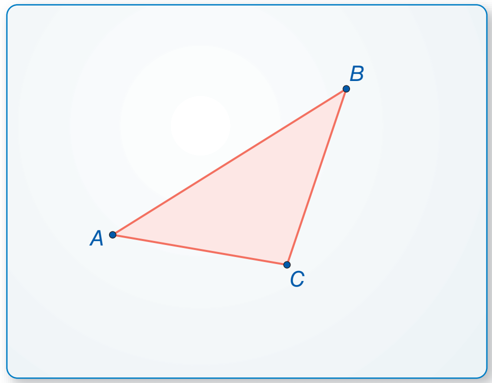
8.1 Similar Polygons

Essential Question How are similar polygons related?

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
G.7.A
G.7.B

EXPLORATION 1 **Comparing Triangles after a Dilation**

Work with a partner. Use dynamic geometry software to draw any $\triangle ABC$. Dilate $\triangle ABC$ to form a similar $\triangle A'B'C'$ using any scale factor k and any center of dilation.



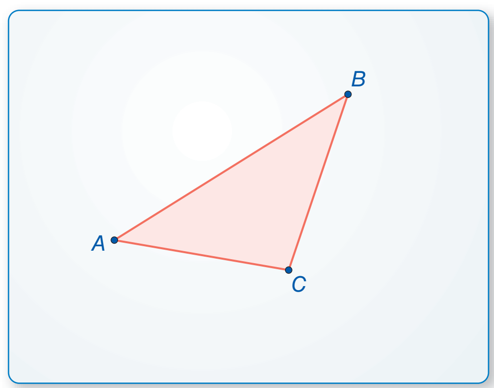
a. Compare the corresponding angles of $\triangle A'B'C'$ and $\triangle ABC$.

b. Find the ratios of the lengths of the sides of $\triangle A'B'C'$ to the lengths of the corresponding sides of $\triangle ABC$. What do you observe?

c. Repeat parts (a) and (b) for several other triangles, scale factors, and centers of dilation. Do you obtain similar results?

EXPLORATION 2 **Comparing Triangles after a Dilation**

Work with a partner. Use dynamic geometry software to draw any $\triangle ABC$. Dilate $\triangle ABC$ to form a similar $\triangle A'B'C'$ using any scale factor k and any center of dilation.



a. Compare the perimeters of $\triangle A'B'C'$ and $\triangle ABC$. What do you observe?

b. Compare the areas of $\triangle A'B'C'$ and $\triangle ABC$. What do you observe?

c. Repeat parts (a) and (b) for several other triangles, scale factors, and centers of dilation. Do you obtain similar results?

ANALYZING MATHEMATICAL RELATIONSHIPS

To be proficient in math, you need to look closely to discern a pattern or structure.

Communicate Your Answer

3. How are similar polygons related?

4. A $\triangle RST$ is dilated by a scale factor of 3 to form $\triangle R'S'T'$. The area of $\triangle RST$ is 1 square inch. What is the area of $\triangle R'S'T'$?

8.1 Lesson

What You Will Learn

- Use similarity statements.
- Find corresponding lengths in similar polygons.
- Find perimeters and areas of similar polygons.
- Decide whether polygons are similar.

Core Vocabulary

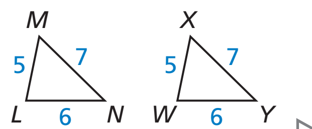
Previous
similar figures
similarity transformation
corresponding parts

Using Similarity Statements

Recall from Section 4.6 that two geometric figures are similar figures if and only if there is a similarity transformation that maps one figure onto the other.

ANALYZING MATHEMATICAL RELATIONSHIPS

Notice that any two congruent figures are also similar. In $\triangle LMN$ and $\triangle WXY$ below, the scale factor is $\frac{5}{5} = \frac{6}{6} = \frac{7}{7} = 1$. So, you can write $\triangle LMN \sim \triangle WXY$ and $\triangle LMN \cong \triangle WXY$.



Core Concept

Corresponding Parts of Similar Polygons

In the diagram below, $\triangle ABC$ is similar to $\triangle DEF$. You can write "$\triangle ABC$ is similar to $\triangle DEF$" as $\triangle ABC \sim \triangle DEF$. A similarity transformation preserves angle measure. So, corresponding angles are congruent. A similarity transformation also enlarges or reduces side lengths by a scale factor k. So, corresponding side lengths are proportional.

