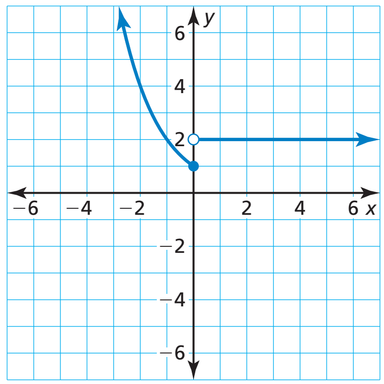
6.8 Piecewise Functions

Essential Question How can you describe a function that is represented by more than one equation?

EXPLORATION 1 Analyzing a Piecewise Function

Work with a partner.



a. Does the graph represent y as a function of x? Justify your conclusion.

b. What is the value of the function when $x = 0$? How can you tell?

c. Find the domain of the exponential piece of the graph.

$$f(x) = 2^{-x}, \text{ if }$$

d. Find the domain of the linear piece of the graph.

$$f(x) = 2, \text{ if }$$

e. Combine the results of parts (c) and (d) to write a single description of the function.

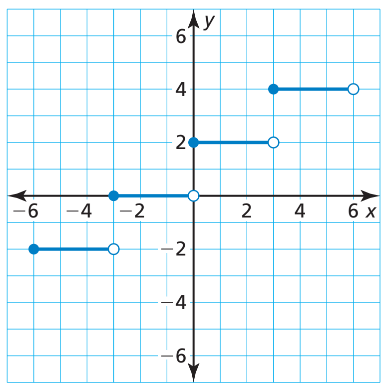
$$f(x) = \begin{cases} 2^{-x}, & \text{if } \\ 2, & \text{if } \end{cases}$$

CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to justify your conclusions and communicate them to others.

EXPLORATION 2 Analyzing a Piecewise Function

Work with a partner.



a. Does the graph represent y as a function of x? Justify your conclusion.

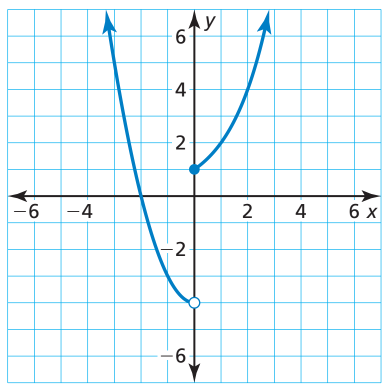
b. Find the domain of each piece of the graph.

$$f(x) = \begin{cases} -2, & \text{if } \\ 0, & \text{if } \\ 2, & \text{if } \\ 4, & \text{if } \end{cases}$$

Communicate Your Answer

3. How can you describe a function that is represented by more than one equation?

4. Find the domain of each piece of the function represented by the graph.



$$f(x) = x^2 - 4, \text{ if }$$

$$f(x) = 2^x, \text{ if }$$

6.8 Lesson

Core Vocabulary

piecewise function, *p. 352*
step function, *p. 354*

What You Will Learn

- Evaluate piecewise functions.
- Graph and analyze piecewise functions.
- Graph and write step functions.

Evaluating Piecewise Functions

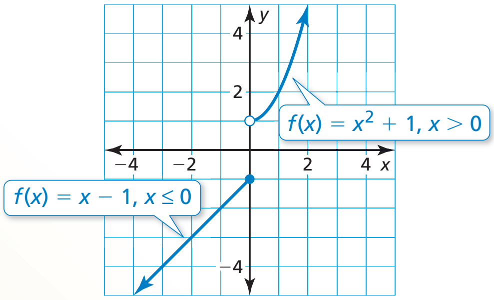
Core Concept

Piecewise Function

A **piecewise function** is a function defined by two or more equations. Each "piece" of the function applies to a different part of its domain. An example is shown below.

$$f(x) = \begin{cases} x - 1, & \text{if } x \le 0 \\ x^2 + 1, & \text{if } x > 0 \end{cases}$$

- The expression $x - 1$ represents the value of f when x is less than or equal to 0.
- The expression $x^2 + 1$ represents the value of f when x is greater than 0.



EXAMPLE 1 Evaluating a Piecewise Function

Evaluate the function f above when (a) $x = 0$ and (b) $x = 4$.

SOLUTION

a. $f(x) = x - 1$ Because $0 \le 0$, use the first equation.

$f(0) = 0 - 1$ Substitute 0 for x.

$f(0) = -1$ Simplify.

