

3.1 Pairs of Lines and Angles



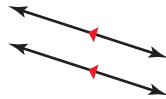
TEXAS ESSENTIAL
KNOWLEDGE AND SKILLS
Preparing for G.5.A

Essential Question What does it mean when two lines are parallel, intersecting, coincident, or skew?

EXPLORATION 1 Points of Intersection

Work with a partner. Write the number of points of intersection of each pair of coplanar lines.

a. parallel lines



b. intersecting lines

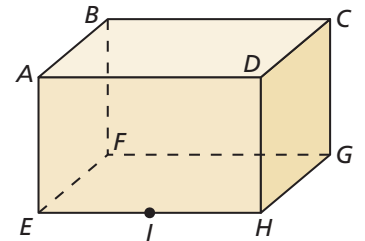


c. coincident lines



EXPLORATION 2 Classifying Pairs of Lines

Work with a partner. The figure shows a right rectangular prism. All its angles are right angles. Classify each of the following pairs of lines as *parallel*, *intersecting*, *coincident*, or *skew*. Justify your answers. (Two lines are **skew lines** when they do not intersect and are not coplanar.)



Pair of Lines	Classification	Reason
a. \overleftrightarrow{AB} and \overleftrightarrow{BC}		
b. \overleftrightarrow{AD} and \overleftrightarrow{BC}		
c. \overleftrightarrow{EI} and \overleftrightarrow{IH}		
d. \overleftrightarrow{BF} and \overleftrightarrow{EH}		
e. \overleftrightarrow{EF} and \overleftrightarrow{CG}		
f. \overleftrightarrow{AB} and \overleftrightarrow{GH}		

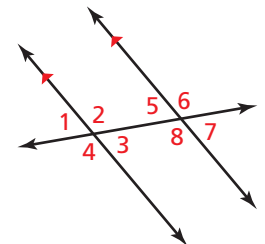
MAKING MATHEMATICAL ARGUMENTS

To be proficient in math, you need to understand and use stated assumptions, definitions, and previously established results.

EXPLORATION 3 Identifying Pairs of Angles

Work with a partner. In the figure, two parallel lines are intersected by a third line called a *transversal*.

- Identify all the pairs of vertical angles. Explain your reasoning.
- Identify all the linear pairs of angles. Explain your reasoning.



Communicate Your Answer

- What does it mean when two lines are parallel, intersecting, coincident, or skew?
- In Exploration 2, find three more pairs of lines that are different from those given. Classify the pairs of lines as *parallel*, *intersecting*, *coincident*, or *skew*. Justify your answers.

3.1 Lesson

Core Vocabulary

parallel lines, p. 126
 skew lines, p. 126
 parallel planes, p. 126
 transversal, p. 128
 corresponding angles, p. 128
 alternate interior angles, p. 128
 alternate exterior angles, p. 128
 consecutive interior angles, p. 128

Previous
 perpendicular lines

What You Will Learn

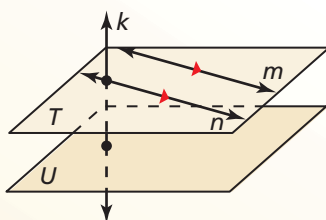
- ▶ Identify lines and planes.
- ▶ Identify parallel and perpendicular lines.
- ▶ Identify pairs of angles formed by transversals.

Identifying Lines and Planes

Core Concept

Parallel Lines, Skew Lines, and Parallel Planes

Two lines that do not intersect are either *parallel lines* or *skew lines*. Two lines are **parallel lines** when they do not intersect and are coplanar. Two lines are **skew lines** when they do not intersect and are not coplanar. Also, two planes that do not intersect are **parallel planes**.



Lines m and n are parallel lines ($m \parallel n$).

Lines m and k are skew lines.

Planes T and U are parallel planes ($T \parallel U$).

Lines k and n are intersecting lines, and there is a plane (not shown) containing them.

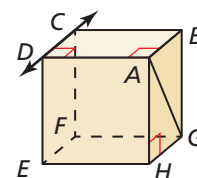
Small directed arrows, as shown in red on lines m and n above, are used to show that lines are parallel. The symbol \parallel means “is parallel to,” as in $m \parallel n$.

