

2.4 Algebraic Reasoning



TEXAS ESSENTIAL KNOWLEDGE AND SKILLS

Preparing for
G.6.A
G.6.B
G.6.D
G.6.E

Essential Question How can algebraic properties help you solve an equation?

EXPLORATION 1 Justifying Steps in a Solution

Work with a partner. In previous courses, you studied different properties, such as the properties of equality and the Distributive, Commutative, and Associative Properties. Write the property that justifies each of the following solution steps.

Algebraic Step	Justification
$2(x + 3) - 5 = 5x + 4$	Write given equation.
$2x + 6 - 5 = 5x + 4$	
$2x + 1 = 5x + 4$	
$2x - 2x + 1 = 5x - 2x + 4$	
$1 = 3x + 4$	
$1 - 4 = 3x + 4 - 4$	
$-3 = 3x$	
$\frac{-3}{3} = \frac{3x}{3}$	
$-1 = x$	
$x = -1$	

ANALYZING MATHEMATICAL RELATIONSHIPS

To be proficient in math, you need to look closely to discern a pattern or structure.

EXPLORATION 2 Stating Algebraic Properties

Work with a partner. The symbols \blacklozenge and \bullet represent addition and multiplication (not necessarily in that order). Determine which symbol represents which operation. Justify your answer. Then state each algebraic property being illustrated.

Example of Property	Name of Property
$5 \blacklozenge 6 = 6 \blacklozenge 5$	
$5 \bullet 6 = 6 \bullet 5$	
$4 \blacklozenge (5 \blacklozenge 6) = (4 \blacklozenge 5) \blacklozenge 6$	
$4 \bullet (5 \bullet 6) = (4 \bullet 5) \bullet 6$	
$0 \blacklozenge 5 = 0$	
$0 \bullet 5 = 5$	
$1 \blacklozenge 5 = 5$	
$4 \blacklozenge (5 \bullet 6) = 4 \blacklozenge 5 \bullet 4 \blacklozenge 6$	

Communicate Your Answer

- How can algebraic properties help you solve an equation?
- Solve $3(x + 1) - 1 = -13$. Justify each step.

2.4 Lesson

Core Vocabulary

Previous
equation
solve an equation
formula

What You Will Learn

- ▶ Use Algebraic Properties of Equality to justify the steps in solving an equation.
- ▶ Use the Distributive Property to justify the steps in solving an equation.
- ▶ Use properties of equality involving segment lengths and angle measures.

Using Algebraic Properties of Equality

When you *solve an equation*, you use properties of real numbers. Segment lengths and angle measures are real numbers, so you can also use these properties to write logical arguments about geometric figures.

Core Concept

Algebraic Properties of Equality

Let a , b , and c be real numbers.

Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $a \cdot c = b \cdot c$, $c \neq 0$.
Division Property of Equality	If $a = b$, then $\frac{a}{c} = \frac{b}{c}$, $c \neq 0$.
Substitution Property of Equality	If $a = b$, then a can be substituted for b (or b for a) in any equation or expression.

REMEMBER

Inverse operations “undo” each other. Addition and subtraction are inverse operations. Multiplication and division are inverse operations.

EXAMPLE 1 Justifying Steps

Solve $3x + 2 = 23 - 4x$. Justify each step.

SOLUTION

Equation	Explanation	Reason
$3x + 2 = 23 - 4x$	Write the equation.	Given
$3x + 2 + 4x = 23 - 4x + 4x$	Add $4x$ to each side.	Addition Property of Equality
$7x + 2 = 23$	Combine like terms.	Simplify.
$7x + 2 - 2 = 23 - 2$	Subtract 2 from each side.	Subtraction Property of Equality
$7x = 21$	Combine constant terms.	Simplify.
$x = 3$	Divide each side by 7.	Division Property of Equality

- ▶ The solution is $x = 3$.

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Solve the equation. Justify each step.

1. $6x - 11 = -35$
2. $-2p - 9 = 10p - 17$
3. $39 - 5z = -1 + 5z$

Using the Distributive Property

Core Concept

Distributive Property

Let a , b , and c be real numbers.

Sum $a(b + c) = ab + ac$

Difference $a(b - c) = ab - ac$

EXAMPLE 2 Using the Distributive Property

Solve $-5(7w + 8) = 30$. Justify each step.

