3.2 Parallel Lines and Transversals

Essential Question When two parallel lines are cut by a transversal, which of the resulting pairs of angles are congruent?

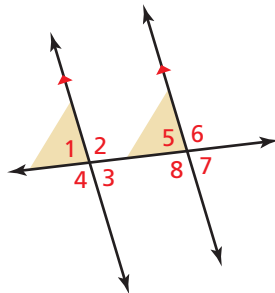### EXPLORATION 1 Exploring Parallel Lines

Work with a partner.
Use dynamic geometry software to draw two parallel lines. Draw a third line that intersects both parallel lines. Find the measures of the eight angles that are formed. What can you conclude?

USING PRECISE MATHEMATICAL LANGUAGE

To be proficient in math, you need to communicate precisely with others.
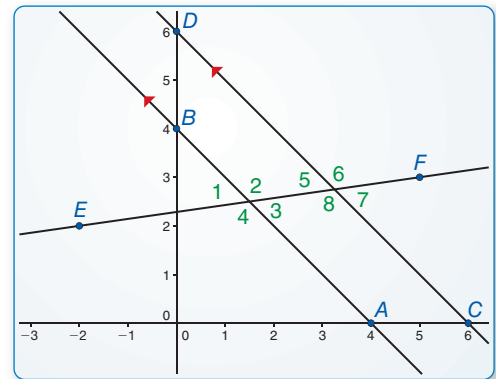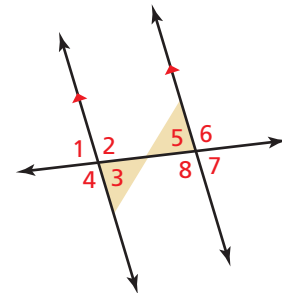
EXPLORATION 2 Writing Conjectures

Work with a partner. Use the results of Exploration 1 to write conjectures about the following pairs of angles formed by two parallel lines and a transversal.
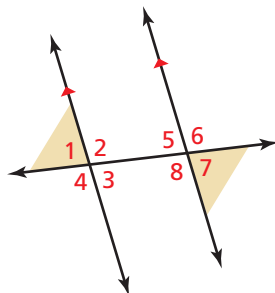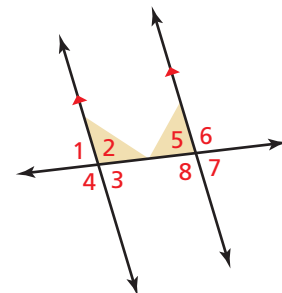
a. corresponding angles

b. alternate interior angles

c. alternate exterior angles

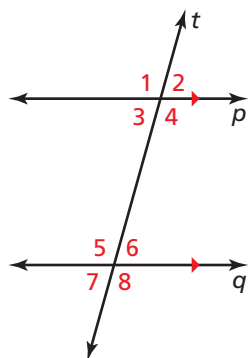**d.** consecutive interior angles

Communicate Your Answer

3. When two parallel lines are cut by a transversal, which of the resulting pairs of angles are congruent?

4. In Exploration 2, $m\angle 1 = 80°$. Find the other angle measures.

Core Vocabulary

Previous
corresponding angles
parallel lines
supplementary angles
vertical angles

What You Will Learn

▶ Use properties of parallel lines.
▶ Prove theorems about parallel lines.
▶ Solve real-life problems.

Using Properties of Parallel Lines

🔄 Theorems

Theorem 3.1 Corresponding Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.

Examples In the diagram at the left, $\angle 2 \cong \angle 6$ and $\angle 3 \cong \angle 7$.

Proof Ex. 36, p. 184

Theorem 3.2 Alternate Interior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

Examples In the diagram at the left, $\angle 3 \cong \angle 6$ and $\angle 4 \cong \angle 5$.

Proof Example 4, p. 134

Theorem 3.3 Alternate Exterior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.

Examples In the diagram at the left, $\angle 1 \cong \angle 8$ and $\angle 2 \cong \angle 7$.

Proof Ex. 15, p. 136

Theorem 3.4 Consecutive Interior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.

Examples In the diagram at the left, $\angle 3$ and $\angle 5$ are supplementary, and $\angle 4$ and $\angle 6$ are supplementary.

Proof Ex. 16, p. 136

ANOTHER WAY

There are many ways to solve Example 1. Another way is to use the Corresponding Angles Theorem to find $m\angle 5$ and then use the Vertical Angles Congruence Theorem (Theorem 2.6) to find $m\angle 4$ and $m\angle 8$.

EXAMPLE 1 Identifying Angles

The measures of three of the numbered angles are 120°. Identify the angles. Explain your reasoning.

