5.3 Proving Triangle Congruence by SAS

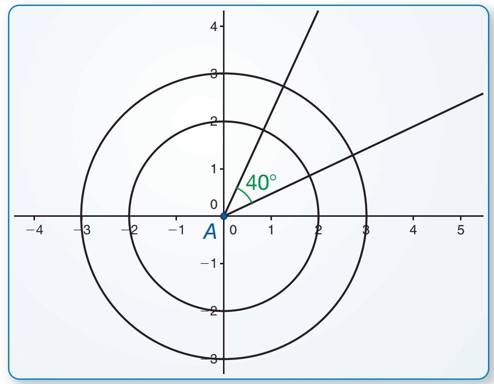
Essential Question What can you conclude about two triangles when you know that two pairs of corresponding sides and the corresponding included angles are congruent?

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
G.5.A
G.6.B

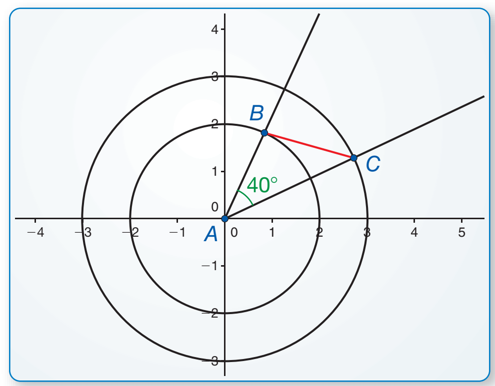
EXPLORATION 1 Drawing Triangles

Work with a partner. Use dynamic geometry software.

a. Construct circles with radii of 2 units and 3 units centered at the origin. Construct a 40° angle with its vertex at the origin. Label the vertex A.



b. Locate the point where one ray of the angle intersects the smaller circle and label this point B. Locate the point where the other ray of the angle intersects the larger circle and label this point C. Then draw $\triangle ABC$.



c. Find BC, $m\angle B$, and $m\angle C$.

d. Repeat parts (a)–(c) several times, redrawing the angle in different positions. Keep track of your results by copying and completing the table below. Write a conjecture about your findings.

SELECTING TOOLS
To be proficient in math, you need to use technology to help visualize the results of varying assumptions, explore consequences, and compare predictions with data.

	A	B	C	AB	AC	BC	$m\angle A$	$m\angle B$	$m\angle C$
1.	(0, 0)			2	3		40°		
2.	(0, 0)			2	3		40°		
3.	(0, 0)			2	3		40°		
4.	(0, 0)			2	3		40°		
5.	(0, 0)			2	3		40°		

Communicate Your Answer

2. What can you conclude about two triangles when you know that two pairs of corresponding sides and the corresponding included angles are congruent?

3. How would you prove your conjecture in Exploration 1(d)?

5.3 Lesson

Core Vocabulary

Previous
congruent figures
rigid motion

STUDY TIP

The *included angle* of two sides of a triangle is the angle formed by the two sides.

What You Will Learn

- Use the Side-Angle-Side (SAS) Congruence Theorem.
- Solve real-life problems.

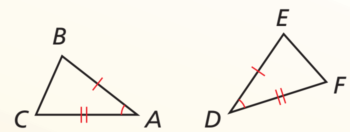
Using the Side-Angle-Side Congruence Theorem

Theorem

Theorem 5.5 Side-Angle-Side (SAS) Congruence Theorem

If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

If $\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, and $\overline{AC} \cong \overline{DF}$, then $\triangle ABC \cong \triangle DEF$.

