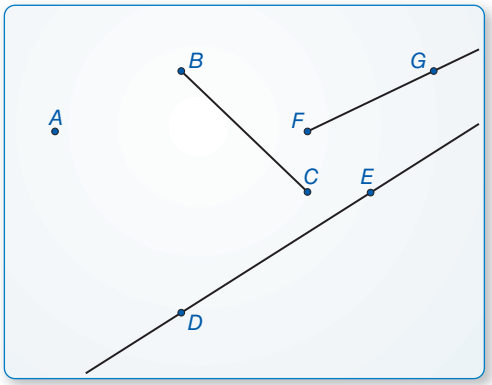
1.1 Points, Lines, and Planes

Essential Question How can you use dynamic geometry software to visualize geometric concepts?

EXPLORATION 1 Using Dynamic Geometry Software

Work with a partner. Use dynamic geometry software to draw several points. Also, draw some lines, line segments, and rays. What is the difference between a line, a line segment, and a ray?

Sample



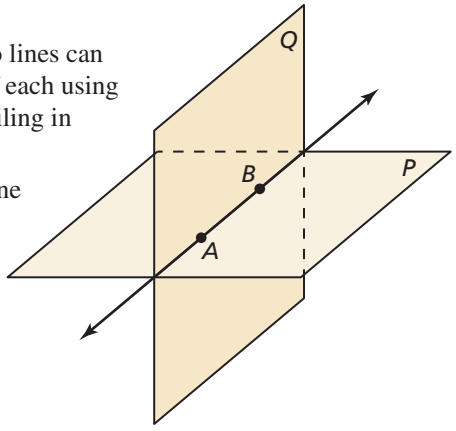
EXPLORATION 2 Intersections of Lines and Planes

Work with a partner.

a. Describe and sketch the ways in which two lines can intersect or not intersect. Give examples of each using the lines formed by the walls, floor, and ceiling in your classroom.

b. Describe and sketch the ways in which a line and a plane can intersect or not intersect. Give examples of each using the walls, floor, and ceiling in your classroom.

c. Describe and sketch the ways in which two planes can intersect or not intersect. Give examples of each using the walls, floor, and ceiling in your classroom.



UNDERSTANDING MATHEMATICAL TERMS

To be proficient in math, you need to understand definitions and previously established results. An appropriate tool, such as a software package, can sometimes help.

EXPLORATION 3 Exploring Dynamic Geometry Software

Work with a partner. Use dynamic geometry software to explore geometry. Use the software to find a term or concept that is unfamiliar to you. Then use the capabilities of the software to determine the meaning of the term or concept.

Communicate Your Answer

4. How can you use dynamic geometry software to visualize geometric concepts?

1.1 Lesson

Core Vocabulary

undefined terms, *p. 4*
point, *p. 4*
line, *p. 4*
plane, *p. 4*
collinear points, *p. 4*
coplanar points, *p. 4*
defined terms, *p. 5*
line segment, or segment, *p. 5*
endpoints, *p. 5*
ray, *p. 5*
opposite rays, *p. 5*
intersection, *p. 6*

What You Will Learn

- Name points, lines, and planes.
- Name segments and rays.
- Sketch intersections of lines and planes.
- Solve real-life problems involving lines and planes.

Using Undefined Terms

In geometry, the words *point*, *line*, and *plane* are **undefined terms**. These words do not have formal definitions, but there is agreement about what they mean.

Core Concept

Undefined Terms: Point, Line, and Plane

Point A **point** has no dimension. A dot represents a point.

point *A*

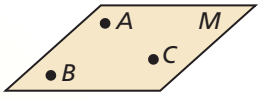
Line A **line** has one dimension. It is represented by a line with two arrowheads, but it extends without end.

Through any two points, there is exactly one line. You can use any two points on a line to name it.

line ℓ, line *AB* ($\overleftrightarrow{AB}$), or line *BA* ($\overleftrightarrow{BA}$)

Plane A **plane** has two dimensions. It is represented by a shape that looks like a floor or a wall, but it extends without end.

Through any three points not on the same line, there is exactly one plane. You can use three points that are not all on the same line to name a plane.



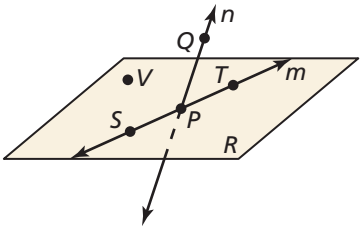
plane *M*, or plane *ABC*

Collinear points are points that lie on the same line. **Coplanar points** are points that lie in the same plane.

