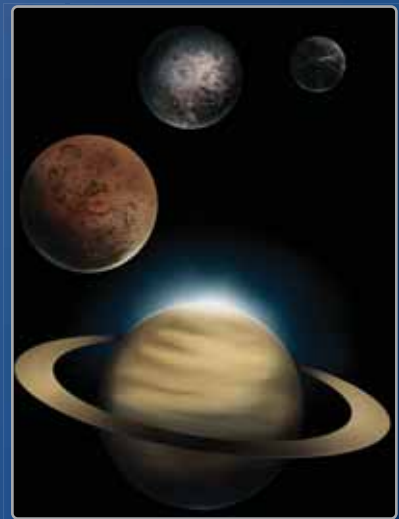


# 10 Circles

- 10.1 Lines and Segments That Intersect Circles
- 10.2 Finding Arc Measures
- 10.3 Using Chords
- 10.4 Inscribed Angles and Polygons
- 10.5 Angle Relationships in Circles
- 10.6 Segment Relationships in Circles
- 10.7 Circles in the Coordinate Plane



Seismograph (p. 582)



Saturn (p. 577)



Car (p. 554)



Dartboard (p. 547)



Bicycle Chain (p. 539)

**Mathematical Thinking:** *Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.*

# Maintaining Mathematical Proficiency

## Multiplying Binomials (A.10.B)

**Example 1** Find the product  $(x + 3)(2x - 1)$ .

$$\begin{aligned}(x + 3)(2x - 1) &= \overset{\text{First}}{x}(\overset{\text{Outer}}{2x}) + \overset{\text{Inner}}{x}(\overset{\text{Last}}{-1}) + \overset{\text{Inner}}{3}(\overset{\text{Outer}}{2x}) + \overset{\text{Last}}{(3)}(\overset{\text{Last}}{-1}) && \text{FOIL Method} \\ &= 2x^2 + (-x) + 6x + (-3) && \text{Multiply.} \\ &= 2x^2 + 5x - 3 && \text{Simplify.}\end{aligned}$$

► The product is  $2x^2 + 5x - 3$ .

**Find the product.**

- $(x + 7)(x + 4)$
- $(a + 1)(a - 5)$
- $(q - 9)(3q - 4)$
- $(2v - 7)(5v + 1)$
- $(4h + 3)(2 + h)$
- $(8 - 6b)(5 - 3b)$

## Solving Quadratic Equations by Completing the Square (A.8.A)

**Example 2** Solve  $x^2 + 8x - 3 = 0$  by completing the square.

$$\begin{aligned}x^2 + 8x - 3 &= 0 && \text{Write original equation.} \\ x^2 + 8x &= 3 && \text{Add 3 to each side.} \\ x^2 + 8x + 4^2 &= 3 + 4^2 && \text{Complete the square by adding } \left(\frac{8}{2}\right)^2, \text{ or } 4^2, \text{ to each side.} \\ (x + 4)^2 &= 19 && \text{Write the left side as a square of a binomial.} \\ x + 4 &= \pm\sqrt{19} && \text{Take the square root of each side.} \\ x &= -4 \pm\sqrt{19} && \text{Subtract 4 from each side.}\end{aligned}$$

► The solutions are  $x = -4 + \sqrt{19} \approx 0.36$  and  $x = -4 - \sqrt{19} \approx -8.36$ .

**Solve the equation by completing the square. Round your answer to the nearest hundredth, if necessary.**

- $x^2 - 2x = 5$
- $r^2 + 10r = -7$
- $w^2 - 8w = 9$
- $p^2 + 10p - 4 = 0$
- $k^2 - 4k - 7 = 0$
- $-z^2 + 2z = 1$

**13. ABSTRACT REASONING** Write an expression that represents the product of two consecutive positive odd integers. Explain your reasoning.

# Mathematical Thinking

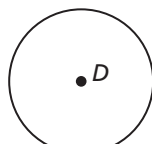
Mathematically proficient students use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. (G.1.B)

## Analyzing Relationships of Circles

### Core Concept

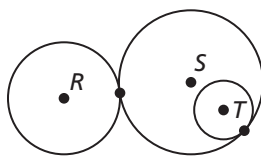
#### Circles and Tangent Circles

A **circle** is the set of all points in a plane that are equidistant from a given point called the **center** of the circle. A circle with center  $D$  is called “circle  $D$ ” and can be written as  $\odot D$ .



circle  $D$ , or  $\odot D$

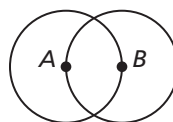
Coplanar circles that intersect in one point are called **tangent circles**.



$\odot R$  and  $\odot S$  are tangent circles.  
 $\odot S$  and  $\odot T$  are tangent circles.

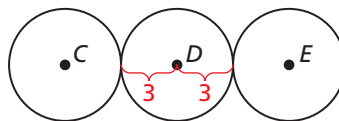
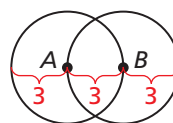
#### EXAMPLE 1 Relationships of Circles and Tangent Circles

- Each circle at the right consists of points that are 3 units from the center. What is the greatest distance from any point on  $\odot A$  to any point on  $\odot B$ ?
- Three circles,  $\odot C$ ,  $\odot D$ , and  $\odot E$ , consist of points that are 3 units from their centers. The centers  $C$ ,  $D$ , and  $E$  of the circles are collinear,  $\odot C$  is tangent to  $\odot D$ , and  $\odot D$  is tangent to  $\odot E$ . What is the distance from  $\odot C$  to  $\odot E$ ?



#### SOLUTION

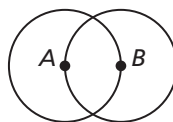
- Because the points on each circle are 3 units from the center, the greatest distance from any point on  $\odot A$  to any point on  $\odot B$  is  $3 + 3 + 3 = 9$  units.
- Because  $C$ ,  $D$ , and  $E$  are collinear,  $\odot C$  is tangent to  $\odot D$ , and  $\odot D$  is tangent to  $\odot E$ , the circles are as shown. So, the distance from  $\odot C$  to  $\odot E$  is  $3 + 3 = 6$  units.



## Monitoring Progress

Let  $\odot A$ ,  $\odot B$ , and  $\odot C$  consist of points that are 3 units from the centers.

- Draw  $\odot C$  so that it passes through points  $A$  and  $B$  in the figure at the right. Explain your reasoning.
- Draw  $\odot A$ ,  $\odot B$ , and  $\odot C$  so that each is tangent to the other two. Draw a larger circle,  $\odot D$ , that is tangent to each of the other three circles. Is the distance from point  $D$  to a point on  $\odot D$  less than, greater than, or equal to 6? Explain.



# 10.1 Lines and Segments That Intersect Circles



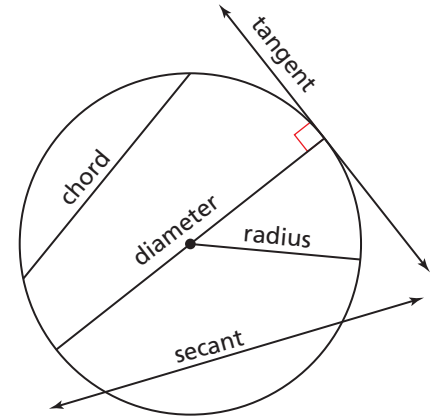
TEXAS ESSENTIAL  
KNOWLEDGE AND SKILLS

G.5.A  
G.12.A

**Essential Question** What are the definitions of the lines and segments that intersect a circle?

## EXPLORATION 1 Lines and Line Segments That Intersect Circles

**Work with a partner.** The drawing at the right shows five lines or segments that intersect a circle. Use the relationships shown to write a definition for each type of line or segment. Then use the Internet or some other resource to verify your definitions.

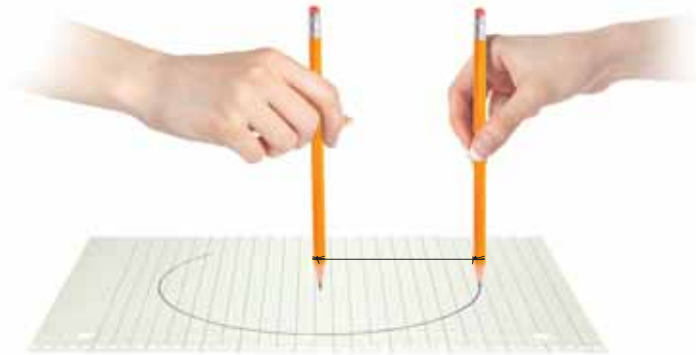


- Chord:
- Secant:
- Tangent:
- Radius:
- Diameter:

## EXPLORATION 2 Using String to Draw a Circle

**Work with a partner.** Use two pencils, a piece of string, and a piece of paper.

- Tie the two ends of the piece of string loosely around the two pencils.
- Anchor one pencil on the paper at the center of the circle. Use the other pencil to draw a circle around the anchor point while using slight pressure to keep the string taut. Do not let the string wind around either pencil.



### REASONING

To be proficient in math, you need to know and flexibly use different properties of operations and objects.

- Explain how the distance between the two pencil points as you draw the circle is related to two of the lines or line segments you defined in Exploration 1.

## Communicate Your Answer

- What are the definitions of the lines and segments that intersect a circle?
- Of the five types of lines and segments in Exploration 1, which one is a subset of another? Explain.
- Explain how to draw a circle with a diameter of 8 inches.

# 10.1 Lesson

## Core Vocabulary

circle, p. 534  
 center, p. 534  
 radius, p. 534  
 chord, p. 534  
 diameter, p. 534  
 secant, p. 534  
 tangent, p. 534  
 point of tangency, p. 534  
 tangent circles, p. 535  
 concentric circles, p. 535  
 common tangent, p. 535

## READING

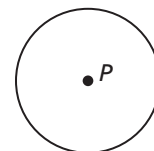
The words “radius” and “diameter” refer to lengths as well as segments. For a given circle, think of a radius and a diameter as segments and the radius and the diameter as lengths.

## What You Will Learn

- ▶ Identify special segments and lines.
- ▶ Draw and identify common tangents.
- ▶ Use properties of tangents.

## Identifying Special Segments and Lines

A **circle** is the set of all points in a plane that are equidistant from a given point called the **center** of the circle. A circle with center  $P$  is called “circle  $P$ ” and can be written as  $\odot P$ .



circle  $P$ , or  $\odot P$

## Core Concept

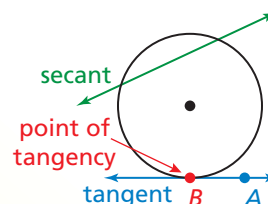
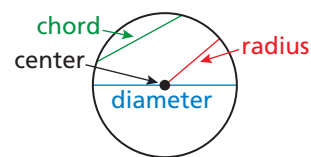
### Lines and Segments That Intersect Circles

A segment whose endpoints are the center and any point on a circle is a **radius**.

A **chord** is a segment whose endpoints are on a circle. A **diameter** is a chord that contains the center of the circle.

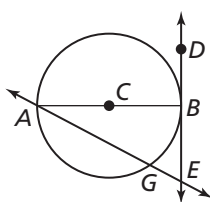
A **secant** is a line that intersects a circle in two points.

A **tangent** is a line in the plane of a circle that intersects the circle in exactly one point, the **point of tangency**. The *tangent ray*  $\overrightarrow{AB}$  and the *tangent segment*  $\overline{AB}$  are also called tangents.



## EXAMPLE 1

### Identifying Special Segments and Lines



Tell whether the line, ray, or segment is best described as a *radius*, *chord*, *diameter*, *secant*, or *tangent* of  $\odot C$ .

- |                          |                          |
|--------------------------|--------------------------|
| a. $\overline{AC}$       | b. $\overline{AB}$       |
| c. $\overrightarrow{DE}$ | d. $\overrightarrow{AE}$ |

### SOLUTION

- a.  $\overline{AC}$  is a radius because  $C$  is the center and  $A$  is a point on the circle.
- b.  $\overline{AB}$  is a diameter because it is a chord that contains the center  $C$ .
- c.  $\overrightarrow{DE}$  is a tangent ray because it is contained in a line that intersects the circle in exactly one point.
- d.  $\overrightarrow{AE}$  is a secant because it is a line that intersects the circle in two points.

## STUDY TIP

In this book, assume that all segments, rays, or lines that appear to be tangent to a circle are tangents.

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1. In Example 1, what word best describes  $\overline{AG}$ ?  $\overline{CB}$ ?
2. In Example 1, name a tangent and a tangent segment.

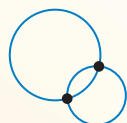
## Drawing and Identifying Common Tangents

### Core Concept

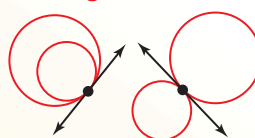
#### Coplanar Circles and Common Tangents

In a plane, two circles can intersect in two points, one point, or no points. Coplanar circles that intersect in one point are called **tangent circles**. Coplanar circles that have a common center are called **concentric circles**.

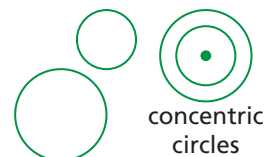
2 points of intersection



1 point of intersection (tangent circles)



no points of intersection

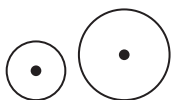


A line or segment that is tangent to two coplanar circles is called a **common tangent**. A *common internal tangent* intersects the segment that joins the centers of the two circles. A *common external tangent* does not intersect the segment that joins the centers of the two circles.

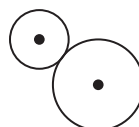
#### EXAMPLE 2 Drawing and Identifying Common Tangents

Tell how many common tangents the circles have and draw them. Use blue to indicate common external tangents and red to indicate common internal tangents.

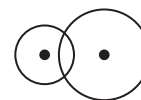
a.



b.



c.



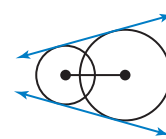
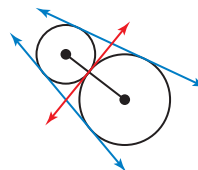
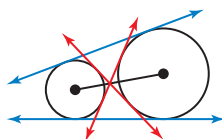
#### SOLUTION

Draw the segment that joins the centers of the two circles. Then draw the common tangents. Use blue to indicate lines that do not intersect the segment joining the centers and red to indicate lines that intersect the segment joining the centers.

a. 4 common tangents

b. 3 common tangents

c. 2 common tangents



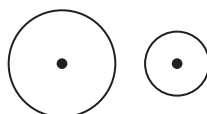
#### Monitoring Progress



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Tell how many common tangents the circles have and draw them. State whether the tangents are external tangents or internal tangents.

3.



4.



5.

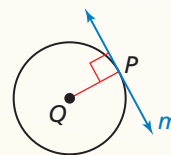


## Using Properties of Tangents

### Theorems

#### Theorem 10.1 Tangent Line to Circle Theorem

In a plane, a line is tangent to a circle if and only if the line is perpendicular to a radius of the circle at its endpoint on the circle.

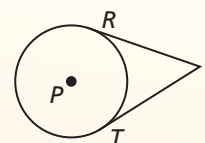


Line  $m$  is tangent to  $\odot Q$  if and only if  $m \perp \overline{QP}$ .

*Proof* Ex. 47, p. 540

#### Theorem 10.2 External Tangent Congruence Theorem

Tangent segments from a common external point are congruent.



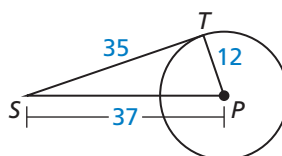
If  $\overline{SR}$  and  $\overline{ST}$  are tangent segments, then  $\overline{SR} \cong \overline{ST}$ .

*Proof* Ex. 46, p. 540

#### EXAMPLE 3

#### Verifying a Tangent to a Circle

Is  $\overline{ST}$  tangent to  $\odot P$ ?



#### SOLUTION

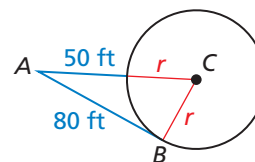
Use the Converse of the Pythagorean Theorem (Theorem 9.2). Because  $12^2 + 35^2 = 37^2$ ,  $\triangle PTS$  is a right triangle and  $\overline{ST} \perp \overline{PT}$ . So,  $\overline{ST}$  is perpendicular to a radius of  $\odot P$  at its endpoint on  $\odot P$ .