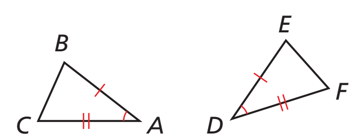


Proof p. 250

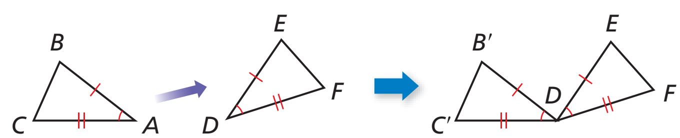
PROOF Side-Angle-Side (SAS) Congruence Theorem

Given $\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, $\overline{AC} \cong \overline{DF}$

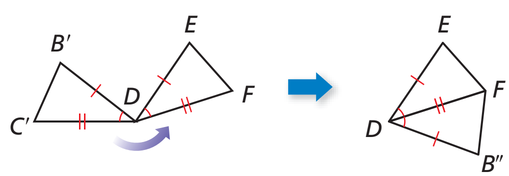
Prove $\triangle ABC \cong \triangle DEF$



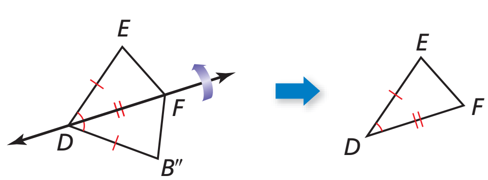
First, translate $\triangle ABC$ so that point A maps to point D, as shown below.



This translation maps $\triangle ABC$ to $\triangle DB'C'$. Next, rotate $\triangle DB'C'$ counterclockwise through $\angle C'DF$ so that the image of $\overrightarrow{DC'}$ coincides with $\overrightarrow{DF}$, as shown below.



Because $\overline{DC'} \cong \overline{DF}$, the rotation maps point C' to point F. So, this rotation maps $\triangle DB'C'$ to $\triangle DB''F$. Now, reflect $\triangle DB''F$ in the line through points D and F, as shown below.



Because points D and F lie on $\overleftrightarrow{DF}$, this reflection maps them onto themselves. Because a reflection preserves angle measure and $\angle B''DF \cong \angle EDF$, the reflection maps $\overrightarrow{DB''}$ to $\overrightarrow{DE}$. Because $\overline{DB''} \cong \overline{DE}$, the reflection maps point B'' to point E. So, this reflection maps $\triangle DB''F$ to $\triangle DEF$.

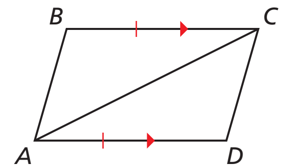
Because you can map $\triangle ABC$ to $\triangle DEF$ using a composition of rigid motions, $\triangle ABC \cong \triangle DEF$.

EXAMPLE 1 Using the SAS Congruence Theorem

Write a proof.

Given $\overline{BC} \cong \overline{DA}$, $\overline{BC} \parallel \overline{AD}$

Prove $\triangle ABC \cong \triangle CDA$



STUDY TIP

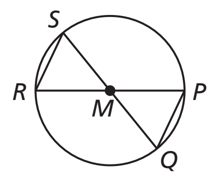
Make your proof easier to read by identifying the steps where you show congruent sides (S) and angles (A).

SOLUTION

STATEMENTS	REASONS
S **1.** $\overline{BC} \cong \overline{DA}$	**1.** Given
2. $\overline{BC} \parallel \overline{AD}$	**2.** Given
A **3.** $\angle BCA \cong \angle DAC$	**3.** Alternate Interior Angles Theorem (Thm. 3.2)
S **4.** $\overline{AC} \cong \overline{CA}$	**4.** Reflexive Property of Congruence (Thm. 2.1)
5. $\triangle ABC \cong \triangle CDA$	**5.** SAS Congruence Theorem

EXAMPLE 2 Using SAS and Properties of Shapes

In the diagram, $\overline{QS}$ and $\overline{RP}$ pass through the center M of the circle. What can you conclude about $\triangle MRS$ and $\triangle MPQ$?



SOLUTION

Because they are vertical angles, $\angle PMQ \cong \angle RMS$. All points on a circle are the same distance from the center, so $\overline{MP}$, $\overline{MQ}$, $\overline{MR}$, and $\overline{MS}$ are all congruent.

▶ So, $\triangle MRS$ and $\triangle MPQ$ are congruent by the SAS Congruence Theorem.

Monitoring Progress

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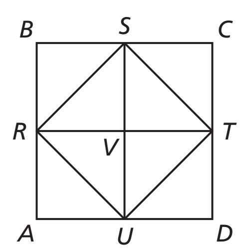
In the diagram, *ABCD* is a square with four congruent sides and four right angles. *R*, *S*, *T*, and *U* are the midpoints of the sides of *ABCD*. Also, $\overline{RT} \perp \overline{SU}$ and $\overline{SV} \cong \overline{VU}$.