Corresponding angles

$\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

Ratios of corresponding side lengths

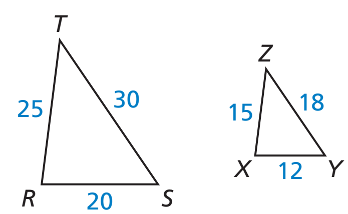
$\frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA} = k$

READING

In a *statement of proportionality*, any pair of ratios forms a true proportion.

EXAMPLE 1 Using Similarity Statements

In the diagram, $\triangle RST \sim \triangle XYZ$.



a. Find the scale factor from $\triangle RST$ to $\triangle XYZ$.

b. List all pairs of congruent angles.

c. Write the ratios of the corresponding side lengths in a *statement of proportionality*.

SOLUTION

a. $\frac{XY}{RS} = \frac{12}{20} = \frac{3}{5}$ $\qquad \frac{YZ}{ST} = \frac{18}{30} = \frac{3}{5}$ $\qquad \frac{ZX}{TR} = \frac{15}{25} = \frac{3}{5}$

So, the scale factor is $\frac{3}{5}$.

b. $\angle R \cong \angle X$, $\angle S \cong \angle Y$, and $\angle T \cong \angle Z$.

c. Because the ratios in part (a) are equal, $\frac{XY}{RS} = \frac{YZ}{ST} = \frac{ZX}{TR}$.

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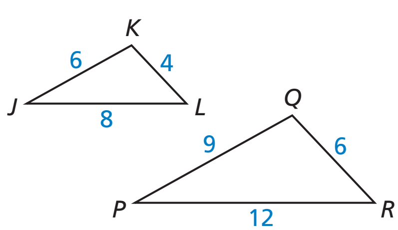


1. In the diagram, $\triangle JKL \sim \triangle PQR$. Find the scale factor from $\triangle JKL$ to $\triangle PQR$. Then list all pairs of congruent angles and write the ratios of the corresponding side lengths in a statement of proportionality.

Finding Corresponding Lengths in Similar Polygons

> **READING**
> Corresponding lengths in similar triangles include side lengths, altitudes, medians, and midsegments.

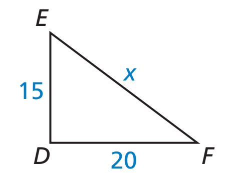
Core Concept

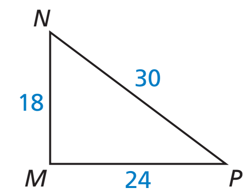
Corresponding Lengths in Similar Polygons

If two polygons are similar, then the ratio of any two corresponding lengths in the polygons is equal to the scale factor of the similar polygons.

EXAMPLE 2 Finding a Corresponding Length

In the diagram, $\triangle DEF \sim \triangle MNP$. Find the value of x.





SOLUTION

The triangles are similar, so the corresponding side lengths are proportional.

$\frac{MN}{DE} = \frac{NP}{EF}$ Write proportion.

$\frac{18}{15} = \frac{30}{x}$ Substitute.

$18x = 450$ Cross Products Property

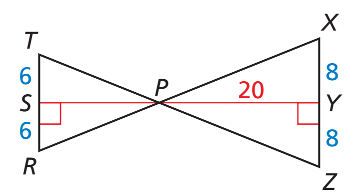
$x = 25$ Solve for x.

▶ The value of x is 25.

> **FORMULATING A PLAN**
> There are several ways to write the proportion. For example, you could write $\frac{DF}{MP} = \frac{EF}{NP}$.

EXAMPLE 3 Finding a Corresponding Length

In the diagram, $\triangle TPR \sim \triangle XPZ$. Find the length of the altitude $\overline{PS}$.



SOLUTION

First, find the scale factor from $\triangle XPZ$ to $\triangle TPR$.

$$\frac{TR}{XZ} = \frac{6+6}{8+8} = \frac{12}{16} = \frac{3}{4}$$

Because the ratio of the lengths of the altitudes in similar triangles is equal to the scale factor, you can write the following proportion.

$\frac{PS}{PY} = \frac{3}{4}$ Write proportion.

$\frac{PS}{20} = \frac{3}{4}$ Substitute 20 for PY.

$PS = 15$ Multiply each side by 20 and simplify.