SOLUTION

Equation	Explanation	Reason
$-5(7w + 8) = 30$	Write the equation.	Given
$-35w - 40 = 30$	Multiply.	Distributive Property
$-35w = 70$	Add 40 to each side.	Addition Property of Equality
$w = -2$	Divide each side by -35 .	Division Property of Equality

► The solution is $w = -2$.

EXAMPLE 3 Solving a Real-Life Problem

You get a raise at your part-time job. To write your raise as a percent, use the formula $p(r + 1) = n$, where p is your previous wage, r is the percent increase (as a decimal), and n is your new wage. Solve the formula for r . What is your raise written as a percent when your hourly wage increases from \$7.25 to \$7.54 per hour?

SOLUTION

Step 1 Solve for r in the formula $p(r + 1) = n$.

Equation	Explanation	Reason
$p(r + 1) = n$	Write the equation.	Given
$pr + p = n$	Multiply.	Distributive Property
$pr = n - p$	Subtract p from each side.	Subtraction Property of Equality
$r = \frac{n - p}{p}$	Divide each side by p .	Division Property of Equality

Step 2 Evaluate $r = \frac{n - p}{p}$ when $n = 7.54$ and $p = 7.25$.

$$r = \frac{n - p}{p} = \frac{7.54 - 7.25}{7.25} = \frac{0.29}{7.25} = 0.04$$

► Your raise is 4%.

REMEMBER

When evaluating expressions, use the order of operations.



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Solve the equation. Justify each step.

4. $3(3x + 14) = -3$

5. $4 = -10b + 6(2 - b)$

6. Solve the formula $A = \frac{1}{2}bh$ for b . Justify each step. Then find the base of a triangle whose area is 952 square feet and whose height is 56 feet.

Using Other Properties of Equality

The following properties of equality are true for all real numbers. Segment lengths and angle measures are real numbers, so these properties of equality are true for all segment lengths and angle measures.

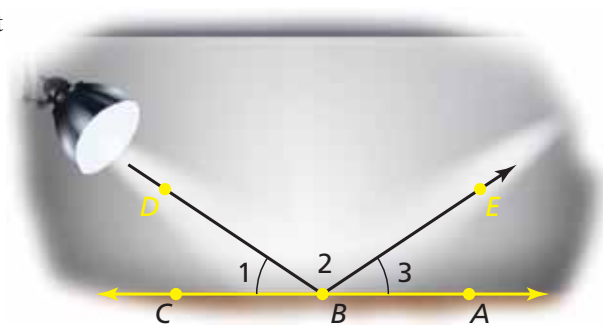
Core Concept

Reflexive, Symmetric, and Transitive Properties of Equality

	Real Numbers	Segment Lengths	Angle Measures
Reflexive Property	$a = a$	$AB = AB$	$m\angle A = m\angle A$
Symmetric Property	If $a = b$, then $b = a$.	If $AB = CD$, then $CD = AB$.	If $m\angle A = m\angle B$, then $m\angle B = m\angle A$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.	If $AB = CD$ and $CD = EF$, then $AB = EF$.	If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

EXAMPLE 4 Using Properties of Equality with Angle Measures

You reflect the beam of a spotlight off a mirror lying flat on a stage, as shown. Determine whether $m\angle DBA = m\angle EBC$.



SOLUTION

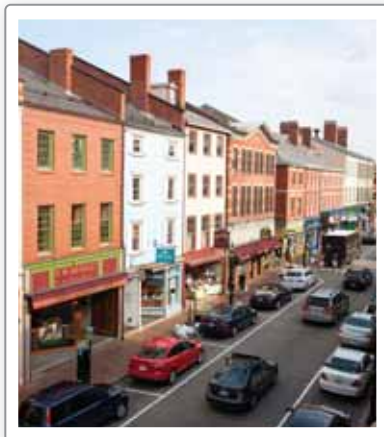
Equation	Explanation	Reason
$m\angle 1 = m\angle 3$	Marked in diagram.	Given
$m\angle DBA = m\angle 3 + m\angle 2$	Add measures of adjacent angles.	Angle Addition Postulate (Post. 1.4)
$m\angle DBA = m\angle 1 + m\angle 2$	Substitute $m\angle 1$ for $m\angle 3$.	Substitution Property of Equality
$m\angle 1 + m\angle 2 = m\angle EBC$	Add measures of adjacent angles.	Angle Addition Postulate (Post. 1.4)
$m\angle DBA = m\angle EBC$	Both measures are equal to the sum $m\angle 1 + m\angle 2$.	Transitive Property of Equality

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Name the property of equality that the statement illustrates.