SOLUTION

By the Alternate Exterior Angles Theorem, $m\angle 8 = 120°$.

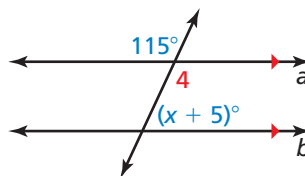$\angle 5$ and $\angle 8$ are vertical angles. Using the Vertical Angles Congruence Theorem (Theorem 2.6), $m\angle 5 = 120°$.

$\angle 5$ and $\angle 4$ are alternate interior angles. By the Alternate Interior Angles Theorem, $\angle 4 = 120°$.

▶ So, the three angles that each have a measure of 120° are $\angle 4$, $\angle 5$, and $\angle 8$.
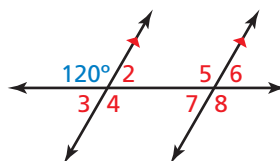
EXAMPLE 2 **Using Properties of Parallel Lines**

Find the value of x.

115°
4
$(x + 5)°$
a
b

SOLUTION

By the Vertical Angles Congruence Theorem (Theorem 2.6), $m\angle 4 = 115°$. Lines a and b are parallel, so you can use the theorems about parallel lines.

$m\angle 4 + (x + 5)° = 180°$	Consecutive Interior Angles Theorem
$115° + (x + 5)° = 180°$	Substitute 115° for $m\angle 4$.
$x + 120 = 180$	Combine like terms.
$x = 60$	Subtract 120 from each side.

▶ So, the value of x is 60.

Check

$115° + (x + 5)° = 180°$

$115 + (60 + 5) \overset{?}{=} 180$

$180 = 180$ ✓

EXAMPLE 3 **Using Properties of Parallel Lines**

Find the value of x.

1 136°
c
$(7x + 9)°$ d
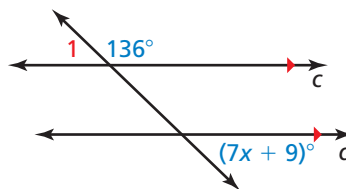
SOLUTION

By the Linear Pair Postulate (Postulate 2.8), $m\angle 1 = 180° - 136° = 44°$. Lines c and d are parallel, so you can use the theorems about parallel lines.

$m\angle 1 = (7x + 9)°$	Alternate Exterior Angles Theorem
$44° = (7x + 9)°$	Substitute 44° for $m\angle 1$.
$35 = 7x$	Subtract 9 from each side.
$5 = x$	Divide each side by 7.

▶ So, the value of x is 5.

Check

$44° = (7x + 9)°$

$44 \overset{?}{=} 7(5) + 9$

$44 = 44$ ✓

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Use the diagram.

1. Given $m\angle 1 = 105°$, find $m\angle 4$, $m\angle 5$, and $m\angle 8$. Tell which theorem you use in each case.

2. Given $m\angle 3 = 68°$ and $m\angle 8 = (2x + 4)°$, what is the value of x? Show your steps.

1 2   5 6
3 4   7 8
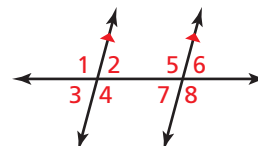
Proving Theorems about Parallel Lines

EXAMPLE 4 **Proving the Alternate Interior Angles Theorem**

Prove that if two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

SOLUTION

Draw a diagram. Label a pair of alternate interior angles as ∠1 and ∠2. You are looking for an angle that is related to both ∠1 and ∠2. Notice that one angle is a vertical angle with ∠2 and a corresponding angle with ∠1. Label it ∠3.

STUDY TIP

Before you write a proof, identify the **Given** and **Prove** statements for the situation described or for any diagram you draw.
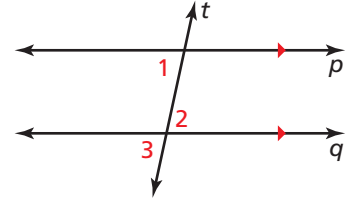
Given $p \parallel q$

Prove $\angle 1 \cong \angle 2$

| STATEMENTS	REASONS
1. $p \parallel q$	1. Given
2. $\angle 1 \cong \angle 3$	2. Corresponding Angles Theorem
3. $\angle 3 \cong \angle 2$	3. Vertical Angles Congruence Theorem (Theorem 2.6)
4. $\angle 1 \cong \angle 2$	4. Transitive Property of Congruence (Theorem 2.2)

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3. In the proof in Example 4, if you use the third statement before the second statement, could you still prove the theorem? Explain.

