5.2 Congruent Polygons

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
G.5.A
G.6.C
G.6.D

ANALYZING MATHEMATICAL RELATIONSHIPS

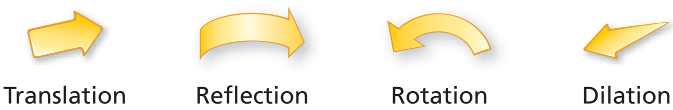
To be proficient in math, you need to look closely to discern a pattern or structure.

Essential Question

Given two congruent triangles, how can you use rigid motions to map one triangle to the other triangle?

EXPLORATION 1 Describing Rigid Motions

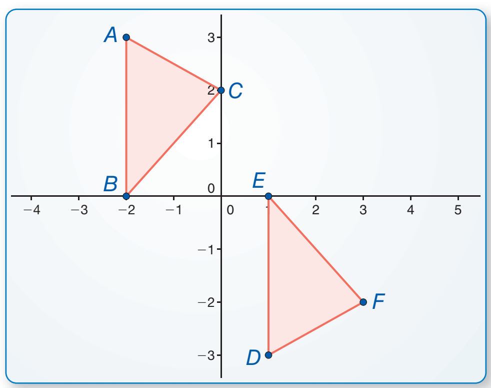
Work with a partner. Of the four transformations you studied in Chapter 4, which are rigid motions? Under a rigid motion, why is the image of a triangle always congruent to the original triangle? Explain your reasoning.



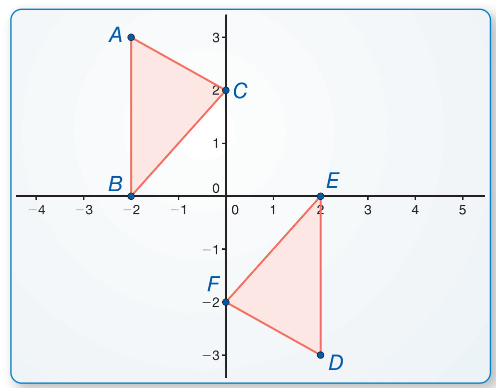
EXPLORATION 2 Finding a Composition of Rigid Motions

Work with a partner. Describe a composition of rigid motions that maps $\triangle ABC$ to $\triangle DEF$. Use dynamic geometry software to verify your answer.

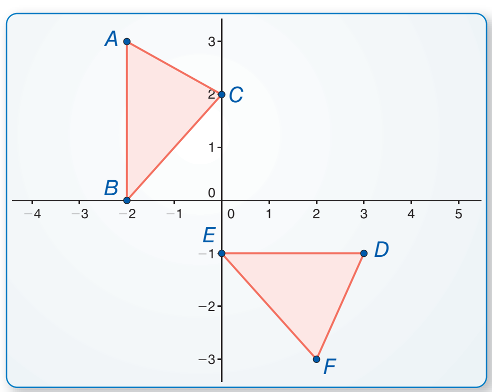
a. $\triangle ABC \cong \triangle DEF$



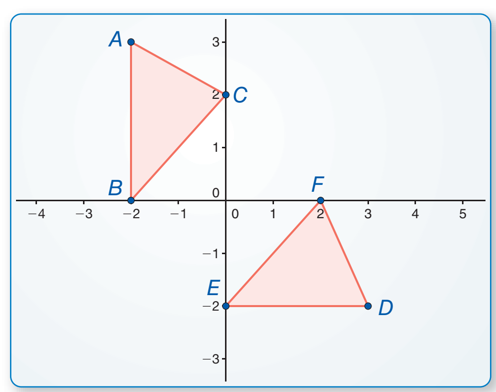
b. $\triangle ABC \cong \triangle DEF$



c. $\triangle ABC \cong \triangle DEF$



d. $\triangle ABC \cong \triangle DEF$



Communicate Your Answer

3. Given two congruent triangles, how can you use rigid motions to map one triangle to the other triangle?

4. The vertices of $\triangle ABC$ are $A(1, 1)$, $B(3, 2)$, and $C(4, 4)$. The vertices of $\triangle DEF$ are $D(2, -1)$, $E(0, 0)$, and $F(-1, 2)$. Describe a composition of rigid motions that maps $\triangle ABC$ to $\triangle DEF$.

5.2 Lesson

What You Will Learn

- Identify and use corresponding parts.
- Use the Third Angles Theorem.