► By the Tangent Line to Circle Theorem,  $\overline{ST}$  is tangent to  $\odot P$ .

#### EXAMPLE 4

#### Finding the Radius of a Circle

In the diagram, point  $B$  is a point of tangency. Find the radius  $r$  of  $\odot C$ .



#### SOLUTION

You know from the Tangent Line to Circle Theorem that  $\overline{AB} \perp \overline{BC}$ , so  $\triangle ABC$  is a right triangle. You can use the Pythagorean Theorem (Theorem 9.1).

$$AC^2 = BC^2 + AB^2 \quad \text{Pythagorean Theorem}$$

$$(r + 50)^2 = r^2 + 80^2 \quad \text{Substitute.}$$

$$r^2 + 100r + 2500 = r^2 + 6400 \quad \text{Multiply.}$$

$$100r = 3900 \quad \text{Subtract } r^2 \text{ and } 2500 \text{ from each side.}$$

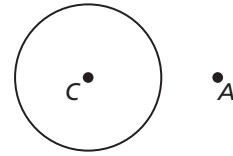
$$r = 39 \quad \text{Divide each side by } 100.$$

► The radius is 39 feet.

## CONSTRUCTION

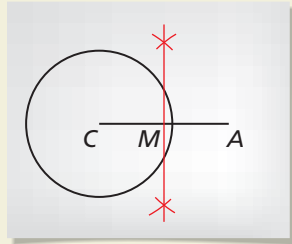
### Constructing a Tangent to a Circle

Given  $\odot C$  and point  $A$ , construct a line tangent to  $\odot C$  that passes through  $A$ . Use a compass and straightedge.



### SOLUTION

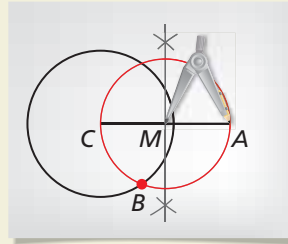
#### Step 1



#### Find a midpoint

Draw  $\overline{AC}$ . Construct the bisector of the segment and label the midpoint  $M$ .

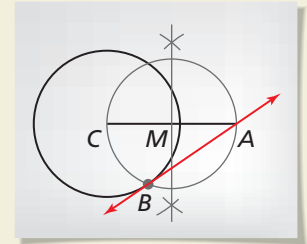
#### Step 2



#### Draw a circle

Construct  $\odot M$  with radius  $MA$ . Label one of the points where  $\odot M$  intersects  $\odot C$  as point  $B$ .

#### Step 3



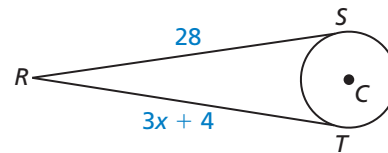
#### Construct a tangent line

Draw  $\overline{AB}$ . It is a tangent to  $\odot C$  that passes through  $A$ .

## EXAMPLE 5

### Using Properties of Tangents

$\overline{RS}$  is tangent to  $\odot C$  at  $S$ , and  $\overline{RT}$  is tangent to  $\odot C$  at  $T$ . Find the value of  $x$ .



### SOLUTION

$$RS = RT$$

External Tangent Congruence Theorem

$$28 = 3x + 4$$

Substitute.

$$8 = x$$

Solve for  $x$ .

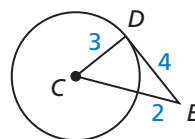
▶ The value of  $x$  is 8.

### Monitoring Progress

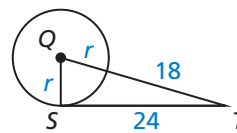


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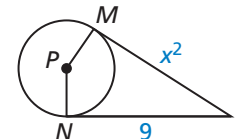
6. Is  $\overline{DE}$  tangent to  $\odot C$ ?



7.  $\overline{ST}$  is tangent to  $\odot Q$ . Find the radius of  $\odot Q$ .



8. Points  $M$  and  $N$  are points of tangency. Find the value(s) of  $x$ .





# 10.1 Exercises

## Vocabulary and Core Concept Check

- WRITING** How are chords and secants alike? How are they different?
- WRITING** Explain how you can determine from the context whether the words *radius* and *diameter* are referring to segments or lengths.
- COMPLETE THE SENTENCE** Coplanar circles that have a common center are called \_\_\_\_\_.
- WHICH ONE DOESN'T BELONG?** Which segment does *not* belong with the other three? Explain your reasoning.

chord

radius

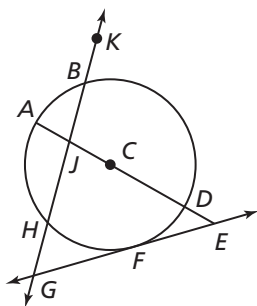
tangent

diameter

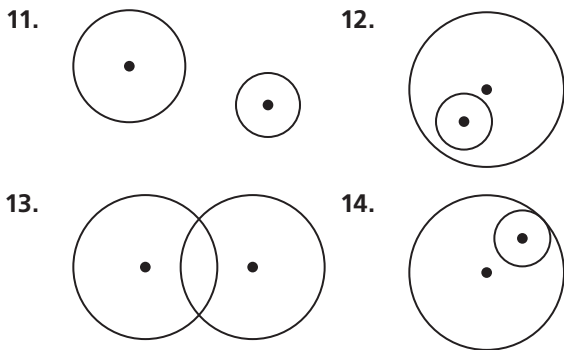
## Monitoring Progress and Modeling with Mathematics

In Exercises 5–10, use the diagram. (See Example 1.)

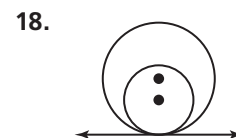
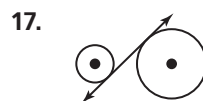
- Name the circle.
- Name two radii.
- Name two chords.
- Name a diameter.
- Name a secant.
- Name a tangent and a point of tangency.



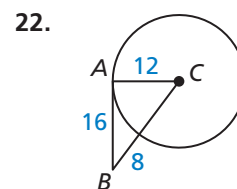
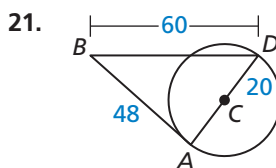
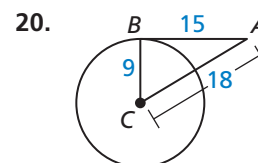
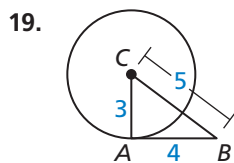
In Exercises 11–14, copy the diagram. Tell how many common tangents the circles have and draw them. (See Example 2.)



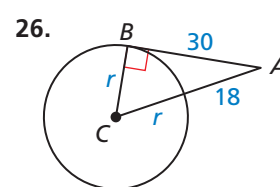
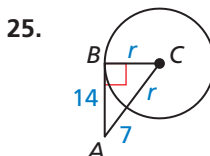
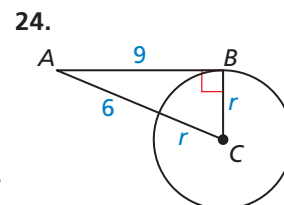
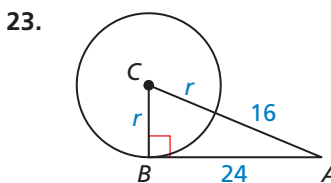
In Exercises 15–18, tell whether the common tangent is *internal* or *external*.



In Exercises 19–22, tell whether  $\overline{AB}$  is tangent to  $\odot C$ . Explain your reasoning. (See Example 3.)



In Exercises 23–26, point  $B$  is a point of tangency. Find the radius  $r$  of  $\odot C$ . (See Example 4.)

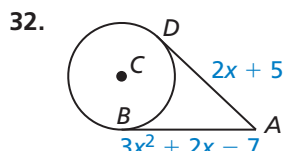
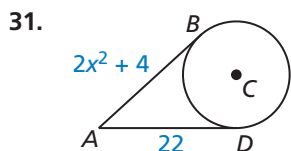
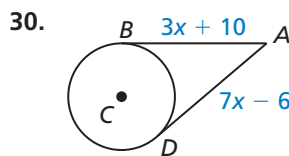
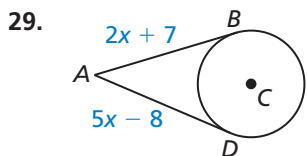


**CONSTRUCTION** In Exercises 27 and 28, construct  $\odot C$  with the given radius and point  $A$  outside of  $\odot C$ . Then construct a line tangent to  $\odot C$  that passes through  $A$ .

27.  $r = 2$  in.

28.  $r = 4.5$  cm

In Exercises 29–32, points  $B$  and  $D$  are points of tangency. Find the value(s) of  $x$ . (See Example 5.)



33. **ERROR ANALYSIS** Describe and correct the error in determining whether  $\overline{XY}$  is tangent to  $\odot Z$ .

Because  $11^2 + 60^2 = 61^2$ ,  $\triangle XYZ$  is a right triangle. So,  $\overline{XY}$  is tangent to  $\odot Z$ .

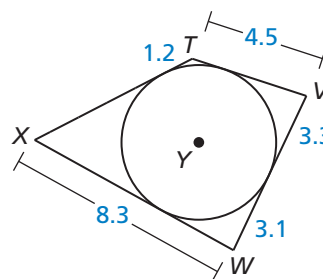
34. **ERROR ANALYSIS** Describe and correct the error in finding the radius of  $\odot T$ .

$39^2 - 36^2 = 15^2$   
So, the radius is 15.

35. **ABSTRACT REASONING** For a point outside of a circle, how many lines exist tangent to the circle that pass through the point? How many such lines exist for a point on the circle? inside the circle? Explain your reasoning.

36. **CRITICAL THINKING** When will two lines tangent to the same circle not intersect? Justify your answer.

37. **USING STRUCTURE** Each side of quadrilateral  $TVWX$  is tangent to  $\odot Y$ . Find the perimeter of the quadrilateral.



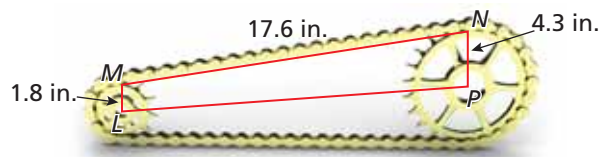
38. **LOGIC** In  $\odot C$ , radii  $\overline{CA}$  and  $\overline{CB}$  are perpendicular.  $\overline{BD}$  and  $\overline{AD}$  are tangent to  $\odot C$ .

- Sketch  $\odot C$ ,  $\overline{CA}$ ,  $\overline{CB}$ ,  $\overline{BD}$ , and  $\overline{AD}$ .
- What type of quadrilateral is  $CADB$ ? Explain your reasoning.

39. **MAKING AN ARGUMENT** Two bike paths are tangent to an approximately circular pond. Your class is building a nature trail that begins at the intersection  $B$  of the bike paths and runs between the bike paths and over a bridge through the center  $P$  of the pond. Your classmate uses the Converse of the Angle Bisector Theorem (Theorem 6.4) to conclude that the trail must bisect the angle formed by the bike paths. Is your classmate correct? Explain your reasoning.

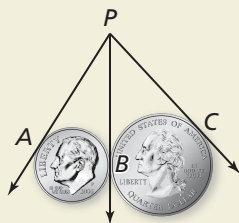


40. **MODELING WITH MATHEMATICS** A bicycle chain is pulled tightly so that  $\overline{MN}$  is a common tangent of the gears. Find the distance  $MN$  between the centers of the gears.

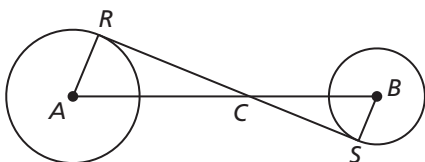


41. **WRITING** Explain why the diameter of a circle is the longest chord of the circle.

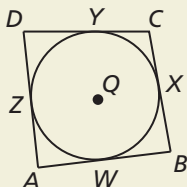
42. **HOW DO YOU SEE IT?** In the figure,  $\overrightarrow{PA}$  is tangent to the dime,  $\overrightarrow{PC}$  is tangent to the quarter, and  $\overrightarrow{PB}$  is a common internal tangent. How do you know that  $\overline{PA} \cong \overline{PB} \cong \overline{PC}$ ?



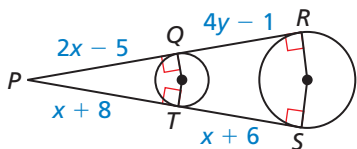
43. **PROOF** In the diagram,  $\overline{RS}$  is a common internal tangent to  $\odot A$  and  $\odot B$ . Prove that  $\frac{AC}{BC} = \frac{RC}{SC}$ .



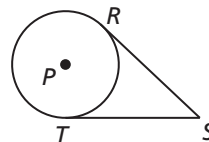
44. **THOUGHT PROVOKING** A polygon is *circumscribed* about a circle when every side of the polygon is tangent to the circle. In the diagram, quadrilateral  $ABCD$  is circumscribed about  $\odot Q$ . Is it always true that  $AB + CD = AD + BC$ ? Justify your answer.



45. **MATHEMATICAL CONNECTIONS** Find the values of  $x$  and  $y$ . Justify your answer.

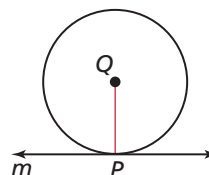


46. **PROVING A THEOREM** Prove the External Tangent Congruence Theorem (Theorem 10.2).



- Given**  $\overline{SR}$  and  $\overline{ST}$  are tangent to  $\odot P$ .  
**Prove**  $\overline{SR} \cong \overline{ST}$

47. **PROVING A THEOREM** Use the diagram to prove each part of the biconditional in the Tangent Line to Circle Theorem (Theorem 10.1).



- a. Prove indirectly that if a line is tangent to a circle, then it is perpendicular to a radius. (*Hint*: If you assume line  $m$  is not perpendicular to  $\overline{QP}$ , then the perpendicular segment from point  $Q$  to line  $m$  must intersect line  $m$  at some other point  $R$ .)

**Given** Line  $m$  is tangent to  $\odot Q$  at point  $P$ .

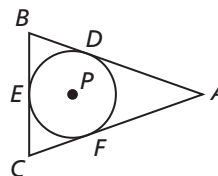
**Prove**  $m \perp \overline{QP}$

- b. Prove indirectly that if a line is perpendicular to a radius at its endpoint, then the line is tangent to the circle.

**Given**  $m \perp \overline{QP}$

**Prove** Line  $m$  is tangent to  $\odot Q$ .

48. **REASONING** In the diagram,  $AB = AC = 12$ ,  $BC = 8$ , and all three segments are tangent to  $\odot P$ . What is the radius of  $\odot P$ ? Justify your answer.

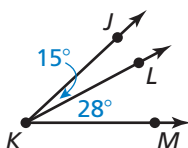


## Maintaining Mathematical Proficiency

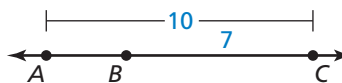
Reviewing what you learned in previous grades and lessons

Find the indicated measure. (Section 1.2 and Section 1.5)

49.  $m\angle JKM$



50.  $AB$



# 10.2 Finding Arc Measures



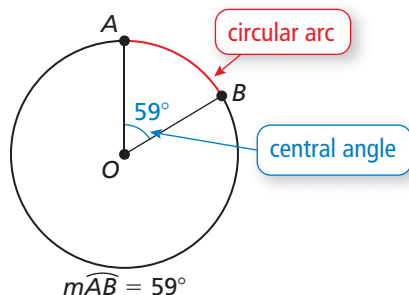
TEXAS ESSENTIAL  
KNOWLEDGE AND SKILLS  
G.3.C  
G.12.A

## Essential Question

How are circular arcs measured?

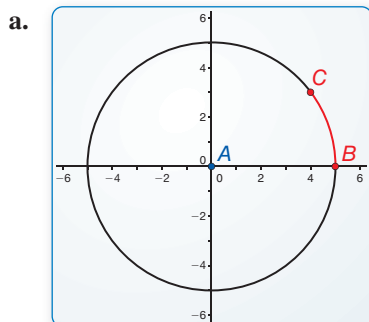
A **central angle** of a circle is an angle whose vertex is the center of the circle. A *circular arc* is a portion of a circle, as shown below. The measure of a circular arc is the measure of its central angle.