Solving Real-Life Problems

EXAMPLE 5 **Solving a Real-life Problem**

When sunlight enters a drop of rain, different colors of light leave the drop at different angles. This process is what makes a rainbow. For violet light, $m\angle 2 = 40°$. What is $m\angle 1$? How do you know?
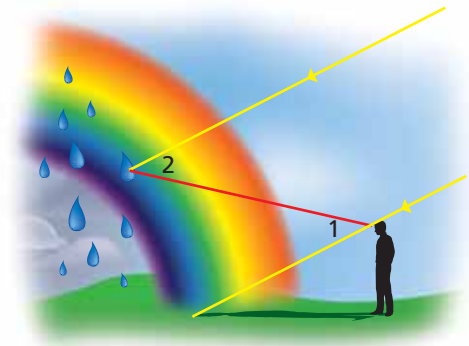
SOLUTION

Because the Sun's rays are parallel, ∠1 and ∠2 are alternate interior angles. By the Alternate Interior Angles Theorem, $\angle 1 \cong \angle 2$.

▶ So, by the definition of congruent angles, $m\angle 1 = m\angle 2 = 40°$.

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4. **WHAT IF?** In Example 5, yellow light leaves a drop at an angle of $m\angle 2 = 41°$. What is $m\angle 1$? How do you know?
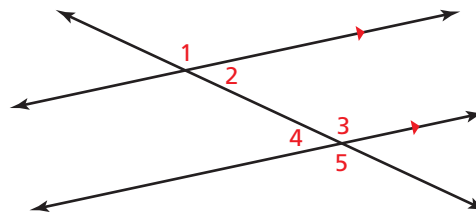
Vocabulary and Core Concept Check

1. **WRITING** How are the Alternate Interior Angles Theorem (Theorem 3.2) and the Alternate Exterior Angles Theorem (Theorem 3.3) alike? How are they different?

2. **WHICH ONE DOESN'T BELONG?** Which pair of angle measures does *not* belong with the other three? Explain.

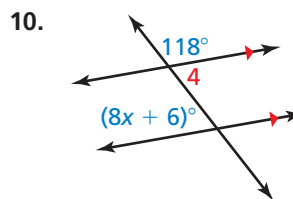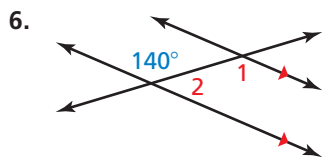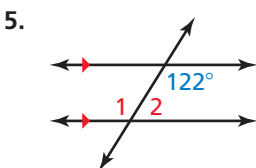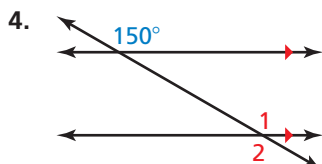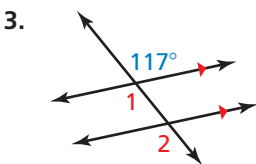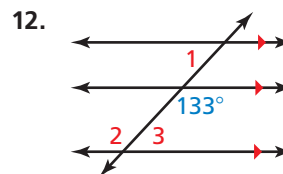| $m\angle 1$ and $m\angle 3$	$m\angle 2$ and $m\angle 4$

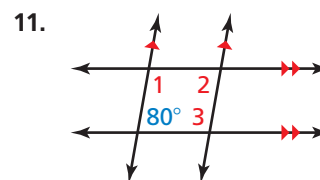| $m\angle 2$ and $m\angle 3$	$m\angle 1$ and $m\angle 5$

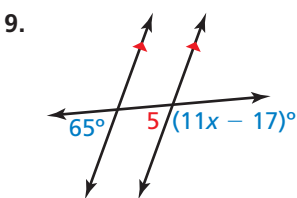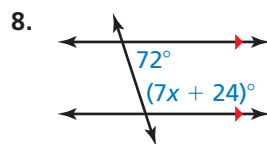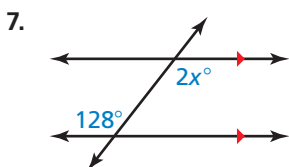## Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find $m\angle 1$ and $m\angle 2$. Tell which theorem you use in each case. *(See Example 1.)*

3.

4.

5.

6.

In Exercises 7–10, find the value of x. Show your steps. *(See Examples 2 and 3.)*

7.

8.

9.

10.

In Exercises 11 and 12, find $m\angle 1$, $m\angle 2$, and $m\angle 3$. Explain your reasoning.

11.

12.

13. **ERROR ANALYSIS** Describe and correct the error in the student's reasoning.