EXAMPLE 1 Naming Points, Lines, and Planes

a. Give two other names for $\overleftrightarrow{PQ}$ and plane *R*.

b. Name three points that are collinear. Name four points that are coplanar.



SOLUTION

a. Other names for $\overleftrightarrow{PQ}$ are $\overleftrightarrow{QP}$ and line *n*. Other names for plane *R* are plane *SVT* and plane *PTV*.

b. Points *S*, *P*, and *T* lie on the same line, so they are collinear. Points *S*, *P*, *T*, and *V* lie in the same plane, so they are coplanar.

Monitoring Progress

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1. Use the diagram in Example 1. Give two other names for $\overleftrightarrow{ST}$. Name a point that is *not* coplanar with points *Q*, *S*, and *T*.

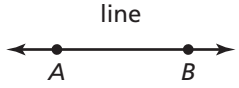
Using Defined Terms

In geometry, terms that can be described using known words such as *point* or *line* are called **defined terms**.

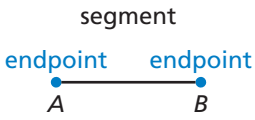
Core Concept

Defined Terms: Segment and Ray

The definitions below use line AB (written as $\overleftrightarrow{AB}$) and points A and B.

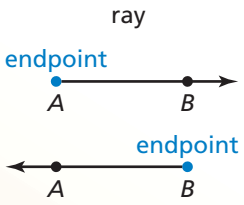


Segment The **line segment** AB, or **segment** AB, (written as $\overline{AB}$) consists of the **endpoints** A and B and all points on $\overleftrightarrow{AB}$ that are between A and B. Note that $\overline{AB}$ can also be named $\overline{BA}$.

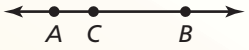


Ray The **ray** AB (written as $\overrightarrow{AB}$) consists of the endpoint A and all points on $\overleftrightarrow{AB}$ that lie on the same side of A as B.

Note that $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are different rays.



Opposite Rays If point C lies on $\overleftrightarrow{AB}$ between A and B, then $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are **opposite rays**.

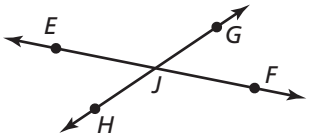


Segments and rays are collinear when they lie on the same line. So, opposite rays are collinear. Lines, segments, and rays are coplanar when they lie in the same plane.

EXAMPLE 2 Naming Segments, Rays, and Opposite Rays

a. Give another name for $\overline{GH}$.

b. Name all rays with endpoint J. Which of these rays are opposite rays?



COMMON ERROR

In Example 2, $\overrightarrow{JG}$ and $\overrightarrow{JF}$ have a common endpoint, but they are not collinear. So, they are *not* opposite rays.

SOLUTION

a. Another name for $\overline{GH}$ is $\overline{HG}$.

b. The rays with endpoint J are $\overrightarrow{JE}$, $\overrightarrow{JG}$, $\overrightarrow{JF}$, and $\overrightarrow{JH}$. The pairs of opposite rays with endpoint J are $\overrightarrow{JE}$ and $\overrightarrow{JF}$, and $\overrightarrow{JG}$ and $\overrightarrow{JH}$.

Monitoring Progress

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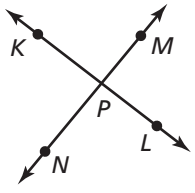
Use the diagram.

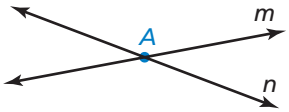


2. Give another name for $\overline{KL}$.

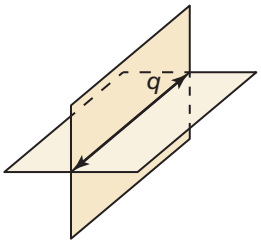
3. Are $\overrightarrow{KP}$ and $\overrightarrow{PK}$ the same ray? Are $\overrightarrow{NP}$ and $\overrightarrow{NM}$ the same ray? Explain.

Sketching Intersections

Two or more geometric figures *intersect* when they have one or more points in common. The **intersection** of the figures is the set of points the figures have in common. Some examples of intersections are shown below.



The intersection of two different lines is a point.



The intersection of two different planes is a line.

EXAMPLE 3 Sketching Intersections of Lines and Planes

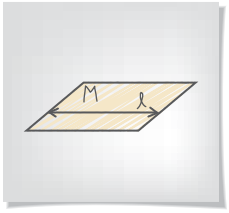
a. Sketch a plane and a line that is in the plane.

b. Sketch a plane and a line that does not intersect the plane.

