
<th>Name of Novel</th>
<th>Release Date</th>
<th>Current Number of Copies Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Number Sense** Describe all numbers that satisfy both inequalities. Include a graph with your description.

44. $4m > -4$ and $3m < 15$
45. $\frac{n}{3} \geq -4$ and $\frac{n}{5} \geq 1$
46. $2x \geq -6$ and $2x \geq 6$
47. $\frac{1}{2} - s > -7$ and $\frac{1}{3} - s < 12$

**Fair Game Review** What you learned in previous grades & lessons

Solve the equation. Check your solution. (Section 3.5)

48. $-2w + 4 = -12$
49. $\frac{v}{5} - 6 = 3$
50. $3(x - 1) = 18$
51. $\frac{m + 200}{4} = 51$
52. **MULTIPLE CHOICE** What is the value of $\frac{2}{3} + \left(-\frac{5}{7}\right)$? (Section 2.2)
   - [A] $-\frac{3}{4}$
   - [B] $-\frac{1}{21}$
   - [C] $\frac{7}{10}$
   - [D] $1\frac{8}{21}$
4.4 Solving Two-Step Inequalities

Essential Question How can you use an inequality to describe the dimensions of a figure?

1 ACTIVITY: Areas and Perimeters of Figures

Work with a partner.

- Use the given condition to choose the inequality that you can use to find the possible values of the variable. Justify your answer.
- Write four values of the variable that satisfy the inequality you chose.

a. You want to find the values of \( x \) so that the area of the rectangle is more than 22 square units.

\[
4x + 12 > 22 \quad 4x + 3 > 22 \\
4x + 12 \geq 22 \quad 2x + 14 > 22
\]

b. You want to find the values of \( x \) so that the perimeter of the rectangle is greater than or equal to 28 units.

\[
x + 7 \geq 28 \quad 4x + 12 \geq 28 \quad 2x + 14 \geq 28 \quad 2x + 14 \leq 28
\]

c. You want to find the values of \( y \) so that the area of the parallelogram is fewer than 41 square units.

\[
5y + 7 < 41 \quad 5y + 35 < 41 \\
5y + 7 \leq 41 \quad 5y + 35 \leq 41
\]

d. You want to find the values of \( z \) so that the area of the trapezoid is at most 100 square units.

\[
5z + 30 \leq 100 \quad 10z + 30 \leq 100 \\
5z + 30 < 100 \quad 10z + 30 < 100
\]

Inequalities
In this lesson, you will
- solve multi-step inequalities.
- solve real-life problems.
Section 4.4  Solving Two-Step Inequalities  147

Work with a partner.

- Use the given condition to choose the inequality that you can use to find the possible values of the variable. Justify your answer.
- Write four values of the variable that satisfy the inequality you chose.

a. You want to find the values of $x$ so that the volume of the rectangular prism is at least 50 cubic units.

$$15x + 30 > 50 \quad x + 10 \geq 50 \quad 15x + 30 \geq 50 \quad 15x + 2 \geq 50$$

b. You want to find the values of $x$ so that the volume of the rectangular prism is no more than 36 cubic units.

$$8x + 4 < 36 \quad 36x + 18 < 36 \quad 2x + 9.5 \leq 36 \quad 36x + 18 \leq 36$$

What Is Your Answer?

3. **IN YOUR OWN WORDS**  How can you use an inequality to describe the dimensions of a figure?

4. Use what you know about solving equations and inequalities to describe how you can solve a two-step inequality. Give an example to support your explanation.

Practice

Use what you learned about solving two-step inequalities to complete Exercises 3 and 4 on page 150.
You can solve two-step inequalities in the same way you solve two-step equations.