If  $m\angle AOB < 180^\circ$ , then the circular arc is called a **minor arc** and is denoted by  $\widehat{AB}$ .

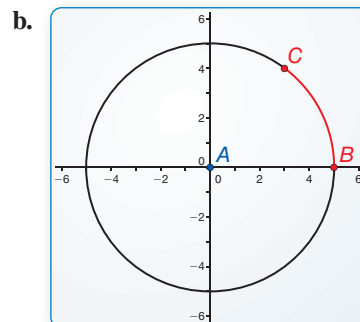


### EXPLORATION 1 Measuring Circular Arcs

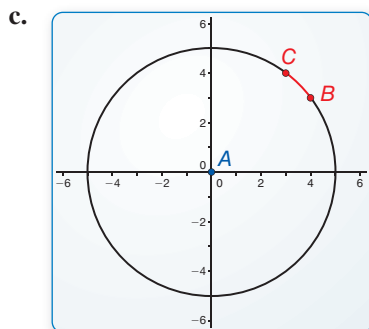
**Work with a partner.** Use dynamic geometry software to find the measure of  $\widehat{BC}$ . Verify your answers using trigonometry.



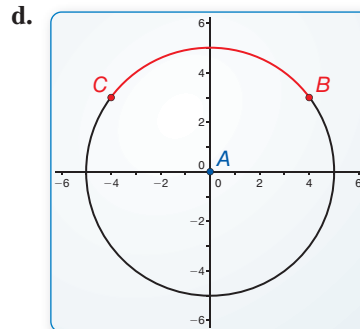
Points  
 $A(0, 0)$   
 $B(5, 0)$   
 $C(4, 3)$



Points  
 $A(0, 0)$   
 $B(5, 0)$   
 $C(3, 4)$



Points  
 $A(0, 0)$   
 $B(4, 3)$   
 $C(3, 4)$



Points  
 $A(0, 0)$   
 $B(4, 3)$   
 $C(-4, 3)$

### SELECTING TOOLS

To be proficient in math, you need to use technological tools to explore and deepen your understanding of concepts.

## Communicate Your Answer

- How are circular arcs measured?
- Use dynamic geometry software to draw a circular arc with the given measure.
  - $30^\circ$
  - $45^\circ$
  - $60^\circ$
  - $90^\circ$

# 10.2 Lesson

## Core Vocabulary

- central angle, p. 542
- minor arc, p. 542
- major arc, p. 542
- semicircle, p. 542
- measure of a minor arc, p. 542
- measure of a major arc, p. 542
- adjacent arcs, p. 543
- congruent circles, p. 544
- congruent arcs, p. 544
- similar arcs, p. 545

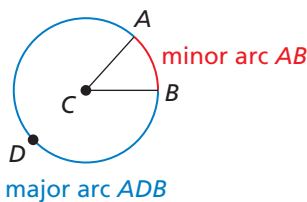
## What You Will Learn

- ▶ Find arc measures.
- ▶ Identify congruent arcs.
- ▶ Prove circles are similar.

## Finding Arc Measures

A **central angle** of a circle is an angle whose vertex is the center of the circle. In the diagram,  $\angle ACB$  is a central angle of  $\odot C$ .

If  $m\angle ACB$  is less than  $180^\circ$ , then the points on  $\odot C$  that lie in the interior of  $\angle ACB$  form a **minor arc** with endpoints  $A$  and  $B$ . The points on  $\odot C$  that do not lie on the minor arc  $AB$  form a **major arc** with endpoints  $A$  and  $B$ . A **semicircle** is an arc with endpoints that are the endpoints of a diameter.



Minor arcs are named by their endpoints. The minor arc associated with  $\angle ACB$  is named  $\widehat{AB}$ . Major arcs and semicircles are named by their endpoints and a point on the arc. The major arc associated with  $\angle ACB$  can be named  $\widehat{ADB}$ .

## STUDY TIP

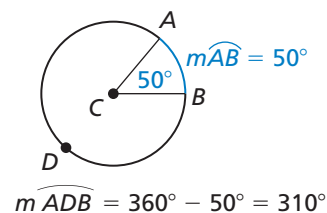
The measure of a minor arc is less than  $180^\circ$ . The measure of a major arc is greater than  $180^\circ$ .

## Core Concept

### Measuring Arcs

The **measure of a minor arc** is the measure of its central angle. The expression  $m\widehat{AB}$  is read as “the measure of arc  $AB$ .”

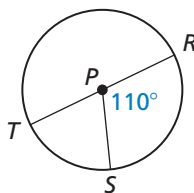
The measure of the entire circle is  $360^\circ$ . The **measure of a major arc** is the difference of  $360^\circ$  and the measure of the related minor arc. The measure of a semicircle is  $180^\circ$ .



### EXAMPLE 1 Finding Measures of Arcs

Find the measure of each arc of  $\odot P$ , where  $\overline{RT}$  is a diameter.

- a.  $\widehat{RS}$
- b.  $\widehat{RTS}$
- c.  $\widehat{RST}$



### SOLUTION

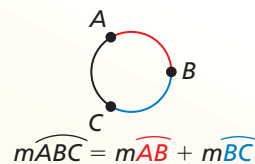
- a.  $\widehat{RS}$  is a minor arc, so  $m\widehat{RS} = m\angle RPS = 110^\circ$ .
- b.  $\widehat{RTS}$  is a major arc, so  $m\widehat{RTS} = 360^\circ - 110^\circ = 250^\circ$ .
- c.  $\overline{RT}$  is a diameter, so  $\widehat{RST}$  is a semicircle, and  $m\widehat{RST} = 180^\circ$ .

Two arcs of the same circle are **adjacent arcs** when they intersect at exactly one point. You can add the measures of two adjacent arcs.

## Postulate

### Postulate 10.1 Arc Addition Postulate

The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.



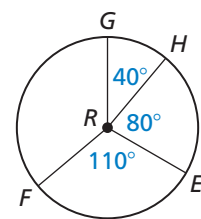
### EXAMPLE 2 Using the Arc Addition Postulate

Find the measure of each arc.

- a.  $\widehat{GE}$       b.  $\widehat{GEF}$       c.  $\widehat{GF}$

#### SOLUTION

- a.  $m\widehat{GE} = m\widehat{GH} + m\widehat{HE} = 40^\circ + 80^\circ = 120^\circ$   
 b.  $m\widehat{GEF} = m\widehat{GE} + m\widehat{EF} = 120^\circ + 110^\circ = 230^\circ$   
 c.  $m\widehat{GF} = 360^\circ - m\widehat{GEF} = 360^\circ - 230^\circ = 130^\circ$



### EXAMPLE 3 Finding Measures of Arcs

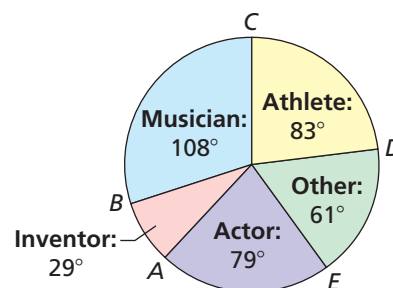
A recent survey asked teenagers whether they would rather meet a famous musician, athlete, actor, inventor, or other person. The circle graph shows the results. Find the indicated arc measures.

- a.  $m\widehat{AC}$       b.  $m\widehat{ACD}$   
 c.  $m\widehat{ADC}$       d.  $m\widehat{EBD}$

#### SOLUTION

- a.  $m\widehat{AC} = m\widehat{AB} + m\widehat{BC}$   
 $= 29^\circ + 108^\circ$   
 $= 137^\circ$   
 b.  $m\widehat{ACD} = m\widehat{AC} + m\widehat{CD}$   
 $= 137^\circ + 83^\circ$   
 $= 220^\circ$   
 c.  $m\widehat{ADC} = 360^\circ - m\widehat{AC}$   
 $= 360^\circ - 137^\circ$   
 $= 223^\circ$   
 d.  $m\widehat{EBD} = 360^\circ - m\widehat{ED}$   
 $= 360^\circ - 61^\circ$   
 $= 299^\circ$

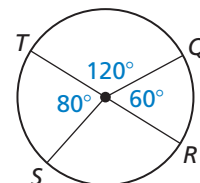
Whom Would You Rather Meet?



## Monitoring Progress Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

Identify the given arc as a *major arc*, *minor arc*, or *semicircle*. Then find the measure of the arc.

1.  $\widehat{TQ}$       2.  $\widehat{QRT}$       3.  $\widehat{TQR}$   
 4.  $\widehat{QS}$       5.  $\widehat{TS}$       6.  $\widehat{RST}$



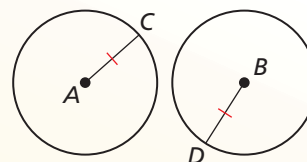
## Identifying Congruent Arcs

Two circles are **congruent circles** if and only if a rigid motion or a composition of rigid motions maps one circle onto the other. This statement is equivalent to the Congruent Circles Theorem below.

### Theorem

#### Theorem 10.3 Congruent Circles Theorem

Two circles are congruent circles if and only if they have the same radius.



*Proof* Ex. 35, p. 548

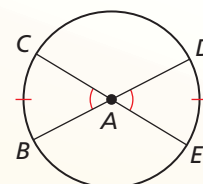
$\odot A \cong \odot B$  if and only if  $\overline{AC} \cong \overline{BD}$ .

Two arcs are **congruent arcs** if and only if they have the same measure and they are arcs of the same circle or of congruent circles.

### Theorem

#### Theorem 10.4 Congruent Central Angles Theorem

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding central angles are congruent.

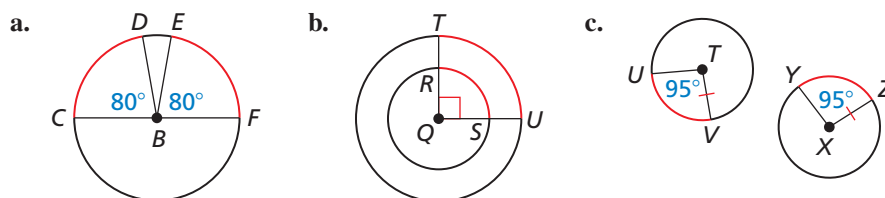


*Proof* Ex. 37, p. 548

$\widehat{BC} \cong \widehat{DE}$  if and only if  $\angle BAC \cong \angle DAE$ .

### EXAMPLE 4 Identifying Congruent Arcs

Tell whether the red arcs are congruent. Explain why or why not.



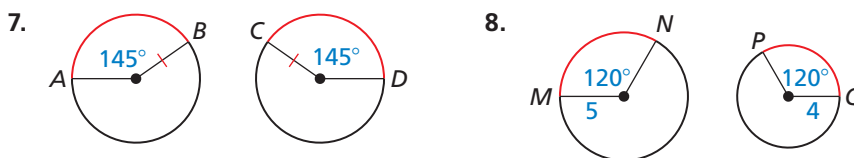
### STUDY TIP

The two circles in part (c) are congruent by the Congruent Circles Theorem because they have the same radius.

### SOLUTION

- $\widehat{CD} \cong \widehat{EF}$  because they are arcs of the same circle and  $m\widehat{CD} = m\widehat{EF}$ . Also,  $\angle CBD \cong \angle FBE$  by the Congruent Central Angles Theorem.
- $\widehat{RS}$  and  $\widehat{TU}$  have the same measure, but are not congruent because they are arcs of circles that are not congruent.
- $\widehat{UV} \cong \widehat{YZ}$  because they are arcs of congruent circles and  $m\widehat{UV} = m\widehat{YZ}$ . Also,  $\angle UTV \cong \angle YXZ$  by the Congruent Central Angles Theorem.

Tell whether the red arcs are congruent. Explain why or why not.



## Proving Circles Are Similar

### Theorem

#### Theorem 10.5 Similar Circles Theorem

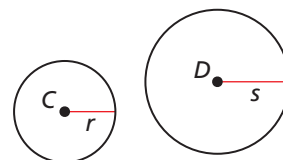
All circles are similar.

*Proof* p. 545; Ex. 33, p. 548

#### **PROOF** Similar Circles Theorem

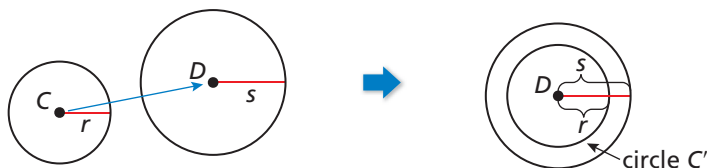
All circles are similar.

**Given**  $\odot C$  with center  $C$  and radius  $r$ ,  
 $\odot D$  with center  $D$  and radius  $s$



**Prove**  $\odot C \sim \odot D$

First, translate  $\odot C$  so that point  $C$  maps to point  $D$ . The image of  $\odot C$  is  $\odot C'$  with center  $D$ . So,  $\odot C'$  and  $\odot D$  are concentric circles.



$\odot C'$  is the set of all points that are  $r$  units from point  $D$ . Dilate  $\odot C'$  using center of dilation  $D$  and scale factor  $\frac{s}{r}$ .



This dilation maps the set of all the points that are  $r$  units from point  $D$  to the set of all points that are  $\frac{s}{r}(r) = s$  units from point  $D$ .  $\odot D$  is the set of all points that are  $s$  units from point  $D$ . So, this dilation maps  $\odot C'$  to  $\odot D$ .

Because a similarity transformation maps  $\odot C$  to  $\odot D$ ,  $\odot C \sim \odot D$ .

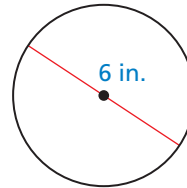
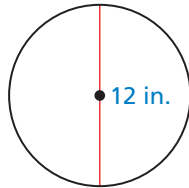
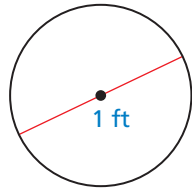
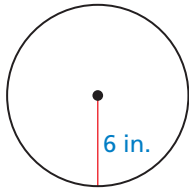
Two arcs are **similar arcs** if and only if they have the same measure. All congruent arcs are similar, but not all similar arcs are congruent. For instance, in Example 4, the pairs of arcs in parts (a), (b), and (c) are similar but only the pairs of arcs in parts (a) and (c) are congruent.



# 10.2 Exercises

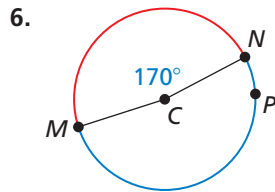
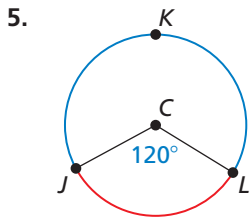
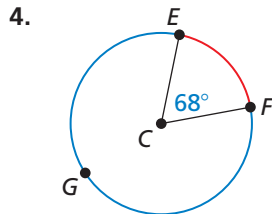
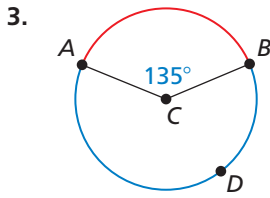
## Vocabulary and Core Concept Check

- VOCABULARY** Copy and complete: If  $\angle ACB$  and  $\angle DCE$  are congruent central angles of  $\odot C$ , then  $\widehat{AB}$  and  $\widehat{DE}$  are \_\_\_\_\_.
- WHICH ONE DOESN'T BELONG?** Which circle does *not* belong with the other three? Explain your reasoning.



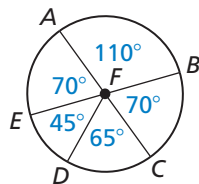
## Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, name the red minor arc and find its measure. Then name the blue major arc and find its measure.



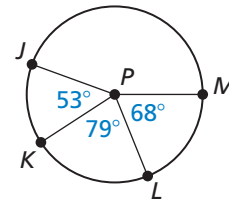
In Exercises 7–14, identify the given arc as a *major arc*, *minor arc*, or *semicircle*. Then find the measure of the arc. (See Example 1.)

