Solving Systems of Linear Inequalities

Essential Question How can you graph a system of three linear inequalities?
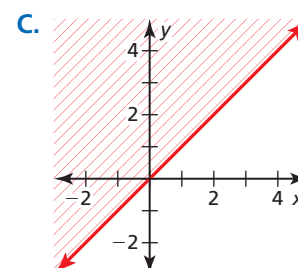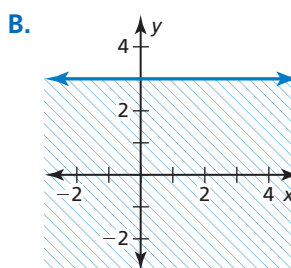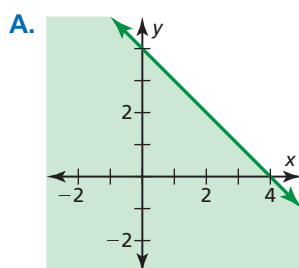
EXPLORATION 1   **Graphing Linear Inequalities**

Work with a partner. Match each linear inequality with its graph. Explain your reasoning.

$x + y \leq 4$ Inequality 1

$x - y \leq 0$ Inequality 2

$y \leq 3$ Inequality 3

A.

B.

C.

EXPLORATION 2   **Graphing a System of Linear Inequalities**

USING PRECISE MATHEMATICAL LANGUAGE

To be proficient in math, you need to explain mathematical ideas using precise mathematical language.

Work with a partner. Consider the linear inequalities given in Exploration 1.

$x + y \leq 4$ Inequality 1

$x - y \leq 0$ Inequality 2

$y \leq 3$ Inequality 3

a. Use three different colors to graph the inequalities in the same coordinate plane. What is the result?

b. Describe each of the shaded regions of the graph. What does the unshaded region represent?
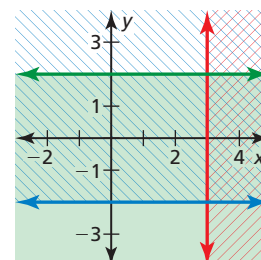
Communicate Your Answer

3. How can you graph a system of three linear inequalities?

4. When graphing a system of three linear inequalities, which region represents the solution of the system?

5. Do you think all systems of three linear inequalities have a solution? Explain your reasoning.

6. Write a system of three linear inequalities represented by the graph.

What You Will Learn

▶ Check solutions of systems of linear inequalities.
▶ Graph systems of linear inequalities.
▶ Write systems of linear inequalities.
▶ Use systems of linear inequalities to solve real-life problems.

Core Vocabulary

system of linear inequalities, *p. 82*
solution of a system of linear inequalities, *p. 82*
graph of a system of linear inequalities, *p. 83*

Previous
linear inequality in two variables

Systems of Linear Inequalities

A **system of linear inequalities** is a set of two or more linear inequalities in the same variables. An example is shown.

$y \leq x + 2$	Inequality 1
$y < -2x + 1$	Inequality 2
$y < -x - 1$	Inequality 3

A **solution of a system of linear inequalities** is an ordered pair that is a solution of each inequality in the system.

EXAMPLE 1   Checking Solutions

Tell whether each ordered pair is a solution of the system of linear inequalities.

$y < 4x$	Inequality 1
$y \geq x - 1$	Inequality 2
$y \leq 2x + 1$	Inequality 3

a. $(1, 0)$ **b.** $(-2, -8)$

SOLUTION

a. Substitute 1 for x and 0 for y in each inequality.

Inequality 1	Inequality 2	Inequality 3
$y < 4x$	$y \geq x - 1$	$y \leq 2x + 1$
$0 \overset{?}{<} 4(1)$	$0 \overset{?}{\geq} 1 - 1$	$0 \overset{?}{\leq} 2(1) + 1$
$0 < 4$ ✓	$0 \geq 0$ ✓	$0 \leq 3$ ✓

▶ Because the ordered pair $(1, 0)$ is a solution of each inequality, it is a solution of the system.

b. Substitute -2 for x and -8 for y in each inequality.