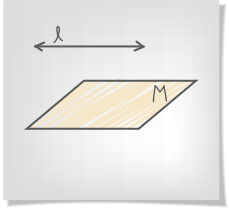
c. Sketch a plane and a line that intersects the plane at a point.

SOLUTION

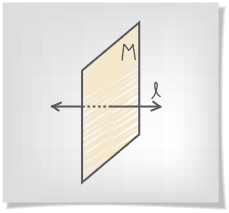
a.



b.



c.



EXAMPLE 4 Sketching Intersections of Planes

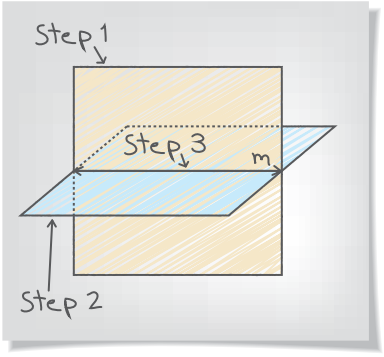
Sketch two planes that intersect in a line.

SOLUTION

Step 1 Draw a vertical plane. Shade the plane.

Step 2 Draw a second plane that is horizontal. Shade this plane a different color. Use dashed lines to show where one plane is hidden.

Step 3 Draw the line of intersection.



Monitoring Progress

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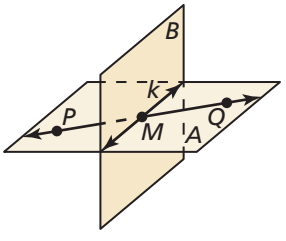
4. Sketch two different lines that intersect a plane at the same point.

Use the diagram.



5. Name the intersection of $\overleftrightarrow{PQ}$ and line k.

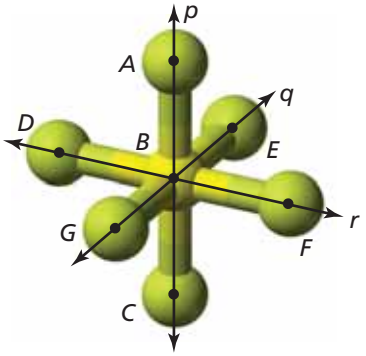
6. Name the intersection of plane A and plane B.

7. Name the intersection of line k and plane A.

Solving Real-Life Problems

EXAMPLE 5 Modeling with Mathematics

The diagram shows a molecule of sulfur hexafluoride, the most potent greenhouse gas in the world. Name two different planes that contain line r.



Electric utilities use sulfur hexafluoride as an insulator. Leaks in electrical equipment contribute to the release of sulfur hexafluoride into the atmosphere.

SOLUTION

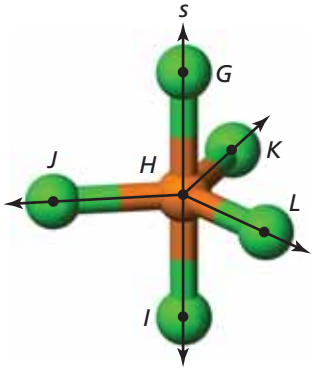
1. **Understand the Problem** In the diagram, you are given three lines, p, q, and r, that intersect at point B. You need to name two different planes that contain line r.
2. **Make a Plan** The planes should contain two points on line r and one point not on line r.
3. **Solve the Problem** Points D and F are on line r. Point E does not lie on line r. So, plane DEF contains line r. Another point that does not lie on line r is C. So, plane CDF contains line r.

   Note that you cannot form a plane through points D, B, and F. By definition, three points that do not lie on the same line form a plane. Points D, B, and F are collinear, so they do *not* form a plane.
4. **Look Back** The question asks for two *different* planes. You need to check whether plane DEF and plane CDF are two unique planes or the same plane named differently. Because point C does not lie on plane DEF, plane DEF and plane CDF are different planes.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Use the diagram that shows a molecule of phosphorus pentachloride.



8. Name two different planes that contain line s.
9. Name three different planes that contain point K.
10. Name two different planes that contain $\overleftrightarrow{HJ}$.

1.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

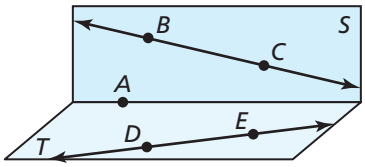
1. **WRITING** Compare collinear points and coplanar points.

2. **WHICH ONE DOESN'T BELONG?** Which term does *not* belong with the other three? Explain your reasoning.

$\overline{AB}$ | plane CDE | $\overleftrightarrow{FG}$ | $\overrightarrow{HI}$

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, use the diagram.



