# 1.1 Interval Notation and Set Notation

**Texas Essential Knowledge and Skills**
Preparing for 2A.6.K, 2A.7.I

## Essential Question

When is it convenient to use set-builder notation to represent a set of numbers?

A collection of objects is called a **set**. You can use braces { } to represent a set by listing its members or by using **set-builder notation** to define the set in terms of the properties of its members. For instance, the set of the numbers 1, 2, and 3 can be denoted as

$\{1, 2, 3\}$ List the members of the set in braces.

and the set of all odd whole numbers can be denoted as

$\{x \mid x \text{ is a whole number and } x \text{ is odd}\}$ Set-builder notation

which is read "The set of all real numbers $x$ such that $x$ is a whole number and $x$ is odd."

If all of the members of a set $A$ are also members of a set $B$, then set $A$ is a **subset** of set $B$.

For instance, if set $A = \{a, b\}$ and set $B = \{a, b, c, d\}$, then set $A$ is a subset of set $B$.

### EXPLORATION 1 Writing Subsets in Set Notation

**Work with a partner.** Write all the nonempty subsets of each set.

**a.** $\{4, 5\}$

**b.** $\{c, d\}$

**c.** $\{2, 4, 6\}$

**d.** $\{e, f, g, h\}$

**ANALYZING MATHEMATICAL RELATIONSHIPS**

To be proficient in math, you need to connect and communicate mathematical ideas.

### EXPLORATION 2 Writing Subsets in Set Notation

**Work with a partner.** Write each given subset of the real numbers in set-builder notation. Describe each set-subset relationship among these sets.

**a.** the integers

**b.** the whole numbers

**c.** the natural numbers

**d.** the rational numbers

**e.** the irrational numbers

**f.** the positive integers

### EXPLORATION 3 Writing Subsets in Set Notation

**Work with a partner.** Write each indicated set of numbers using either braces to list its members or set-builder notation. Explain your choice of notation.

**a.** the whole numbers 50 through 54

**b.** the real numbers 0 through 4

**c.** the prime whole numbers

**d.** the integers $-100$ through 100

## Communicate Your Answer

**4.** When is it convenient to use set-builder notation to represent a set of numbers?

**5.** What are some relationships between subsets of the real numbers?

# 1.1 Lesson

## Core Vocabulary

set, *p. 4*
subset, *p. 4*
endpoints, *p. 4*
bounded interval, *p. 4*
unbounded interval, *p. 5*
set-builder notation, *p. 6*

## What You Will Learn

- Represent intervals using interval notation.
- Represent intervals using set-builder notation.

## Using Interval Notation

In mathematics, a collection of objects is called a **set**. You can use braces { } to represent a set by listing its members or *elements*. For instance, the set

$\{1, 2, 3\}$ A set with three members

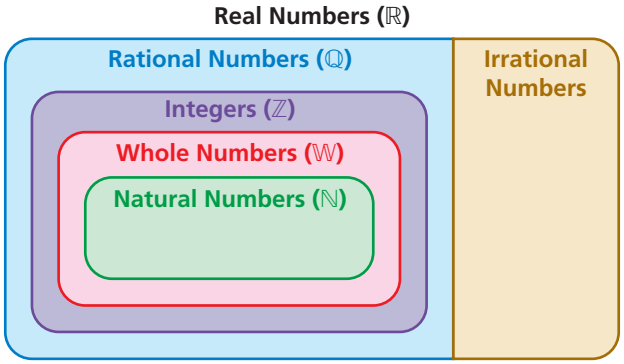
contains the three numbers 1, 2, and 3. Many sets are also described in words, such as the set of real numbers.

If all the members of a set $A$ are also members of a set $B$, then set $A$ is a **subset** of set $B$. The set of natural numbers $\{1, 2, 3, 4, \ldots\}$ is a subset of the set of real numbers. The diagram shows several important subsets of the real numbers.

**UNDERSTANDING MATHEMATICAL TERMS**

The symbols represent subsets of the real numbers.

$\mathbb{R}$: Real numbers
$\mathbb{Q}$: Rational numbers
$\mathbb{Z}$: Integers
$\mathbb{W}$: Whole numbers
$\mathbb{N}$: Natural numbers

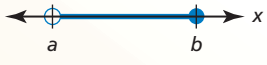



Many subsets of the real numbers can be represented as *intervals* on the real number line.

## Core Concept

### Bounded Intervals on the Real Number Line

Let $a$ and $b$ be two real numbers such that $a < b$. Then $a$ and $b$ are the **endpoints** of four different **bounded intervals** on the real number line, as shown below. A bracket or closed circle indicates that the endpoint is included in the interval and a parenthesis or open circle indicates that the endpoint is not included in the interval.

| *Inequality* | *Interval Notation* | *Graph* |
|---|---|---|
| $a \le x \le b$ | $[a, b]$ |  |
| $a < x < b$ | $(a, b)$ |  |
| $a \le x < b$ | $[a, b)$ |  |
| $a < x \le b$ | $(a, b]$ |  |



The length of any bounded interval, $[a, b]$, $(a, b)$, $[a, b)$, or $(a, b]$, is the distance between its endpoints: $b - a$. Any bounded interval has a *finite* length. An interval that does not have a finite length is called *unbounded* or *infinite*.