▶ The length of the altitude $\overline{PS}$ is 15.

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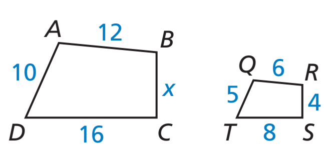
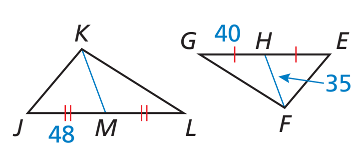
2. Find the value of x.



$ABCD \sim QRST$

3. Find KM.



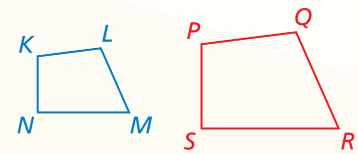
$\triangle JKL \sim \triangle EFG$

Finding Perimeters and Areas of Similar Polygons

Theorem

Theorem 8.1 Perimeters of Similar Polygons

If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.



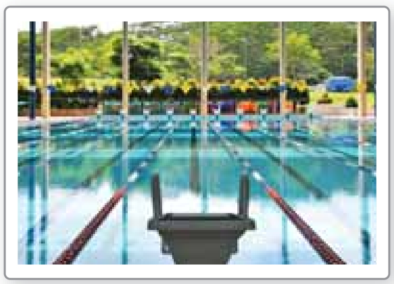
If $KLMN \sim PQRS$, then $\dfrac{PQ + QR + RS + SP}{KL + LM + MN + NK} = \dfrac{PQ}{KL} = \dfrac{QR}{LM} = \dfrac{RS}{MN} = \dfrac{SP}{NK}$.

Proof Ex. 52, p. 430; *BigIdeasMath.com*

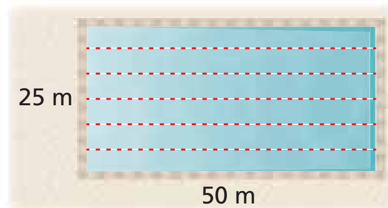
ANALYZING MATHEMATICAL RELATIONSHIPS

When two similar polygons have a scale factor of k, the ratio of their perimeters is equal to k.

EXAMPLE 4 Modeling with Mathematics



A town plans to build a new swimming pool. An Olympic pool is rectangular with a length of 50 meters and a width of 25 meters. The new pool will be similar in shape to an Olympic pool but will have a length of 40 meters. Find the perimeters of an Olympic pool and the new pool.



SOLUTION

1. **Understand the Problem** You are given the length and width of a rectangle and the length of a similar rectangle. You need to find the perimeters of both rectangles.
2. **Make a Plan** Find the scale factor of the similar rectangles and find the perimeter of an Olympic pool. Then use the Perimeters of Similar Polygons Theorem to write and solve a proportion to find the perimeter of the new pool.
3. **Solve the Problem** Because the new pool will be similar to an Olympic pool, the scale factor is the ratio of the lengths, $\frac{40}{50} = \frac{4}{5}$. The perimeter of an Olympic pool is $2(50) + 2(25) = 150$ meters. Write and solve a proportion to find the perimeter x of the new pool.

$$\frac{x}{150} = \frac{4}{5} \quad \text{Perimeters of Similar Polygons Theorem}$$

$$x = 120 \quad \text{Multiply each side by 150 and simplify.}$$

► So, the perimeter of an Olympic pool is 150 meters, and the perimeter of the new pool is 120 meters.

4. **Look Back** Check that the ratio of the perimeters is equal to the scale factor.

$$\frac{120}{150} = \frac{4}{5} \checkmark$$

STUDY TIP

You can also write the scale factor as a decimal. In Example 4, you can write the scale factor as 0.8 and multiply by 150 to get $x = 0.8(150) = 120$.

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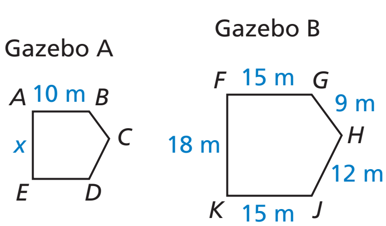
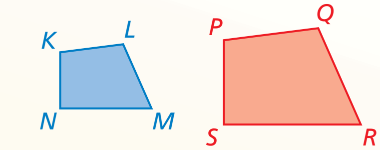
4. The two gazebos shown are similar pentagons. Find the perimeter of Gazebo A.