❌

$\angle 9 \cong \angle 10$ by the Corresponding Angles Theorem (Theorem 3.1).
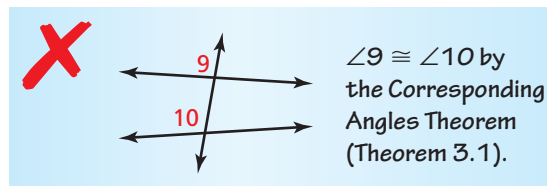
14. HOW DO YOU SEE IT?
Use the diagram.
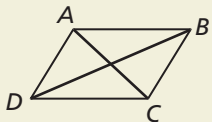
a. Name two pairs of congruent angles when $\overline{AD}$ and $\overline{BC}$ are parallel. Explain your reasoning.

b. Name two pairs of supplementary angles when $\overline{AB}$ and $\overline{DC}$ are parallel. Explain your reasoning.

PROVING A THEOREM In Exercises 15 and 16, prove the theorem. *(See Example 4.)*

15. Alternate Exterior Angles Theorem (Thm. 3.3)

16. Consecutive Interior Angles Theorem (Thm. 3.4)

17. PROBLEM SOLVING
A group of campers tie up their food between two parallel trees, as shown. The rope is pulled taut, forming a straight line. Find $m\angle 2$. Explain your reasoning. *(See Example 5.)*

18. DRAWING CONCLUSIONS You are designing a box like the one shown.

a. The measure of $\angle 1$ is 70°. Find $m\angle 2$ and $m\angle 3$.

b. Explain why $\angle ABC$ is a straight angle.

c. If $m\angle 1$ is 60°, will $\angle ABC$ still be a straight angle? Will the opening of the box be *more steep* or *less steep*? Explain.

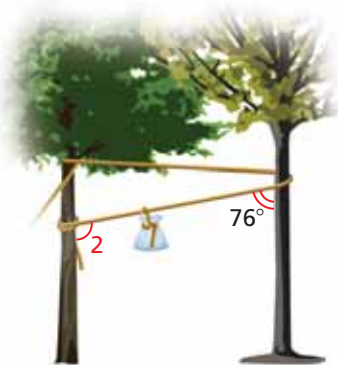**19. CRITICAL THINKING** Is it possible for consecutive interior angles to be congruent? Explain.

20. THOUGHT PROVOKING The postulates and theorems in this book represent Euclidean geometry. In spherical geometry, all points are points on the surface of a sphere. A line is a circle on the sphere whose diameter is equal to the diameter of the sphere. In spherical geometry, is it possible that a transversal intersects two parallel lines? Explain your reasoning.

MATHEMATICAL CONNECTIONS In Exercises 21 and 22, write and solve a system of linear equations to find the values of *x* and *y*.

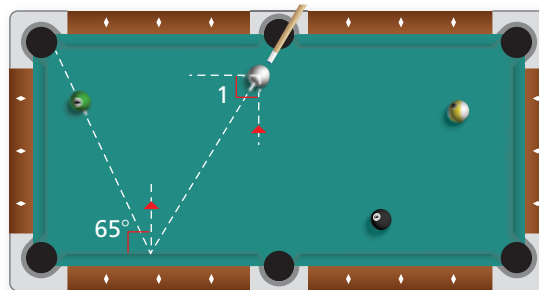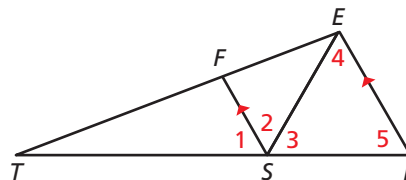**21.**
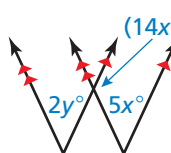
22.
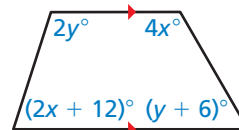
23. MAKING AN ARGUMENT During a game of pool, your friend claims to be able to make the shot shown in the diagram by hitting the cue ball so that $m\angle 1 = 25°$. Is your friend correct? Explain your reasoning.

24. REASONING In the diagram, $\angle 4 \cong \angle 5$ and $\overline{SE}$ bisects $\angle RSF$. Find $m\angle 1$. Explain your reasoning.

Maintaining Mathematical Proficiency
Reviewing what you learned in previous grades and lessons

Write the converse of the conditional statement. Decide whether it is true or false. *(Section 2.1)*

25. If two angles are vertical angles, then they are congruent.

26. If you go to the zoo, then you will see a tiger.

27. If two angles form a linear pair, then they are supplementary.

28. If it is warm outside, then we will go to the park.