The value of f is -1 when $x = 0$.

b. $f(x) = x^2 + 1$ Because $4 > 0$, use the second equation.

$f(4) = (4)^2 + 1$ Substitute 4 for x.

$f(4) = 17$ Simplify.

The value of f is 17 when $x = 4$.

Monitoring Progress

Evaluate the function.

$$f(x) = \begin{cases} x + 4, & \text{if } x < -2 \\ 2^{-x}, & \text{if } -2 \le x \le 5 \\ 3x^2, & \text{if } x > 5 \end{cases}$$

1. $f(-8)$ **2.** $f(-2)$

3. $f(0)$ **4.** $f(3)$

5. $f(5)$ **6.** $f(10)$

Graphing and Analyzing Piecewise Functions

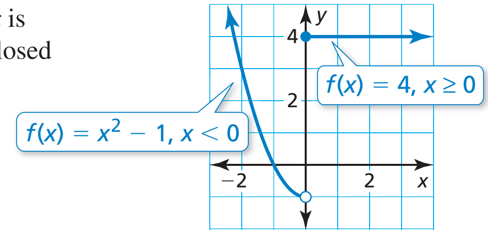
EXAMPLE 2 **Graphing a Piecewise Function**

Graph $y = \begin{cases} x^2 - 1, & \text{if } x < 0 \\ 4, & \text{if } x \geq 0 \end{cases}$. Describe the domain and range.

SOLUTION

Step 1 Graph $y = x^2 - 1$ for $x < 0$. Because x is not equal to 0, use an open circle at $(0, -1)$.

Step 2 Graph $y = 4$ for $x \geq 0$. Because x is greater than or equal to 0, use a closed circle at $(0, 4)$.



▶ The domain is $\{x \mid -\infty < x < \infty\}$.

The range is $\{y \mid y > -1\}$.

Monitoring Progress

Graph the function. Describe the domain and range.

7. $y = \begin{cases} 2x + 1, & \text{if } x \leq 0 \\ x^2, & \text{if } x > 0 \end{cases}$

8. $y = \begin{cases} -3, & \text{if } x \leq -1 \\ 2^x - 1, & \text{if } x > -1 \end{cases}$

EXAMPLE 3 **Analyzing a Piecewise Function**

In Example 2, identify the intercept(s) of the graph of the function, and the interval(s) on which the function is increasing, decreasing, or constant.

SOLUTION

An x-intercept of a graph occurs when $y = 0$. So, the x-intercept is -1. The y-intercept of a graph occurs when $x = 0$. Because there is an open circle at $(0, -1)$ and a closed circle at $(0, 4)$, the y-intercept is 4.

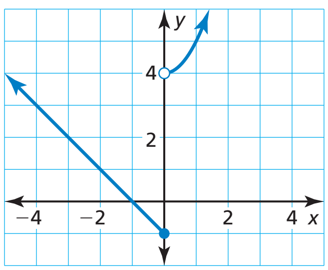
The function is decreasing when $x < 0$ and the function is constant when $x \geq 0$.

▶ So, the x-intercept is -1 and the y-intercept is 4. The function is decreasing on the interval $(-\infty, 0)$ and constant on the interval $(0, \infty)$.

Monitoring Progress

Identify the intercept(s) of the graph of the function, and the intervals(s) on which the function is increasing, decreasing, or constant.

9.



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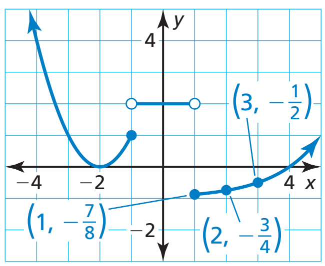
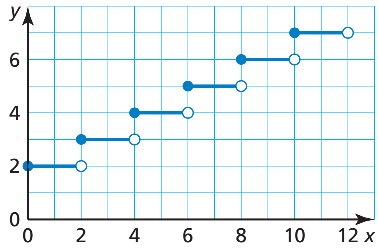


Graphing and Writing Step Functions

A **step function** is a piecewise function defined by a constant value over each part of its domain. The graph of a step function consists of a series of line segments.

STUDY TIP

The graph of a step function looks like a staircase.



$$f(x) = \begin{cases} 2, & \text{if } 0 \le x < 2 \\ 3, & \text{if } 2 \le x < 4 \\ 4, & \text{if } 4 \le x < 6 \\ 5, & \text{if } 6 \le x < 8 \\ 6, & \text{if } 8 \le x < 10 \\ 7, & \text{if } 10 \le x < 12 \end{cases}$$

EXAMPLE 4 Graphing and Writing a Step Function

You rent a karaoke machine for 5 days. The rental company charges \$50 for the first day and \$25 for each additional day. Write and graph a step function that represents the relationship between the number x of days and the total cost y (in dollars) of renting the karaoke machine.