Theorem

Theorem 8.2 Areas of Similar Polygons

If two polygons are similar, then the ratio of their areas is equal to the squares of the ratios of their corresponding side lengths.



If $KLMN \sim PQRS$, then $\dfrac{\text{Area of } PQRS}{\text{Area of } KLMN} = \left(\dfrac{PQ}{KL}\right)^2 = \left(\dfrac{QR}{LM}\right)^2 = \left(\dfrac{RS}{MN}\right)^2 = \left(\dfrac{SP}{NK}\right)^2$.

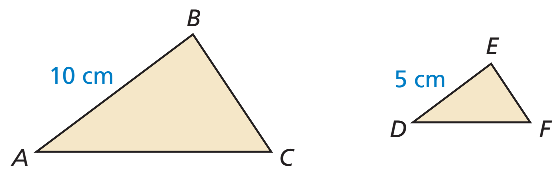
Proof Ex. 53, p. 430; *BigIdeasMath.com*

ANALYZING MATHEMATICAL RELATIONSHIPS

When two similar polygons have a scale factor of k, the ratio of their areas is equal to k^2.

EXAMPLE 5 Finding Areas of Similar Polygons

In the diagram, $\triangle ABC \sim \triangle DEF$. Find the area of $\triangle DEF$.



Area of $\triangle ABC = 36 \text{ cm}^2$

SOLUTION

Because the triangles are similar, the ratio of the area of $\triangle ABC$ to the area of $\triangle DEF$ is equal to the square of the ratio of AB to DE. Write and solve a proportion to find the area of $\triangle DEF$. Let A represent the area of $\triangle DEF$.

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = \left(\frac{AB}{DE}\right)^2$$ Areas of Similar Polygons Theorem

$$\frac{36}{A} = \left(\frac{10}{5}\right)^2$$ Substitute.

$$\frac{36}{A} = \frac{100}{25}$$ Square the right side of the equation.

$$36 \cdot 25 = 100 \cdot A$$ Cross Products Property

$$900 = 100A$$ Simplify.

$$9 = A$$ Solve for A.

The area of $\triangle DEF$ is 9 square centimeters.

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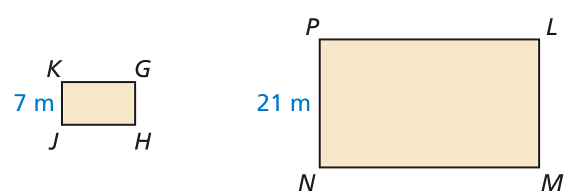
5. In the diagram, $GHJK \sim LMNP$. Find the area of $LMNP$.

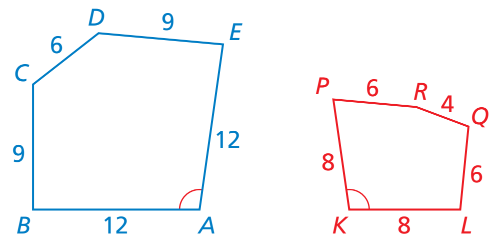


Area of $GHJK = 84 \text{ m}^2$

Deciding Whether Polygons Are Similar

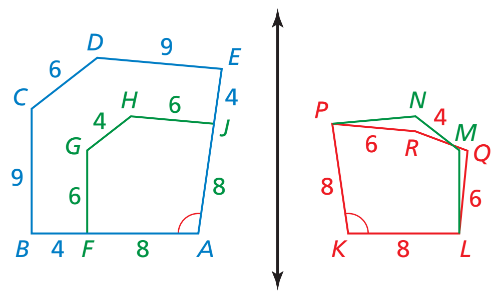
EXAMPLE 6 Deciding Whether Polygons Are Similar

Decide whether *ABCDE* and *KLQRP* are similar. Explain your reasoning.



SOLUTION

Corresponding sides of the pentagons are proportional with a scale factor of $\frac{2}{3}$. However, this does not necessarily mean the pentagons are similar. A dilation with center *A* and scale factor $\frac{2}{3}$ moves *ABCDE* onto *AFGHJ*. Then a reflection moves *AFGHJ* onto *KLMNP*.



KLMNP does not exactly coincide with *KLQRP*, because not all the corresponding angles are congruent. (Only $\angle A$ and $\angle K$ are congruent.)

▶ Because angle measure is not preserved, the two pentagons are not similar.