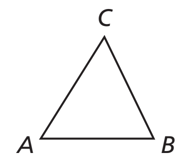


1. Prove that $\triangle SVR \cong \triangle UVR$.

2. Prove that $\triangle BSR \cong \triangle DUT$.

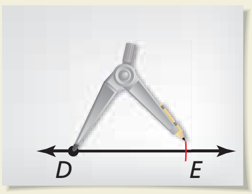
CONSTRUCTION Copying a Triangle Using SAS

Construct a triangle that is congruent to $\triangle ABC$ using the SAS Congruence Theorem. Use a compass and straightedge.



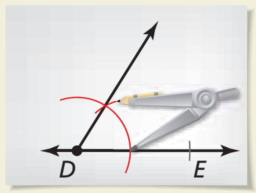
SOLUTION

Step 1



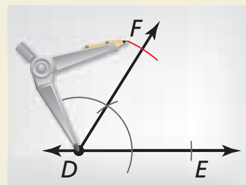
Construct a side Construct $\overline{DE}$ so that it is congruent to $\overline{AB}$.

Step 2



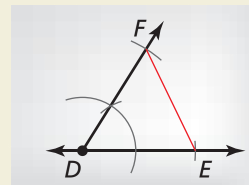
Construct an angle Construct $\angle D$ with vertex D and side $\overrightarrow{DE}$ so that it is congruent to $\angle A$.

Step 3



Construct a side Construct $\overline{DF}$ so that it is congruent to $\overline{AC}$.

Step 4

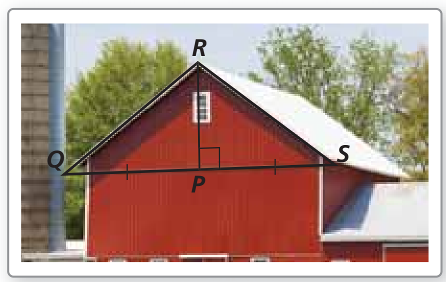


Draw a triangle Draw $\triangle DEF$. By the SAS Congruence Theorem, $\triangle ABC \cong \triangle DEF$.

Solving Real-Life Problems

EXAMPLE 3 Solving a Real-Life Problem

You are making a canvas sign to hang on the triangular portion of the barn wall shown in the picture. You think you can use two identical triangular sheets of canvas. You know that $\overline{RP} \perp \overline{QS}$ and $\overline{PQ} \cong \overline{PS}$. Use the SAS Congruence Theorem to show that $\triangle PQR \cong \triangle PSR$.



SOLUTION

You are given that $\overline{PQ} \cong \overline{PS}$. By the Reflexive Property of Congruence (Theorem 2.1), $\overline{RP} \cong \overline{RP}$. By the definition of perpendicular lines, both $\angle RPQ$ and $\angle RPS$ are right angles, so they are congruent. So, two pairs of sides and their included angles are congruent.

▶ $\triangle PQR$ and $\triangle PSR$ are congruent by the SAS Congruence Theorem.

Monitoring Progress

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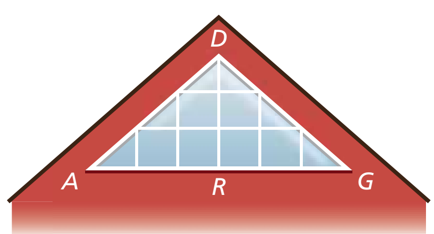
3. You are designing the window shown in the photo. You want to make $\triangle DRA$ congruent to $\triangle DRG$. You design the window so that $\overline{DA} \cong \overline{DG}$ and $\angle ADR \cong \angle GDR$. Use the SAS Congruence Theorem to prove $\triangle DRA \cong \triangle DRG$.



5.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

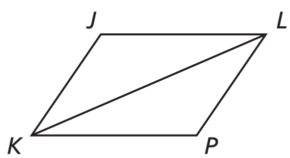
Vocabulary and Core Concept Check

1. **WRITING** What is an included angle?