Segments and rays are parallel when they lie in parallel lines. A line is parallel to a plane when the line is in a plane parallel to the given plane. In the diagram above, line n is parallel to plane U .

EXAMPLE 1 Identifying Lines and Planes

Think of each segment in the figure as part of a line. Which line(s) or plane(s) appear to fit the description?

- a. line(s) parallel to \overleftrightarrow{CD} and containing point A
- b. line(s) skew to \overleftrightarrow{CD} and containing point A
- c. line(s) perpendicular to \overleftrightarrow{CD} and containing point A
- d. plane(s) parallel to plane EFG and containing point A



SOLUTION

- a. \overleftrightarrow{AB} , \overleftrightarrow{HG} , and \overleftrightarrow{EF} all appear parallel to \overleftrightarrow{CD} , but only \overleftrightarrow{AB} contains point A .
- b. Both \overleftrightarrow{AG} and \overleftrightarrow{AH} appear skew to \overleftrightarrow{CD} and contain point A .
- c. \overleftrightarrow{BC} , \overleftrightarrow{AD} , \overleftrightarrow{DE} , and \overleftrightarrow{FC} all appear perpendicular to \overleftrightarrow{CD} , but only \overleftrightarrow{AD} contains point A .
- d. Plane ABC appears parallel to plane EFG and contains point A .

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1. Look at the diagram in Example 1. Name the line(s) through point F that appear skew to \overleftrightarrow{EH} .

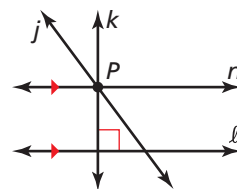
REMEMBER

Recall that if two lines intersect to form a right angle, then they are perpendicular lines.



Identifying Parallel and Perpendicular Lines

Two distinct lines in the same plane either are parallel, like line ℓ and line n , or intersect in a point, like line j and line n .



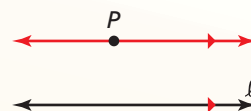
Through a point not on a line, there are infinitely many lines. Exactly one of these lines is parallel to the given line, and exactly one of them is perpendicular to the given line. For example, line k is the line through point P perpendicular to line ℓ , and line n is the line through point P parallel to line ℓ .

Postulates

Postulate 3.1 Parallel Postulate

If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line.

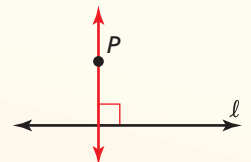
There is exactly one line through P parallel to ℓ .



Postulate 3.2 Perpendicular Postulate

If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line.

There is exactly one line through P perpendicular to ℓ .



EXAMPLE 2

Identifying Parallel and Perpendicular Lines

The given line markings show how the roads in a town are related to one another.

- Name a pair of parallel lines.
- Name a pair of perpendicular lines.
- Is $\overleftrightarrow{FE} \parallel \overleftrightarrow{AC}$? Explain.

SOLUTION

- $\overleftrightarrow{MD} \parallel \overleftrightarrow{FE}$
- $\overleftrightarrow{MD} \perp \overleftrightarrow{BF}$
- \overleftrightarrow{FE} is not parallel to \overleftrightarrow{AC} , because \overleftrightarrow{MD} is parallel to \overleftrightarrow{FE} , and by the Parallel Postulate, there is exactly one line parallel to \overleftrightarrow{FE} through M .



Monitoring Progress



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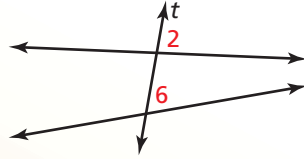
- In Example 2, can you use the Perpendicular Postulate to show that \overleftrightarrow{AC} is not perpendicular to \overleftrightarrow{BF} ? Explain why or why not.

Identifying Pairs of Angles

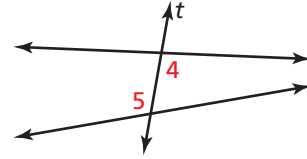
A **transversal** is a line that intersects two or more coplanar lines at different points.