Core Vocabulary

corresponding parts, *p. 244*

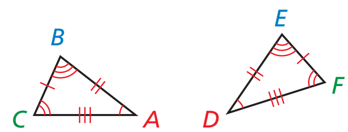
Previous

congruent figures

Identifying and Using Corresponding Parts

Recall that two geometric figures are congruent if and only if a rigid motion or a composition of rigid motions maps one of the figures onto the other. A rigid motion maps each part of a figure to a **corresponding part** of its image. Because rigid motions preserve length and angle measure, corresponding parts of congruent figures are congruent. In congruent polygons, this means that the *corresponding sides* and the *corresponding angles* are congruent.

When $\triangle DEF$ is the image of $\triangle ABC$ after a rigid motion or a composition of rigid motions, you can write congruence statements for the corresponding angles and corresponding sides.



Corresponding angles

$\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

Corresponding sides

$\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DF}$

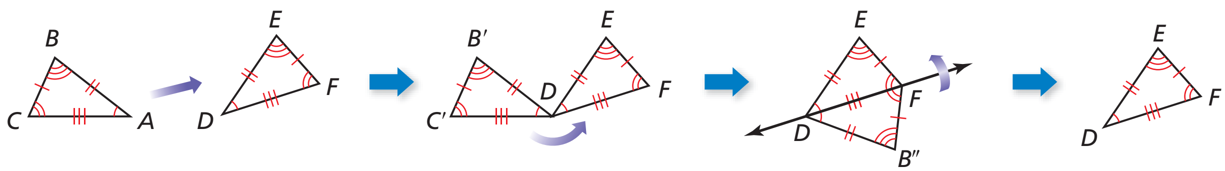
STUDY TIP

Notice that both of the following statements are true.

1. If two triangles are congruent, then all their corresponding parts are congruent.
2. If all the corresponding parts of two triangles are congruent, then the triangles are congruent.

When you write a congruence statement for two polygons, always list the corresponding vertices in the same order. You can write congruence statements in more than one way. Two possible congruence statements for the triangles above are $\triangle ABC \cong \triangle DEF$ or $\triangle BCA \cong \triangle EFD$.

When all the corresponding parts of two triangles are congruent, you can show that the triangles are congruent. Using the triangles above, first translate $\triangle ABC$ so that point A maps to point D. This translation maps $\triangle ABC$ to $\triangle DB'C'$. Next, rotate $\triangle DB'C'$ counterclockwise through $\angle C'DF$ so that the image of $\overrightarrow{DC'}$ coincides with $\overrightarrow{DF}$. Because $\overline{DC'} \cong \overline{DF}$, the rotation maps point C' to point F. So, this rotation maps $\triangle DB'C'$ to $\triangle DB''F$.

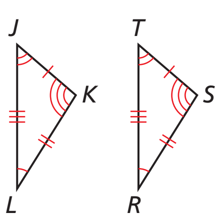


Now, reflect $\triangle DB''F$ in the line through points D and F. This reflection maps the sides and angles of $\triangle DB''F$ to the corresponding sides and corresponding angles of $\triangle DEF$, so $\triangle ABC \cong \triangle DEF$.

So, to show that two triangles are congruent, it is sufficient to show that their corresponding parts are congruent. In general, this is true for all polygons.

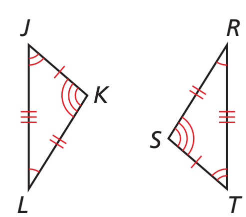
REASONING

To help you identify corresponding parts, rotate $\triangle TSR$.



EXAMPLE 1 Identifying Corresponding Parts

Write a congruence statement for the triangles.
Identify all pairs of congruent corresponding parts.



SOLUTION

The diagram indicates that $\triangle JKL \cong \triangle TSR$.

Corresponding angles $\angle J \cong \angle T$, $\angle K \cong \angle S$, $\angle L \cong \angle R$

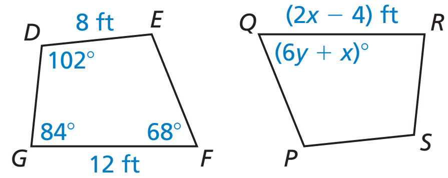
Corresponding sides $\overline{JK} \cong \overline{TS}$, $\overline{KL} \cong \overline{SR}$, $\overline{LJ} \cong \overline{RT}$

EXAMPLE 2 Using Properties of Congruent Figures

In the diagram, $DEFG \cong SPQR$.

a. Find the value of x.

b. Find the value of y.



SOLUTION

a. You know that $\overline{FG} \cong \overline{QR}$.

