TEXAS ESSENTIAL KNOWLEDGE AND SKILLS

2A.2.A
2A.6.C

Essential Question What are the characteristics of some of the basic parent functions?

An **absolute value function** is a function that contains an absolute value expression. The parent absolute value function is
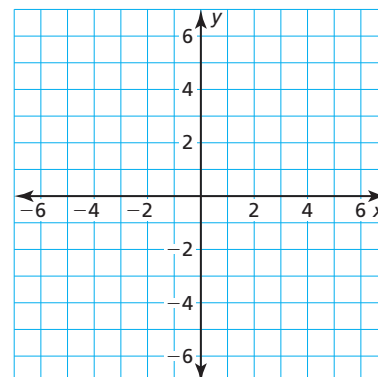
$$f(x) = |x|.$$ Parent absolute value function

EXPLORATION 1 **Graphing the Parent Absolute Value Function**

Work with a partner. Complete the table. Then use the values in the table to sketch the graph of the parent absolute value function

$$f(x) = |x|.$$

| x	−6	−4	−2	0	2	4	6
f(x)							

EXPLORATION 2 **Identifying Basic Parent Functions**

Work with a partner. Graphs of four basic parent functions are shown below. Classify each function as *constant*, *linear*, *quadratic*, or *exponential*. Justify your reasoning.

JUSTIFYING THE SOLUTION

To be proficient in math, you need to justify your conclusions and communicate them clearly to others.

a.

b.

c.

d.
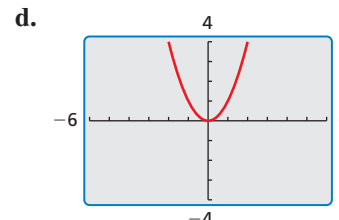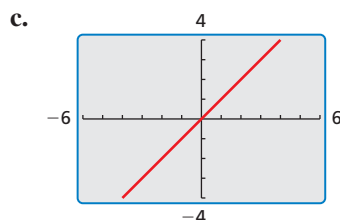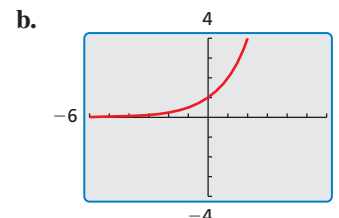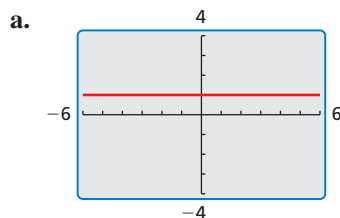
Communicate Your Answer

3. What are the characteristics of some of the basic parent functions?

4. Write an equation for each function whose graph is shown in Exploration 2. Then use a graphing calculator to verify that your equations are correct.

What You Will Learn
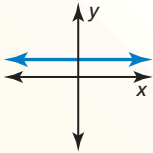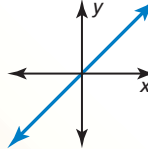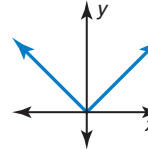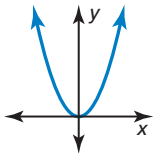
▶ Identify families of functions.

▶ Describe transformations of parent functions.

▶ Describe combinations of transformations.

Identifying Function Families

Functions that belong to the same *family* share key characteristics. The **parent function** is the most basic function in a family. Functions in the same family are *transformations* of their parent function.

Core Concept

Parent Functions

Family	Constant	Linear	Absolute Value	Quadratic
Rule	$f(x) = 1$	$f(x) = x$	$f(x) = |x|$	$f(x) = x^2$
Graph				
Domain	All real numbers	All real numbers	All real numbers	All real numbers
Range	$y = 1$	All real numbers	$y \geq 0$	$y \geq 0$

ANALYZING MATHEMATICAL RELATIONSHIPS

You can also use function rules to identify functions. The only variable term in *f* is an $|x|$-term, so it is an absolute value function.
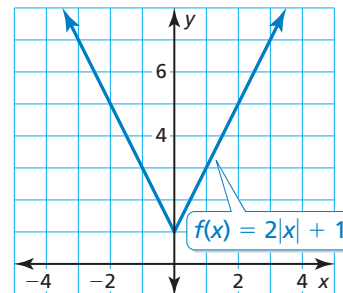
EXAMPLE 1 **Identifying a Function Family**

Identify the function family to which *f* belongs. Compare the graph of *f* to the graph of its parent function.

SOLUTION

The graph of *f* is V-shaped, so *f* is an absolute value function.

