3.3 Proofs with Parallel Lines

Texas Essential Knowledge and Skills
G.5.B
G.5.C
G.6.A

MAKING MATHEMATICAL ARGUMENTS
To be proficient in math, you need to make conjectures and build a logical progression of statements to explore the truth of your conjectures.

Essential Question

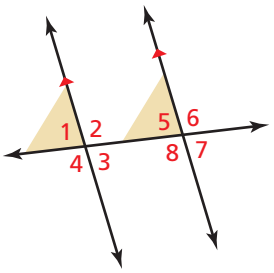
For which of the theorems involving parallel lines and transversals is the converse true?

EXPLORATION 1 Exploring Converses

Work with a partner. Write the converse of each conditional statement. Draw a diagram to represent the converse. Determine whether the converse is true. Justify your conclusion.

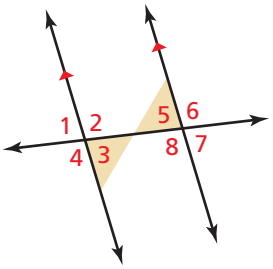
a. Corresponding Angles Theorem (Theorem 3.1)
If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.

Converse



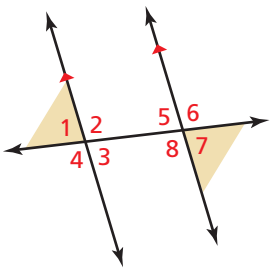
b. Alternate Interior Angles Theorem (Theorem 3.2)
If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

Converse



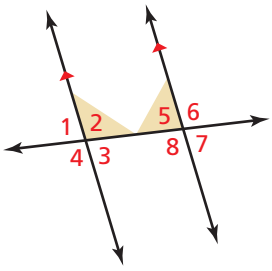
c. Alternate Exterior Angles Theorem (Theorem 3.3)
If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.

Converse



d. Consecutive Interior Angles Theorem (Theorem 3.4)
If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.

Converse



Communicate Your Answer

2. For which of the theorems involving parallel lines and transversals is the converse true?
3. In Exploration 1, explain how you would prove any of the theorems that you found to be true.

3.3 Lesson

Core Vocabulary

Previous

converse
parallel lines
transversal
corresponding angles
congruent
alternate interior angles
alternate exterior angles
consecutive interior angles

What You Will Learn

- Use the Corresponding Angles Converse.
- Construct parallel lines.
- Prove theorems about parallel lines.
- Use the Transitive Property of Parallel Lines.

Using the Corresponding Angles Converse

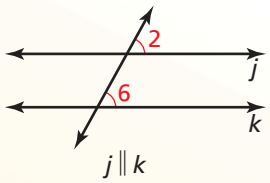
Theorem 3.5 below is the converse of the Corresponding Angles Theorem (Theorem 3.1). Similarly, the other theorems about angles formed when parallel lines are cut by a transversal have true converses. Remember that the converse of a true conditional statement is not necessarily true, so you must prove each converse of a theorem.

Theorem

Theorem 3.5 Corresponding Angles Converse

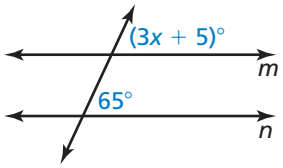
If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are parallel.

Proof Ex. 36, p. 184



EXAMPLE 1 Using the Corresponding Angles Converse

Find the value of x that makes $m \parallel n$.



SOLUTION

Lines m and n are parallel when the marked corresponding angles are congruent.

$(3x + 5)° = 65°$ Use the Corresponding Angles Converse to write an equation.

$3x = 60$ Subtract 5 from each side.

$x = 20$ Divide each side by 3.

- So, lines m and n are parallel when $x = 20$.

Monitoring Progress

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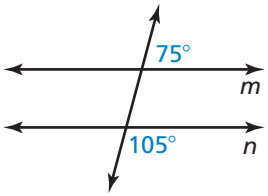
1. Is there enough information in the diagram to conclude that $m \parallel n$? Explain.