Inequality 1	Inequality 2	Inequality 3
$y < 4x$	$y \geq x - 1$	$y \leq 2x + 1$
$-8 \overset{?}{<} 2(-2)$	$-8 \overset{?}{\geq} -2 - 1$	$-8 \overset{?}{\leq} 2(-2) + 1$
$-8 < -4$ ✓	$-8 \not\geq -3$ ✗	$-8 \leq -3$ ✓

▶ Because $(-2, -8)$ is not a solution of each inequality, it is *not* a solution of the system.

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Tell whether the ordered pair is a solution of the system of linear inequalities.

1. $(2, 1)$;
$$y < 3$$
$$y > -x + 3$$
$$y \geq x - 4$$

2. $(-2, 3)$;
$$y \leq -3x + 2$$
$$y \geq 2x - 5$$
$$y < x + 6$$

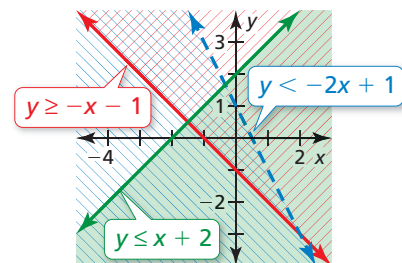# Graphing Systems of Linear Inequalities

The **graph of a system of linear inequalities** is the graph of all the solutions of the system.

Core Concept

Graphing a System of Linear Inequalities

Step 1 Graph each inequality in the same coordinate plane.

Step 2 Find the intersection of the half-planes that are solutions of the inequalities. This intersection is the graph of the system.

$y \geq -x - 1$

$y < -2x + 1$

$y \leq x + 2$

EXAMPLE 2   Graphing a System of Linear Inequalities

Graph the system of linear inequalities.

$y > -2x - 5$ Inequality 1

$y \leq x + 3$ Inequality 2

$y \geq x - 2$ Inequality 3

SOLUTION

Step 1 Graph each inequality.

Step 2 Find the intersection of the half-planes. One solution is $(2, 3)$.

$(2, 3)$

Check

Verify that $(2, 3)$ is a solution of each inequality.

Inequality 1

$y > -2x - 5$

$3 \overset{?}{>} -2(2) - 5$

$3 > -9$ ✓

Inequality 2

$y \leq x + 3$

$3 \overset{?}{\leq} 2 + 3$

$3 \leq 5$ ✓

Inequality 3

$y \geq x - 2$

$3 \overset{?}{\geq} 2 - 2$

$3 \geq 0$ ✓
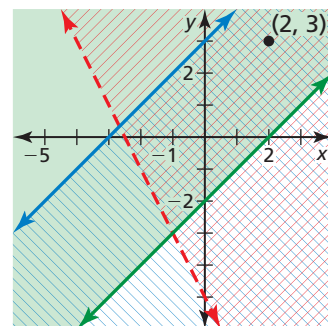
EXAMPLE 3   Graphing a System of Linear Inequalities (No Solution)

Graph the system of linear inequalities.

$2x + 3y < 6$ Inequality 1

$y \geq -\frac{2}{3}x + 4$ Inequality 2

$-2x + y < -2$ Inequality 3

SOLUTION

Step 1 Graph each inequality.

Step 2 Find the intersection of the half-planes. Notice that there is no region shaded red, blue, and green.

▶   So, the system has no solution.
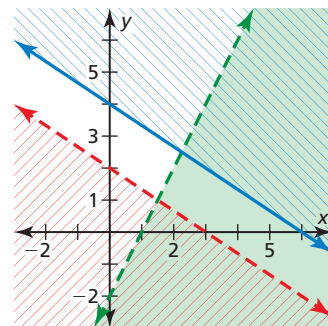
Graph the system of linear inequalities.

3. $y < 3x - 2$ **4.** $x + y > -3$ **5.** $2x - \frac{1}{2}y \geq 4$

$y > -x + 4$ $-6x + y < 1$ $4x - y \leq 5$

$y \leq 2$ $x + y \geq 8$ $x + y \geq 10$

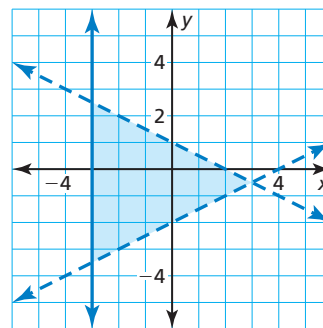## Writing Systems of Linear Inequalities

EXAMPLE 4 **Writing a System of Linear Inequalities**

Write a system of linear inequalities represented by the graph.