## Core Concept

### Unbounded Intervals on the Real Number Line

Let $a$ and $b$ be real numbers. Each interval on the real number line shown below is called an **unbounded interval**.

| *Inequality* | *Interval Notation* | *Graph* |
|---|---|---|
| $x \geq a$ | $[a, \infty)$ |  |
| $x > a$ | $(a, \infty)$ |  |
| $x \leq b$ | $(-\infty, b]$ |  |
| $x < b$ | $(-\infty, b)$ |  |
| | $(-\infty, \infty)$ |  |

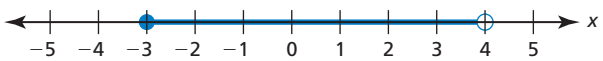
The symbols $\infty$ (*infinity*) and $-\infty$ (*negative infinity*) are used to represent the unboundedness of intervals such as $[7, \infty)$ and $(-\infty, 7]$. Because these symbols do not represent real numbers, they are always enclosed by a parenthesis.

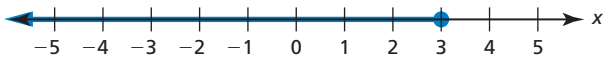
### EXAMPLE 1 Writing Interval Notation

Write each interval in interval notation.

**a.** $-2 \leq x \leq 3$

**b.** $x > -1$

**c.** 

**d.** 

### SOLUTION

**a.** The graph of $-2 \leq x \leq 3$ is the bounded interval $[-2, 3]$.

**b.** The graph of $x > -1$ is the unbounded interval $(-1, \infty)$.

**c.** The graph represents all the real numbers between $-3$ and 4, including the endpoint $-3$. This is the bounded interval $[-3, 4)$.

**d.** The graph represents all the real numbers less than or equal to 3. This is the unbounded interval $(-\infty, 3]$.

## Monitoring Progress

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**Write the interval in interval notation.**

**1.** $-7 < x < -4$

**2.** $x \leq 5$

**3.** 

## Using Set-Builder Notation

Another way to represent intervals is to write them in *set-builder notation*.

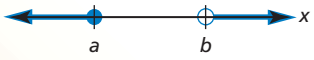
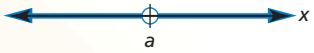
### Core Concept

#### Set-Builder Notation

**Set-builder notation** uses symbols to define a set in terms of the properties of the members of the set.

**Set-builder notation** $\{x \mid x < b\}$

**Words** the set of all real numbers $x$ such that $x$ is less than $b$

| *Set-builder Notation* | *Graph* |
|---|---|
| $\{x \mid x \leq a \text{ or } x > b\}$ |   |
| $\{x \mid x \neq a\}$ |   |

### EXAMPLE 2 Using Set-Builder Notation

Sketch the graph of each set of numbers.

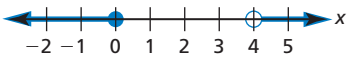
**a.** $\{x \mid 2 < x \leq 5\}$

**b.** $\{x \mid x \leq 0 \text{ or } x > 4\}$

**SOLUTION**

**a.** The real numbers in the set satisfy both $x > 2$ *and* $x \leq 5$.


**b.** The real numbers in the set satisfy either $x \leq 0$ *or* $x > 4$.




**UNDERSTANDING MATHEMATICAL TERMS**

The symbol $\in$ denotes membership in a set. The expression $x \in \mathbb{Z}$ means that $x$ is a member (or *element*) of the set of integers.

### EXAMPLE 3 Writing Set-Builder Notation

Write the set of numbers in set-builder notation.

**a.** the set of all integers greater than 5

**b.** $(-\infty, -1)$ or $(-1, \infty)$

**SOLUTION**

**a.** $x$ is greater than 5 and $x$ is an integer.

▶ $\{x \mid x > 5 \text{ and } x \in \mathbb{Z}\}$

**b.** $x$ can be any real number except $-1$.

▶ $\{x \mid x \neq -1\}$

### Monitoring Progress

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**Sketch the graph of the set of numbers.**

**4.** $\{x \mid -6 < x \leq -2\}$

**5.** $\{x \mid x \leq 0 \text{ or } x \geq 10\}$

**Write the set of numbers in set-builder notation.**

**6.** $(-\infty, -1]$ or $(1, \infty)$

**7.** the set of all integers except $-4$

# 1.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

## Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** Two real numbers $a$ and $b$ are the _____ of four different _____ intervals on the real number line.