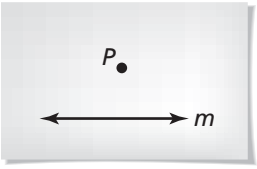
2. Explain why the Corresponding Angles Converse is the converse of the Corresponding Angles Theorem (Theorem 3.1).

Constructing Parallel Lines

The Corresponding Angles Converse justifies the construction of parallel lines, as shown below.

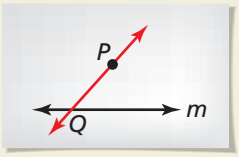
CONSTRUCTION **Constructing Parallel Lines**

Use a compass and straightedge to construct a line through point P that is parallel to line m.



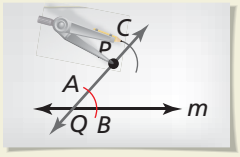
SOLUTION

Step 1



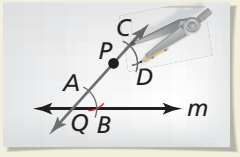
Draw a point and line Start by drawing point P and line m. Choose a point Q anywhere on line m and draw $\overleftrightarrow{QP}$.

Step 2



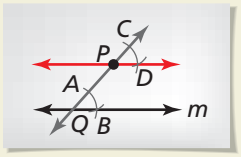
Draw arcs Draw an arc with center Q that crosses $\overleftrightarrow{QP}$ and line m. Label points A and B. Using the same compass setting, draw an arc with center P. Label point C.

Step 3



Copy angle Draw an arc with radius AB and center A. Using the same compass setting, draw an arc with center C. Label the intersection D.

Step 4

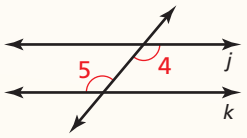


Draw parallel lines Draw $\overleftrightarrow{PD}$. This line is parallel to line m.

Theorems

Theorem 3.6 Alternate Interior Angles Converse

If two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

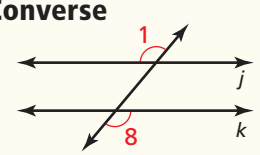


$j \parallel k$

Proof Example 2, p. 140

Theorem 3.7 Alternate Exterior Angles Converse

If two lines are cut by a transversal so the alternate exterior angles are congruent, then the lines are parallel.

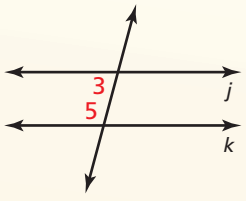


$j \parallel k$

Proof Ex. 11, p. 142

Theorem 3.8 Consecutive Interior Angles Converse

If two lines are cut by a transversal so the consecutive interior angles are supplementary, then the lines are parallel.



If $\angle 3$ and $\angle 5$ are supplementary, then $j \parallel k$.

Proof Ex. 12, p. 142

Proving Theorems about Parallel Lines

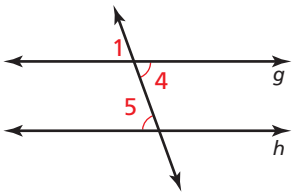
EXAMPLE 2 Proving the Alternate Interior Angles Converse

Prove that if two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

SOLUTION

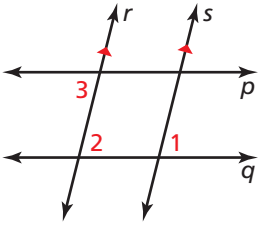
Given $\angle 4 \cong \angle 5$

Prove $g \parallel h$



STATEMENTS	REASONS
1. $\angle 4 \cong \angle 5$	**1.** Given
2. $\angle 1 \cong \angle 4$	**2.** Vertical Angles Congruence Theorem (Theorem 2.6)
3. $\angle 1 \cong \angle 5$	**3.** Transitive Property of Congruence (Theorem 2.2)
4. $g \parallel h$	**4.** Corresponding Angles Converse

EXAMPLE 3 Determining Whether Lines Are Parallel



In the diagram, $r \parallel s$ and $\angle 1$ is congruent to $\angle 3$. Prove $p \parallel q$.

SOLUTION

Look at the diagram to make a plan. The diagram suggests that you look at angles 1, 2, and 3. Also, you may find it helpful to focus on one pair of lines and one transversal at a time.