- $\widehat{BC}$
- $\widehat{DC}$
- $\widehat{ED}$
- $\widehat{AE}$
- $\widehat{EAB}$
- $\widehat{ABC}$
- $\widehat{BAC}$
- $\widehat{EBD}$

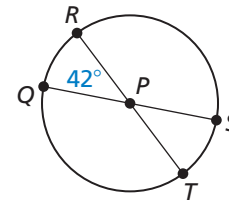


In Exercises 15 and 16, find the measure of each arc. (See Example 2.)

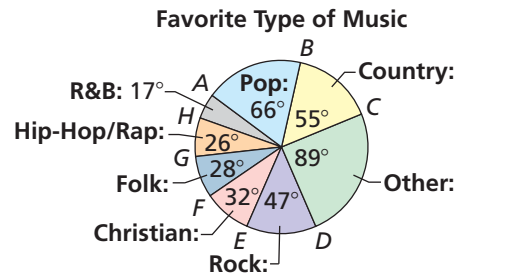
- $\widehat{JL}$
  - $\widehat{KM}$
  - $\widehat{JLM}$
  - $\widehat{JM}$



- $\widehat{RS}$
  - $\widehat{QRS}$
  - $\widehat{QST}$
  - $\widehat{QT}$

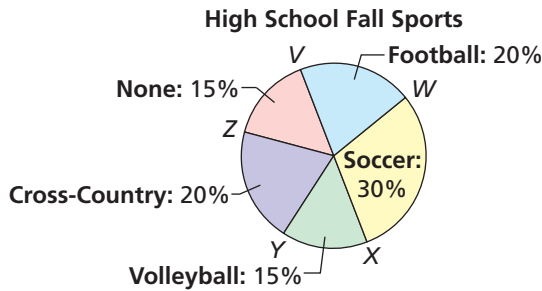


- MODELING WITH MATHEMATICS** A recent survey asked high school students their favorite type of music. The results are shown in the circle graph. Find each indicated arc measure. (See Example 3.)



- $m\widehat{AE}$
- $m\widehat{ACE}$
- $m\widehat{GDC}$
- $m\widehat{BHC}$
- $m\widehat{FD}$
- $m\widehat{FBD}$

18. **ABSTRACT REASONING** The circle graph shows the percentages of students enrolled in fall sports at a high school. Is it possible to find the measure of each minor arc? If so, find the measure of the arc for each category shown. If not, explain why it is not possible.



In Exercises 19–22, tell whether the red arcs are congruent. Explain why or why not. (See Example 4.)

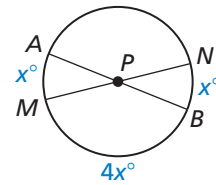
- 19.
- 20.
- 21.
- 22.
- 23.
- 24.

**MATHEMATICAL CONNECTIONS** In Exercises 23 and 24, find the value of  $x$ . Then find the measure of the red arc.

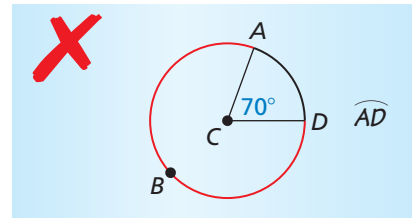
- 23.
- 24.

25. **MAKING AN ARGUMENT** Your friend claims that any two arcs with the same measure are similar. Your cousin claims that any two arcs with the same measure are congruent. Who is correct? Explain.

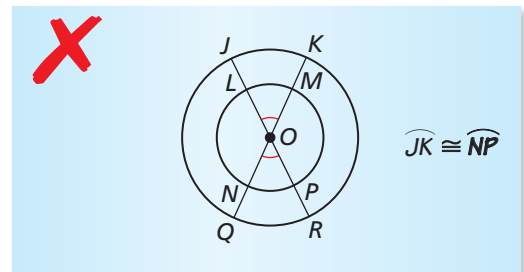
26. **MAKING AN ARGUMENT** Your friend claims that there is not enough information given to find the value of  $x$ . Is your friend correct? Explain your reasoning.



27. **ERROR ANALYSIS** Describe and correct the error in naming the red arc.



28. **ERROR ANALYSIS** Describe and correct the error in naming congruent arcs.



29. **ATTENDING TO PRECISION** Two diameters of  $\odot P$  are  $\overline{AB}$  and  $\overline{CD}$ . Find  $m\widehat{ACD}$  and  $m\widehat{AC}$  when  $m\widehat{AD} = 20^\circ$ .
30. **REASONING** In  $\odot R$ ,  $m\widehat{AB} = 60^\circ$ ,  $m\widehat{BC} = 25^\circ$ ,  $m\widehat{CD} = 70^\circ$ , and  $m\widehat{DE} = 20^\circ$ . Find two possible measures of  $\widehat{AE}$ .
31. **MODELING WITH MATHEMATICS** On a regulation dartboard, the outermost circle is divided into twenty congruent sections. What is the measure of each arc in this circle?



32. **MODELING WITH MATHEMATICS** You can use the time zone wheel to find the time in different locations across the world. For example, to find the time in Tokyo when it is 4 P.M. in San Francisco, rotate the small wheel until 4 P.M. and San Francisco line up, as shown. Then look at Tokyo to see that it is 9 A.M. there.

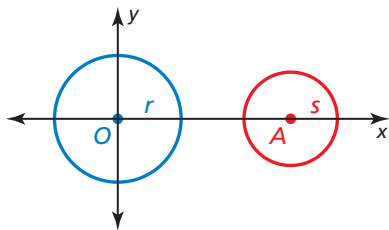


- What is the arc measure between each time zone on the wheel?
- What is the measure of the minor arc from the Tokyo zone to the Anchorage zone?
- If two locations differ by  $180^\circ$  on the wheel, then it is 3 P.M. at one location when it is \_\_\_\_\_ at the other location.

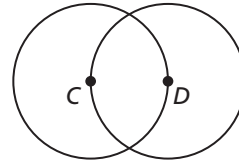
33. **PROVING A THEOREM** Write a coordinate proof of the Similar Circles Theorem (Theorem 10.5).

**Given**  $\odot O$  with center  $O(0, 0)$  and radius  $r$ ,  
 $\odot A$  with center  $A(a, 0)$  and radius  $s$

**Prove**  $\odot O \sim \odot A$



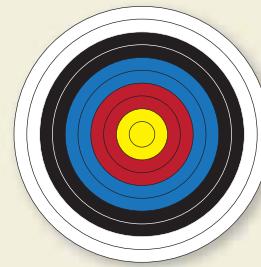
34. **ABSTRACT REASONING** Is there enough information to tell whether  $\odot C \cong \odot D$ ? Explain your reasoning.



35. **PROVING A THEOREM** Use the diagram on page 544 to prove each part of the biconditional in the Congruent Circles Theorem (Theorem 10.3).

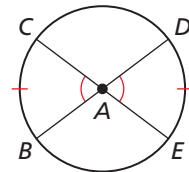
- a. **Given**  $\overline{AC} \cong \overline{BD}$       **Prove**  $\odot A \cong \odot B$   
b. **Given**  $\odot A \cong \odot B$       **Prove**  $\overline{AC} \cong \overline{BD}$

36. **HOW DO YOU SEE IT?** Are the circles on the target *similar* or *congruent*? Explain your reasoning.



37. **PROVING A THEOREM** Use the diagram to prove each part of the biconditional in the Congruent Central Angles Theorem (Theorem 10.4).

- a. **Given**  $\angle BAC \cong \angle DAE$   
**Prove**  $\widehat{BC} \cong \widehat{DE}$   
b. **Given**  $\widehat{BC} \cong \widehat{DE}$   
**Prove**  $\angle BAC \cong \angle DAE$

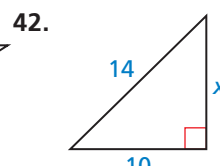
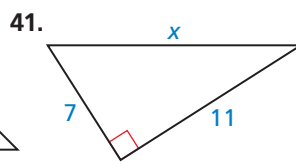
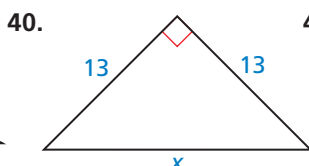
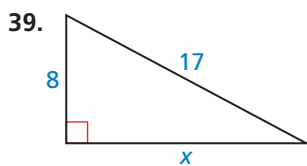


38. **THOUGHT PROVOKING** Write a formula for the length of a circular arc. Justify your answer.

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the value of  $x$ . Tell whether the side lengths form a Pythagorean triple. (Section 9.1)



# 10.3 Using Chords



TEXAS ESSENTIAL  
KNOWLEDGE AND SKILLS  
G.5.A  
G.12.A

## ANALYZING MATHEMATICAL RELATIONSHIPS

To be proficient in math,  
you need to look closely  
to discern a pattern  
or structure.

**Essential Question** What are two ways to determine when a chord is a diameter of a circle?

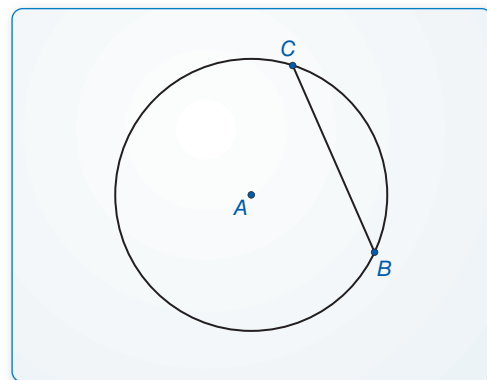
### EXPLORATION 1 Drawing Diameters

**Work with a partner.** Use dynamic geometry software to construct a circle of radius 5 with center at the origin. Draw a diameter that has the given point as an endpoint. Explain how you know that the chord you drew is a diameter.

- a.  $(4, 3)$       b.  $(0, 5)$       c.  $(-3, 4)$       d.  $(-5, 0)$

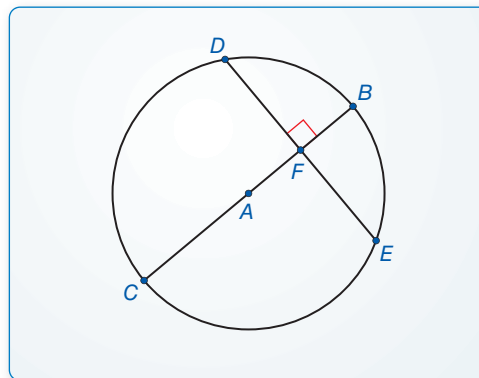
### EXPLORATION 2 Writing a Conjecture about Chords

**Work with a partner.** Use dynamic geometry software to construct a chord  $\overline{BC}$  of a circle  $A$ . Construct a chord on the perpendicular bisector of  $\overline{BC}$ . What do you notice? Change the original chord and the circle several times. Are your results always the same? Use your results to write a conjecture.



### EXPLORATION 3 A Chord Perpendicular to a Diameter

**Work with a partner.** Use dynamic geometry software to construct a diameter  $\overline{BC}$  of a circle  $A$ . Then construct a chord  $\overline{DE}$  perpendicular to  $\overline{BC}$  at point  $F$ . Find the lengths  $DF$  and  $EF$ . What do you notice? Change the chord perpendicular to  $\overline{BC}$  and the circle several times. Do you always get the same results? Write a conjecture about a chord that is perpendicular to a diameter of a circle.



## Communicate Your Answer

4. What are two ways to determine when a chord is a diameter of a circle?

# 10.3 Lesson

## What You Will Learn

► Use chords of circles to find lengths and arc measures.

### Core Vocabulary

#### Previous

chord  
arc  
diameter

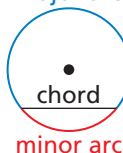
### Using Chords of Circles

Recall that a *chord* is a segment with endpoints on a circle. Because its endpoints lie on the circle, any chord divides the circle into two arcs. A diameter divides a circle into two semicircles. Any other chord divides a circle into a minor arc and a major arc.

semicircle

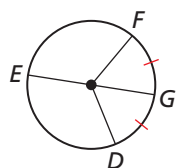


major arc



### READING

If  $\widehat{GD} \cong \widehat{GF}$ , then the point  $G$ , and any line, segment, or ray that contains  $G$ , bisects  $\widehat{FD}$ .

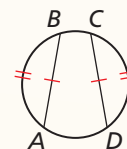


$\overline{EG}$  bisects  $\widehat{FD}$ .

### Theorems

#### Theorem 10.6 Congruent Corresponding Chords Theorem

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

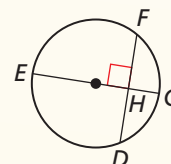


$\widehat{AB} \cong \widehat{CD}$  if and only if  $\overline{AB} \cong \overline{CD}$ .

*Proof* Ex. 19, p. 554

#### Theorem 10.7 Perpendicular Chord Bisector Theorem

If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

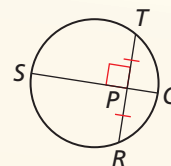


If  $\overline{EG}$  is a diameter and  $\overline{EG} \perp \overline{DF}$ , then  $\widehat{HD} \cong \widehat{HF}$  and  $\widehat{GD} \cong \widehat{GF}$ .

*Proof* Ex. 22, p. 554

#### Theorem 10.8 Perpendicular Chord Bisector Converse

If one chord of a circle is a perpendicular bisector of another chord, then the first chord is a diameter.

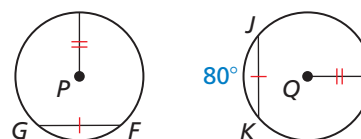


If  $\overline{QS}$  is a perpendicular bisector of  $\overline{TR}$ , then  $\overline{QS}$  is a diameter of the circle.

*Proof* Ex. 23, p. 554

### EXAMPLE 1 Using Congruent Chords to Find an Arc Measure

In the diagram,  $\odot P \cong \odot Q$ ,  $\overline{FG} \cong \overline{JK}$ , and  $m\widehat{JK} = 80^\circ$ . Find  $m\widehat{FG}$ .



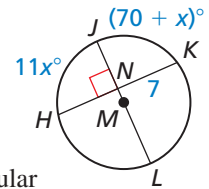
#### SOLUTION

Because  $\overline{FG}$  and  $\overline{JK}$  are congruent chords in congruent circles, the corresponding minor arcs  $\widehat{FG}$  and  $\widehat{JK}$  are congruent by the Congruent Corresponding Chords Theorem.

► So,  $m\widehat{FG} = m\widehat{JK} = 80^\circ$ .

### EXAMPLE 2 Using a Diameter

- a. Find  $\overline{HK}$ .                      b. Find  $m\widehat{HK}$ .



#### SOLUTION

- a. Diameter  $\overline{JL}$  is perpendicular to  $\overline{HK}$ . So, by the Perpendicular Chord Bisector Theorem,  $\overline{JL}$  bisects  $\overline{HK}$ , and  $HN = NK$ .

► So,  $HK = 2(NK) = 2(7) = 14$ .

- b. Diameter  $\overline{JL}$  is perpendicular to  $\overline{HK}$ . So, by the Perpendicular Chord Bisector Theorem,  $\overline{JL}$  bisects  $\widehat{HK}$ , and  $m\widehat{HJ} = m\widehat{JK}$ .

$$m\widehat{HJ} = m\widehat{JK} \quad \text{Perpendicular Chord Bisector Theorem}$$

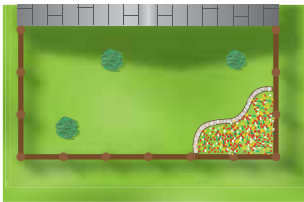
$$11x^\circ = (70 + x)^\circ \quad \text{Substitute.}$$

$$10x = 70 \quad \text{Subtract } x \text{ from each side.}$$

$$x = 7 \quad \text{Divide each side by 10.}$$

► So,  $m\widehat{HJ} = m\widehat{JK} = (70 + x)^\circ = (70 + 7)^\circ = 77^\circ$ , and  $m\widehat{HK} = 2(m\widehat{HJ}) = 2(77^\circ) = 154^\circ$ .

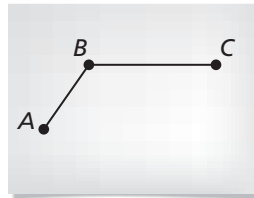
### EXAMPLE 3 Using Perpendicular Bisectors



Three bushes are arranged in a garden, as shown. Where should you place a sprinkler so that it is the same distance from each bush?