2. **COMPLETE THE SENTENCE** If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then ____________.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, name the included angle between the pair of sides given.



3. $\overline{JK}$ and $\overline{KL}$

4. $\overline{PK}$ and $\overline{LK}$

5. $\overline{LP}$ and $\overline{LK}$

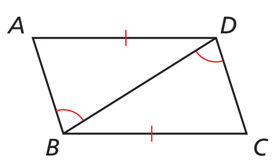
6. $\overline{JL}$ and $\overline{JK}$

7. $\overline{KL}$ and $\overline{JL}$

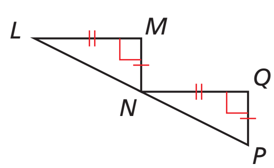
8. $\overline{KP}$ and $\overline{PL}$

In Exercises 9–14, decide whether enough information is given to prove that the triangles are congruent using the SAS Congruence Theorem (Theorem 5.5). Explain.

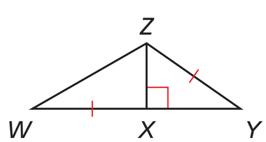
9. $\triangle ABD$, $\triangle CDB$



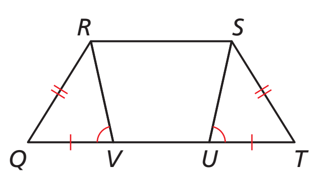
10. $\triangle LMN$, $\triangle NQP$



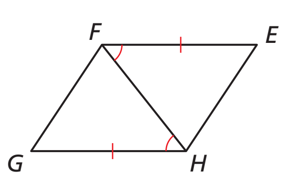
11. $\triangle YXZ$, $\triangle WXZ$



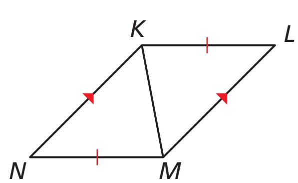
12. $\triangle QRV$, $\triangle TSU$



13. $\triangle EFH$, $\triangle GHF$



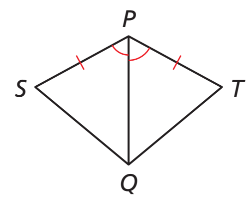
14. $\triangle KLM$, $\triangle MNK$



In Exercises 15–18, write a proof. *(See Example 1.)*

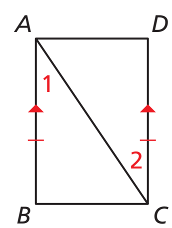
15. **Given** $\overline{PQ}$ bisects $\angle SPT$, $\overline{SP} \cong \overline{TP}$

Prove $\triangle SPQ \cong \triangle TPQ$



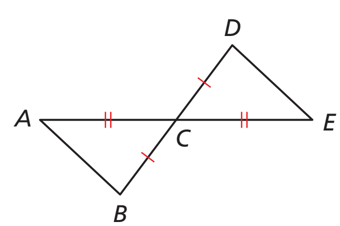
16. **Given** $\overline{AB} \cong \overline{CD}$, $\overline{AB} \parallel \overline{CD}$

Prove $\triangle ABC \cong \triangle CDA$



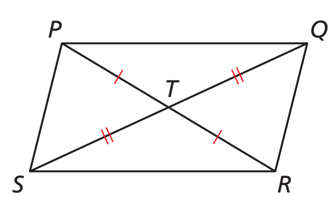
17. **Given** C is the midpoint of $\overline{AE}$ and $\overline{BD}$.

Prove $\triangle ABC \cong \triangle EDC$



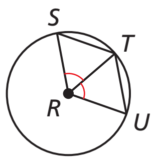
18. **Given** $\overline{PT} \cong \overline{RT}$, $\overline{QT} \cong \overline{ST}$

Prove $\triangle PQT \cong \triangle RST$

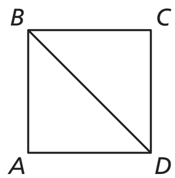


In Exercises 19–22, use the given information to name two triangles that are congruent. Explain your reasoning. *(See Example 2.)*

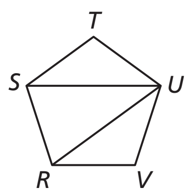
19. $\angle SRT \cong \angle URT$, and R is the center of the circle.