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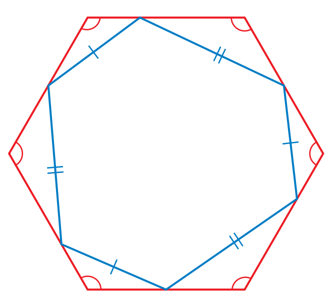
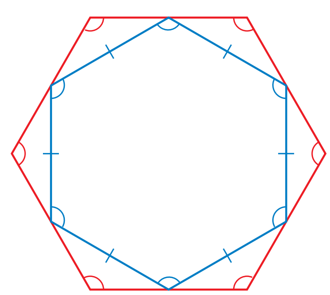
Refer to the floor tile designs below. In each design, the red shape is a regular hexagon.



Tile Design 1



Tile Design 2

6. Decide whether the hexagons in Tile Design 1 are similar. Explain.

7. Decide whether the hexagons in Tile Design 2 are similar. Explain.

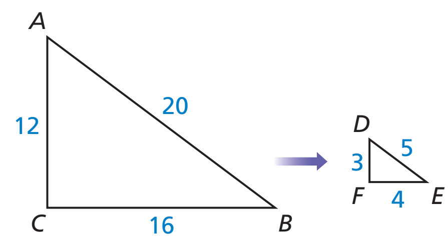
8.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** For two figures to be similar, the corresponding angles must be ____________, and the corresponding side lengths must be _________________.

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.



$\triangle ABC \sim \triangle DEF$

What is the scale factor?

What is the ratio of their areas?

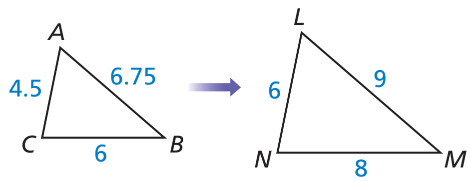
What is the ratio of their corresponding side lengths?

What is the ratio of their perimeters?

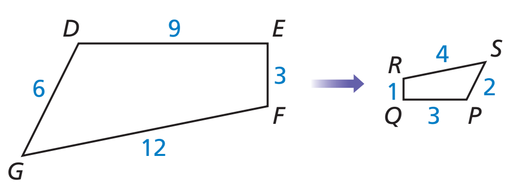
Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, find the scale factor. Then list all pairs of congruent angles and write the ratios of the corresponding side lengths in a statement of proportionality. *(See Example 1.)*

3. $\triangle ABC \sim \triangle LMN$

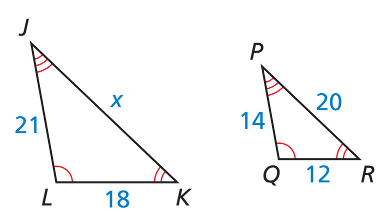


4. $DEFG \sim PQRS$



In Exercises 5–8, the polygons are similar. Find the value of *x*. *(See Example 2.)*

5.



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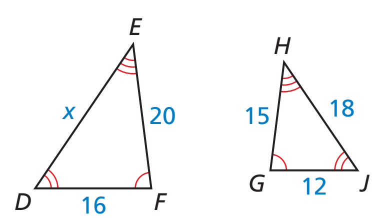

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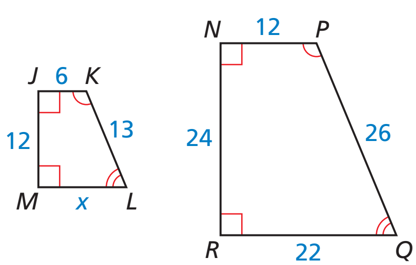

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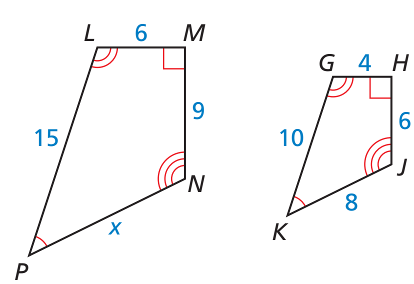
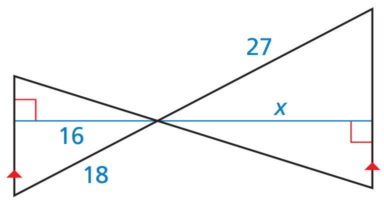
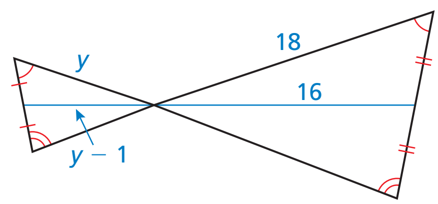


In Exercises 9 and 10, the black triangles are similar. Identify the type of segment shown in blue and find the value of the variable. *(See Example 3.)*

9.