Core Concept

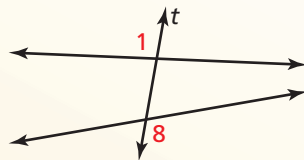
Angles Formed by Transversals



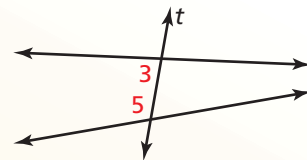
Two angles are **corresponding angles** when they have corresponding positions. For example, $\angle 2$ and $\angle 6$ are above the lines and to the right of the transversal t .



Two angles are **alternate interior angles** when they lie between the two lines and on opposite sides of the transversal t .



Two angles are **alternate exterior angles** when they lie outside the two lines and on opposite sides of the transversal t .

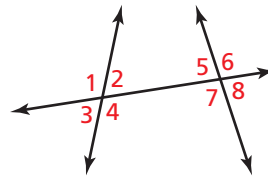


Two angles are **consecutive interior angles** when they lie between the two lines and on the same side of the transversal t .

EXAMPLE 3 Identifying Pairs of Angles

Identify all pairs of angles of the given type.

- a. corresponding
- b. alternate interior
- c. alternate exterior
- d. consecutive interior

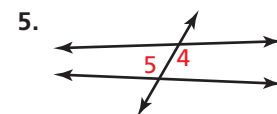
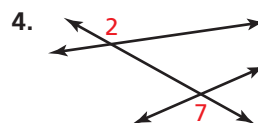
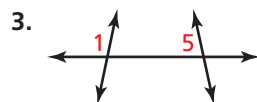


SOLUTION

- | | | | |
|---|---|---|---|
| a. $\angle 1$ and $\angle 5$
$\angle 2$ and $\angle 6$
$\angle 3$ and $\angle 7$
$\angle 4$ and $\angle 8$ | b. $\angle 2$ and $\angle 7$
$\angle 4$ and $\angle 5$ | c. $\angle 1$ and $\angle 8$
$\angle 3$ and $\angle 6$ | d. $\angle 2$ and $\angle 5$
$\angle 4$ and $\angle 7$ |
|---|---|---|---|

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Classify the pair of numbered angles.



3.1 Exercises

Vocabulary and Core Concept Check

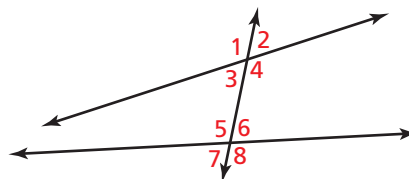
- COMPLETE THE SENTENCE** Two lines that do not intersect and are also not parallel are _____ lines.
- WHICH ONE DOESN'T BELONG?** Which angle pair does *not* belong with the other three? Explain your reasoning.

∠2 and ∠3

∠4 and ∠5

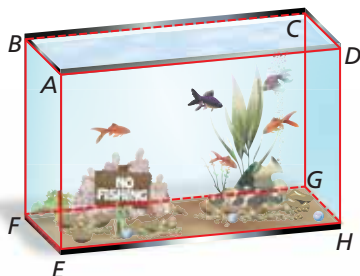
∠1 and ∠8

∠2 and ∠7



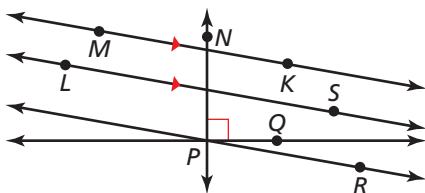
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, think of each segment in the diagram as part of a line. All the angles are right angles. Which line(s) or plane(s) contain point *B* and appear to fit the description? (See Example 1.)



- line(s) parallel to \overleftrightarrow{CD}
- line(s) perpendicular to \overleftrightarrow{CD}
- line(s) skew to \overleftrightarrow{CD}
- plane(s) parallel to plane *CDH*

In Exercises 7–10, use the diagram. (See Example 2.)

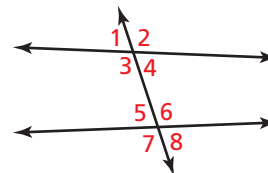


- Name a pair of parallel lines.
- Name a pair of perpendicular lines.