SOLUTION

Inequality 1 The vertical boundary line passes through $(-3, 0)$. So, an equation of the line is $x = -3$. The shaded region is to the *right* of the *solid* boundary line, so the inequality is $x \geq -3$.

Inequality 2 The slope of the boundary line is $\frac{1}{2}$, and the y-intercept is -2. So, an equation of the line is $y = \frac{1}{2}x - 2$. The shaded region is *above* the *dashed* boundary line, so the inequality is $y > \frac{1}{2}x - 2$.

Inequality 3 The slope of the boundary line is $-\frac{1}{2}$, and the y-intercept is 1. So, an equation of the line is $y = -\frac{1}{2}x + 1$. The shaded region is *below* the *dashed* boundary line, so the inequality is $y < -\frac{1}{2}x + 1$.

▶ The system of inequalities represented by the graph is

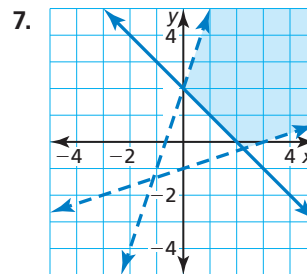$x \geq -3$ Inequality 1
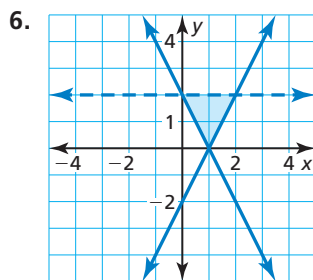
$y > \frac{1}{2}x - 2$ Inequality 2

$y < -\frac{1}{2}x + 1.$ Inequality 3

Write a system of inequalities represented by the graph.

6.

7.

Solving Real-Life Problems

EXAMPLE 5 **Applying Mathematics**

A discount shoe store is having a sale, as described in the advertisement shown. Use the information in the ad to write and graph a system of inequalities that represents the regular and possible sale prices. How much can regularly priced shoes cost on sale?
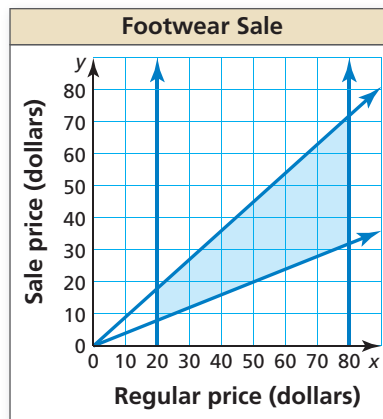
SOLUTION

1. **Understand the Problem** You know the range of regular prices and the range of discounts. You are asked to write and graph a system that represents the situation and determine how much regularly priced shoes cost on sale.

2. **Make a Plan** Use the given information to write a system of inequalities. Then graph the system and identify an ordered pair in the solution region.

3. **Solve the Problem** Write a system of inequalities. Let x be the regular price and let y be the sale price.

$x \geq 20$	Regular price must be at least \$20.
$x \leq 80$	Regular price must be at most \$80.
$y \geq 0.4x$	Sale price is at least $(100 - 60)\% = 40\%$ of regular price.
$y \leq 0.9x$	Sale price is at most $(100 - 10)\% = 90\%$ of regular price.

Graph the system.

Check

$x \geq 20$

$50 \geq 20$ ✓

$x \leq 80$

$50 \leq 80$ ✓

$y \geq 0.4x$

$30 \overset{?}{\geq} 0.4(50)$

$30 \geq 20$ ✓

$y \leq 0.9x$

$30 \overset{?}{\leq} 0.9(50)$

$30 \leq 45$ ✓

Footwear Sale

Sale price (dollars) vs. Regular price (dollars)

From the graph, you can see that one ordered pair in the solution region is (50, 30).