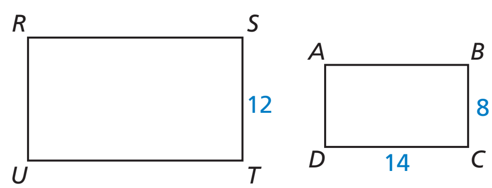


10.



In Exercises 11 and 12, *RSTU* ~ *ABCD*. Find the ratio of their perimeters.

11.



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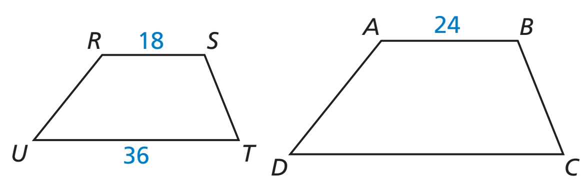


In Exercises 13–16, two polygons are similar. The perimeter of one polygon and the ratio of the corresponding side lengths are given. Find the perimeter of the other polygon.

13. perimeter of smaller polygon: 48 cm; ratio: $\frac{2}{3}$

14. perimeter of smaller polygon: 66 ft; ratio: $\frac{3}{4}$

15. perimeter of larger polygon: 120 yd; ratio: $\frac{1}{6}$

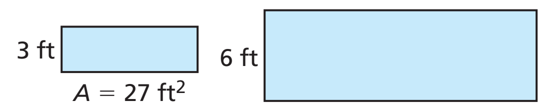
16. perimeter of larger polygon: 85 m; ratio: $\frac{2}{5}$

17. **MODELING WITH MATHEMATICS** A school gymnasium is being remodeled. The basketball court will be similar to an NCAA basketball court, which has a length of 94 feet and a width of 50 feet. The school plans to make the width of the new court 45 feet. Find the perimeters of an NCAA court and of the new court in the school. *(See Example 4.)*

18. **MODELING WITH MATHEMATICS** Your family has decided to put a rectangular patio in your backyard, similar to the shape of your backyard. Your backyard has a length of 45 feet and a width of 20 feet. The length of your new patio is 18 feet. Find the perimeters of your backyard and of the patio.

In Exercises 19–22, the polygons are similar. The area of one polygon is given. Find the area of the other polygon. *(See Example 5.)*

19.



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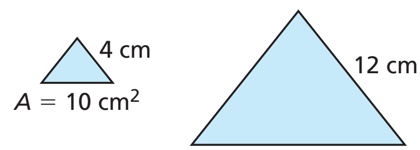

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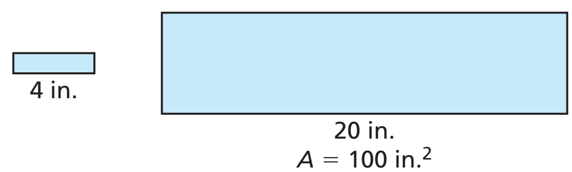

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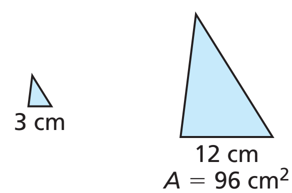
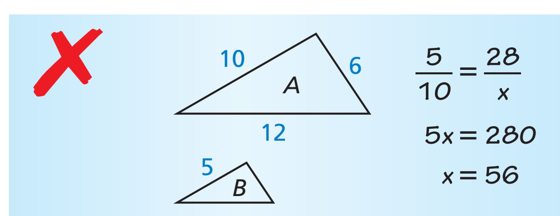
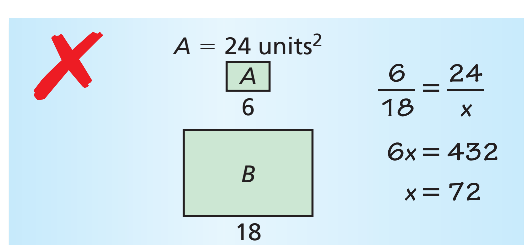


23. **ERROR ANALYSIS** Describe and correct the error in finding the perimeter of triangle B. The triangles are similar.