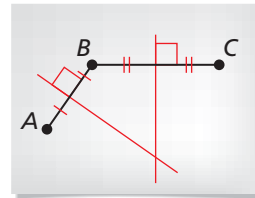
#### SOLUTION

Step 1



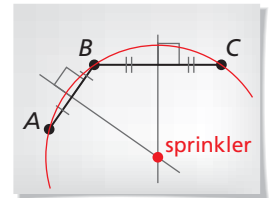
Label the bushes  $A$ ,  $B$ , and  $C$ , as shown. Draw segments  $\overline{AB}$  and  $\overline{BC}$ .

Step 2



Draw the perpendicular bisectors of  $\overline{AB}$  and  $\overline{BC}$ . By the Perpendicular Bisector Converse, these lie on diameters of the circle containing  $A$ ,  $B$ , and  $C$ .

Step 3

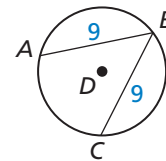


Find the point where the perpendicular bisectors intersect. This is the center of the circle, which is equidistant from points  $A$ ,  $B$ , and  $C$ .

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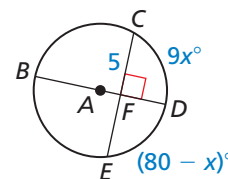
In Exercises 1 and 2, use the diagram of  $\odot D$ .

- If  $m\widehat{AB} = 110^\circ$ , find  $m\widehat{BC}$ .
- If  $m\widehat{AC} = 150^\circ$ , find  $m\widehat{AB}$ .



In Exercises 3 and 4, find the indicated length or arc measure.

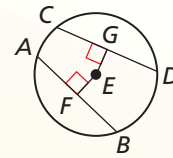
- $CE$
- $m\widehat{CE}$



## Theorem

### Theorem 10.9 Equidistant Chords Theorem

In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

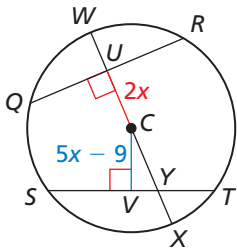


$\overline{AB} \cong \overline{CD}$  if and only if  $EF = EG$ .

*Proof* Ex. 25, p. 554

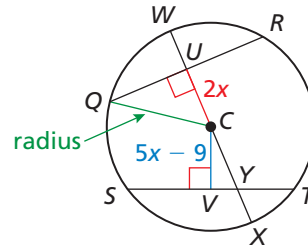
### EXAMPLE 4 Using Congruent Chords to Find a Circle's Radius

In the diagram,  $QR = ST = 16$ ,  $CU = 2x$ , and  $CV = 5x - 9$ . Find the radius of  $\odot C$ .



#### SOLUTION

Because  $\overline{CQ}$  is a segment whose endpoints are the center and a point on the circle, it is a radius of  $\odot C$ . Because  $\overline{CU} \perp \overline{QR}$ ,  $\triangle QUC$  is a right triangle. Apply properties of chords to find the lengths of the legs of  $\triangle QUC$ .



**Step 1** Find  $CU$ .

Because  $\overline{QR}$  and  $\overline{ST}$  are congruent chords,  $\overline{QR}$  and  $\overline{ST}$  are equidistant from  $C$  by the Equidistant Chords Theorem. So,  $CU = CV$ .

$$CU = CV \quad \text{Equidistant Chords Theorem}$$

$$2x = 5x - 9 \quad \text{Substitute.}$$

$$x = 3 \quad \text{Solve for } x.$$

So,  $CU = 2x = 2(3) = 6$ .

**Step 2** Find  $QU$ .

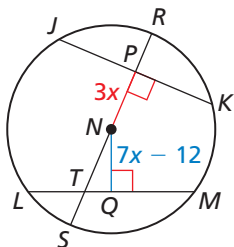
Because diameter  $\overline{WX} \perp \overline{QR}$ ,  $\overline{WX}$  bisects  $\overline{QR}$  by the Perpendicular Chord Bisector Theorem.

$$\text{So, } QU = \frac{1}{2}(16) = 8.$$

**Step 3** Find  $CQ$ .

Because the lengths of the legs are  $CU = 6$  and  $QU = 8$ ,  $\triangle QUC$  is a right triangle with the Pythagorean triple 6, 8, 10. So,  $CQ = 10$ .

► So, the radius of  $\odot C$  is 10 units.



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5. In the diagram,  $JK = LM = 24$ ,  $NP = 3x$ , and  $NQ = 7x - 12$ . Find the radius of  $\odot N$ .

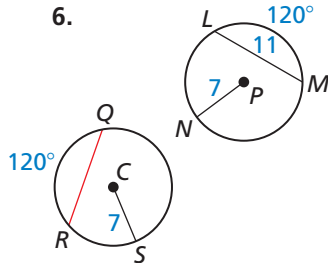
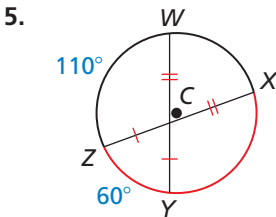
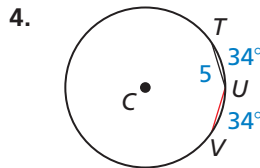
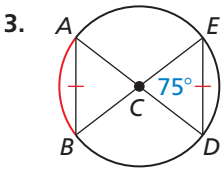
# 10.3 Exercises

## Vocabulary and Core Concept Check

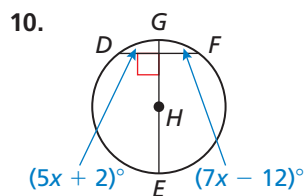
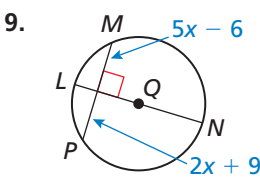
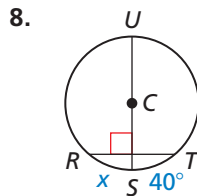
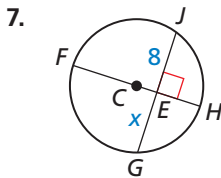
- WRITING** Describe what it means to bisect a chord.
- WRITING** Two chords of a circle are perpendicular and congruent. Does one of them have to be a diameter? Explain your reasoning.

## Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the measure of the red arc or chord in  $\odot C$ . (See Example 1.)



In Exercises 7–10, find the value of  $x$ . (See Example 2.)



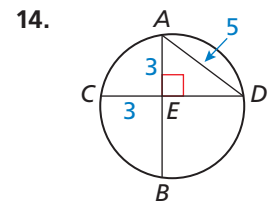
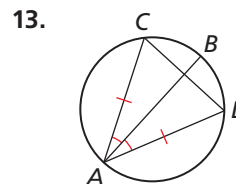
11. **ERROR ANALYSIS** Describe and correct the error in reasoning.

Because  $\overline{AC}$  bisects  $\overline{DB}$ ,  $\overline{BC} \cong \overline{CD}$ .

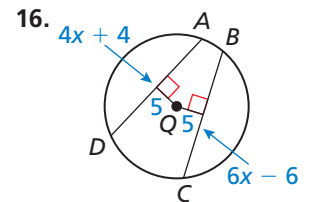
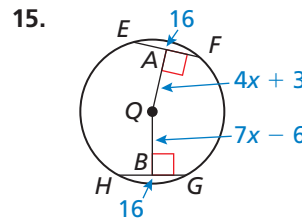
12. **PROBLEM SOLVING** In the cross section of the submarine shown, the control panels are parallel and the same length. Describe a method you can use to find the center of the cross section. Justify your method. (See Example 3.)



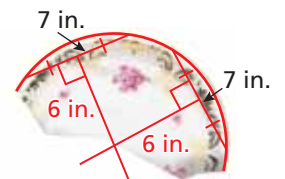
In Exercises 13 and 14, determine whether  $\overline{AB}$  is a diameter of the circle. Explain your reasoning.



In Exercises 15 and 16, find the radius of  $\odot Q$ . (See Example 4.)

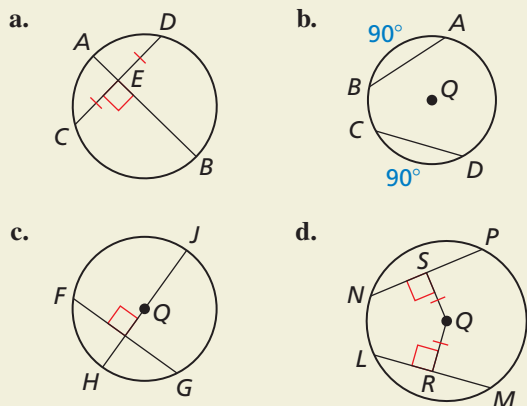


17. **PROBLEM SOLVING** An archaeologist finds part of a circular plate. What was the diameter of the plate to the nearest tenth of an inch? Justify your answer.

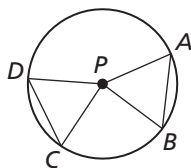




18. **HOW DO YOU SEE IT?** What can you conclude from each diagram? Name a theorem that justifies your answer.



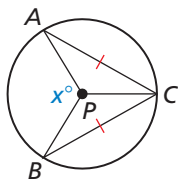
19. **PROVING A THEOREM** Use the diagram to prove each part of the biconditional in the Congruent Corresponding Chords Theorem (Theorem 10.6).



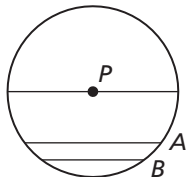
- a. **Given**  $\overline{AB}$  and  $\overline{CD}$  are congruent chords.  
**Prove**  $\widehat{AB} \cong \widehat{CD}$
- b. **Given**  $\widehat{AB} \cong \widehat{CD}$   
**Prove**  $\overline{AB} \cong \overline{CD}$

20. **MATHEMATICAL CONNECTIONS**

In  $\odot P$ , all the arcs shown have integer measures. Show that  $x$  must be even.



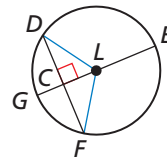
21. **REASONING** In  $\odot P$ , the lengths of the parallel chords are 20, 16, and 12. Find  $m\widehat{AB}$ . Explain your reasoning.



22. **PROVING A THEOREM** Use congruent triangles to prove the Perpendicular Chord Bisector Theorem (Theorem 10.7).

**Given**  $\overline{EG}$  is a diameter of  $\odot L$ .  
 $\overline{EG} \perp \overline{DF}$

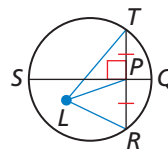
**Prove**  $\overline{DC} \cong \overline{FC}$ ,  $\widehat{DG} \cong \widehat{FG}$



23. **PROVING A THEOREM** Write a proof of the Perpendicular Chord Bisector Converse (Theorem 10.8).

**Given**  $\overline{QS}$  is a perpendicular bisector of  $\overline{RT}$ .

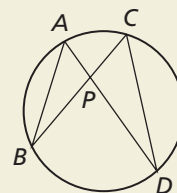
**Prove**  $\overline{QS}$  is a diameter of the circle  $L$ .



(Hint: Plot the center  $L$  and draw  $\triangle LPT$  and  $\triangle LPR$ .)

24. **THOUGHT PROVOKING**

Consider two chords that intersect at point  $P$ . Do you think that  $\frac{AP}{BP} = \frac{CP}{DP}$ ? Justify your answer.



25. **PROVING A THEOREM** Use the diagram with the Equidistant Chords Theorem (Theorem 10.9) on page 552 to prove both parts of the biconditional of this theorem.

26. **MAKING AN ARGUMENT** A car is designed so that the rear wheel is only partially visible below the body of the car. The bottom edge of the panel is parallel to the ground. Your friend claims that the point where the tire touches the ground bisects  $\widehat{AB}$ . Is your friend correct? Explain your reasoning.



## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the missing interior angle measure. (Section 7.1)

27. Quadrilateral  $JKLM$  has angle measures  $m\angle J = 32^\circ$ ,  $m\angle K = 25^\circ$ , and  $m\angle L = 44^\circ$ . Find  $m\angle M$ .
28. Pentagon  $PQRST$  has angle measures  $m\angle P = 85^\circ$ ,  $m\angle Q = 134^\circ$ ,  $m\angle R = 97^\circ$ , and  $m\angle S = 102^\circ$ . Find  $m\angle T$ .

# 10.1–10.3 What Did You Learn?

## Core Vocabulary

circle, *p.* 534  
center, *p.* 534  
radius, *p.* 534  
chord, *p.* 534  
diameter, *p.* 534  
secant, *p.* 534  
tangent, *p.* 534

point of tangency, *p.* 534  
tangent circles, *p.* 535  
concentric circles, *p.* 535  
common tangent, *p.* 535  
central angle, *p.* 542  
minor arc, *p.* 542  
major arc, *p.* 542

semicircle, *p.* 542  
measure of a minor arc, *p.* 542  
measure of a major arc, *p.* 542  
adjacent arcs, *p.* 543  
congruent circles, *p.* 544  
congruent arcs, *p.* 544  
similar arcs, *p.* 545

## Core Concepts

### Section 10.1

Lines and Segments That Intersect Circles, *p.* 534  
Coplanar Circles and Common Tangents, *p.* 535  
Theorem 10.1 Tangent Line to Circle Theorem, *p.* 536

Theorem 10.2 External Tangent Congruence Theorem,  
*p.* 536

### Section 10.2

Measuring Arcs, *p.* 542  
Postulate 10.1 Arc Addition Postulate, *p.* 543  
Theorem 10.3 Congruent Circles Theorem, *p.* 544

Theorem 10.4 Congruent Central Angles Theorem,  
*p.* 544

Theorem 10.5 Similar Circles Theorem, *p.* 545

### Section 10.3

Theorem 10.6 Congruent Corresponding Chords  
Theorem, *p.* 550  
Theorem 10.7 Perpendicular Chord Bisector  
Theorem, *p.* 550

Theorem 10.8 Perpendicular Chord Bisector Converse,  
*p.* 550

Theorem 10.9 Equidistant Chords Theorem, *p.* 552

## Mathematical Thinking

1. Explain how separating quadrilateral  $TVWX$  into several segments helped you solve Exercise 37 on page 539.
2. In Exercise 30 on page 547, what two cases did you consider to reach your answers? Are there any other cases? Explain your reasoning.
3. Explain how you used inductive reasoning to solve Exercise 24 on page 554.

### Study Skills

## Keeping Your Mind Focused While Completing Homework

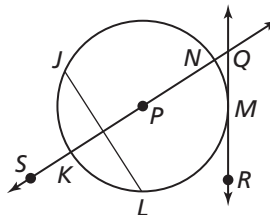
- Before doing homework, review the Concept boxes and Examples. Talk through the Examples out loud.
- Complete homework as though you were also preparing for a quiz. Memorize the different types of problems, formulas, rules, and so on.



# 10.1–10.3 Quiz

In Exercises 1–6, use the diagram. (Section 10.1)

- Name the circle.
- Name a radius.
- Name a diameter.
- Name a chord.
- Name a secant.
- Name a tangent.

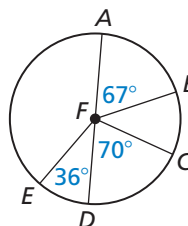


Find the value of  $x$ . (Section 10.1)

- 
- 

Identify the given arc as a *major arc*, *minor arc*, or *semicircle*. Then find the measure of the arc. (Section 10.2)

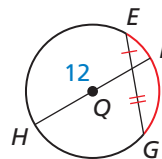
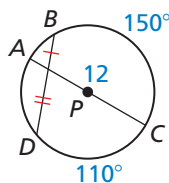
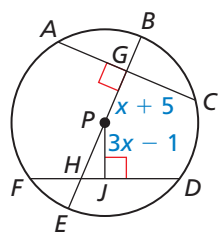
- $\widehat{AE}$
- $\widehat{BC}$
- $\widehat{AC}$
- $\widehat{ACD}$
- $\widehat{ACE}$
- $\widehat{BEC}$



Tell whether the red arcs are congruent. Explain why or why not. (Section 10.2)

- 
- 

17. Find the measure of the red arc in  $\odot Q$ . (Section 10.3)



- In the diagram,  $AC = FD = 30$ ,  $PG = x + 5$ , and  $PJ = 3x - 1$ . Find the radius of  $\odot P$ . (Section 10.3)

- A circular clock can be divided into 12 congruent sections. (Section 10.2)
  - Find the measure of each arc in this circle.
  - Find the measure of the minor arc formed by the hour and minute hands when the time is 7:00.
  - Find a time at which the hour and minute hands form an arc that is congruent to the arc in part (b).