- If $m\angle 6 = m\angle 7$, then $m\angle 7 = m\angle 6$.
- $34^\circ = 34^\circ$
- $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 5$. So, $m\angle 1 = m\angle 5$.

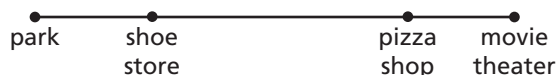
EXAMPLE 5 Modeling with Mathematics



A park, a shoe store, a pizza shop, and a movie theater are located in order on a city street. The distance between the park and the shoe store is the same as the distance between the pizza shop and the movie theater. Show that the distance between the park and the pizza shop is the same as the distance between the shoe store and the movie theater.

SOLUTION

- Understand the Problem** You know that the locations lie in order and that the distance between two of the locations (park and shoe store) is the same as the distance between the other two locations (pizza shop and movie theater). You need to show that two of the other distances are the same.
- Make a Plan** Draw and label a diagram to represent the situation.



Modify your diagram by letting the points P , S , Z , and M represent the park, the shoe store, the pizza shop, and the movie theater, respectively. Show any mathematical relationships.



Use the Segment Addition Postulate (Postulate 1.2) to show that $PZ = SM$.

3. Solve the Problem

Equation	Explanation	Reason
$PS = ZM$	Marked in diagram.	Given
$PZ = PS + SZ$	Add lengths of adjacent segments.	Segment Addition Postulate (Post. 1.2)
$SM = SZ + ZM$	Add lengths of adjacent segments.	Segment Addition Postulate (Post. 1.2)
$PS + SZ = ZM + SZ$	Add SZ to each side of $PS = ZM$.	Addition Property of Equality
$PZ = SM$	Substitute PZ for $PS + SZ$ and SM for $ZM + SZ$.	Substitution Property of Equality

- Look Back** Reread the problem. Make sure your diagram is drawn precisely using the given information. Check the steps in your solution.

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Name the property of equality that the statement illustrates.

- If $JK = KL$ and $KL = 16$, then $JK = 16$.
- $PQ = ST$, so $ST = PQ$.
- $ZY = ZY$
- In Example 5, a hot dog stand is located halfway between the shoe store and the pizza shop, at point H . Show that $PH = HM$.

Vocabulary and Core Concept Check

- VOCABULARY** The statement “The measure of an angle is equal to itself” is true because of what property?
- DIFFERENT WORDS, SAME QUESTION** Which is different? Find both answers.

What property justifies the following statement?

If $c = d$, then $d = c$.

If $JK = LM$, then $LM = JK$.

If $e = f$ and $f = g$, then $e = g$.

If $m\angle R = m\angle S$, then $m\angle S = m\angle R$.

Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, write the property that justifies each step.

3.	$3x - 12 = 7x + 8$	Given
	$-4x - 12 = 8$	_____
	$-4x = 20$	_____
	$x = -5$	_____

4.	$5(x - 1) = 4x + 13$	Given
	$5x - 5 = 4x + 13$	_____
	$x - 5 = 13$	_____
	$x = 18$	_____

In Exercises 5–14, solve the equation. Justify each step. (See Examples 1 and 2.)

- | | |
|-----------------------------|-----------------------|
| 5. $5x - 10 = -40$ | 6. $6x + 17 = -7$ |
| 7. $2x - 8 = 6x - 20$ | 8. $4x + 9 = 16 - 3x$ |
| 9. $5(3x - 20) = -10$ | |
| 10. $3(2x + 11) = 9$ | |
| 11. $2(-x - 5) = 12$ | |
| 12. $44 - 2(3x + 4) = -18x$ | |
| 13. $4(5x - 9) = -2(x + 7)$ | |
| 14. $3(4x + 7) = 5(3x + 3)$ | |

In Exercises 15–20, solve the equation for y . Justify each step. (See Example 3.)