The graph is shifted up and is narrower than the graph of the parent absolute value function. The domain of each function is all real numbers, but the range of *f* is $\{y \mid y \geq 1\}$ and the range of the parent absolute value function is $\{y \mid y \geq 0\}$.

$f(x) = 2|x| + 1$

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1. Identify the function family to which *g* belongs. Compare the graph of *g* to the graph of its parent function.

$g(x) = \frac{1}{4}(x - 3)^2$
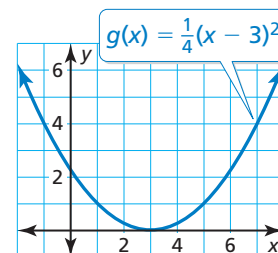
Describing Transformations

A **transformation** changes the size, shape, position, or orientation of a graph. A **translation** is a transformation that shifts a graph horizontally and/or vertically but does not change its size, shape, or orientation.
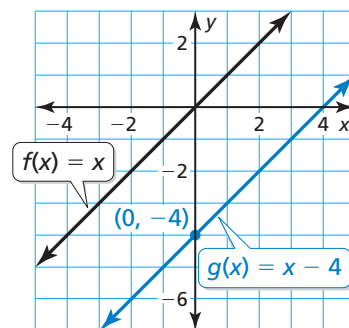
EXAMPLE 2 Graphing and Describing Translations

Graph $g(x) = x - 4$ and its parent function. Then describe the transformation.

SOLUTION

The function g is a linear function with a slope of 1 and a y-intercept of -4. So, draw a line through the point $(0, -4)$ with a slope of 1.

The graph of g is 4 units below the graph of the parent linear function f.

▶ So, the graph of $g(x) = x - 4$ is a vertical translation 4 units down of the graph of the parent linear function.

A **reflection** is a transformation that flips a graph over a line called the *line of reflection*. A reflected point is the same distance from the line of reflection as the original point but on the opposite side of the line.

EXAMPLE 3 Graphing and Describing Reflections

Graph $p(x) = -x^2$ and its parent function. Then describe the transformation.

SOLUTION

The function p is a quadratic function. Use a table of values to graph each function.

x	$y = x^2$	$y = -x^2$
-2	4	-4
-1	1	-1
0	0	0
1	1	-1
2	4	-4
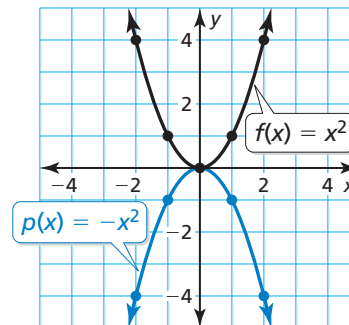
The graph of p is the graph of the parent function flipped over the x-axis.

▶ So, $p(x) = -x^2$ is a reflection in the x-axis of the parent quadratic function.

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Graph the function and its parent function. Then describe the transformation.

2. $g(x) = x + 3$ **3.** $h(x) = (x - 2)^2$ **4.** $n(x) = -|x|$

Another way to transform the graph of a function is to multiply all of the y-coordinates by the same positive factor (other than 1). When the factor is greater than 1, the transformation is a **vertical stretch**. When the factor is greater than 0 and less than 1, it is a **vertical shrink**.

EXAMPLE 4   **Graphing and Describing Stretches and Shrinks**

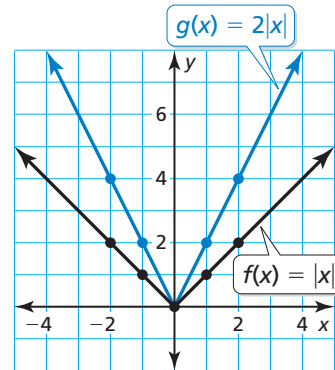Graph each function and its parent function. Then describe the transformation.

a. $g(x) = 2|x|$ **b.** $h(x) = \frac{1}{2}x^2$
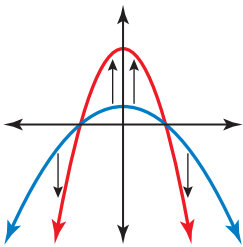
SOLUTION

a. The function g is an absolute value function. Use a table of values to graph the functions.

x	$y = |x|$	$y = 2|x|$
-2	2	4
-1	1	2
0	0	0
1	1	2
2	2	4

REASONING

To visualize a vertical stretch, imagine *pulling* the points away from the x-axis.

The y-coordinate of each point on g is two times the y-coordinate of the corresponding point on the parent function.

