

APPLYING

MATHEMATICS

conclusions.

To be proficient in math, you

need to analyze relationships

mathematically to draw

Essential Question How can you find the surface area of a prism or

a cylinder?

Recall that the **surface area** of a polyhedron is the sum of the areas of its faces. The **lateral area** of a polyhedron is the sum of the areas of its lateral faces.

EXPLORATION 1

Finding a Formula for Surface Area

Work with a partner. Consider the polyhedron shown.

- a. Identify the polyhedron. Then sketch its net so that the lateral faces form a rectangle with the same height *h* as the polyhedron. What types of figures make up the net?
- **b.** Write an expression that represents the perimeter *P* of the base of the polyhedron. Show how you can use P to write an expression that represents the lateral area L of the polyhedron.



radius, r

height, h

c. Let B represent the area of a base of the polyhedron. Write a formula for the surface area S.

EXPLORATION 2

Work with a partner. Consider the solid shown.

- a. Identify the solid. Then sketch its net. What types of figures make up the net?
- **b.** Write an expression that represents the perimeter *P* of the base of the solid. Show how you can use P to write an expression that represents the lateral area L of the solid.
- c. Write an expression that represents the area B of a base of the solid.
- **d.** Write a formula for the surface area S.

Communicate Your Answer

- **3.** How can you find the surface area of a prism or a cylinder?
- **4.** Consider the rectangular prism shown.
 - a. Find the surface area of the rectangular prism by drawing its net and finding the sum of the areas of its faces.
 - **b.** Find the surface area of the rectangular prism by using the formula you wrote in Exploration 1.
 - c. Compare your answers to parts (a) and (b). What do you notice?



Finding a Formula for Surface Area

12.2 Lesson

Core Vocabulary

lateral faces, p. 646 lateral edges, p. 646 surface area, p. 646 lateral area, p. 646 net, p. 646 right prism, p. 646 oblique prism, p. 647 oblique cylinder, p. 647

Previous

prism bases of a prism cylinder composite solid

What You Will Learn

- Find lateral areas and surface areas of right prisms.
- Find lateral areas and surface areas of right cylinders.
- Use surface areas of right prisms and right cylinders.

Finding Lateral Areas and Surface Areas of Right Prisms

Recall that a *prism* is a polyhedron with two congruent faces, called *bases*, that lie in parallel planes. The other faces, called **lateral faces**, are parallelograms formed by connecting the corresponding vertices of the bases. The segments connecting these vertices are **lateral edges**. Prisms are classified by the shapes of their bases.



The **surface area** of a polyhedron is the sum of the areas of its faces. The **lateral area** of a polyhedron is the sum of the areas of its lateral faces.

Imagine that you cut some edges of a polyhedron and unfold it. The two-dimensional representation of the faces is called a **net**. The surface area of a prism is equal to the area of its net.

The height of a prism is the perpendicular distance between its bases. In a **right prism**, each lateral edge is perpendicular to both bases. A prism with lateral edges that are not perpendicular to the bases is an **oblique prism**.





Oblique triangular prism

height

G Core Concept

Lateral Area and Surface Area of a Right Prism

For a right prism with base perimeter P, base apothem a, height h, and base area B, the lateral area L and surface area S are as follows.

Lateral area L = PhSurface area S = 2B + L= aP + Ph





Finding Lateral Area and Surface Area

Find the lateral area and the surface area of the right pentagonal prism.

SOLUTION

L = Ph

= (35.25)(9)

= 317.25

S = aP + Ph

Find the apothem and perimeter of a base.

Find the lateral area and the surface area.

 $=(\sqrt{23.574375})(35.25) + 317.25$

$$a = \sqrt{6^2 - 3.525^2} = \sqrt{23.574375}$$
$$P = 5(7.05) = 35.25$$



Substitute.

Substitute.

Multiply.

7.05 ft 9 ft

Formula for lateral area of a right prism

Formula for surface area of a right prism

6 ft



Throughout this chapter, round lateral areas, surface areas, and volumes to the nearest hundredth, if necessary.



right cylinder



oblique cylinder

 ≈ 488.40 Use a calculator. The lateral area is 317.25 square feet and the surface area is about 488.40 square feet.



1. Find the lateral area and the surface area of a right rectangular prism with a height of 7 inches, a length of 3 inches, and a width of 4 inches.