Plan for Proof

a. Look at $\angle 1$ and $\angle 2$. $\angle 1 \cong \angle 2$ because $r \parallel s$.

b. Look at $\angle 2$ and $\angle 3$. If $\angle 2 \cong \angle 3$, then $p \parallel q$.

Plan for Action

a. It is given that $r \parallel s$, so by the Corresponding Angles Theorem (Theorem 3.1), $\angle 1 \cong \angle 2$.

b. It is also given that $\angle 1 \cong \angle 3$. Then $\angle 2 \cong \angle 3$ by the Transitive Property of Congruence (Theorem 2.2).

▶ So, by the Alternate Interior Angles Converse, $p \parallel q$.

Monitoring Progress

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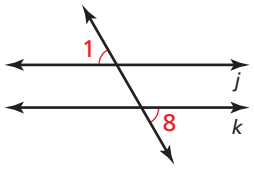
3. If you use the diagram below to prove the Alternate Exterior Angles Converse, what **Given** and **Prove** statements would you use?



4. Copy and complete the following paragraph proof of the Alternate Interior Angles Converse using the diagram in Example 2.

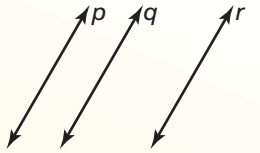
It is given that $\angle 4 \cong \angle 5$. By the _____, $\angle 1 \cong \angle 4$. Then by the Transitive Property of Congruence (Theorem 2.2), _____. So, by the _____, $g \parallel h$.

Using the Transitive Property of Parallel Lines

Theorem

Theorem 3.9 Transitive Property of Parallel Lines

If two lines are parallel to the same line, then they are parallel to each other.

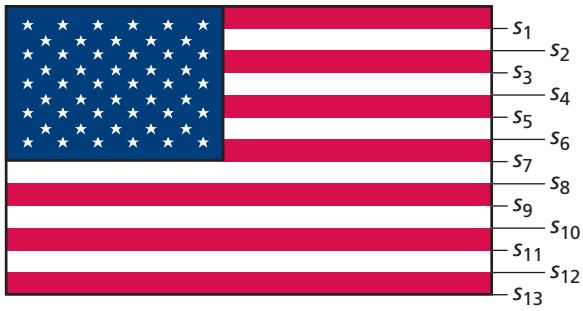


If $p \parallel q$ and $q \parallel r$, then $p \parallel r$.

Proof Ex. 39, p. 144; Ex. 33, p. 160

EXAMPLE 4 Using the Transitive Property of Parallel Lines

The flag of the United States has 13 alternating red and white stripes. Each stripe is parallel to the stripe immediately below it. Explain why the top stripe is parallel to the bottom stripe.



SOLUTION

You can name the stripes from top to bottom as $s_1, s_2, s_3, \ldots, s_{13}$. Each stripe is parallel to the one immediately below it, so $s_1 \parallel s_2$, $s_2 \parallel s_3$, and so on. Then $s_1 \parallel s_3$ by the Transitive Property of Parallel Lines. Similarly, because $s_3 \parallel s_4$, it follows that $s_1 \parallel s_4$. By continuing this reasoning, $s_1 \parallel s_{13}$.

▶ So, the top stripe is parallel to the bottom stripe.

Monitoring Progress

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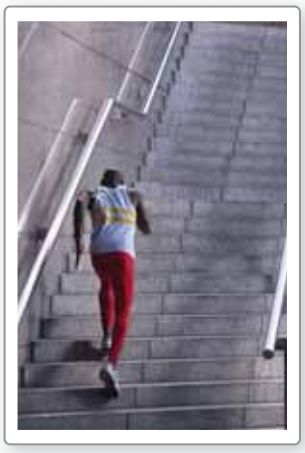
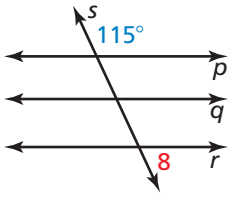
5. Each step is parallel to the step immediately above it. The bottom step is parallel to the ground. Explain why the top step is parallel to the ground.