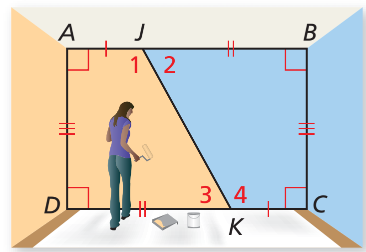
$$FG = QR$$
$$12 = 2x - 4$$
$$16 = 2x$$
$$8 = x$$

b. You know that $\angle F \cong \angle Q$.

$$m\angle F = m\angle Q$$
$$68^\circ = (6y + x)^\circ$$
$$68 = 6y + 8$$
$$10 = y$$

EXAMPLE 3 Showing That Figures Are Congruent

You divide the wall into orange and blue sections along $\overline{JK}$. Will the sections of the wall be the same size and shape? Explain.



SOLUTION

From the diagram, $\angle A \cong \angle C$ and $\angle D \cong \angle B$ because all right angles are congruent. Also, by the Lines Perpendicular to a Transversal Theorem (Thm. 3.12), $\overline{AB} \parallel \overline{DC}$. Then $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$ by the Alternate Interior Angles Theorem (Thm. 3.2). So, all pairs of corresponding angles are congruent. The diagram shows $\overline{AJ} \cong \overline{CK}$, $\overline{KD} \cong \overline{JB}$, and $\overline{DA} \cong \overline{BC}$. By the Reflexive Property of Congruence (Thm. 2.1), $\overline{JK} \cong \overline{KJ}$. So, all pairs of corresponding sides are congruent. Because all corresponding parts are congruent, $AJKD \cong CKJB$.

▶ Yes, the two sections will be the same size and shape.

Monitoring Progress

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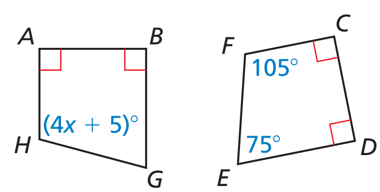
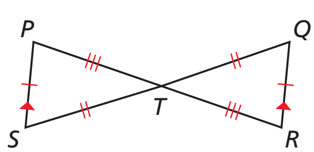
In the diagram, $ABGH \cong CDEF$.



1. Identify all pairs of congruent corresponding parts.
2. Find the value of x.
3. In the diagram at the left, show that $\triangle PTS \cong \triangle RTQ$.



Theorem

Theorem 5.3 Properties of Triangle Congruence

Triangle congruence is reflexive, symmetric, and transitive.

Reflexive For any triangle $\triangle ABC$, $\triangle ABC \cong \triangle ABC$.

Symmetric If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$.

Transitive If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.

Proof BigIdeasMath.com

STUDY TIP

The properties of congruence that are true for segments and angles are also true for triangles.

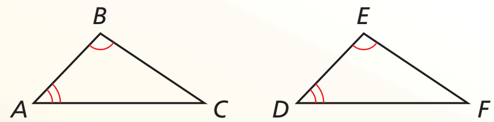
Using the Third Angles Theorem

Theorem

Theorem 5.4 Third Angles Theorem

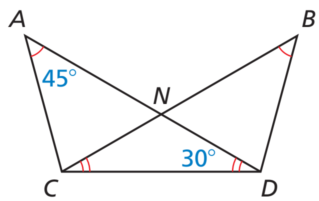
If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.

Proof Ex. 19, p. 248



If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\angle C \cong \angle F$.

EXAMPLE 4 Using the Third Angles Theorem



Find $m\angle BDC$.

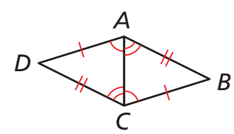
SOLUTION

$\angle A \cong \angle B$ and $\angle ADC \cong \angle BCD$, so by the Third Angles Theorem, $\angle ACD \cong \angle BDC$. By the Triangle Sum Theorem (Theorem 5.1), $m\angle ACD = 180° - 45° - 30° = 105°$.

▶ So, $m\angle BDC = m\angle ACD = 105°$ by the definition of congruent angles.

EXAMPLE 5 Proving That Triangles Are Congruent

Use the information in the figure to prove that $\triangle ACD \cong \triangle CAB$.



SOLUTION

Given $\overline{AD} \cong \overline{CB}$, $\overline{DC} \cong \overline{BA}$, $\angle ACD \cong \angle CAB$, $\angle CAD \cong \angle ACB$

Prove $\triangle ACD \cong \triangle CAB$

Plan for Proof
a. Use the Reflexive Property of Congruence (Thm. 2.1) to show that $\overline{AC} \cong \overline{CA}$.
b. Use the Third Angles Theorem to show that $\angle B \cong \angle D$.