Finding Lateral Areas and Surface Areas of **Right Cylinders**

Recall that a *cylinder* is a solid with congruent circular bases that lie in parallel planes. The height of a cylinder is the perpendicular distance between its bases. The radius of a base is the *radius* of the cylinder. In a **right cylinder**, the segment joining the centers of the bases is perpendicular to the bases. In an **oblique cylinder**, this segment is *not* perpendicular to the bases.

The lateral area of a cylinder is the area of its curved surface. For a right cylinder, it is equal to the product of the circumference and the height, or $2\pi rh$. The surface area of a cylinder is equal to the sum of the lateral area and the areas of the two bases.

Core Concept

Lateral Area and Surface Area of a Right Cylinder





Finding Lateral Area and Surface Area

Find the lateral area and the surface area of the right cylinder.

SOLUTION

Find the lateral area and the surface area.

$L = 2\pi rh$	Formula for lateral area of a right cylinder	
$= 2\pi(4)(8)$	Substitute.	
$= 64\pi$	Simplify.	
≈ 201.06	Use a calculator.	
$S = 2\pi r^2 + 2\pi rh$	Formula for surface area of a right cylinder	
$=2\pi(4)^2+64\pi$	Substitute.	
$=96\pi$	Simplify.	
≈ 301.59	Use a calculator.	



The lateral area is 64π , or about 201.06 square meters. The surface area is 96π , or about 301.59 square meters.

EXAMPLE 3

Solving a Real-Life Problem

You are designing a label for the cylindrical soup can shown. The label will cover the lateral area of the can. Find the minimum amount of material needed for the label.

SOLUTION

Find the radius of a base.

$$r = \frac{1}{2}(9) = 4.5$$

Find the lateral area.

$L = 2\pi rh$	Formula for lateral area of a right cylinder	
$= 2\pi(4.5)(12)$	Substitute.	
$= 108 \pi$	Simplify.	
≈ 339.29	Use a calculator.	

You need a minimum of about 339.29 square centimeters of material.

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2. Find the lateral area and the surface area of the right cylinder.



3. WHAT IF? In Example 3, you change the design of the can so that the diameter is 12 centimeters. Find the minimum amount of material needed for the label.



-9 cm-

12 cm

Using Surface Areas of Right Prisms and Right Cylinders

EXAMPLE 4

PLE 4 Finding the Surface Area of a Composite Solid



Find the lateral area and the surface area of the composite solid.

SOLUTION

Lateral area of solid = Lateral area of cylinder + Lateral area of prism = $2\pi rh + Ph$ = $2\pi(6)(12) + 14(12)$ = $144\pi + 168$ ≈ 620.39 Surface area of solid = Lateral area of solid + $2 \cdot (\text{Area of a base} - \text{Area of a base} + 144\pi + 168 + 2(\pi r^2 - \ell w))$ = $144\pi + 168 + 2[\pi(6)^2 - 4(3)]$ = $216\pi + 144$

≈ 822.58

The lateral area is about 620.39 square meters and the surface area is about 822.58 square meters.

EXAMPLE 5 Changing Dimensions in a Solid

Describe how doubling all the linear dimensions affects the surface area of the right cylinder.

SOLUTION

	Before change	After change
Dimensions	r = 2 ft, $h = 8$ ft	r = 4 ft, $h = 16$ ft
Surface area	$S = 2\pi r^2 + 2\pi rh$ = $2\pi (2)^2 + 2\pi (2)(8)$	$S = 2\pi r^2 + 2\pi rh$ = $2\pi (4)^2 + 2\pi (4)(16)$
	$=40\pi\mathrm{ft}^2$	$= 160\pi \mathrm{ft}^2$



Doubling all the linear dimensions results in a surface area that is $\frac{160\pi}{40\pi} = 4 = 2^2$ times the original surface area.

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- 4. Find the lateral area and the surface area of the composite solid at the left.
- 5. In Example 5, describe how multiplying all the linear dimensions by $\frac{1}{2}$ affects the surface area of the right cylinder.



12.2 Exercises

-Vocabulary and Core Concept Check

- 1. VOCABULARY Sketch a right triangular prism. Identify the bases, lateral faces, and lateral edges.
- 2. WRITING Explain how the formula S = 2B + L applies to finding the surface area of both a right prism and a right cylinder.

Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, find the surface area of the solid formed by the net.



In Exercises 5–8, find the lateral area and the surface area of the right prism. (*See Example 1.*)



- **7.** A regular pentagonal prism has a height of 3.5 inches and a base edge length of 2 inches.
- **8.** A regular hexagonal prism has a height of 80 feet and a base edge length of 40 feet.