**EXAMPLE 1 Solving Two-Step Inequalities**

a. Solve \(5x - 4 \geq 11\). Graph the solution.

\[
5x - 4 \geq 11 \\
+ 4 + 4 \\
5x \geq 15 \\
\text{Addition Property of Inequality} \\
\text{Simplify.}
\]

\[
\frac{5x}{5} \geq \frac{15}{5} \\
\text{Division Property of Inequality} \\
x \geq 3 \\
\text{Simplify.}
\]

\[x \geq 3\]

The solution is \(x \geq 3\).

b. Solve \(\frac{b}{-3} + 4 < 13\). Graph the solution.

\[
\frac{b}{-3} + 4 < 13 \\
- 4 - 4 \\
\frac{b}{-3} < 9 \\
\text{Subtraction Property of Inequality} \\
\text{Simplify.}
\]

\[-3 \cdot \frac{b}{-3} > -3 \cdot 9 \\
\text{Use the Multiplication Property of Inequality.} \\
\text{Reverse the inequality symbol.} \\
b > -27 \\
\text{Simplify.}
\]

\[b > -27\]

The solution is \(b > -27\).

**On Your Own**

Solve the inequality. Graph the solution.

1. \(6y - 7 > 5\) 
2. \(4 - 3d \geq 19\) 
3. \(\frac{w}{-4} + 8 > 9\)
EXAMPLE 2  Graphing an Inequality

Which graph represents the solution of \(-7(x + 3) \leq 28\)?

\[
\begin{align*}
-7(x + 3) & \leq 28 \\
-7x - 21 & \leq 28 \\
\text{Step 1: Undo the subtraction.} & \quad + 21 \quad + 21 \\
\text{Step 2: Undo the multiplication.} & \quad \frac{-7x}{-7} \geq \frac{49}{-7} \\
& \quad x \geq -7
\end{align*}
\]

The correct answer is (B).

EXAMPLE 3  Real-Life Application

A contestant in a weight-loss competition wants to lose an average of at least 8 pounds per month during a 5-month period. How many pounds must the contestant lose in the fifth month to meet the goal?

Write and solve an inequality. Let \(x\) be the number of pounds lost in the fifth month.

\[
\begin{align*}
12 + 9 + 5 + 8 + x & \geq 5 \\
\frac{34 + x}{5} & \geq 8 \\
\frac{5 \cdot 34 + x}{5} & \geq 5 \cdot 8 \\
34 + x & \geq 40 \\
x & \geq 6
\end{align*}
\]

So, the contestant must lose at least 6 pounds to meet the goal.

On Your Own

Solve the inequality. Graph the solution.

4. \(2(k - 5) < 6\)  
5. \(-4(n - 10) < 32\)  
6. \(-3 \leq 0.5(8 + y)\)

7. WHAT IF? In Example 3, the contestant wants to lose an average of at least 9 pounds per month. How many pounds must the contestant lose in the fifth month to meet the goal?
4.4 Exercises

Vocabulary and Concept Check

1. **WRITING** Compare and contrast solving two-step inequalities and solving two-step equations.

2. **OPEN-ENDED** Describe how to solve the inequality $3(a + 5) < 9$.

Practice and Problem Solving

Match the inequality with its graph.

3. $\frac{t}{3} - 1 \geq -3$
   - **A.**
   - **B.**
   - **C.**

4. $5x + 7 \leq 32$
   - **A.**
   - **B.**
   - **C.**

Solve the inequality. Graph the solution.

5. $8y - 5 < 3$
6. $3p + 2 \geq -10$
7. $2 > 8 - \frac{4}{3}h$
8. $-2 > \frac{m}{6} - 7$
9. $-1.2b - 5.3 \geq 1.9$
10. $-1.3 \geq 2.9 - 0.6r$

11. **ERROR ANALYSIS** Describe and correct the error in solving the inequality.
    
    \[ \frac{x}{3} + 4 < 6 \]
    \[ x + 4 < 18 \]
    \[ x < 14 \]

Solve the inequality. Graph the solution.