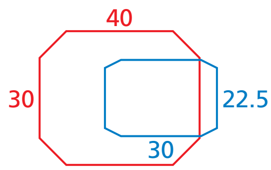


24. **ERROR ANALYSIS** Describe and correct the error in finding the area of rectangle B. The rectangles are similar.

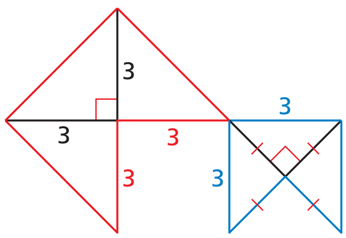


In Exercises 25 and 26, decide whether the red and blue polygons are similar. *(See Example 6.)*

25.



26.

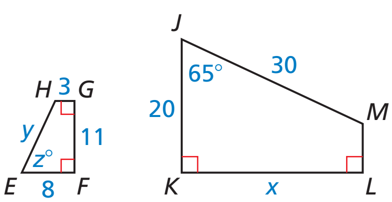


27. REASONING Triangles ABC and DEF are similar. Which statement is correct? Select all that apply.

Ⓐ $\frac{BC}{EF} = \frac{AC}{DF}$ Ⓑ $\frac{AB}{DE} = \frac{CA}{FE}$

Ⓒ $\frac{AB}{EF} = \frac{BC}{DE}$ Ⓓ $\frac{CA}{FD} = \frac{BC}{EF}$

ANALYZING RELATIONSHIPS In Exercises 28–34, $JKLM \sim EFGH$.



28. Find the scale factor of $JKLM$ to $EFGH$.

29. Find the scale factor of $EFGH$ to $JKLM$.

30. Find the values of x, y, and z.

31. Find the perimeter of each polygon.

32. Find the ratio of the perimeters of $JKLM$ to $EFGH$.

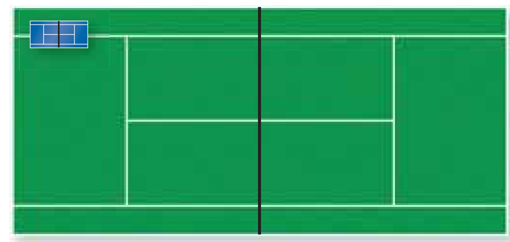
33. Find the area of each polygon.

34. Find the ratio of the areas of $JKLM$ to $EFGH$.

35. USING STRUCTURE Rectangle A is similar to rectangle B. Rectangle A has side lengths of 6 and 12. Rectangle B has a side length of 18. What are the possible values for the length of the other side of rectangle B? Select all that apply.

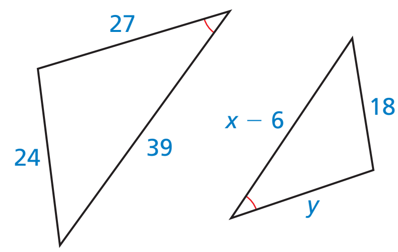
Ⓐ 6 Ⓑ 9 Ⓒ 24 Ⓓ 36

36. DRAWING CONCLUSIONS In table tennis, the table is a rectangle 9 feet long and 5 feet wide. A tennis court is a rectangle 78 feet long and 36 feet wide. Are the two surfaces similar? Explain. If so, find the scale factor of the tennis court to the table.

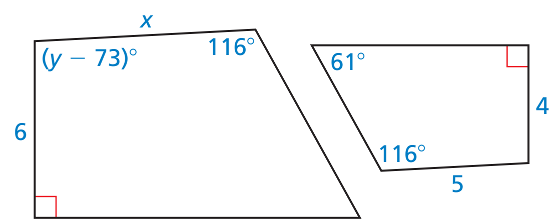


MATHEMATICAL CONNECTIONS In Exercises 37 and 38, the two polygons are similar. Find the values of x and y.

37.



38.



ATTENDING TO PRECISION In Exercises 39–42, the figures are similar. Find the missing corresponding side length.

39. Figure A has a perimeter of 72 meters and one of the side lengths is 18 meters. Figure B has a perimeter of 120 meters.

40. Figure A has a perimeter of 24 inches. Figure B has a perimeter of 36 inches and one of the side lengths is 12 inches.