SOLUTION

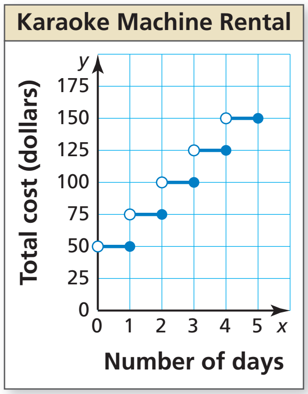
Step 1 Use a table to organize the information.

Number of days	Total cost (dollars)
$0 < x \le 1$	50
$1 < x \le 2$	75
$2 < x \le 3$	100
$3 < x \le 4$	125
$4 < x \le 5$	150

Step 2 Write the step function.

$$f(x) = \begin{cases} 50, & \text{if } 0 < x \le 1 \\ 75, & \text{if } 1 < x \le 2 \\ 100, & \text{if } 2 < x \le 3 \\ 125, & \text{if } 3 < x \le 4 \\ 150, & \text{if } 4 < x \le 5 \end{cases}$$

Step 3 Graph the step function.



Monitoring Progress

11. A landscaper rents a wood chipper for 4 days. The rental company charges \$100 for the first day and \$50 for each additional day. Write and graph a step function that represents the relationship between the number x of days and the total cost y (in dollars) of renting the chipper.

6.8 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. VOCABULARY Compare piecewise functions and step functions.

2. WRITING Describe how to write a step function given its graph.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, evaluate the function. *(See Example 1.)*

$$f(x) = \begin{cases} 2x^2 - 1, & \text{if } x < -2 \\ 2x + 1, & \text{if } x \geq -2 \end{cases}$$

$$g(x) = \begin{cases} -3x, & \text{if } x \leq -1 \\ 3x, & \text{if } -1 < x < 2 \\ x^2 - 5, & \text{if } x \geq 2 \end{cases}$$

3. $f(-3)$

4. $f(-2)$

5. $f(0)$

6. $f(5)$

7. $g(-1)$

8. $g(0)$

9. $g(2)$

10. $g(5)$

In Exercises 11–16, graph the function. Describe the domain and range. *(See Example 2.)*

11. $y = \begin{cases} -x^2, & \text{if } x < 2 \\ x - 6, & \text{if } x \geq 2 \end{cases}$

12. $y = \begin{cases} 2x^2, & \text{if } x \leq 0 \\ -2x^2, & \text{if } x > 0 \end{cases}$

13. $y = \begin{cases} -3x - 2, & \text{if } x \leq -1 \\ 2^x + 2, & \text{if } x > -1 \end{cases}$

14. $y = \begin{cases} x^2 - 3, & \text{if } x < 4 \\ 4x - 4, & \text{if } x \geq 4 \end{cases}$

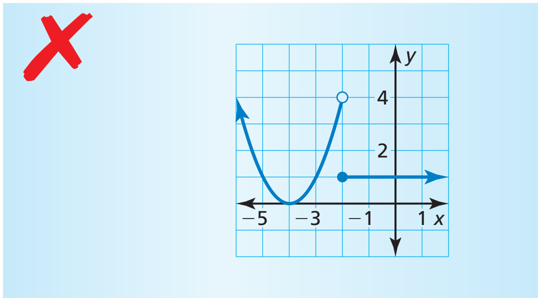
15. $y = \begin{cases} 2^{-x} - 8, & \text{if } x < -3 \\ x - 1, & \text{if } -3 \leq x \leq 3 \\ -2x^2 + 8, & \text{if } x > 3 \end{cases}$

16. $y = \begin{cases} 2x^2 + 1, & \text{if } x \leq -1 \\ -2^x + 2, & \text{if } -1 < x < 2 \\ x^2 + 2^x, & \text{if } x \geq 2 \end{cases}$

17. ERROR ANALYSIS Describe and correct the error in finding $f(1)$ when $f(x) = \begin{cases} 3x^2, & \text{if } x < 1 \\ x - 15, & \text{if } x \geq 1 \end{cases}$.