12. $5(g + 4) > 15$
13. $4(w - 6) \leq -12$
14. $-8 \leq \frac{2}{5}(k - 2)$
15. $-\frac{1}{4}(d + 1) < 2$
16. $7.2 > 0.9(n + 8.6)$
17. $20 \geq -3.2(c - 4.3)$

18. **UNICYCLE** The first jump in a unicycle high-jump contest is shown. The bar is raised 2 centimeters after each jump. Solve the inequality $2n + 10 \geq 26$ to find the number of additional jumps needed to meet or exceed the goal of clearing a height of 26 centimeters.

150 Chapter 4 Inequalities
Solve the inequality. Graph the solution.

19. \(9x - 4x + 4 \geq 36 - 12\)

20. \(3d - 7d + 2.8 < 5.8 - 27\)

21. **SCUBA DIVER** A scuba diver is at an elevation of \(-38\) feet. The diver starts moving at a rate of \(-12\) feet per minute. Write and solve an inequality that represents how long it will take the diver to reach an elevation deeper than \(-200\) feet.

22. **KILLER WHALES** A killer whale has eaten 75 pounds of fish today. It needs to eat at least 140 pounds of fish each day.
   a. A bucket holds 15 pounds of fish. Write and solve an inequality that represents how many more buckets of fish the whale needs to eat.
   b. Should the whale eat four or five more buckets of fish? Explain.

23. **REASONING** A student theater charges $9.50 per ticket.
   a. The theater has already sold 70 tickets. Write and solve an inequality that represents how many more tickets the theater needs to sell to earn at least $1000.
   b. The theater increases the ticket price by $1. Without solving an inequality, describe how this affects the total number of tickets needed to earn at least $1000.

24. **Problem Solving** For what values of \(r\) will the area of the shaded region be greater than or equal to 12 square units?

---

**Fair Game Review** What you learned in previous grades & lessons

Find the missing values in the ratio table. Then write the equivalent ratios.

(Skills Review Handbook)

25. | Flutes | 7 | 28 | Clarinets | 4 | 12 |
26. | Boys | 6 | 3 | Girls | 10 | 50 |

27. **MULTIPLE CHOICE** What is the volume of the cube?
   (Skills Review Handbook)
   
   - A 8 ft³
   - B 16 ft³
   - C 24 ft³
   - D 32 ft³
4.3–4.4 Quiz

Solve the inequality. Graph the solution.  
(Section 4.3 and Section 4.4)

1. \(3p \leq 18\)

2. \(2x > -\frac{3}{5}\)

3. \(\frac{r}{3} \geq -5\)

4. \(-\frac{z}{8} < 1.5\)

5. \(3n + 2 \leq 11\)

6. \(-2 < 5 - \frac{k}{2}\)

7. \(1.3m - 3.8 < -1.2\)

8. \(4.8 \geq 0.3(12 - y)\)

Write the word sentence as an inequality. Then solve the inequality.  
(Section 4.3)

9. The quotient of a number and 5 is less than 4.

10. Six times a number is at least \(-14\).

11. **PEPPERS** You have $18 to buy peppers. Peppers cost $1.50 each. Write and solve an inequality that represents the number of peppers you can buy.  
(Section 4.3)

12. **MOVIES** You have a gift card worth $90. You want to buy several movies that cost $12 each. Write and solve an inequality that represents the number of movies you can buy and still have at least $30 on the gift card.  
(Section 4.4)

13. **CHOCOLATES** Your class sells boxes of chocolates to raise $500 for a field trip. You earn $6.25 for each box of chocolates sold. Write and solve an inequality that represents the number of boxes your class must sell to meet or exceed the fundraising goal.  
(Section 4.3)

14. **FENCE** You want to put up a fence that encloses a triangular region with an area greater than or equal to 60 square feet. What is the least possible value of \(c\)? Explain.  
(Section 4.3)
Review Key Vocabulary

inequality, p. 126
solution set, p. 126
solution of an inequality, p. 126
graph of an inequality, p. 127

4.1 Writing and Graphing Inequalities (pp. 124–129)

a. Six plus a number \( x \) is at most \(-\frac{1}{4}\). Write this word sentence as an inequality.

\[
6 + x \leq -\frac{1}{4}.
\]

An inequality is \( 6 + x \leq -\frac{1}{4} \).

b. Graph \( m > 3 \).