▶ So, the graph of $g(x) = 2|x|$ is a vertical stretch of the graph of the parent absolute value function.

b. The function h is a quadratic function. Use a table of values to graph the functions.

x	$y = x^2$	$y = \frac{1}{2}x^2$
-2	4	2
-1	1	$\frac{1}{2}$
0	0	0
1	1	$\frac{1}{2}$
2	4	2

To visualize a vertical shrink, imagine *pushing* the points toward the x-axis.
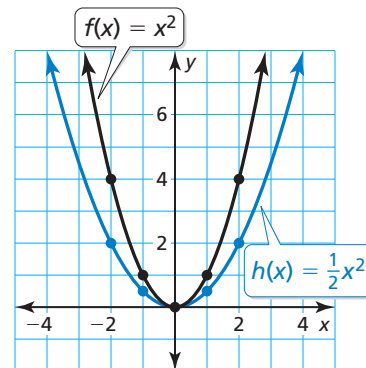
The y-coordinate of each point on h is one-half of the y-coordinate of the corresponding point on the parent function.

▶ So, the graph of $h(x) = \frac{1}{2}x^2$ is a vertical shrink of the graph of the parent quadratic function.

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Graph the function and its parent function. Then describe the transformation.

    5. $g(x) = 3x$ **6.** $h(x) = \frac{3}{2}x^2$ **7.** $c(x) = 0.2|x|$

Combinations of Transformations

You can use more than one transformation to change the graph of a function.

EXAMPLE 5 **Describing Combinations of Transformations**

Use a graphing calculator to graph $g(x) = -|x + 5| - 3$ and its parent function.
Then describe the transformations.

SOLUTION

The function g is an absolute value function.

▶ The graph shows that $g(x) = -|x + 5| - 3$
is a reflection in the x-axis followed by a
translation 5 units left and 3 units down of the
graph of the parent absolute value function.
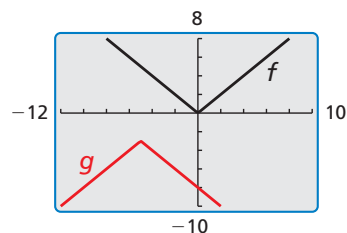
EXAMPLE 6 **Modeling with Mathematics**

Time (seconds), x	Height (feet), y
0	8
0.5	20
1	24
1.5	20
2	8

The table shows the height y of a dirt bike x seconds after jumping off a ramp. What
type of function can you use to model the data? Estimate the height after 1.75 seconds.

SOLUTION

1. **Understand the Problem** You are asked to identify the type of function that can
   model the table of values and then to find the height at a specific time.

2. **Make a Plan** Create a scatter plot of the data. Then use the relationship shown in
   the scatter plot to estimate the height after 1.75 seconds.

3. **Solve the Problem** Create a scatter plot.

   The data appear to lie on a curve that resembles
   a quadratic function. Sketch the curve.

   ▶ So, you can model the data with a quadratic
   function. The graph shows that the height is
   about 15 feet after 1.75 seconds.
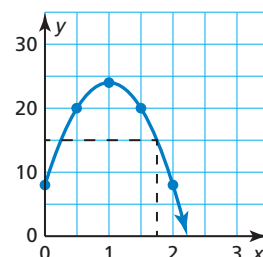
4. **Look Back** To check that your solution is reasonable, analyze the values in the
   table. Notice that the heights decrease after 1 second. Because 1.75 is between
   1.5 and 2, the height must be between 20 feet and 8 feet.

   $$8 < 15 < 20 \checkmark$$

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Use a graphing calculator to graph the function and its parent function.
Then describe the transformations.

8. $h(x) = -\frac{1}{4}x + 5$

9. $d(x) = 3(x - 5)^2 - 1$

10. The table shows the amount of fuel in a chainsaw over time. What type of
function can you use to model the data? When will the tank be empty?

| Time (minutes), x	0	10	20	30	40
Fuel remaining (fluid ounces), y	15	12	9	6	3

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The function $f(x) = x^2$ is the _____ of $f(x) = 2x^2 - 3$.

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

What are the coordinates of the vertices after a reflection in the *x*-axis, followed by a translation 2 units right?

What are the coordinates of the vertices after a translation 6 units up and 2 units right?