- | | |
|-------------------------|--|
| 15. $5x + y = 18$ | 16. $-4x + 2y = 8$ |
| 17. $2y + 0.5x = 16$ | 18. $\frac{1}{2}x - \frac{3}{4}y = -2$ |
| 19. $12 - 3y = 30x + 6$ | 20. $3x + 7 = -7 + 9y$ |

In Exercises 21–24, solve the equation for the given variable. Justify each step. (See Example 3.)

- | | |
|----------------------------|------------------------------------|
| 21. $C = 2\pi r$; r | 22. $I = Prt$; P |
| 23. $S = 180(n - 2)$; n | 24. $S = 2\pi r^2 + 2\pi rh$; h |


In Exercises 25–32, name the property of equality that the statement illustrates.


- If $x = y$, then $3x = 3y$.
- If $AM = MB$, then $AM + 5 = MB + 5$.
- $x = x$
- If $x = y$, then $y = x$.
- $m\angle Z = m\angle Z$
- If $m\angle A = 29^\circ$ and $m\angle B = 29^\circ$, then $m\angle A = m\angle B$.
- If $AB = LM$, then $LM = AB$.
- If $BC = XY$ and $XY = 8$, then $BC = 8$.

In Exercises 33–40, use the property to copy and complete the statement.

33. Substitution Property of Equality:
If $AB = 20$, then $AB + CD = \underline{\hspace{2cm}}$.
34. Symmetric Property of Equality:
If $m\angle 1 = m\angle 2$, then $\underline{\hspace{2cm}}$.
35. Addition Property of Equality:
If $AB = CD$, then $AB + EF = \underline{\hspace{2cm}}$.
36. Multiplication Property of Equality:
If $AB = CD$, then $5 \cdot AB = \underline{\hspace{2cm}}$.
37. Subtraction Property of Equality:
If $LM = XY$, then $LM - GH = \underline{\hspace{2cm}}$.
38. Distributive Property:
If $5(x + 8) = 2$, then $\underline{\hspace{1cm}} + \underline{\hspace{1cm}} = 2$.
39. Transitive Property of Equality:
If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $\underline{\hspace{2cm}}$.
40. Reflexive Property of Equality:
 $m\angle ABC = \underline{\hspace{2cm}}$.

ERROR ANALYSIS In Exercises 41 and 42, describe and correct the error in solving the equation.

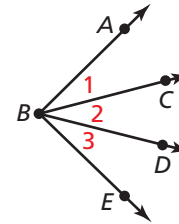
41.  $7x = x + 24$ Given
 $8x = 24$ Addition Property of Equality
 $x = 3$ Division Property of Equality

42.  $6x + 14 = 32$ Given
 $6x = 18$ Division Property of Equality
 $x = 3$ Simplify.

43. **REWRITING A FORMULA** The formula for the perimeter P of a rectangle is $P = 2\ell + 2w$, where ℓ is the length and w is the width. Solve the formula for ℓ . Justify each step. Then find the length of a rectangular lawn with a perimeter of 32 meters and a width of 5 meters.

44. **REWRITING A FORMULA** The formula for the area A of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$, where h is the height and b_1 and b_2 are the lengths of the two bases. Solve the formula for b_1 . Justify each step. Then find the length of one of the bases of the trapezoid when the area of the trapezoid is 91 square meters, the height is 7 meters, and the length of the other base is 20 meters.

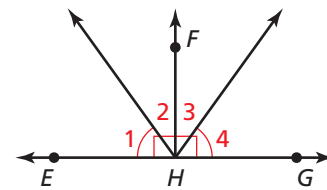
45. **ANALYZING RELATIONSHIPS** In the diagram, $m\angle ABD = m\angle CBE$. Show that $m\angle 1 = m\angle 3$. (See Example 4.)



46. **ANALYZING RELATIONSHIPS** In the diagram, $AC = BD$. Show that $AB = CD$. (See Example 5.)



47. **ANALYZING RELATIONSHIPS** Copy and complete the table to show that $m\angle 2 = m\angle 3$.