# 10.4 Inscribed Angles and Polygons

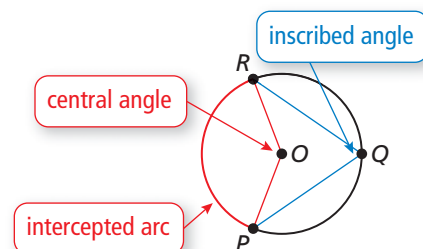


TEXAS ESSENTIAL  
KNOWLEDGE AND SKILLS

G.5.A  
G.12.A

**Essential Question** How are inscribed angles related to their intercepted arcs? How are the angles of an inscribed quadrilateral related to each other?

An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. An arc that lies between two lines, rays, or segments is called an **intercepted arc**. A polygon is an **inscribed polygon** when all its vertices lie on a circle.



## USING PRECISE MATHEMATICAL LANGUAGE

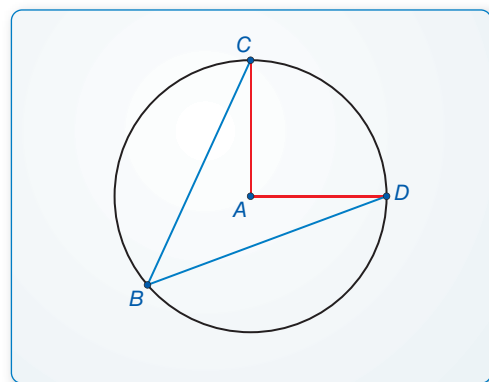
To be proficient in math, you need to communicate precisely with others.

### EXPLORATION 1 Inscribed Angles and Central Angles

**Work with a partner.** Use dynamic geometry software.

- Construct an inscribed angle in a circle. Then construct the corresponding central angle.
- Measure both angles. How is the inscribed angle related to its intercepted arc?
- Repeat parts (a) and (b) several times. Record your results in a table. Write a conjecture about how an inscribed angle is related to its intercepted arc.

#### Sample

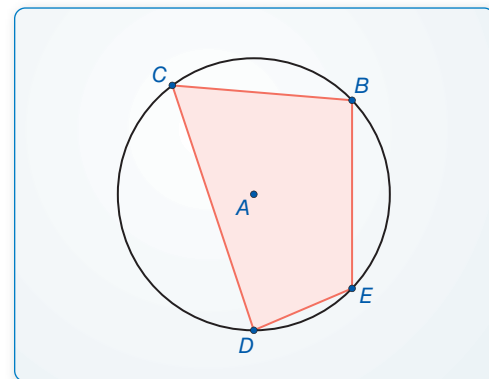


### EXPLORATION 2 A Quadrilateral with Inscribed Angles

**Work with a partner.** Use dynamic geometry software.

- Construct a quadrilateral with each vertex on a circle.
- Measure all four angles. What relationships do you notice?
- Repeat parts (a) and (b) several times. Record your results in a table. Then write a conjecture that summarizes the data.

#### Sample



## Communicate Your Answer

- How are inscribed angles related to their intercepted arcs? How are the angles of an inscribed quadrilateral related to each other?
- Quadrilateral  $EFGH$  is inscribed in  $\odot C$ , and  $m\angle E = 80^\circ$ . What is  $m\angle G$ ? Explain.

# 10.4 Lesson

## Core Vocabulary

inscribed angle, p. 558  
 intercepted arc, p. 558  
 subtend, p. 558  
 inscribed polygon, p. 560  
 circumscribed circle, p. 560

## What You Will Learn

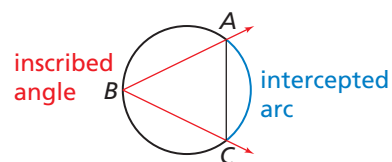
- ▶ Use inscribed angles.
- ▶ Use inscribed polygons.

## Using Inscribed Angles

### Core Concept

#### Inscribed Angle and Intercepted Arc

An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. An arc that lies between two lines, rays, or segments is called an **intercepted arc**. If the endpoints of a chord or arc lie on the sides of an inscribed angle, then the chord or arc is said to **subtend** the angle.

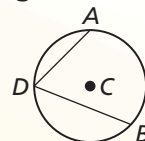


$\angle B$  intercepts  $\widehat{AC}$ .  
 $\widehat{AC}$  subtends  $\angle B$ .  
 $\overline{AC}$  subtends  $\angle B$ .

### Theorem

#### Theorem 10.10 Measure of an Inscribed Angle Theorem

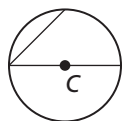
The measure of an inscribed angle is one-half the measure of its intercepted arc.



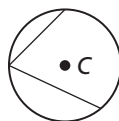
$$m\angle ADB = \frac{1}{2}m\widehat{AB}$$

*Proof* Ex. 37, p. 564

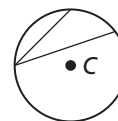
The proof of the Measure of an Inscribed Angle Theorem involves three cases.



**Case 1** Center  $C$  is on a side of the inscribed angle.



**Case 2** Center  $C$  is inside the inscribed angle.

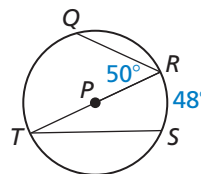


**Case 3** Center  $C$  is outside the inscribed angle.

### EXAMPLE 1 Using Inscribed Angles

Find the indicated measure.

- a.  $m\angle T$
- b.  $m\widehat{QR}$



#### SOLUTION

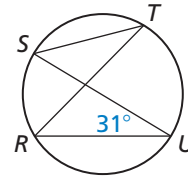
a.  $m\angle T = \frac{1}{2}m\widehat{RS} = \frac{1}{2}(48^\circ) = 24^\circ$

b.  $m\widehat{TQ} = 2m\angle R = 2 \cdot 50^\circ = 100^\circ$

Because  $\widehat{QRS}$  is a semicircle,  $m\widehat{QR} = 180^\circ - m\widehat{TQ} = 180^\circ - 100^\circ = 80^\circ$ .

**EXAMPLE 2****Finding the Measure of an Intercepted Arc**

Find  $m\widehat{RS}$  and  $m\angle STR$ . What do you notice about  $\angle STR$  and  $\angle RUS$ ?

**SOLUTION**

From the Measure of an Inscribed Angle Theorem, you know that  $m\widehat{RS} = 2m\angle RUS = 2(31^\circ) = 62^\circ$ .

Also,  $m\angle STR = \frac{1}{2}m\widehat{RS} = \frac{1}{2}(62^\circ) = 31^\circ$ .

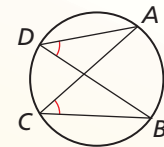
► So,  $\angle STR \cong \angle RUS$ .

Example 2 suggests the Inscribed Angles of a Circle Theorem.

## Theorem

**Theorem 10.11 Inscribed Angles of a Circle Theorem**

If two inscribed angles of a circle intercept the same arc, then the angles are congruent.

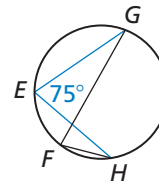


$$\angle ADB \cong \angle ACB$$

*Proof* Ex. 38, p. 564

**EXAMPLE 3****Finding the Measure of an Angle**

Given  $m\angle E = 75^\circ$ , find  $m\angle F$ .

**SOLUTION**

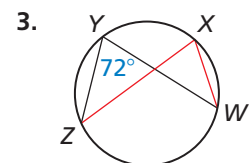
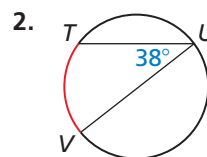
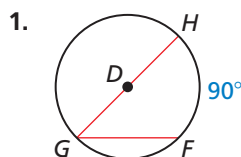
Both  $\angle E$  and  $\angle F$  intercept  $\widehat{GH}$ . So,  $\angle E \cong \angle F$  by the Inscribed Angles of a Circle Theorem.

► So,  $m\angle F = m\angle E = 75^\circ$ .

**Monitoring Progress**

Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

Find the measure of the red arc or angle.

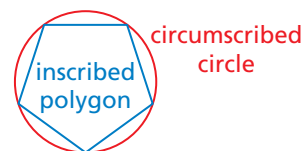


## Using Inscribed Polygons

### Core Concept

#### Inscribed Polygon

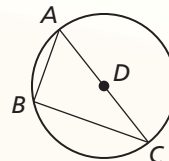
A polygon is an **inscribed polygon** when all its vertices lie on a circle. The circle that contains the vertices is a **circumscribed circle**.



### Theorems

#### Theorem 10.12 Inscribed Right Triangle Theorem

If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.

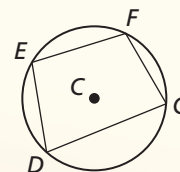


*Proof* Ex. 39, p. 564

$m\angle ABC = 90^\circ$  if and only if  $\overline{AC}$  is a diameter of the circle.

#### Theorem 10.13 Inscribed Quadrilateral Theorem

A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.

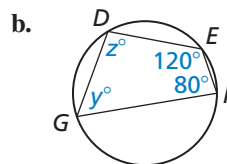
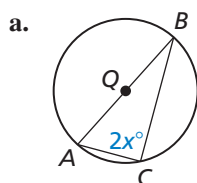


*Proof* Ex. 40, p. 564

$D, E, F,$  and  $G$  lie on  $\odot C$  if and only if  $m\angle D + m\angle F = m\angle E + m\angle G = 180^\circ$ .

### EXAMPLE 4 Using Inscribed Polygons

Find the value of each variable.



#### SOLUTION

- a.  $\overline{AB}$  is a diameter. So,  $\angle C$  is a right angle, and  $m\angle C = 90^\circ$  by the Inscribed Right Triangle Theorem.

$$2x^\circ = 90^\circ$$

$$x = 45$$

► The value of  $x$  is 45.

- b.  $DEFG$  is inscribed in a circle, so opposite angles are supplementary by the Inscribed Quadrilateral Theorem.

$$m\angle D + m\angle F = 180^\circ$$

$$z + 80 = 180$$

$$z = 100$$

$$m\angle E + m\angle G = 180^\circ$$

$$120 + y = 180$$

$$y = 60$$

► The value of  $z$  is 100 and the value of  $y$  is 60.

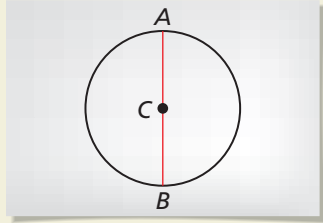
## CONSTRUCTION

### Constructing a Square Inscribed in a Circle

Given  $\odot C$ , construct a square inscribed in a circle.

#### SOLUTION

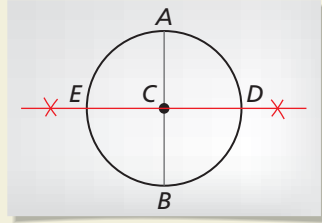
**Step 1**



#### Draw a diameter

Draw any diameter. Label the endpoints  $A$  and  $B$ .

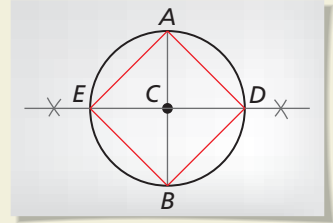
**Step 2**



#### Construct a perpendicular bisector

Construct the perpendicular bisector of the diameter. Label the points where it intersects  $\odot C$  as points  $D$  and  $E$ .

**Step 3**



#### Form a square

Connect points  $A$ ,  $D$ ,  $B$ , and  $E$  to form a square.

### EXAMPLE 5

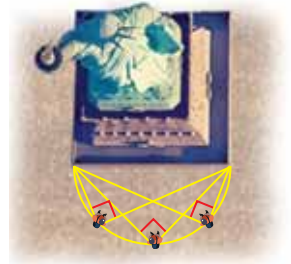
#### Using a Circumscribed Circle

Your camera has a  $90^\circ$  field of vision, and you want to photograph the front of a statue. You stand at a location in which the front of the statue is all that appears in your camera's field of vision, as shown. You want to change your location. Where else can you stand so that the front of the statue is all that appears in your camera's field of vision?



#### SOLUTION

From the Inscribed Right Triangle Theorem, you know that if a right triangle is inscribed in a circle, then the hypotenuse of the triangle is a diameter of the circle. So, draw the circle that has the front of the statue as a diameter.



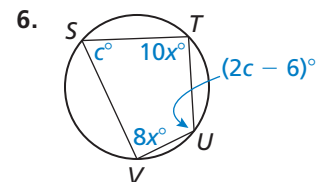
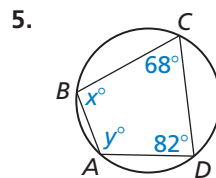
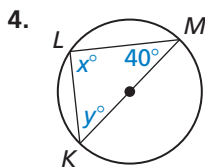
- ▶ The statue fits perfectly within your camera's  $90^\circ$  field of vision from any point on the semicircle in front of the statue.

### Monitoring Progress



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Find the value of each variable.



7. In Example 5, explain how to find locations where the front and left side of the statue are all that appears in your camera's field of vision.



# 10.4 Exercises

## Vocabulary and Core Concept Check

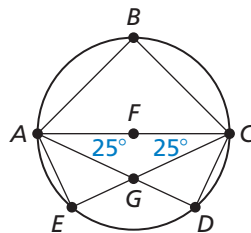
- VOCABULARY** If a circle is circumscribed about a polygon, then the polygon is an \_\_\_\_\_.
- DIFFERENT WORDS, SAME QUESTION** Which is different?  
Find “both” answers.

Find  $m\angle ABC$ .

Find  $m\angle AGC$ .

Find  $m\angle AEC$ .

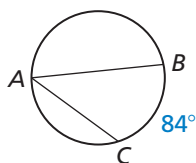
Find  $m\angle ADC$ .



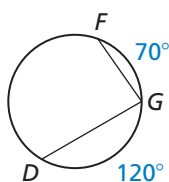
## Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, find the indicated measure.  
(See Examples 1 and 2.)

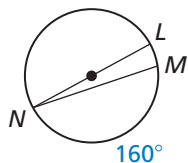
3.  $m\angle A$



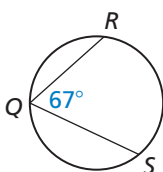
4.  $m\angle G$



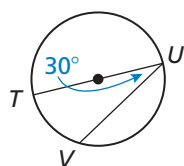
5.  $m\angle N$



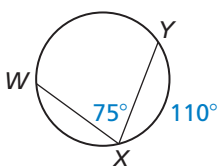
6.  $m\widehat{RS}$



7.  $m\widehat{VU}$

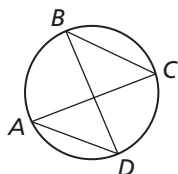


8.  $m\widehat{WX}$

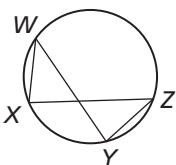


In Exercises 9 and 10, name two pairs of congruent angles.

9.

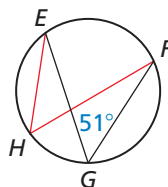


10.

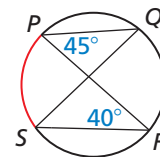


In Exercises 11 and 12, find the measure of the red arc or angle. (See Example 3.)

11.

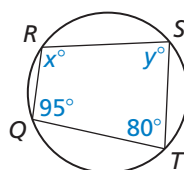


12.

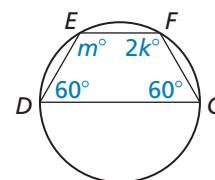


In Exercises 13–16, find the value of each variable.  
(See Example 4.)

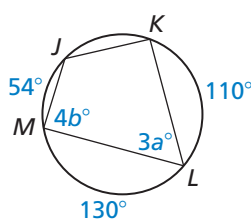
13.



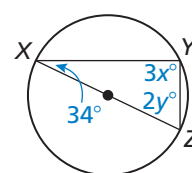
14.