What are the coordinates of the vertices after a translation 2 units right, followed by a reflection in the *x*-axis?
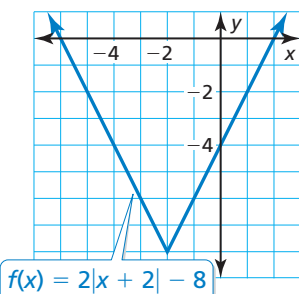
What are the coordinates of the vertices after a translation 6 units up, followed by a reflection in the *x*-axis?
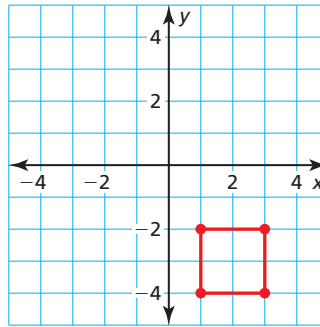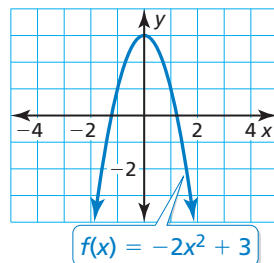

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, identify the function family to which *f* belongs. Compare the graph of *f* to the graph of its parent function. *(See Example 1.)*
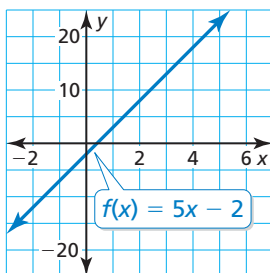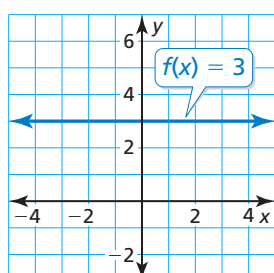
3.

$f(x) = 2|x + 2| - 8$

4.

$f(x) = -2x^2 + 3$

5.

$f(x) = 5x - 2$

6.

$f(x) = 3$

7. **MODELING WITH MATHEMATICS** At 8:00 A.M., the temperature is 43°F. The temperature increases 2°F each hour for the next 7 hours. Graph the temperatures over time *t* ($t = 0$ represents 8:00 A.M.). What type of function can you use to model the data? Explain.

8. **MODELING WITH MATHEMATICS** You purchase a car from a dealership for $10,000. The trade-in value of the car each year after the purchase is given by the function $f(x) = 10,000 - 250x^2$. What type of function can you use to model the data?

In Exercises 9–18, graph the function and its parent function. Then describe the transformation. *(See Examples 2 and 3.)*

9. $g(x) = x + 4$

10. $f(x) = x - 6$

11. $f(x) = x^2 - 1$

12. $h(x) = (x + 4)^2$

13. $g(x) = |x - 5|$

14. $f(x) = 4 + |x|$

15. $h(x) = -x^2$

16. $g(x) = -x$

17. $f(x) = 3$

18. $f(x) = -2$

In Exercises 19–26, graph the function and its parent function. Then describe the transformation. *(See Example 4.)*

19. $f(x) = \frac{1}{3}x$

20. $g(x) = 4x$

21. $f(x) = 2x^2$

22. $h(x) = \frac{1}{3}x^2$

23. $h(x) = \frac{3}{4}x$

24. $g(x) = \frac{4}{3}x$

25. $h(x) = 3|x|$

26. $f(x) = \frac{1}{2}|x|$

In Exercises 27–34, use a graphing calculator to graph the function and its parent function. Then describe the transformations. *(See Example 5.)*

27. $f(x) = 3x + 2$

28. $h(x) = -x + 5$

29. $h(x) = -3|x| - 1$

30. $f(x) = \frac{3}{4}|x| + 1$

31. $g(x) = \frac{1}{2}x^2 - 6$

32. $f(x) = 4x^2 - 3$

33. $f(x) = -(x + 3)^2 + \frac{1}{4}$
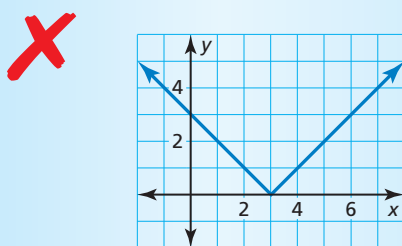
34. $g(x) = -|x - 1| - \frac{1}{2}$

ERROR ANALYSIS In Exercises 35 and 36, identify and correct the error in describing the transformation of the parent function.

35. ✗
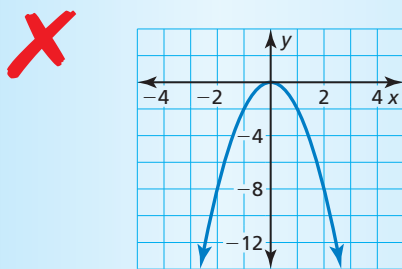
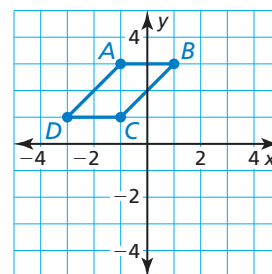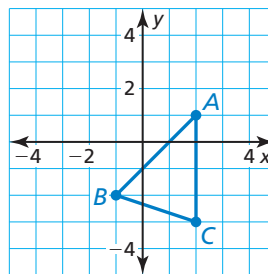The graph is a reflection in the x-axis and a vertical shrink of the parent quadratic function.