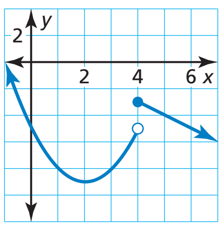
$$\begin{aligned} f(1) &= 3(1)^2 \\ &= 3 \end{aligned}$$

18. ERROR ANALYSIS Describe and correct the error in graphing $y = \begin{cases} (x + 4)^2, & \text{if } x \leq -2 \\ 1, & \text{if } x > -2 \end{cases}$.



Identify the intercept(s) of the function, and the interval(s) on which the function is increasing, decreasing, or constant. *(See Example 3.)*

19.



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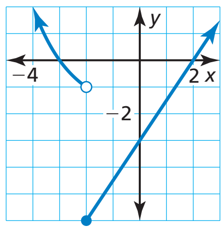

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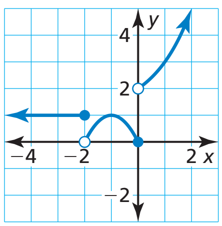

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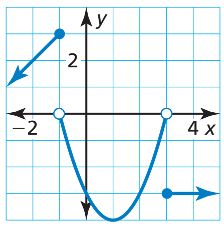
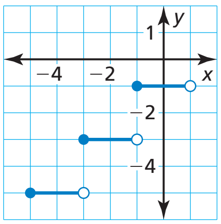


In Exercises 23 and 24, write a step function for the graph.

23.



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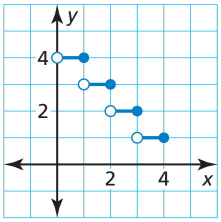


In Exercises 25 and 26, graph the step function. Describe the domain and range.

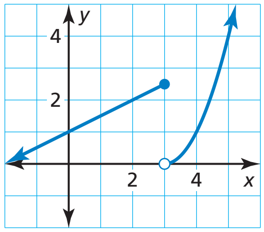
25. $f(x) = \begin{cases} -4, & \text{if } 1 < x \leq 2 \\ -6, & \text{if } 2 < x \leq 3 \\ -8, & \text{if } 3 < x \leq 4 \\ -10, & \text{if } 4 < x \leq 5 \end{cases}$

26. $f(x) = \begin{cases} -2, & \text{if } -6 \leq x < -5 \\ -1, & \text{if } -5 \leq x < -3 \\ 0, & \text{if } -3 \leq x < -2 \\ 1, & \text{if } -2 \leq x < 0 \end{cases}$

27. MODELING WITH MATHEMATICS The cost to join an intramural sports league is \$180 per team and includes the first five team members. For each additional team member, there is a \$30 fee. You plan to have nine people on your team. Write and graph a step function that represents the relationship between the number p of people on your team and the total cost of joining the league. *(See Example 4.)*

28. MODELING WITH MATHEMATICS The rates for a parking garage are shown. Write and graph a step function that represents the relationship between the number x of hours a car is parked in the garage and the total cost of parking in the garage for 1 day.

29. REASONING The piecewise function f consists of two "pieces," one linear piece and one quadratic piece. The graph of f is shown.



a. What is the value of $f(-10)$?

b. What is the value of $f(8)$?

30. USING STRUCTURE The output y of the *greatest integer function* is the greatest integer less than or equal to the input value x. This function is written as $f(x) = [\![x]\!]$. Graph the function for $-4 \leq x < 4$. Is it a piecewise function? a step function? Explain.

31. THOUGHT PROVOKING Explain why

$$y = \begin{cases} 2x - 2, & \text{if } x \leq 3 \\ -2^x, & \text{if } x \geq 3 \end{cases}$$

does not represent a function. How can you redefine y so that it does represent a function?

32. CRITICAL THINKING Describe how the graph of each piecewise function changes when < is replaced with ≤ and ≥ is replaced with >. Do the domain and range change? Explain.

a. $f(x) = \begin{cases} x^2 + 2, & \text{if } x < 2 \\ 3^{-x} + 1, & \text{if } x \geq 2 \end{cases}$

b. $f(x) = \begin{cases} \frac{1}{2}x + \frac{3}{2}, & \text{if } x < 1 \\ 3x^2 - 1, & \text{if } x \geq 1 \end{cases}$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write an equation that relates x and y.

33.

x	−8	−4	0	4	8
y	−6	−3	0	3	6

34.

x	−4	−2	0	2	4
y	6	3	0	−3	−6

Factor the polynomial completely.

35. $15x^3 - 14x^2 - 16x$

36. $x^3 + 2x^2 - 15x - 36$

37. $x^4 + x^3 + 2x - 4$