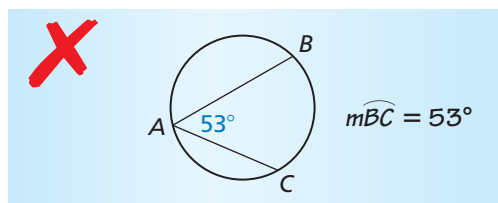
15.



16.



17. **ERROR ANALYSIS** Describe and correct the error in finding  $m\widehat{BC}$ .

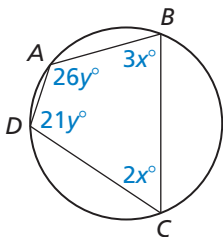


18. **MODELING WITH MATHEMATICS** A carpenter's square is an L-shaped tool used to draw right angles. You need to cut a circular piece of wood into two semicircles. How can you use the carpenter's square to draw a diameter on the circular piece of wood? (See Example 5.)

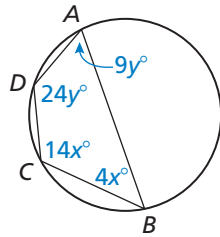


**MATHEMATICAL CONNECTIONS** In Exercises 19–21, find the values of  $x$  and  $y$ . Then find the measures of the interior angles of the polygon.

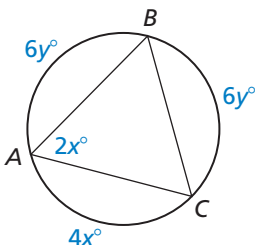
19.



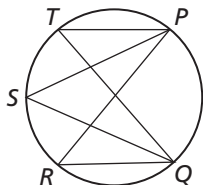
20.



21.



22. **MAKING AN ARGUMENT** Your friend claims that  $\angle PTQ \cong \angle PSQ \cong \angle PRQ$ . Is your friend correct? Explain your reasoning.

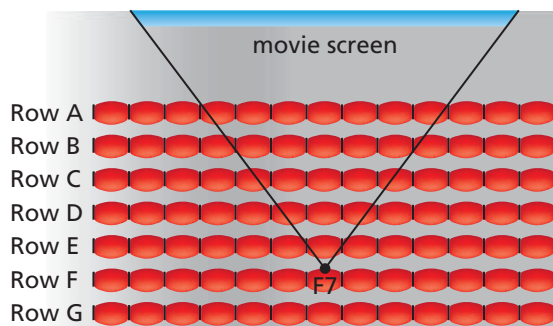


23. **CONSTRUCTION** Construct an equilateral triangle inscribed in a circle.
24. **CONSTRUCTION** The side length of an inscribed regular hexagon is equal to the radius of the circumscribed circle. Use this fact to construct a regular hexagon inscribed in a circle.

**REASONING** In Exercises 25–30, determine whether a quadrilateral of the given type can always be inscribed inside a circle. Explain your reasoning.

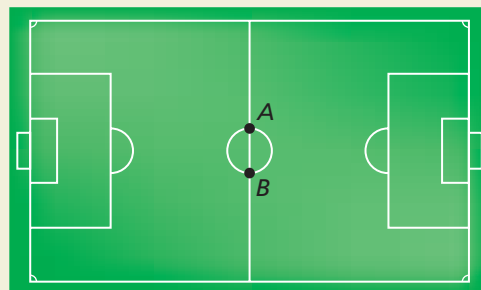
25. square  
26. rectangle  
27. parallelogram  
28. kite  
29. rhombus  
30. isosceles trapezoid
31. **MODELING WITH MATHEMATICS** Three moons, A, B, and C, are in the same circular orbit 100,000 kilometers above the surface of a planet. The planet is 20,000 kilometers in diameter and  $m\angle ABC = 90^\circ$ . Draw a diagram of the situation. How far is moon A from moon C?

32. **MODELING WITH MATHEMATICS** At the movie theater, you want to choose a seat that has the best viewing angle, so that you can be close to the screen and still see the whole screen without moving your eyes. You previously decided that seat F7 has the best viewing angle, but this time someone else is already sitting there. Where else can you sit so that your seat has the same viewing angle as seat F7? Explain.



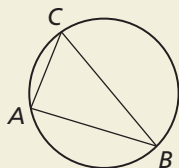
33. **WRITING** A right triangle is inscribed in a circle, and the radius of the circle is given. Explain how to find the length of the hypotenuse.

34. **HOW DO YOU SEE IT?** Let point  $Y$  represent your location on the soccer field below. What type of angle is  $\angle AYB$  if you stand anywhere on the circle except at point  $A$  or point  $B$ ?



35. **WRITING** Explain why the diagonals of a rectangle inscribed in a circle are diameters of the circle.

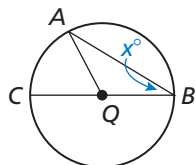
36. **THOUGHT PROVOKING** The figure shows a circle that is circumscribed about  $\triangle ABC$ . Is it possible to circumscribe a circle about any triangle? Justify your answer.



37. **PROVING A THEOREM** If an angle is inscribed in  $\odot Q$ , the center  $Q$  can be on a side of the inscribed angle, inside the inscribed angle, or outside the inscribed angle. Prove each case of the Measure of an Inscribed Angle Theorem (Theorem 10.10).

a. **Case 1**

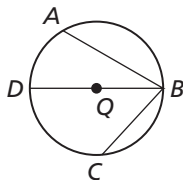
**Given**  $\angle ABC$  is inscribed in  $\odot Q$ .  
Let  $m\angle B = x^\circ$ .  
Center  $Q$  lies on  $\overline{BC}$ .



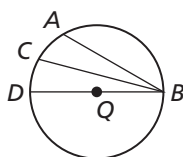
**Prove**  $m\angle ABC = \frac{1}{2} m\widehat{AC}$

(Hint: Show that  $\triangle AQB$  is isosceles. Then write  $m\widehat{AC}$  in terms of  $x$ .)

b. **Case 2** Use the diagram and auxiliary line to write **Given** and **Prove** statements for Case 2. Then write a proof.



c. **Case 3** Use the diagram and auxiliary line to write **Given** and **Prove** statements for Case 3. Then write a proof.

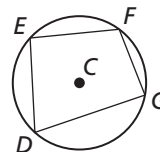


38. **PROVING A THEOREM** Write a paragraph proof of the Inscribed Angles of a Circle Theorem (Theorem 10.11). First, draw a diagram and write **Given** and **Prove** statements.

39. **PROVING A THEOREM** The Inscribed Right Triangle Theorem (Theorem 10.12) is written as a conditional statement and its converse. Write a plan for proof for each statement.

40. **PROVING A THEOREM** Copy and complete the paragraph proof of the Inscribed Quadrilateral Theorem (Theorem 10.13).

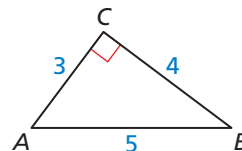
**Given**  $\odot C$  with inscribed quadrilateral  $DEFG$



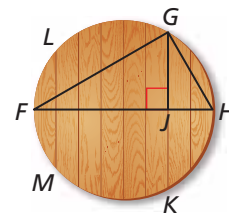
**Prove**  $m\angle D + m\angle F = 180^\circ$ ,  
 $m\angle E + m\angle G = 180^\circ$

By the Arc Addition Postulate (Postulate 10.1),  $m\widehat{EFG} + \underline{\hspace{1cm}} = 360^\circ$  and  $m\widehat{FGD} + m\widehat{DEF} = 360^\circ$ . Using the            Theorem,  $m\widehat{EDG} = 2m\angle F$ ,  $m\widehat{EFG} = 2m\angle D$ ,  $m\widehat{DEF} = 2m\angle G$ , and  $m\widehat{FGD} = 2m\angle E$ . By the Substitution Property of Equality,  $2m\angle D + \underline{\hspace{1cm}} = 360^\circ$ , so       . Similarly,       .

41. **CRITICAL THINKING** In the diagram,  $\angle C$  is a right angle. If you draw the smallest possible circle through  $C$  tangent to  $\overline{AB}$ , the circle will intersect  $\overline{AC}$  at  $J$  and  $\overline{BC}$  at  $K$ . Find the exact length of  $\overline{JK}$ .



42. **CRITICAL THINKING** You are making a circular cutting board. To begin, you glue eight 1-inch boards together, as shown. Then you draw and cut a circle with an 8-inch diameter from the boards.



- $\overline{FH}$  is a diameter of the circular cutting board. Write a proportion relating  $GJ$  and  $JH$ . State a theorem to justify your answer.
- Find  $FJ$ ,  $JH$ , and  $GJ$ . What is the length of the cutting board seam labeled  $\overline{GK}$ ?

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. Check your solution. (Skills Review Handbook)

43.  $3x = 145$

44.  $\frac{1}{2}x = 63$

45.  $240 = 2x$

46.  $75 = \frac{1}{2}(x - 30)$

# 10.5 Angle Relationships in Circles



TEXAS ESSENTIAL  
KNOWLEDGE AND SKILLS

G.5.A  
G.12.A

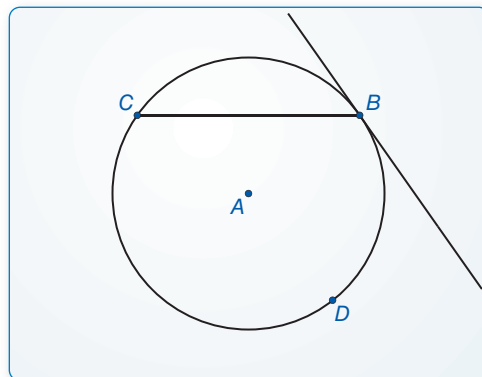
**Essential Question** When a chord intersects a tangent line or another chord, what relationships exist among the angles and arcs formed?

## EXPLORATION 1 Angles Formed by a Chord and Tangent Line

**Work with a partner.** Use dynamic geometry software.

- Construct a chord in a circle. At one of the endpoints of the chord, construct a tangent line to the circle.
- Find the measures of the two angles formed by the chord and the tangent line.
- Find the measures of the two circular arcs determined by the chord.
- Repeat parts (a)–(c) several times. Record your results in a table. Then write a conjecture that summarizes the data.

**Sample**



### MAKING MATHEMATICAL ARGUMENTS

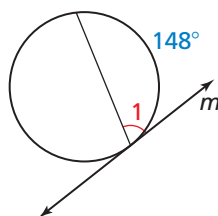
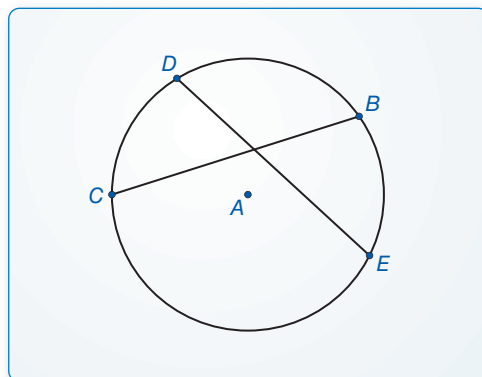
To be proficient in math, you need to understand and use stated assumptions, definitions, and previously established results.

## EXPLORATION 2 Angles Formed by Intersecting Chords

**Work with a partner.** Use dynamic geometry software.

- Construct two chords that intersect inside a circle.
- Find the measure of one of the angles formed by the intersecting chords.
- Find the measures of the arcs intercepted by the angle in part (b) and its vertical angle. What do you observe?
- Repeat parts (a)–(c) several times. Record your results in a table. Then write a conjecture that summarizes the data.

**Sample**



### Communicate Your Answer

- When a chord intersects a tangent line or another chord, what relationships exist among the angles and arcs formed?
- Line  $m$  is tangent to the circle in the figure at the left. Find the measure of  $\angle 1$ .
- Two chords intersect inside a circle to form a pair of vertical angles with measures of  $55^\circ$ . Find the sum of the measures of the arcs intercepted by the two angles.

# 10.5 Lesson

## Core Vocabulary

circumscribed angle, p. 568

Previous  
tangent  
chord  
secant

## What You Will Learn

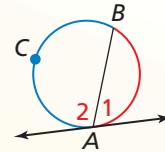
- ▶ Find angle and arc measures.
- ▶ Use circumscribed angles.

## Finding Angle and Arc Measures

### Theorem

#### Theorem 10.14 Tangent and Intersected Chord Theorem

If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one-half the measure of its intercepted arc.

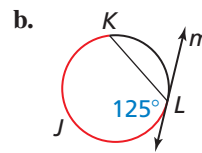
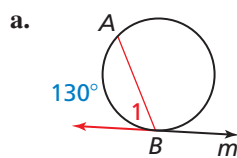


$$m\angle 1 = \frac{1}{2}m\widehat{AB} \quad m\angle 2 = \frac{1}{2}m\widehat{BCA}$$

Proof Ex. 33, p. 572

### EXAMPLE 1 Finding Angle and Arc Measures

Line  $m$  is tangent to the circle. Find the measure of the red angle or arc.



### SOLUTION

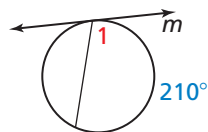
a.  $m\angle 1 = \frac{1}{2}(130^\circ) = 65^\circ$

b.  $m\widehat{KJL} = 2(125^\circ) = 250^\circ$

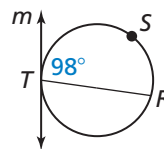
### Monitoring Progress Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

Line  $m$  is tangent to the circle. Find the indicated measure.

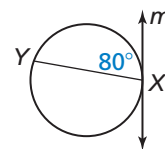
1.  $m\angle 1$



2.  $m\widehat{RST}$



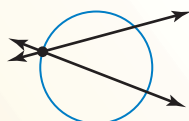
3.  $m\widehat{XY}$



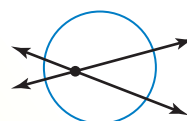
### Core Concept

#### Intersecting Lines and Circles

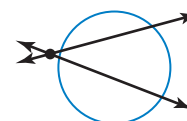
If two nonparallel lines intersect a circle, there are three places where the lines can intersect.



on the circle



inside the circle

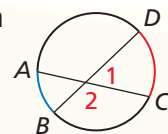


outside the circle

# Theorems

## Theorem 10.15 Angles Inside the Circle Theorem

If two chords intersect *inside* a circle, then the measure of each angle is one-half the *sum* of the measures of the arcs intercepted by the angle and its vertical angle.



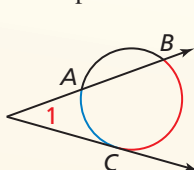
$$m\angle 1 = \frac{1}{2}(m\widehat{DC} + m\widehat{AB}),$$

$$m\angle 2 = \frac{1}{2}(m\widehat{AD} + m\widehat{BC})$$

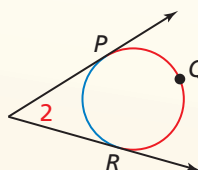
*Proof* Ex. 35, p. 572

## Theorem 10.16 Angles Outside the Circle Theorem

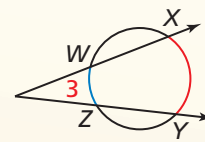
If a tangent and a secant, two tangents, or two secants intersect *outside* a circle, then the measure of the angle formed is one-half the *difference* of the measures of the intercepted arcs.



$$m\angle 1 = \frac{1}{2}(m\widehat{BC} - m\widehat{AC})$$



$$m\angle 2 = \frac{1}{2}(m\widehat{PQR} - m\widehat{PR})$$

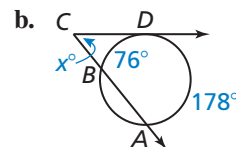
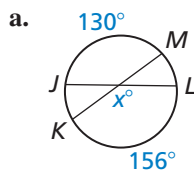


$$m\angle 3 = \frac{1}{2}(m\widehat{XY} - m\widehat{WZ})$$

*Proof* Ex. 37, p. 572

### EXAMPLE 2 Finding an Angle Measure

Find the value of  $x$ .



### SOLUTION

- a. The chords  $\overline{JL}$  and  $\overline{KM}$  intersect inside the circle. Use the Angles Inside the Circle Theorem.

$$x^\circ = \frac{1}{2}(m\widehat{JM} + m\widehat{LK})$$

$$x^\circ = \frac{1}{2}(130^\circ + 156^\circ)$$

$$x = 143$$

► So, the value of  $x$  is 143.