36. ✗

The graph is a translation 3 units right of the parent absolute value function, so the function is $f(x) = |x + 3|$.

MATHEMATICAL CONNECTIONS In Exercises 37 and 38, find the coordinates of the figure after the transformation.

37. Translate 2 units down.

38. Reflect in the x-axis.

USING TOOLS In Exercises 39–44, identify the function family and describe the domain and range. Use a graphing calculator to verify your answer.

39. $g(x) = |x + 2| - 1$

40. $h(x) = |x - 3| + 2$

41. $g(x) = 3x + 4$

42. $f(x) = -4x + 11$

43. $f(x) = 5x^2 - 2$

44. $f(x) = -2x^2 + 6$

45. **MODELING WITH MATHEMATICS** The table shows the speeds of a car as it travels through an intersection with a stop sign. What type of function can you use to model the data? Estimate the speed of the car when it is 20 yards past the intersection. *(See Example 6.)*

Displacement from sign (yards), x	Speed (miles per hour), y
−100	40
−50	20
−10	4
0	0
10	4
50	20
100	40

46. **THOUGHT PROVOKING** In the same coordinate plane, sketch the graph of the parent quadratic function and the graph of a quadratic function that has no x-intercepts. Describe the transformation(s) of the parent function.

47. **USING STRUCTURE** Graph the functions $f(x) = |x - 4|$ and $g(x) = |x| - 4$. Are they equivalent? Explain.

48. HOW DO YOU SEE IT? Consider the graphs of f, g, and h.
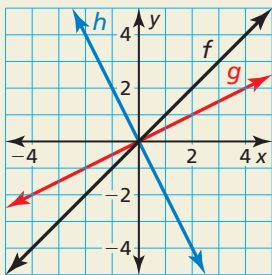
a. Does the graph of g represent a vertical stretch or a vertical shrink of the graph of f? Explain your reasoning.

b. Describe how to transform the graph of f to obtain the graph of h.

49. MAKING AN ARGUMENT Your friend says two different translations of the graph of the parent linear function can result in the graph of $f(x) = x - 2$. Is your friend correct? Explain.

50. DRAWING CONCLUSIONS A person swims at a constant speed of 1 meter per second. What type of function can be used to model the distance the swimmer travels? If the person has a 10-meter head start, what type of transformation does this represent? Explain.

51. PROBLEM SOLVING You are playing basketball with your friends. The height (in feet) of the ball above the ground t seconds after a shot is modeled by the function $f(t) = -16t^2 + 32t + 5.2$.

a. Without graphing, identify the type of function that models the height of the basketball.

b. What is the value of t when the ball is released from your hand? Explain your reasoning.

c. How many feet above the ground is the ball when it is released from your hand? Explain.

52. MODELING WITH MATHEMATICS The table shows the battery lives of a computer over time. What type of function can you use to model the data? Interpret the meaning of the x-intercept in this situation.

Time (hours), x	Battery life remaining, y
1	80%
3	40%
5	0%
6	20%
8	60%

53. REASONING Compare each function with its parent function. State whether it contains a *horizontal translation*, *vertical translation*, *both*, or *neither*. Explain your reasoning.

a. $f(x) = 2|x| - 3$ b. $f(x) = (x - 8)^2$

c. $f(x) = |x + 2| + 4$ d. $f(x) = 4x^2$

54. CRITICAL THINKING Use the values -1, 0, 1, and 2 in the correct box so the graph of each function intersects the x-axis. Explain your reasoning.

a. $f(x) = 3x^{\boxed{}} + 1$ b. $f(x) = |2x - 6| - \boxed{}$

c. $f(x) = \boxed{} x^2 + 1$ d. $f(x) = \boxed{}$

Maintaining Mathematical Proficiency
Reviewing what you learned in previous grades and lessons

Determine whether the ordered pair is a solution of the equation. *(Skills Review Handbook)*

55. $f(x) = x - 3$; $(5, 2)$ **56.** $f(x) = x - 4$; $(12, 8)$

57. $f(x) = 2x + 4$; $(5, 10)$ **58.** $f(x) = 3x + 9$; $(7, 28)$

Find the x-intercept and the y-intercept of the graph of the equation. *(Skills Review Handbook)*

59. $y = x$ **60.** $y = x + 2$

61. $3x + y = 1$ **62.** $x - 2y = 8$