41. Figure A has an area of 48 square feet and one of the side lengths is 6 feet. Figure B has an area of 75 square feet.

42. Figure A has an area of 18 square feet. Figure B has an area of 98 square feet and one of the side lengths is 14 feet.

CRITICAL THINKING **In Exercises 43–48, tell whether the polygons are *always, sometimes,* or *never* similar.**

43. two isosceles triangles

44. two isosceles trapezoids

45. two rhombuses

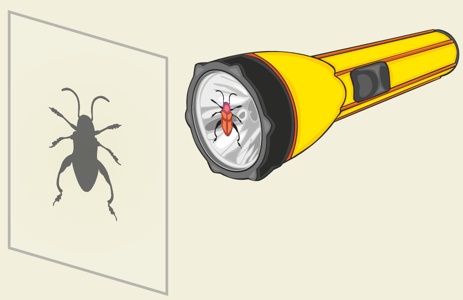
46. two squares

47. two regular polygons

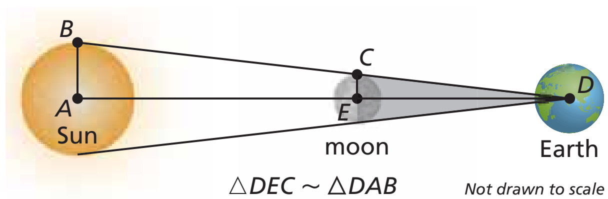
48. a right triangle and an equilateral triangle

49. **MAKING AN ARGUMENT** Your sister claims that when the side lengths of two rectangles are proportional, the two rectangles must be similar. Is she correct? Explain your reasoning.

50. **HOW DO YOU SEE IT?** You shine a flashlight directly on an object to project its image onto a parallel screen. Will the object and the image be similar? Explain your reasoning.



51. **MODELING WITH MATHEMATICS** During a total eclipse of the Sun, the moon is directly in line with the Sun and blocks the Sun's rays. The distance DA between Earth and the Sun is 93,000,000 miles, the distance DE between Earth and the moon is 240,000 miles, and the radius AB of the Sun is 432,500 miles. Use the diagram and the given measurements to estimate the radius EC of the moon.

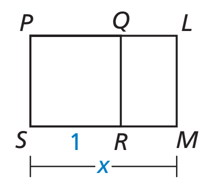


52. **PROVING A THEOREM** Prove the Perimeters of Similar Polygons Theorem (Theorem 8.1) for similar rectangles. Include a diagram in your proof.

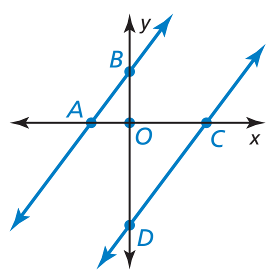
53. **PROVING A THEOREM** Prove the Areas of Similar Polygons Theorem (Theorem 8.2) for similar rectangles. Include a diagram in your proof.

54. **THOUGHT PROVOKING** The postulates and theorems in this book represent Euclidean geometry. In spherical geometry, all points are points on the surface of a sphere. A line is a circle on the sphere whose diameter is equal to the diameter of the sphere. A plane is the surface of the sphere. In spherical geometry, is it possible that two triangles are similar but not congruent? Explain your reasoning.

55. **CRITICAL THINKING** In the diagram, *PQRS* is a square, and $PLMS \sim LMRQ$. Find the exact value of x. This value is called the *golden ratio*. Golden rectangles have their length and width in this ratio. Show that the similar rectangles in the diagram are golden rectangles.



56. **MATHEMATICAL CONNECTIONS** The equations of the lines shown are $y = \frac{4}{3}x + 4$ and $y = \frac{4}{3}x - 8$. Show that $\triangle AOB \sim \triangle COD$.

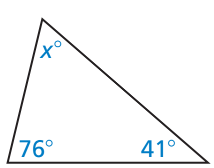


Maintaining Mathematical Proficiency

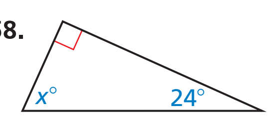
Reviewing what you learned in previous grades and lessons

Find the value of x. *(Section 5.1)*

57.



58.



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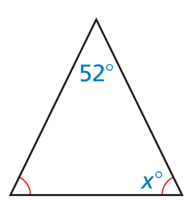

60.