▶ So, a \$50 pair of shoes could cost \$30 on sale.

4. **Look Back** Check your solution by substituting $x = 50$ and $y = 30$ in each inequality in the system, as shown.

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8. Identify and interpret another solution of Example 5.

9. **WHAT IF?** Suppose all the shoes were sold except those regularly priced from \$60 to \$80. How does this change the system? Is (50, 30) still a solution? Explain.
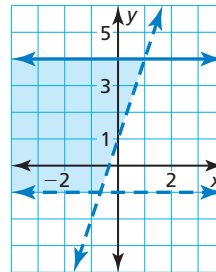
Vocabulary and Core Concept Check

1. **VOCABULARY** What must be true in order for an ordered pair to be a solution of a system of linear inequalities?

2. **WHICH ONE DOESN'T BELONG?** Use the graph shown. Which ordered pair does *not* belong with the other three? Explain your reasoning.

$(-3, 0)$ $(-3, 2)$

$(0, 1)$ $(-1, 4)$

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, tell whether the ordered pair is a solution of the system of linear inequalities. *(See Example 1.)*

3. $(1, 2)$; $\begin{aligned} y &\geq 5x - 6 \\ y &\leq 3x + 1 \\ y &> -x + 2 \end{aligned}$

4. $(8, -2)$; $\begin{aligned} y &< 2x + 5 \\ y &\geq -4x - 1 \\ y &< 2x - 2 \end{aligned}$

5. $(0, 0)$; $\begin{aligned} y &< 3x + 12 \\ x - y &< 4 \\ y &> -\tfrac{1}{2}x \end{aligned}$

6. $(-2, 5)$; $\begin{aligned} y &\leq x + 3 \\ y &\geq -x \\ 3x - y &> -6 \end{aligned}$

7. $(2, 1)$; $\begin{aligned} x + y &\leq 4 \\ x + y &> -1 \\ x - y &\geq -2 \\ x - y &\leq 2 \end{aligned}$

8. $(1, 3)$; $\begin{aligned} 2x + y &< 5 \\ 2x + y &\geq -3 \\ 3x - y &> -4 \\ 3x - y &\leq 3 \end{aligned}$

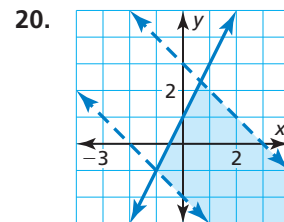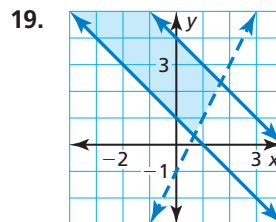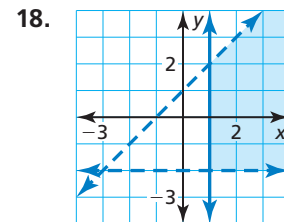**In Exercises 9–16, graph the system of linear inequalities.** *(See Examples 2 and 3.)*

9. $\begin{aligned} y &< 4 \\ x &> -3 \\ y &> x \end{aligned}$

10. $\begin{aligned} y &\geq 1 \\ x &\leq 6 \\ y &< 2x - 5 \end{aligned}$

11. $\begin{aligned} y &< 2x + 4 \\ y &\geq 4x - 3 \\ y &< 2x - 3 \end{aligned}$

12. $\begin{aligned} y &< 2x + 1 \\ y &< -3x - 1 \\ y &> -3x + 2 \end{aligned}$

13. $\begin{aligned} 2x - 3y &> -6 \\ 5x - 3y &< 3 \\ x + 3y &> -3 \end{aligned}$

14. $\begin{aligned} x - 4y &> 0 \\ x + y &\leq 1 \\ x + 3y &> -1 \end{aligned}$

15. $\begin{aligned} y &\geq 0 \\ x &\leq 9 \\ x + y &< 15 \\ y &< x \end{aligned}$

16. $\begin{aligned} y &< 5 \\ y &> -6 \\ 2x + y &\geq -1 \\ y &\leq x + 3 \end{aligned}$

In Exercises 17–22, write a system of linear inequalities represented by the graph. *(See Example 4.)*

17.