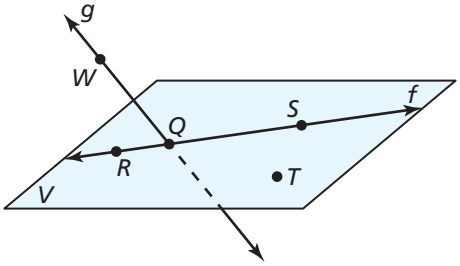
3. Name four points.

4. Name two lines.

5. Name the plane that contains points A, B, and C.

6. Name the plane that contains points A, D, and E.

In Exercises 7–10, use the diagram. *(See Example 1.)*



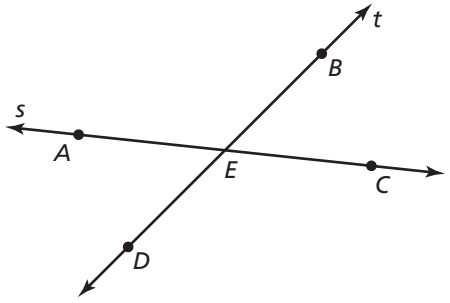
7. Give two other names for $\overleftrightarrow{WQ}$.

8. Give another name for plane V.

9. Name three points that are collinear. Then name a fourth point that is not collinear with these three points.

10. Name a point that is not coplanar with R, S, and T.

In Exercises 11–16, use the diagram. *(See Example 2.)*



11. What is another name for $\overline{BD}$?

12. What is another name for $\overline{AC}$?

13. What is another name for ray $\overrightarrow{AE}$?

14. Name all rays with endpoint E.

15. Name two pairs of opposite rays.

16. Name one pair of rays that are not opposite rays.

In Exercises 17–24, sketch the figure described. *(See Examples 3 and 4.)*

17. plane P and line ℓ intersecting at one point

18. plane K and line m intersecting at all points on line m

19. $\overrightarrow{AB}$ and $\overleftrightarrow{AC}$

20. $\overrightarrow{MN}$ and $\overrightarrow{NX}$

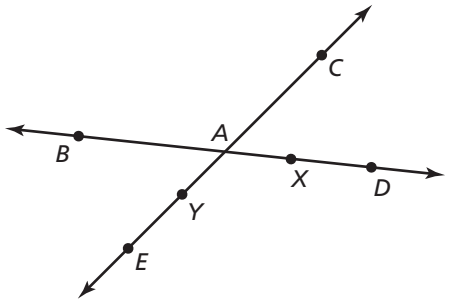
21. plane M and $\overrightarrow{NB}$ intersecting at B

22. plane M and $\overrightarrow{NB}$ intersecting at A

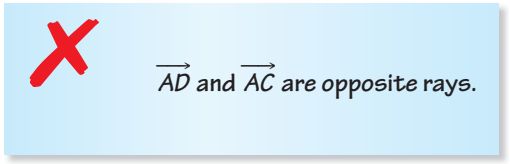
23. plane A and plane B not intersecting

24. plane C and plane D intersecting at $\overleftrightarrow{XY}$

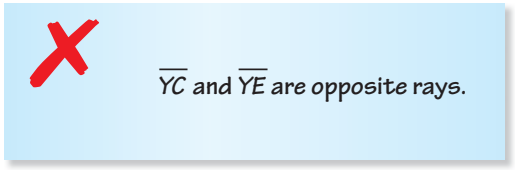
ERROR ANALYSIS **In Exercises 25 and 26, describe and correct the error in naming opposite rays in the diagram.**



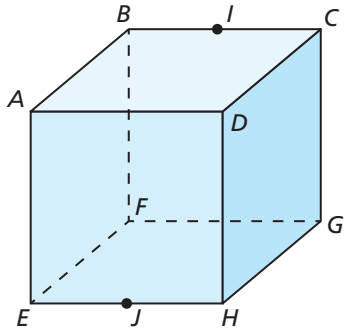
25.



26.



In Exercises 27–34, use the diagram.



27. Name a point that is collinear with points E and H.

28. Name a point that is collinear with points B and I.

29. Name a point that is not collinear with points E and H.

30. Name a point that is not collinear with points B and I.

31. Name a point that is coplanar with points D, A, and B.

32. Name a point that is coplanar with points C, G, and F.

33. Name the intersection of plane AEH and plane FBE.

34. Name the intersection of plane BGF and plane HDG.

In Exercises 35–38, name the geometric term modeled by the object.

35.

36.

37.

38.

In Exercises 39–44, use the diagram to name all the points that are not coplanar with the given points.