6. In the diagram below, $p \parallel q$ and $q \parallel r$. Find $m\angle 8$. Explain your reasoning.



3.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

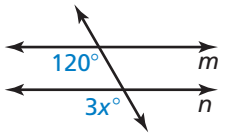
Vocabulary and Core Concept Check

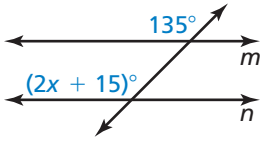
1. **VOCABULARY** Two lines are cut by a transversal. Which angle pairs must be congruent for the lines to be parallel?

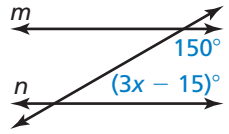
2. **WRITING** Use the theorems from Section 3.2 and the converses of those theorems in this section to write three biconditional statements about parallel lines and transversals.

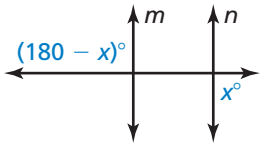
Monitoring Progress and Modeling with Mathematics

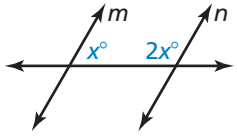
In Exercises 3–8, find the value of x that makes $m \parallel n$. Explain your reasoning. *(See Example 1.)*

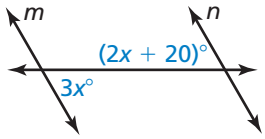
3. 

4. 

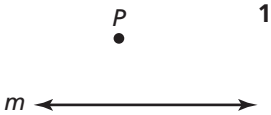
5. 

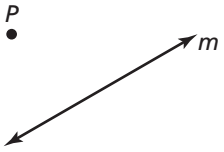
6. 

7. 

8. 

In Exercises 9 and 10, use a compass and straightedge to construct a line through point P that is parallel to line m.

9. 

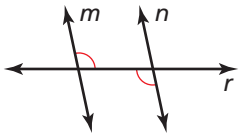
10. 

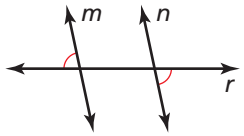
PROVING A THEOREM **In Exercises 11 and 12, prove the theorem.** *(See Example 2.)*

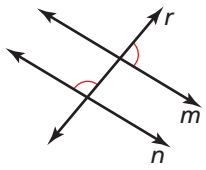
11. Alternate Exterior Angles Converse (Theorem 3.7)

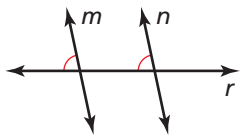
12. Consecutive Interior Angles Converse (Theorem 3.8)

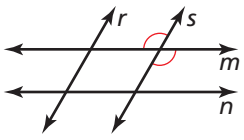
In Exercises 13–18, decide whether there is enough information to prove that $m \parallel n$. If so, state the theorem you would use. *(See Example 3.)*

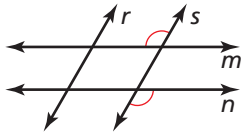
13. 

14. 

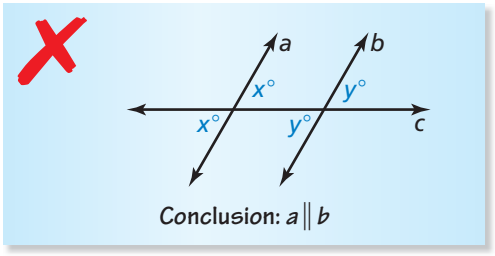
15. 

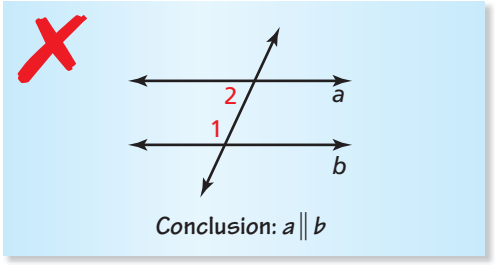
16. 

17. 

18. 

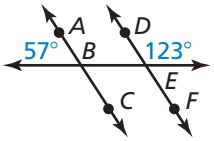
ERROR ANALYSIS **In Exercises 19 and 20, describe and correct the error in the reasoning.**

19. 