18.

19.

20.

21.

22.
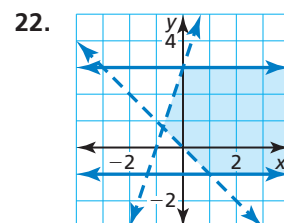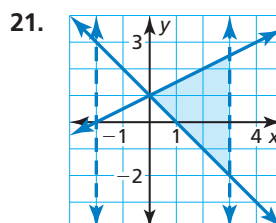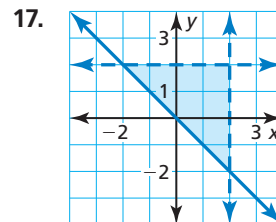
ERROR ANALYSIS In Exercises 23 and 24, describe and correct the error in graphing the system of linear inequalities.

23.

✗

$y \leq 3$

$y > 1$

$-y \geq -2x$

24.

✗

$y \leq 3$

$x + y \geq 5$

$x > -1$
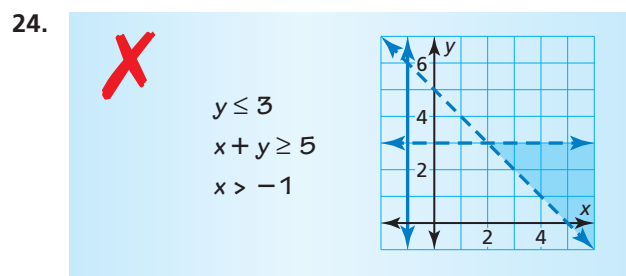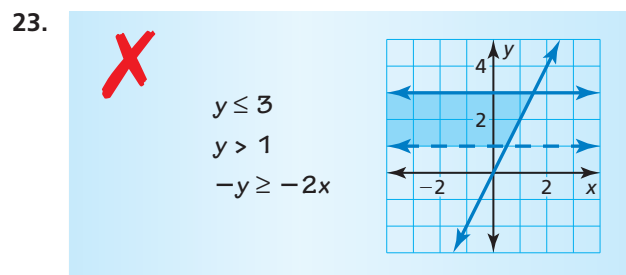
25. MODELING WITH MATHEMATICS You are buying movie passes and gift cards as prizes for an event. You need at least five movie passes and two gift cards. A movie pass costs $6 and a gift card costs $10. The most you can spend is $70. *(See Example 5.)*

   a. Write and graph a system of linear inequalities that represents the situation.

   b. Identify and interpret two possible solutions.

26. MODELING WITH MATHEMATICS The Junior-Senior Prom Committee consists of five to eight representatives from the junior and senior classes. The committee must include at least two juniors and at least three seniors.

   a. Write and graph a system of linear inequalities that represents the situation.

   b. Identify and interpret two possible solutions.

27. PROBLEM SOLVING An online media store is having a sale, as described in the advertisement shown. Use the information in the ad to write and graph a system of inequalities for the regular video game prices and possible sale prices. Then use the graph to estimate the range of possible sale prices for games that are regularly priced at $20.

ONE DAY SALE!

**SAVE 30%–70%
ON ALL
VIDEO GAMES**

(REGULAR PRICE: $20–$50)

28. PROBLEM SOLVING In baseball, the strike zone is a rectangle the width of home plate that extends from the batter's knees to a point halfway between the shoulders S and the top T of the uniform pants. The width of home plate is 17 inches. A batter's knees are 20 inches above the ground and the point halfway between his shoulders and the top of his pants is 42 inches above the ground. Write a system of inequalities that represents the strike zone.
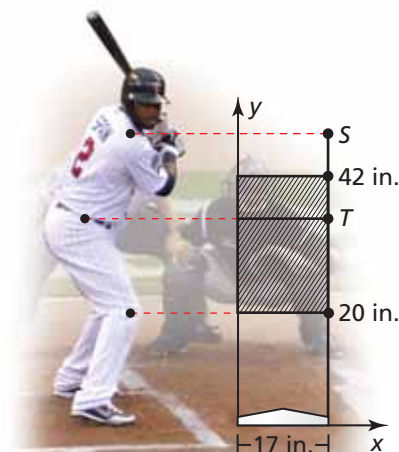
Not drawn to scale

29. MATHEMATICAL CONNECTIONS The following points are the vertices of a rectangle.

$$(-3, 3), (4, -2), (-3, -2), (4, 3)$$

   a. Write a system of linear inequalities that represents the points inside the rectangle.

   b. Find the area of the rectangle.