Plan in Action

	STATEMENTS	REASONS
	1. $\overline{AD} \cong \overline{CB}$, $\overline{DC} \cong \overline{BA}$	**1.** Given
a.	**2.** $\overline{AC} \cong \overline{CA}$	**2.** Reflexive Property of Congruence (Theorem 2.1)
	3. $\angle ACD \cong \angle CAB$, $\angle CAD \cong \angle ACB$	**3.** Given
b.	**4.** $\angle B \cong \angle D$	**4.** Third Angles Theorem
	5. $\triangle ACD \cong \triangle CAB$	**5.** All corresponding parts are congruent.

Monitoring Progress

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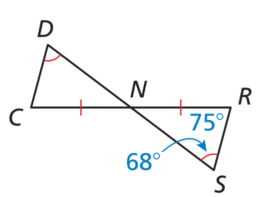


Use the diagram.

4. Find $m\angle DCN$.

5. What additional information is needed to conclude that $\triangle NDC \cong \triangle NSR$?

5.2 Exercises

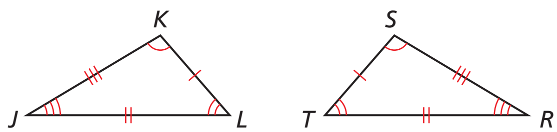
Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. WRITING Based on this lesson, what information do you need to prove that two triangles are congruent? Explain your reasoning.

2. DIFFERENT WORDS, SAME QUESTION Which is different? Find "both" answers.

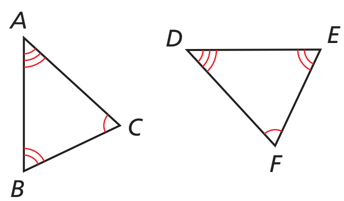
Is $\triangle JKL \cong \triangle RST$?	Is $\triangle KJL \cong \triangle SRT$?
Is $\triangle JLK \cong \triangle STR$?	Is $\triangle LKJ \cong \triangle TSR$?



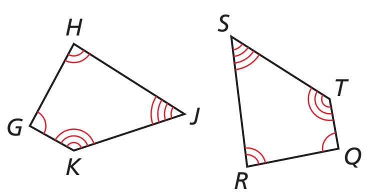
Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, identify all pairs of congruent corresponding parts. Then write another congruence statement for the polygons. *(See Example 1.)*

3. $\triangle ABC \cong \triangle DEF$



4. $GHJK \cong QRST$



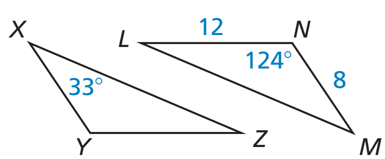
In Exercises 5–8, $\triangle XYZ \cong \triangle MNL$. Copy and complete the statement.

5. $m\angle Y =$ ______

6. $m\angle M =$ ______

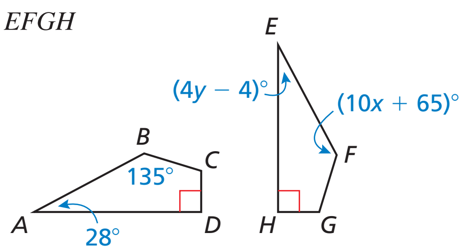
7. $m\angle Z =$ ______

8. $XY =$ ______

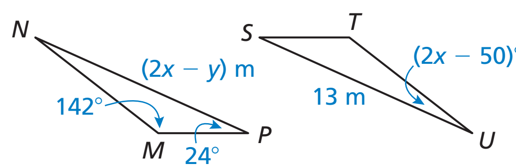


In Exercises 9 and 10, find the values of x and y. *(See Example 2.)*

9. $ABCD \cong EFGH$

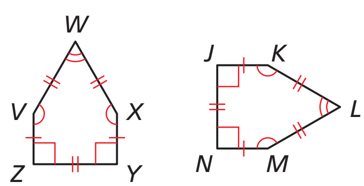


10. $\triangle MNP \cong \triangle TUS$

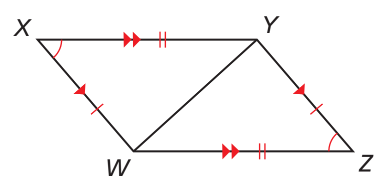


In Exercises 11 and 12, show that the polygons are congruent. Explain your reasoning. *(See Example 3.)*

11.