20. 

In Exercises 21–24, are $\overleftrightarrow{AC}$ and $\overleftrightarrow{DF}$ parallel? Explain your reasoning.

21.



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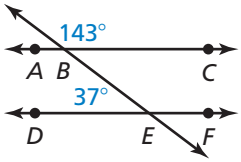

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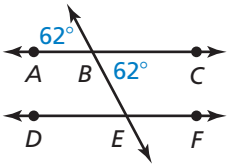

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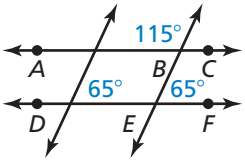
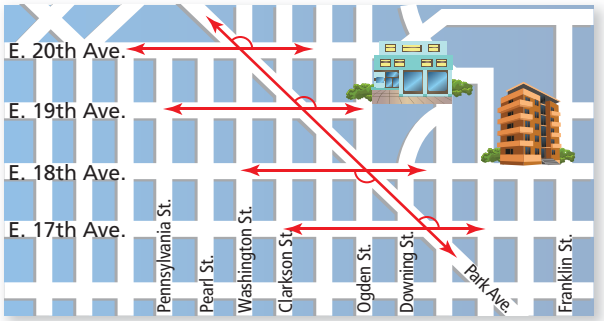
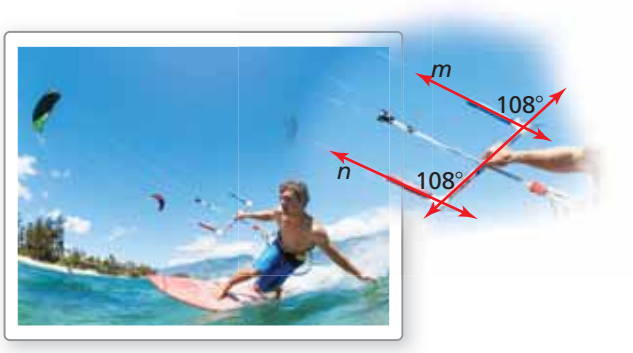


25. ANALYZING RELATIONSHIPS The map shows part of Denver, Colorado. Use the markings on the map. Are the numbered streets parallel to one another? Explain your reasoning. *(See Example 4.)*

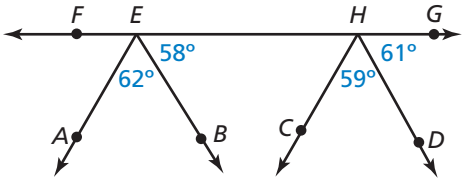


26. ANALYZING RELATIONSHIPS Each rung of the ladder is parallel to the rung directly above it. Explain why the top rung is parallel to the bottom rung.

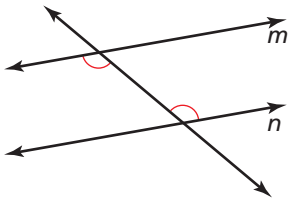
27. MODELING WITH MATHEMATICS The diagram of the control bar of the kite shows the angles formed between the control bar and the kite lines. How do you know that n is parallel to m?



28. REASONING Use the diagram. Which rays are parallel? Which rays are not parallel? Explain your reasoning.



29. ATTENDING TO PRECISION Use the diagram. Which theorems allow you to conclude that $m \parallel n$? Select all that apply. Explain your reasoning.



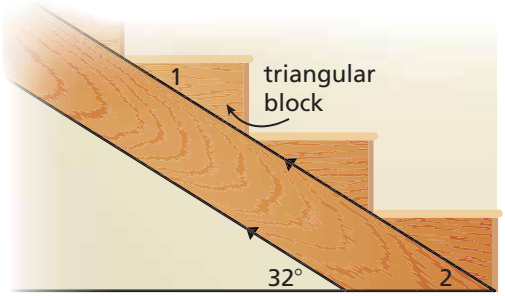
Ⓐ Corresponding Angles Converse (Thm. 3.5)

Ⓑ Alternate Interior Angles Converse (Thm. 3.6)

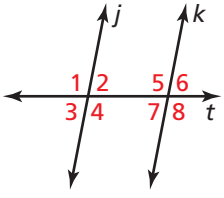
Ⓒ Alternate Exterior Angles Converse (Thm. 3.7)

Ⓓ Consecutive Interior Angles Converse (Thm. 3.8)

30. MODELING WITH MATHEMATICS One way to build stairs is to attach triangular blocks to an angled support, as shown. The sides of the angled support are parallel. If the support makes a 32° angle with the floor, what must $m\angle 1$ be so the top of the step will be parallel to the floor? Explain your reasoning.