30. MATHEMATICAL CONNECTIONS The following points are the vertices of a triangle.

$$(0, -2), (4, 6), (4, -2)$$

   a. Write a system of linear inequalities that represents the points inside the triangle.

   b. Find the area of the triangle.

31. USING EQUATIONS Which quadrant of the coordinate plane contains no solutions of the system of linear inequalities?

$$y \geq 4x + 1$$
$$2x + y < 5$$
$$y \geq -3$$

   Ⓐ Quadrant I   Ⓑ Quadrant II

   Ⓒ Quadrant III   Ⓓ Quadrant IV

32. OPEN-ENDED Write a system of three linear inequalities that has $(-2, 1)$ as a solution.

33. PROBLEM SOLVING A book on the care of tropical fish states that the pH level of the water should be between 8.0 and 8.3 and the temperature of the water should be between 76°F and 80°F. Let x be the pH level and y be the temperature. Write and graph a system of inequalities that describes the proper pH level and temperature of the water. Compare this graph with the graph you would obtain if the temperatures were given in degrees Celsius.

34. HOW DO YOU SEE IT? The graphs of three linear equations are shown.
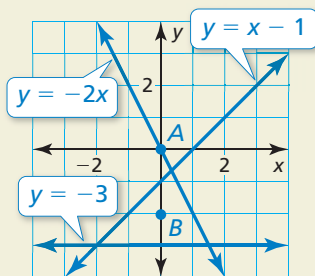
Replace the equal signs with inequality symbols to create a system of linear inequalities that has point B as a solution, but not point A. Explain your reasoning.

$$y \quad\boxed{\phantom{<}}\quad x - 1$$
$$y \quad\boxed{\phantom{<}}\quad -2x$$
$$y \quad\boxed{\phantom{<}}\quad -3$$

35. REASONING Write a system of three linear inequalities for which the solution set consists of the points on the line $y = 5x - 2$. Justify your answer.

36. THOUGHT PROVOKING Is it possible for a system of linear inequalities to have a solution consisting of a single point in a coordinate plane? If so, give an example. If not, explain why.

37. MAKING AN ARGUMENT Your friend says that a system of three linear inequalities with three parallel boundary lines has no solution. Is your friend correct? Justify your answer.

38. MULTIPLE REPRESENTATIONS A person's theoretical maximum heart rate y (in beats per minute) is given by $y = 220 - x$, where x is the person's age in years ($20 \le x \le 65$). When a person exercises, it is recommended that the person strive for a heart rate that is at least 50% of the maximum and at most 75% of the maximum.

a. Write a system of linear inequalities that describes the given information.

b. Graph the system you wrote in part (a).

c. A 40-year-old person has a heart rate of 158 beats per minute when exercising. Is the person's heart rate in the target zone? Explain.

39. USING TOOLS Use a graphing calculator to sketch a graph of each system.

a. $y \le |x|$
$y \ge -|x|$

b. $y > |2x|$
$y < -|2x| + 4$

c. $y \le |x - 2|$
$y > |x| - 2$

d. $y < |x - 3| + 2$
$y \ge |x - 3| - 1$

40. CRITICAL THINKING Write a system of linear inequalities that represents the graph of $y > |x - 2|$.

Maintaining Mathematical Proficiency
Reviewing what you learned in previous grades and lessons

Find the product. *(Skills Review Handbook)*

41. $(x - 2)^2$ **42.** $(3m + 1)^2$ **43.** $(2z - 5)^2$ **44.** $(4 - y)^2$

Write a function g described by the given transformation of $f(x) = |x| - 5$. *(Section 1.3)*

45. translation 2 units to the left
46. reflection in the x-axis
47. translation 4 units up
48. vertical stretch by a factor of 3