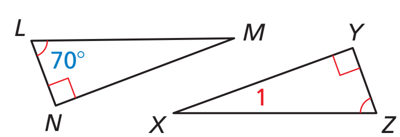


12.

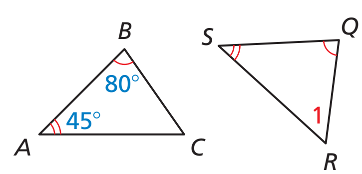


In Exercises 13 and 14, find $m\angle 1$. *(See Example 4.)*

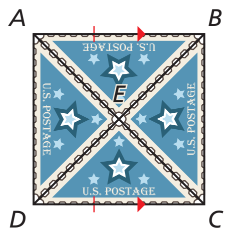
13.



14.



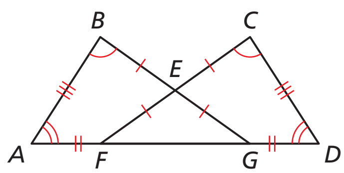
15. PROOF Triangular postage stamps, like the ones shown, are highly valued by stamp collectors. Prove that $\triangle AEB \cong \triangle CED$. *(See Example 5.)*



Given $\overline{AB} \parallel \overline{DC}$, $\overline{AB} \cong \overline{DC}$, E is the midpoint of $\overline{AC}$ and $\overline{BD}$.

Prove $\triangle AEB \cong \triangle CED$

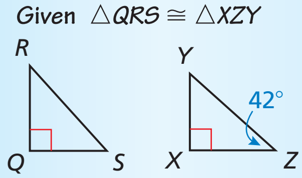
16. PROOF Use the information in the figure to prove that $\triangle ABG \cong \triangle DCF$.



ERROR ANALYSIS In Exercises 17 and 18, describe and correct the error.

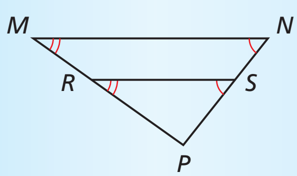
17.

Given $\triangle QRS \cong \triangle XZY$



$\angle S \cong \angle Z$
$m\angle S = m\angle Z$
$m\angle S = 42°$

18.



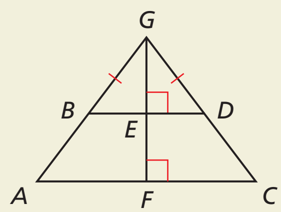
$\triangle MNP \cong \triangle RSP$ because the corresponding angles are congruent.

19. PROVING A THEOREM Prove the Third Angles Theorem (Theorem 5.4) by using the Triangle Sum Theorem (Theorem 5.1).

20. THOUGHT PROVOKING Draw a triangle. Copy the triangle multiple times to create a rug design made of congruent triangles. Which property guarantees that all the triangles are congruent?

21. REASONING $\triangle JKL$ is congruent to $\triangle XYZ$. Identify all pairs of congruent corresponding parts.

22. HOW DO YOU SEE IT? In the diagram, $ABEF \cong CDEF$.



a. Explain how you know that $\overline{BE} \cong \overline{DE}$ and $\angle ABE \cong \angle CDE$.

b. Explain how you know that $\angle GBE \cong \angle GDE$.

c. Explain how you know that $\angle GEB \cong \angle GED$.

d. Do you have enough information to prove that $\triangle BEG \cong \triangle DEG$? Explain.

MATHEMATICAL CONNECTIONS In Exercises 23 and 24, use the given information to write and solve a system of linear equations to find the values of x and y.

23. $\triangle LMN \cong \triangle PQR$, $m\angle L = 40°$, $m\angle M = 90°$, $m\angle P = (17x - y)°$, $m\angle R = (2x + 4y)°$

24. $\triangle STU \cong \triangle XYZ$, $m\angle T = 28°$, $m\angle U = (4x + y)°$, $m\angle X = 130°$, $m\angle Y = (8x - 6y)°$

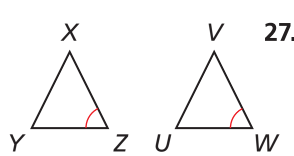
25. PROOF Prove that the criteria for congruent triangles in this lesson is equivalent to the definition of congruence in terms of rigid motions.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

What can you conclude from the diagram? *(Section 1.6)*

26.



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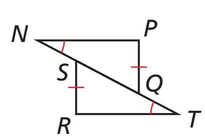
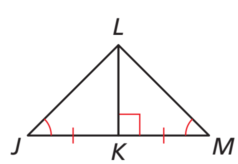


28.



29.