Exercises:

Write the word sentence as an inequality.

1. A number \( w \) is greater than \(-3\).

2. A number \( y \) minus \(\frac{1}{2}\) is no more than \(-\frac{3}{2}\).

Tell whether the given value is a solution of the inequality.

3. \( 5 + j > 8; j = 7 \)

4. \( 6 \div n \leq -5; n = -3 \)

Graph the inequality on a number line.

5. \( q > -1.3 \)

6. \( s < \frac{3}{4} \)

7. BUMPER CARS You must be at least 42 inches tall to ride the bumper cars at an amusement park. Write an inequality that represents this situation.
4.2 Solving Inequalities Using Addition or Subtraction  (pp. 130–135)

Solve $-5 < m - 3$. Graph the solution.

$-5 < m - 3$

- Write the inequality.

$+3 +3$

$-2 < m$

- Addition Property of Inequality

Simplify.

$\therefore$ The solution is $m > -2$.

The solution is $m > -2$.

$\therefore m = -3$ is not a solution.

$\therefore m = 3$ is a solution.

Exercises:

Solve the inequality. Graph the solution.

8. $d + 12 < 19$

9. $t - 4 \leq -14$

10. $-8 \leq z + 6.4$

4.3 Solving Inequalities Using Multiplication or Division  (pp. 138–145)

Solve $\frac{c}{-3} \geq -2$. Graph the solution.

$\frac{c}{-3} \geq -2$

- Write the inequality.

$-3 \cdot \frac{c}{-3} \leq -3 \cdot (-2)$

- Use the Multiplication Property of Inequality.

$\therefore c \leq 6$

Reverse the inequality symbol.

$\therefore$ The solution is $c \leq 6$.

The solution is $c \leq 6$.

$\therefore c = 3$ is a solution.

$\therefore c = 9$ is not a solution.

Exercises:

Solve the inequality. Graph the solution.

11. $6q < -18$

12. $-\frac{r}{3} \leq 6$

13. $-4 > -\frac{4}{3}s$
4.4 Solving Two-Step Inequalities (pp. 146–151)

a. Solve \( 6x - 8 \leq 10 \). Graph the solution.

\[
6x - 8 \leq 10 \\
\quad + 8 \quad + 8 \\
6x \leq 18 \\
\quad \div 6 \quad \div 6 \\
x \leq 3
\]

The solution is \( x \leq 3 \).

b. Solve \( \frac{q}{-4} + 7 < 11 \). Graph the solution.

\[
\frac{q}{-4} + 7 < 11 \\
\quad -7 \quad -7 \\
\frac{q}{-4} < 4 \\
\quad \cdot (-4) \quad \cdot (-4) \\
q > -16 \\
\]

The solution is \( q > -16 \).

Exercises

Solve the inequality. Graph the solution.

14. \( 3x + 4 > 16 \)  
15. \( \frac{z}{-2} - 6 \leq -2 \)  
16. \( -2t - 5 < 9 \)

17. \( 7(q + 2) < -77 \)  
18. \( \frac{1}{3}(p + 9) \leq 4 \)  
19. \( 1.2(j + 3.5) \geq 4.8 \)
Write the word sentence as an inequality.

1. A number \( k \) plus 19.5 is less than or equal to 40.

2. A number \( q \) multiplied by \( \frac{1}{4} \) is greater than \(-16\).

Tell whether the given value is a solution of the inequality.

3. \( n - 3 \leq 4; n = 7 \)

4. \( -\frac{3}{7}m < 1; m = -7 \)

5. \( -4c \geq 7; c = -2 \)

6. \( -2.4m > -6.8; m = -3 \)

Solve the inequality. Graph the solution.