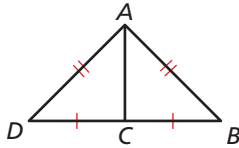


Equation	Reason
$m\angle 1 = m\angle 4$, $m\angle EHF = 90^\circ$, $m\angle GHF = 90^\circ$	Given
$m\angle EHF = m\angle GHF$	
$m\angle EHF = m\angle 1 + m\angle 2$ $m\angle GHF = m\angle 3 + m\angle 4$	
$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	
	Substitution Property of Equality
$m\angle 2 = m\angle 3$	

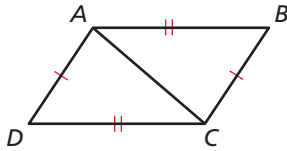
48. **WRITING** Compare the Reflexive Property of Equality with the Symmetric Property of Equality. How are the properties similar? How are they different?

REASONING In Exercises 49 and 50, show that the perimeter of $\triangle ABC$ is equal to the perimeter of $\triangle ADC$.

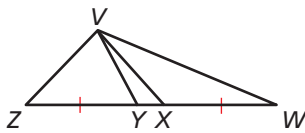
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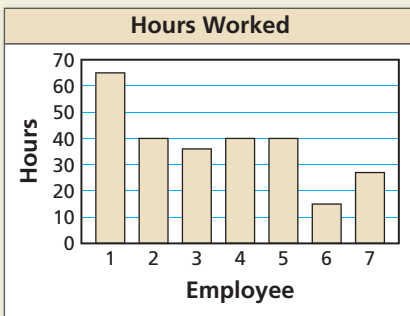
50.



51. **MATHEMATICAL CONNECTIONS** In the figure, $\overline{ZY} \cong \overline{XW}$, $ZX = 5x + 17$, $YW = 10 - 2x$, and $YX = 3$. Find ZY and XW .



52. **HOW DO YOU SEE IT?** The bar graph shows the number of hours each employee works at a grocery store. Give an example of the Reflexive, Symmetric, and Transitive Properties of Equality.



53. **ATTENDING TO PRECISION** Which of the following statements illustrate the Symmetric Property of Equality? Select all that apply.

- (A) If $AC = RS$, then $RS = AC$.
- (B) If $x = 9$, then $9 = x$.
- (C) If $AD = BC$, then $DA = CB$.
- (D) $AB = BA$
- (E) If $AB = LM$ and $LM = RT$, then $AB = RT$.
- (F) If $XY = EF$, then $FE = XY$.

54. **THOUGHT PROVOKING** Write examples from your everyday life to help you remember the Reflexive, Symmetric, and Transitive Properties of Equality. Justify your answers.

55. **MULTIPLE REPRESENTATIONS** The formula to convert a temperature in degrees Fahrenheit ($^{\circ}F$) to degrees Celsius ($^{\circ}C$) is $C = \frac{5}{9}(F - 32)$.

- a. Solve the formula for F . Justify each step.
- b. Make a table that shows the conversion to Fahrenheit for each temperature: $0^{\circ}C$, $20^{\circ}C$, $32^{\circ}C$, and $41^{\circ}C$.
- c. Use your table to graph the temperature in degrees Fahrenheit as a function of the temperature in degrees Celsius. Is this a linear function?

56. **REASONING** Select all the properties that would also apply to inequalities. Explain your reasoning.

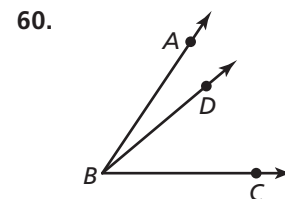
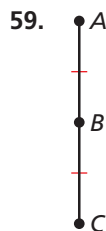
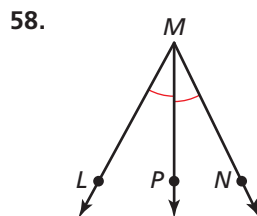
- (A) Addition Property
- (B) Subtraction Property
- (C) Substitution Property
- (D) Reflexive Property
- (E) Symmetric Property
- (F) Transitive Property

Maintaining Mathematical Proficiency Reviewing what you learned in previous grades and lessons

Name the definition, property, or postulate that is represented by each diagram.
(Section 1.2, Section 1.3, and Section 1.5)



$$XY + YZ = XZ$$



$$m\angle ABD + m\angle DBC = m\angle ABC$$