39. N, K, and L

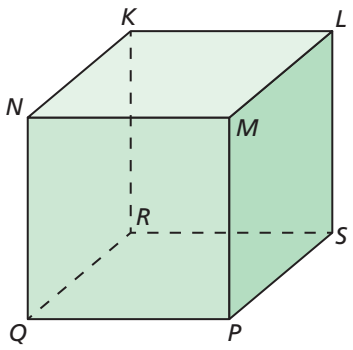
40. P, Q, and N

41. P, Q, and R

42. R, K, and N

43. P, S, and K

44. Q, K, and L



45. **CRITICAL THINKING** Given two points on a line and a third point not on the line, is it possible to draw a plane that includes the line and the third point? Explain your reasoning.

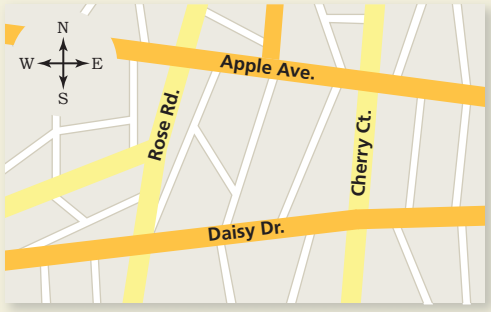
46. **CRITICAL THINKING** Is it possible for one point to be in two different planes? Explain your reasoning.

47. **REASONING** Explain why a four-legged chair may rock from side to side even if the floor is level. Would a three-legged chair on the same level floor rock from side to side? Why or why not?

48. **THOUGHT PROVOKING** You are designing the living room of an apartment. Counting the floor, walls, and ceiling, you want the design to contain at least eight different planes. Draw a diagram of your design. Label each plane in your design.

49. **LOOKING FOR STRUCTURE** Two coplanar intersecting lines will always intersect at one point. What is the greatest number of intersection points that exist if you draw four coplanar lines? Explain.

50. **HOW DO YOU SEE IT?** You and your friend walk in opposite directions, forming opposite rays. You were originally on the corner of Apple Avenue and Cherry Court.



a. Name two possibilities of the road and direction you and your friend may have traveled.

b. Your friend claims he went north on Cherry Court, and you went east on Apple Avenue. Make an argument as to why you know this could not have happened.

MATHEMATICAL CONNECTIONS **In Exercises 51–54, graph the inequality on a number line. Tell whether the graph is a *segment*, a *ray* or *rays*, a *point*, or a *line*.**

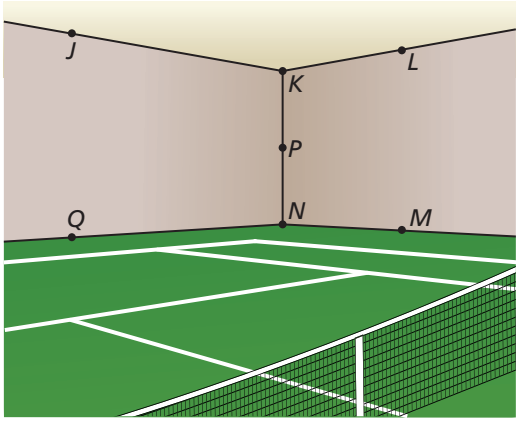
51. $x \leq 3$

52. $-7 \leq x \leq 4$

53. $x \geq 5$ or $x \leq -2$

54. $|x| \leq 0$

55. **MODELING WITH MATHEMATICS** Use the diagram.



a. Name two points that are collinear with P.

b. Name two planes that contain J.

c. Name all the points that are in more than one plane.

CRITICAL THINKING **In Exercises 56–63, complete the statement with *always*, *sometimes*, or *never*. Explain your reasoning.**

56. A line _____ has endpoints.

57. A line and a point _____ intersect.

58. A plane and a point _____ intersect.

59. Two planes _____ intersect in a line.

60. Two points _____ determine a line.

61. Any three points _____ determine a plane.

62. Any three points not on the same line _____ determine a plane.

63. Two lines that are not parallel _____ intersect.

64. **ABSTRACT REASONING** Is it possible for three planes to never intersect? intersect in one line? intersect in one point? Sketch the possible situations.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the absolute value. *(Skills Review Handbook)*

65. $|6 + 2|$

66. $|3 - 9|$

67. $|-8 - 2|$

68. $|7 - 11|$

Solve the equation. *(Skills Review Handbook)*

69. $18 + x = 43$

70. $36 + x = 20$

71. $x - 15 = 7$

72. $x - 23 = 19$