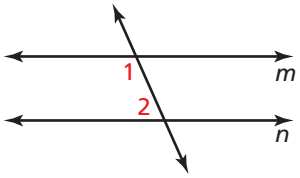
31. ABSTRACT REASONING In the diagram, how many angles must be given to determine whether $j \parallel k$? Give four examples that would allow you to conclude that $j \parallel k$ using the theorems from this lesson.



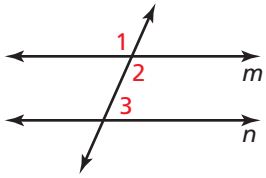
32. THOUGHT PROVOKING Draw a diagram of at least two lines cut by at least one transversal. Mark your diagram so that it cannot be proven that any lines are parallel. Then explain how your diagram would need to change in order to prove that lines are parallel.

PROOF In Exercises 33–36, write a proof.

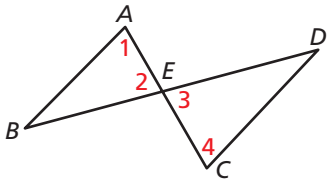
33. Given $m\angle 1 = 115°, m\angle 2 = 65°$
Prove $m \parallel n$



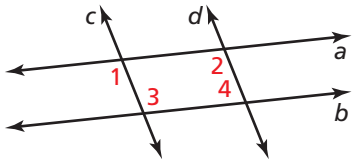
34. Given $\angle 1$ and $\angle 3$ are supplementary.
Prove $m \parallel n$



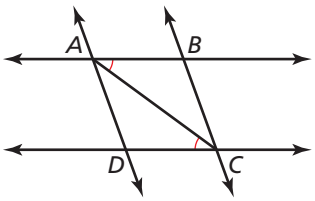
35. Given $\angle 1 \cong \angle 2, \angle 3 \cong \angle 4$
Prove $\overline{AB} \parallel \overline{CD}$



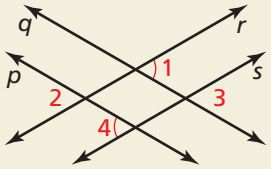
36. Given $a \parallel b, \angle 2 \cong \angle 3$
Prove $c \parallel d$



37. MAKING AN ARGUMENT Your classmate decided that $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$ based on the diagram. Is your classmate correct? Explain your reasoning.



38. HOW DO YOU SEE IT? Are the markings on the diagram enough to conclude that any lines are parallel? If so, which ones? If not, what other information is needed?



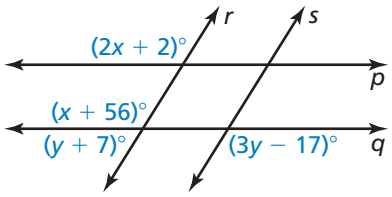
39. PROVING A THEOREM Use these steps to prove the Transitive Property of Parallel Lines Theorem (Theorem 3.9).

a. Copy the diagram with the Transitive Property of Parallel Lines Theorem on page 141.

b. Write the **Given** and **Prove** statements.

c. Use the properties of angles formed by parallel lines cut by a transversal to prove the theorem.

40. MATHEMATICAL CONNECTIONS Use the diagram.



a. Find the value of x that makes $p \parallel q$.

b. Find the value of y that makes $r \parallel s$.

c. Can r be parallel to s and can p be parallel to q at the same time? Explain your reasoning.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Use the Distance Formula to find the distance between the two points. *(Section 1.2)*

41. (1, 3) and (−2, 9)

42. (−3, 7) and (8, −6)

43. (5, −4) and (0, 8)

44. (13, 1) and (9, −4)