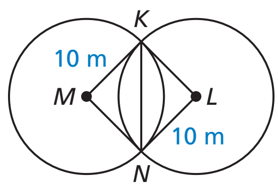
20. $ABCD$ is a square with four congruent sides and four congruent angles.



21. $RSTUV$ is a regular pentagon.

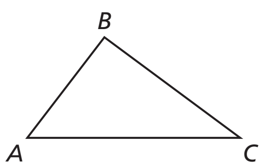


22. $\overline{MK} \perp \overline{MN}$, $\overline{KL} \perp \overline{NL}$, and M and L are centers of circles.

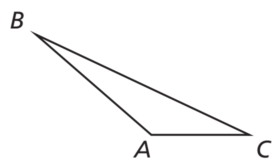


CONSTRUCTION **In Exercises 23 and 24, construct a triangle that is congruent to $\triangle ABC$ using the SAS Congruence Theorem (Theorem 5.5).**

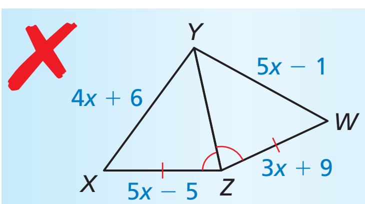
23.



24.



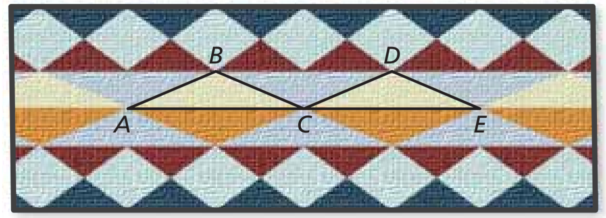
25. **ERROR ANALYSIS** Describe and correct the error in finding the value of x.



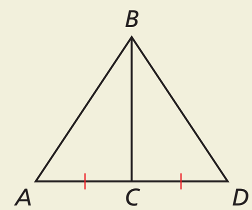
$4x + 6 = 3x + 9$
$x + 6 = 9$
$x = 3$

26. **WRITING** Describe the relationship between your conjecture in Exploration 1(d) on page 249 and the Side-Angle-Side (SAS) Congruence Theorem (Thm. 5.5).

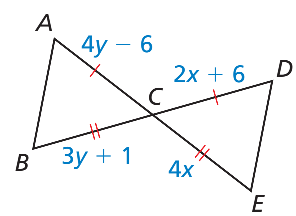
27. **PROOF** The Navajo rug is made of isosceles triangles. You know $\angle B \cong \angle D$. Use the SAS Congruence Theorem (Theorem 5.5) to show that $\triangle ABC \cong \triangle CDE$. *(See Example 3.)*



28. **HOW DO YOU SEE IT?** What additional information do you need to prove that $\triangle ABC \cong \triangle DBC$?

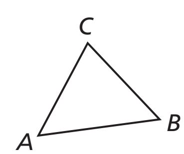


29. **MATHEMATICAL CONNECTIONS** Prove that $\triangle ABC \cong \triangle DEC$. Then find the values of x and y.



30. **THOUGHT PROVOKING** There are six possible subsets of three sides or angles of a triangle: SSS, SAS, SSA, AAA, ASA, and AAS. Which of these correspond to congruence theorems? For those that do not, give a counterexample.

31. **MAKING AN ARGUMENT** Your friend claims it is possible to construct a triangle congruent to $\triangle ABC$ by first constructing $\overline{AB}$ and $\overline{AC}$, and then copying $\angle C$. Is your friend correct? Explain your reasoning.



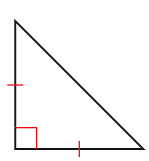
32. **PROVING A THEOREM** Prove the Reflections in Intersecting Lines Theorem (Theorem 4.3).

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Classify the triangle by its sides and by measuring its angles. *(Section 5.1)*

33.



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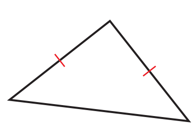

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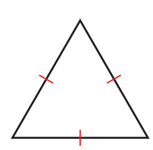

36.