2. **WHICH ONE DOESN'T BELONG?** The graph of which set of numbers does not belong with the other three? Explain.

| | |
|---|---|
| $(-3, 5]$ | $x > -3$ and $x \leq 5$ |
| $\{x \mid -3 < x \leq 5\}$ | the set of all integers greater than $-3$ and less than or equal to 5 |

## Monitoring Progress and Modeling with Mathematics

**In Exercises 3–6, use braces to list the elements of the set.**

3. the set of whole numbers less than 10

4. the set of odd whole numbers less than 24

5. the set of integers greater than 50

6. the set of integers less than $-8$

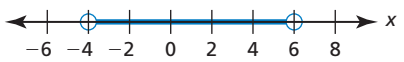
**In Exercises 7–16, write the interval in interval notation.** *(See Example 1.)*

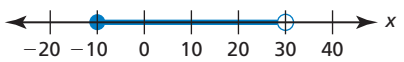
7. $3 < x < 9$

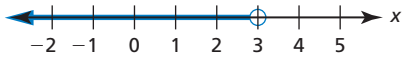
8. $-5 < x < 20$

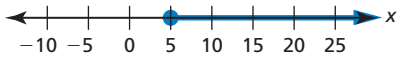
9. $x \geq -13$

10. $x \leq 58$

11. 


12. 


13. 


14. 


15. the real numbers from $-10$ through 10

16. the real numbers between 110 and 220

**In Exercises 17–20, sketch the graph of the set of numbers.** *(See Example 2.)*

17. $\{x \mid 3 < x < 12\}$

18. $\{x \mid -10 \leq x \leq 15\}$

19. $\{x \mid x < 5 \text{ or } x > 10\}$

20. $\{x \mid x \neq 4\}$

**In Exercises 21–28, write the set of numbers in set-builder notation.** *(See Example 3.)*

21. $[-5, 16)$

22. $(22, 98]$

23. $(-\infty, -4]$ or $[4, \infty)$

24. $(-\infty, 5]$ or $[14, \infty)$

25. the set of all integers less than $-20$

26. the set of all real numbers greater than 19 and less than 32

27. the set of all real numbers except 100

28. the set of all whole numbers except 50

29. **ERROR ANALYSIS** Describe and correct the error in rewriting the interval $(-\infty, -8]$ in set-builder notation.

✗ $\{x \mid x < -8\}$

30. **ERROR ANALYSIS** Describe and correct the error in rewriting the interval $[-7, 24)$ in set-builder notation.

✗ $\{x \mid x \geq -7 \text{ or } x < 24\}$

**31. MODELING WITH MATHEMATICS** The elevation relative to sea level in the United States ranges from $-282$ feet in Death Valley, California to 20,320 feet on Mount McKinley, Alaska. Write the range of elevations in interval notation and in set-builder notation.

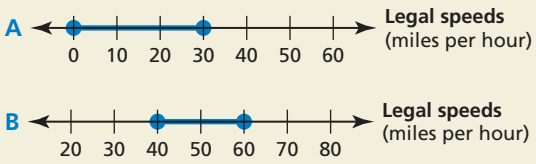
**32. MODELING WITH MATHEMATICS** The the main floor of an auditorium ranges from 6 feet below the stage to 8 feet above the stage. The floor of the balcony ranges from 26 to 37 feet above the stage. Write the range of the floor levels relative to the stage in interval notation and in set-builder notation.

**33. MAKING AN ARGUMENT** Your friend says that it is impossible to write the set

$$\{x \mid x \geq 30 \text{ and } x \leq 60, \text{ and } x \in \mathbb{W}\}$$

in interval notation. Is your friend correct? Explain.

**34. HOW DO YOU SEE IT?** The graphs show the legal driving speeds (in miles per hour) on two different roadways.




**a.** Write the legal driving speeds shown in Graph A in interval notation, in set-builder notation, and in words.

**b.** Write the legal driving speeds shown in Graph B in interval notation, in set-builder notation, and in words.

**c.** One of the roadways is a state highway and the other is a residential street. Which graph represents each roadway?

**35. NUMBER SENSE** Write each set using braces to list the elements, in interval notation, and in set-builder notation. If not possible, explain why.

**a.** the set of even whole numbers

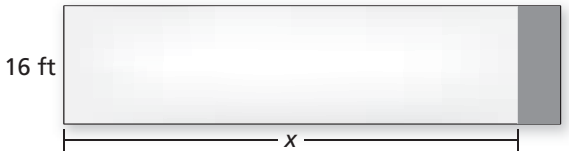
**b.** the set of real numbers less than $-4$

**c.** the set of real numbers 10 or more units from 50

**36. THOUGHT PROVOKING** Explain how you can add the same number to each member of the set of whole numbers to produce another important subset of the real numbers.

**37. MATHEMATICAL CONNECTIONS** You are marking a rectangular paintball zone that must be 34 meters wide and have a perimeter of at least 140 meters but not more than 260 meters. Find the interval for the length $x$ of the rectangular paintball zone.

**38. MATHEMATICAL CONNECTIONS** You have 20 gallons of roof coating to apply to the roof of a mobile home that is 16 feet wide. Twenty gallons covers 760 to 1000 square feet. Find the interval for the length $x$ that you will cover before you need to buy more roof coating.




## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

**Complete the table of values for the function $f$. Then graph the function.** *(Skills Review Handbook)*

**39.** $f(x) = 4x$

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $f(x)$ | | | | | |

**40.** $f(x) = 2x + 2$

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $f(x)$ | | | | | |

**41.** $f(x) = 3x^2$

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $f(x)$ | | | | | |

**42.** $f(x) = 2x^2 - 3$

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $f(x)$ | | | | | |