7. \( w + 4 \leq 3 \)

8. \( x - 4 > -6 \)

9. \( -\frac{2}{9} + y \leq \frac{5}{9} \)

10. \( -6z \geq 36 \)

11. \( -5.2 \geq \frac{P}{4} \)

12. \( 4k - 8 \geq 20 \)

13. \( \frac{4}{7} - b \geq \frac{-1}{7} \)

14. \( -0.6 > -0.3(d + 6) \)

15. **GUMBALLS** You have $2.50. Each gumball in a gumball machine costs $0.25. Write and solve an inequality that represents the number of gumballs you can buy.

16. **PARTY** You can spend no more than $100 on a party you are hosting. The cost per guest is $8.

   a. Write and solve an inequality that represents the number of guests you can invite to the party.

   b. What is the greatest number of guests that you can invite to the party? Explain your reasoning.

17. **BASEBALL CARDS** You have $30 to buy baseball cards. Each pack of cards costs $5. Write and solve an inequality that represents the number of packs of baseball cards you can buy and still have at least $10 left.
1. What is the value of the expression below when $x = -5$, $y = 3$, and $z = -1$?
\[
\frac{x^2 - 3y}{z}
\]
A. $-34$  
B. $-16$  
C. $16$  
D. $34$

2. What is the value of the expression below?
\[
\frac{3 \cdot 2}{8 \cdot 5}
\]
F. $\frac{20}{3}$  
G. $\frac{16}{15}$  
H. $\frac{15}{16}$  
I. $\frac{3}{20}$

3. Which graph represents the inequality below?
\[
\frac{x}{-4} - 8 \geq -9
\]
A.  
B.  
C.  
D.  

4. Which value of $p$ makes the equation below true?
\[
5(p + 6) = 25
\]
F. $-1$  
G. $\frac{34}{5}$  
H. $11$  
I. $14$
5. You set up the lemonade stand. Your profit is equal to your revenue from lemonade sales minus your cost to operate the stand. Your cost is $8. How many cups of lemonade must you sell to earn a profit of $30?

A. 4
B. 44
C. 60
D. 76

6. Which value is a solution of the inequality below?

\[ 3 - 2y < 7 \]

F. \(-6\)
G. \(-3\)
H. \(-2\)
I. \(-1\)

7. What value of \(y\) makes the equation below true?

\[ 12 - 3y = -6 \]

8. What is the mean distance of the four points from \(-3\)?

A. \(-\frac{1}{2}\)
B. \(2\frac{1}{2}\)
C. 3
D. \(7\frac{1}{8}\)
9. Martin graphed the solution of the inequality \(-4x + 18 > 6\) in the box below.

![Graph showing solution to the inequality]

What should Martin do to correct the error that he made?

F. Use an open circle at 3 and shade to the left of 3.
G. Use an open circle at 3 and shade to the right of 3.
H. Use a closed circle and shade to the right of 3.
I. Use an open circle and shade to the left of \(-3\).

10. What is the value of the expression below?

\[
\frac{5}{12} - \frac{7}{8}
\]

11. You are selling T-shirts to raise money for a charity. You sell the T-shirts for $10 each.

Part A You have already sold 2 T-shirts. How many more T-shirts must you sell to raise at least $500? Explain.

Part B Your friend is raising money for the same charity and has not sold any T-shirts previously. He sells the T-shirts for $8 each. What is the total number of T-shirts he must sell to raise at least $500? Explain.

Part C Who has to sell more T-shirts in total? How many more? Explain.

12. Which expression is equivalent to the expression below?

\[
-\frac{2}{3} - \left(-\frac{4}{9}\right)
\]

A. \(\frac{1}{3} + \frac{1}{9}\)  
B. \(-\frac{2}{3} \times \left(-\frac{1}{3}\right)\)  
C. \(-\frac{1}{3} - \frac{7}{9}\)  
D. \(\frac{3}{2} \div \left(-\frac{1}{3}\right)\)