9. Is $\overleftrightarrow{PN} \parallel \overleftrightarrow{KM}$? Explain.

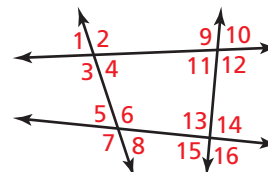
10. Is $\overleftrightarrow{PR} \perp \overleftrightarrow{NP}$? Explain.

In Exercises 11–14, identify all pairs of angles of the given type. (See Example 3.)




- corresponding
- alternate interior
- alternate exterior
- consecutive interior


USING STRUCTURE In Exercises 15–18, classify the angle pair as *corresponding*, *alternate interior*, *alternate exterior*, or *consecutive interior* angles.



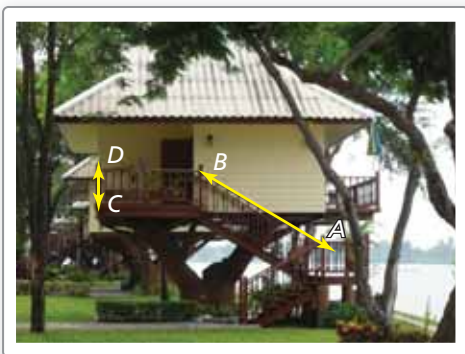
- ∠5 and ∠1
- ∠6 and ∠13
- ∠11 and ∠13
- ∠2 and ∠11

ERROR ANALYSIS In Exercises 19 and 20, describe and correct the error in the conditional statement about lines.

19.  If two lines do not intersect, then they are parallel.

20.  If there is a line and a point not on the line, then there is exactly one line through the point that intersects the given line.

21. **MODELING WITH MATHEMATICS** Use the photo to decide whether the statement is true or false. Explain your reasoning.



- The plane containing the floor of the tree house is parallel to the ground.
- The lines containing the railings of the staircase, such as \overleftrightarrow{AB} , are skew to all lines in the plane containing the ground.
- All the lines containing the balusters, such as \overleftrightarrow{CD} , are perpendicular to the plane containing the floor of the tree house.

22. **THOUGHT PROVOKING** If two lines are intersected by a third line, is the third line necessarily a transversal? Justify your answer with a diagram.

23. **MATHEMATICAL CONNECTIONS** Two lines are cut by a transversal. Is it possible for all eight angles formed to have the same measure? Explain your reasoning.

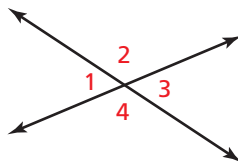
Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Use the diagram to find the measures of all the angles. (Section 2.6)

30. $m\angle 1 = 76^\circ$

31. $m\angle 2 = 159^\circ$



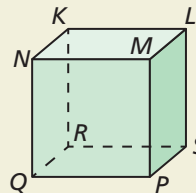
24. **HOW DO YOU SEE IT?** Think of each segment in the figure as part of a line.

a. Which lines are parallel to \overleftrightarrow{NQ} ?

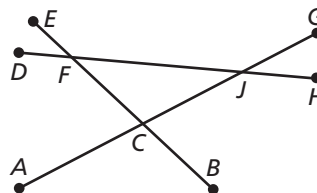
b. Which lines intersect \overleftrightarrow{NQ} ?

c. Which lines are skew to \overleftrightarrow{NQ} ?

d. Should you have named all the lines on the cube in parts (a)–(c) except \overleftrightarrow{NQ} ? Explain.



In Exercises 25–28, copy and complete the statement. List all possible correct answers.



25. $\angle BCG$ and ____ are corresponding angles.

26. $\angle BCG$ and ____ are consecutive interior angles.

27. $\angle FCJ$ and ____ are alternate interior angles.

28. $\angle FCA$ and ____ are alternate exterior angles.

29. **MAKING AN ARGUMENT** Your friend claims the uneven parallel bars in gymnastics are not really parallel. She says one is higher than the other, so they cannot be in the same plane. Is she correct? Explain.