In Exercises 9–12, find the lateral area and the surface area of the right cylinder. (*See Example 2.*)



- **11.** A right cylinder has a diameter of 24 millimeters and a height of 40 millimeters.
- **12.** A right cylinder has a radius of 2.5 feet and a height of 7.5 feet.

13. MODELING WITH MATHEMATICS The inside of the cylindrical swimming pool shown must be covered with a vinyl liner. The liner must cover the side and bottom of the swimming pool. What is the minimum amount of vinyl needed for the liner? (*See Example 3.*)



14. MODELING WITH MATHEMATICS The tent shown has fabric covering all four sides and the floor. What is the minimum amount of fabric needed to 6 ft



5 ft

8 ft

area of the composite solid. (See Example 4.)



19. ERROR ANALYSIS Describe and correct the error in finding the surface area of the right cylinder.



20. ERROR ANALYSIS Describe and correct the error in finding the surface area of the composite solid.



In Exercises 21–24, describe how the change affects the surface area of the right prism or right cylinder. (*See Example 5.*)



In Exercises 25 and 26, find the height of the right prism or right cylinder.



- **27. MATHEMATICAL CONNECTIONS** A cube has a surface area of 343 square inches. Write and solve an equation to find the length of each edge of the cube.
- **28. MATHEMATICAL CONNECTIONS** A right cylinder has a surface area of 108π square meters. The radius of the cylinder is twice its height. Write and solve an equation to find the height of the cylinder.
- **29. MODELING WITH MATHEMATICS** A company makes two types of recycling bins, as shown. Both types of bins have an open top. Which recycling bin requires more material to make? Explain.



- **30. MODELING WITH MATHEMATICS** You are painting a rectangular room that is 13 feet long, 9 feet wide, and 8.5 feet high. There is a window that is 2.5 feet wide and 5 feet high on one wall. On another wall, there is a door that is 4 feet wide and 7 feet high. A gallon of paint covers 350 square feet. How many gallons of paint do you need to cover the four walls with one coat of paint, not including the window and door?
- **31. ANALYZING RELATIONSHIPS** Which creates a greater surface area, doubling the radius of a cylinder or doubling the height of a cylinder? Explain your reasoning.
- **32. MAKING AN ARGUMENT** You cut a cylindrical piece of lead, forming two congruent cylindrical pieces of lead. Your friend claims the surface area of each smaller piece is exactly half the surface area of the original piece. Is your friend correct? Explain your reasoning.
- **33. USING STRUCTURE** The right triangular prisms shown have the same surface area. Find the height *h* of prism B.



- **34. USING STRUCTURE** The lateral surface area of a regular pentagonal prism is 360 square feet. The height of the prism is twice the length of one of the edges of the base. Find the surface area of the prism.
- **35. ANALYZING RELATIONSHIPS** Describe how multiplying all the linear dimensions of the right rectangular prism by each given value affects the surface area of the prism.



36. HOW DO YOU SEE IT?

a. 2

An open gift box is shown.

- a. Why is the area of the net of the box larger than the minimum amount of wrapping paper needed to cover the closed box?
- **b.** When wrapping the box, why would you want to use more than the minimum amount of paper needed?
- **37. REASONING** Consider a cube that is built using 27 unit cubes, as shown.
 - **a.** Find the surface area of the solid formed when the red unit cubes are removed from the solid shown.



d. *n*

- **b.** Find the surface area of the solid formed when the blue unit cubes are removed from the solid shown.
- **c.** Explain why your answers are different in parts (a) and (b).

- **38. THOUGHT PROVOKING** You have 24 cube-shaped building blocks with edge lengths of 1 unit. What arrangement of blocks gives you a rectangular prism with the least surface area? Justify your answer.
- **39. USING STRUCTURE** Sketch the net of the oblique rectangular prism shown. Then find the surface area.



40. WRITING Use the diagram to write a formula that can be used to find the surface area *S* of any cylindrical ring where $0 < r_2 < r_1$.



- **41. USING STRUCTURE** The *diagonal* of a cube is a segment whose endpoints are vertices that are not on the same face. Find the surface area of a cube with a diagonal length of 8 units.
- **42. USING STRUCTURE** A cuboctahedron has 6 square faces and 8 equilateral triangular faces, as shown. A cuboctahedron can be made by slicing off the corners of a cube.
 - **a.** Sketch a net for the cuboctahedron.
 - b. Each edge of a cuboctahedron has a length of 5 millimeters. Find its surface area.



Maintaining Mathematical Proficiency Reviewing what you learned in previous grades and lessons