- b. The tangent  $\overline{CD}$  and the secant  $\overline{CB}$  intersect outside the circle. Use the Angles Outside the Circle Theorem.

$$m\angle BCD = \frac{1}{2}(m\widehat{AD} - m\widehat{BD})$$

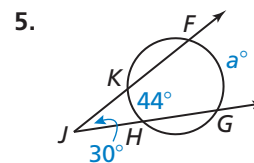
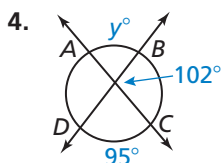
$$x^\circ = \frac{1}{2}(178^\circ - 76^\circ)$$

$$x = 51$$

► So, the value of  $x$  is 51.

### Monitoring Progress Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

Find the value of the variable.

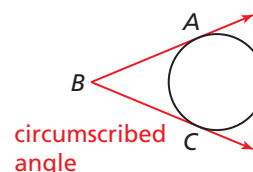


## Using Circumscribed Angles

### Core Concept

#### Circumscribed Angle

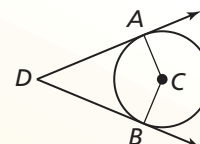
A **circumscribed angle** is an angle whose sides are tangent to a circle.



### Theorem

#### Theorem 10.17 Circumscribed Angle Theorem

The measure of a circumscribed angle is equal to  $180^\circ$  minus the measure of the central angle that intercepts the same arc.

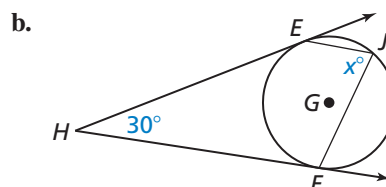
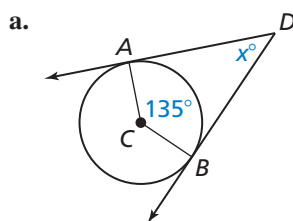


*Proof* Ex. 38, p. 572

$$m\angle ADB = 180^\circ - m\angle ACB$$

### EXAMPLE 3 Finding Angle Measures

Find the value of  $x$ .



#### SOLUTION

- a. By definition,  $m\widehat{AB} = m\angle ACB = 135^\circ$ . Use the Circumscribed Angle Theorem to find  $m\angle ADB$ .

$$m\angle ADB = 180^\circ - m\angle ACB$$

Circumscribed Angle Theorem

$$x^\circ = 180^\circ - 135^\circ$$

Substitute.

$$x = 45$$

Subtract.

► So, the value of  $x$  is 45.

- b. Use the Measure of an Inscribed Angle Theorem (Theorem 10.10) and the Circumscribed Angle Theorem to find  $m\angle EJF$ .

$$m\angle EJF = \frac{1}{2}m\widehat{EF}$$

Measure of an Inscribed Angle Theorem

$$m\angle EJF = \frac{1}{2}m\angle EGF$$

Definition of minor arc

$$m\angle EJF = \frac{1}{2}(180^\circ - m\angle EHF)$$

Circumscribed Angle Theorem

$$m\angle EJF = \frac{1}{2}(180^\circ - 30^\circ)$$

Substitute.

$$x = \frac{1}{2}(180 - 30)$$

Substitute.

$$x = 75$$

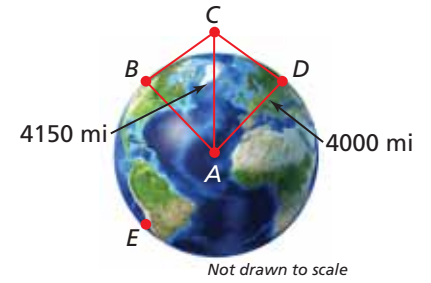
Simplify.

► So, the value of  $x$  is 75.

### EXAMPLE 4

### Modeling with Mathematics

The northern lights are bright flashes of colored light between 50 and 200 miles above Earth. A flash occurs 150 miles above Earth at point  $C$ . What is the measure of  $\widehat{BD}$ , the portion of Earth from which the flash is visible? (Earth's radius is approximately 4000 miles.)



### SOLUTION

- Understand the Problem** You are given the approximate radius of Earth and the distance above Earth that the flash occurs. You need to find the measure of the arc that represents the portion of Earth from which the flash is visible.
- Make a Plan** Use properties of tangents, triangle congruence, and angles outside a circle to find the arc measure.
- Solve the Problem** Because  $\overline{CB}$  and  $\overline{CD}$  are tangents,  $\overline{CB} \perp \overline{AB}$  and  $\overline{CD} \perp \overline{AD}$  by the Tangent Line to Circle Theorem (Theorem 10.1). Also,  $\overline{BC} \cong \overline{DC}$  by the External Tangent Congruence Theorem (Theorem 10.2), and  $\overline{CA} \cong \overline{CA}$  by the Reflexive Property of Congruence (Theorem 2.1). So,  $\triangle ABC \cong \triangle ADC$  by the Hypotenuse-Leg Congruence Theorem (Theorem 5.9). Because corresponding parts of congruent triangles are congruent,  $\angle BCA \cong \angle DCA$ . Solve right  $\triangle CBA$  to find that  $m\angle BCA \approx 74.5^\circ$ . So,  $m\angle BCD \approx 2(74.5^\circ) = 149^\circ$ .

### COMMON ERROR

Because the value for  $m\angle BCD$  is an approximation, use the symbol  $\approx$  instead of  $=$ .

$$\begin{aligned} m\angle BCD &= 180^\circ - m\angle BAD && \text{Circumscribed Angle Theorem} \\ m\angle BCD &= 180^\circ - m\widehat{BD} && \text{Definition of minor arc} \\ 149^\circ &\approx 180^\circ - m\widehat{BD} && \text{Substitute.} \\ 31^\circ &\approx m\widehat{BD} && \text{Solve for } m\widehat{BD}. \end{aligned}$$

► The measure of the arc from which the flash is visible is about  $31^\circ$ .

- Look Back** You can use inverse trigonometric ratios to find  $m\angle BAC$  and  $m\angle DAC$ .

$$m\angle BAC = \cos^{-1}\left(\frac{4000}{4150}\right) \approx 15.5^\circ$$

$$m\angle DAC = \cos^{-1}\left(\frac{4000}{4150}\right) \approx 15.5^\circ$$

So,  $m\angle BAD \approx 15.5^\circ + 15.5^\circ = 31^\circ$ , and therefore  $m\widehat{BD} \approx 31^\circ$ .

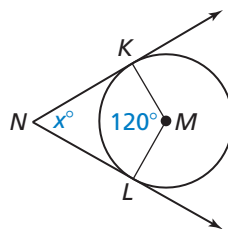
### Monitoring Progress



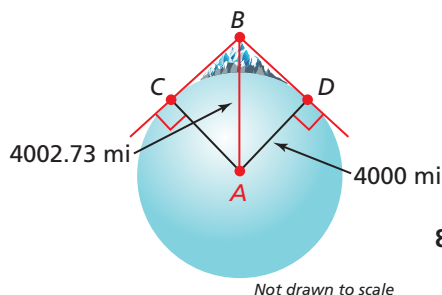
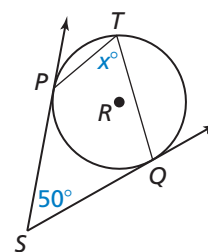
Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

Find the value of  $x$ .

6.



7.



- You are on top of Mount Rainier on a clear day. You are about 2.73 miles above sea level at point  $B$ . Find  $m\widehat{CD}$ , which represents the part of Earth that you can see.



# 10.5 Exercises

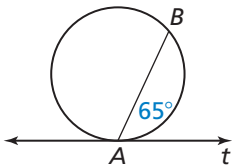
## Vocabulary and Core Concept Check

- COMPLETE THE SENTENCE** Points  $A, B, C,$  and  $D$  are on a circle, and  $\overleftrightarrow{AB}$  intersects  $\overleftrightarrow{CD}$  at point  $P$ . If  $m\angle APC = \frac{1}{2}(m\widehat{BD} - m\widehat{AC})$ , then point  $P$  is \_\_\_\_\_ the circle.
- WRITING** Explain how to find the measure of a circumscribed angle.

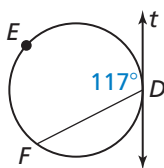
## Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, line  $t$  is tangent to the circle. Find the indicated measure. (See Example 1.)

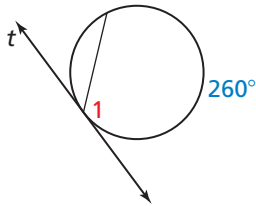
3.  $m\widehat{AB}$



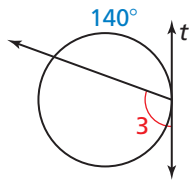
4.  $m\widehat{DEF}$



5.  $m\angle 1$

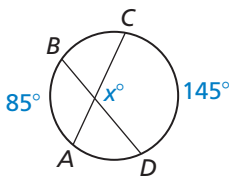


6.  $m\angle 3$

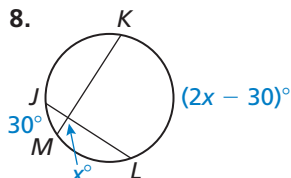


In Exercises 7–14, find the value of  $x$ . (See Examples 2 and 3.)

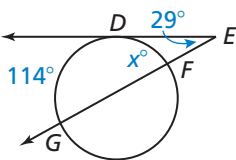
7.



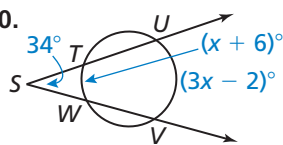
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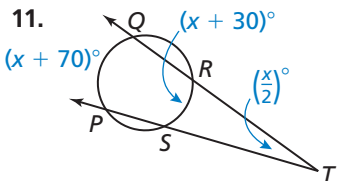
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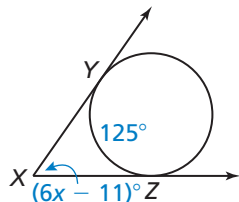
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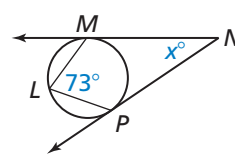
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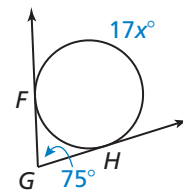
12.



13.

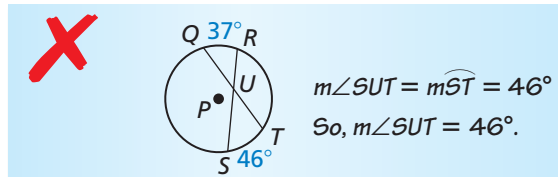


14.

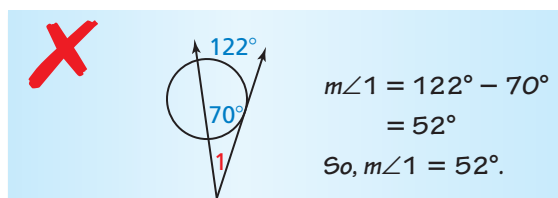


**ERROR ANALYSIS** In Exercises 15 and 16, describe and correct the error in finding the angle measure.

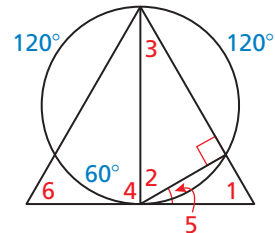
15.



16.



In Exercises 17–22, find the indicated angle measure. Justify your answer.



17.  $m\angle 1$

18.  $m\angle 2$

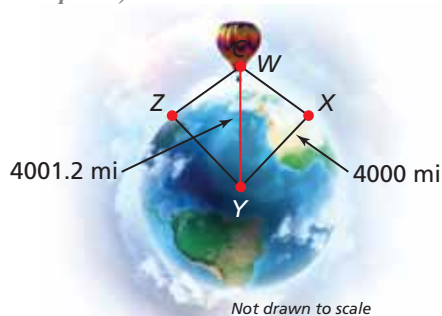
19.  $m\angle 3$

20.  $m\angle 4$

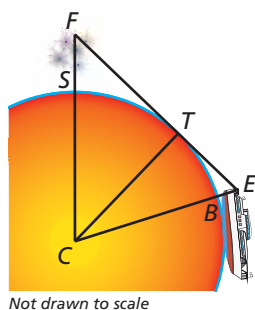
21.  $m\angle 5$

22.  $m\angle 6$

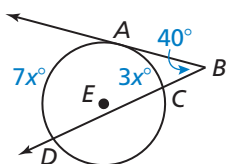
23. **PROBLEM SOLVING** You are flying in a hot air balloon about 1.2 miles above the ground. Find the measure of the arc that represents the part of Earth you can see. The radius of Earth is about 4000 miles. (See Example 4.)



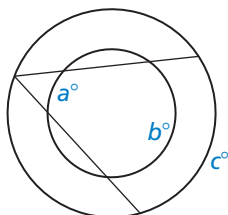
24. **PROBLEM SOLVING** You are watching fireworks over San Diego Bay  $S$  as you sail away in a boat. The highest point the fireworks reach  $F$  is about 0.2 mile above the bay. Your eyes  $E$  are about 0.01 mile above the water. At point  $B$  you can no longer see the fireworks because of the curvature of Earth. The radius of Earth is about 4000 miles, and  $\overline{FE}$  is tangent to Earth at point  $T$ . Find  $m\widehat{SB}$ . Round your answer to the nearest tenth.



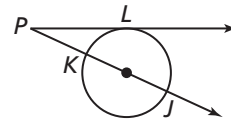
25. **MATHEMATICAL CONNECTIONS** In the diagram,  $\overrightarrow{BA}$  is tangent to  $\odot E$ . Write an algebraic expression for  $m\widehat{CD}$  in terms of  $x$ . Then find  $m\widehat{CD}$ .



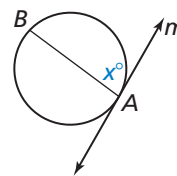
26. **MATHEMATICAL CONNECTIONS** The circles in the diagram are concentric. Write an algebraic expression for  $c$  in terms of  $a$  and  $b$ .



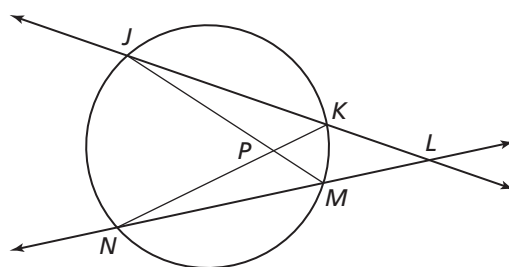
27. **ABSTRACT REASONING** In the diagram,  $\overrightarrow{PL}$  is tangent to the circle, and  $\overline{KJ}$  is a diameter. What is the range of possible angle measures of  $\angle LPJ$ ? Explain your reasoning.



28. **ABSTRACT REASONING** In the diagram,  $\overline{AB}$  is any chord that is not a diameter of the circle. Line  $m$  is tangent to the circle at point  $A$ . What is the range of possible values of  $x$ ? Explain your reasoning. (The diagram is not drawn to scale.)



29. **PROOF** In the diagram,  $\overrightarrow{JL}$  and  $\overrightarrow{NL}$  are secant lines that intersect at point  $L$ . Prove that  $m\angle JPN > m\angle JLN$ .



30. **MAKING AN ARGUMENT** Your friend claims that it is possible for a circumscribed angle to have the same measure as its intercepted arc. Is your friend correct? Explain your reasoning.

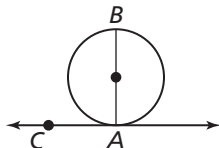
31. **REASONING** Points  $A$  and  $B$  are on a circle, and  $t$  is a tangent line containing  $A$  and another point  $C$ .

- Draw two diagrams that illustrate this situation.
- Write an equation for  $m\widehat{AB}$  in terms of  $m\angle BAC$  for each diagram.
- For what measure of  $\angle BAC$  can you use either equation to find  $m\widehat{AB}$ ? Explain.

32. **REASONING**  $\triangle XYZ$  is an equilateral triangle inscribed in  $\odot P$ .  $\overline{AB}$  is tangent to  $\odot P$  at point  $X$ ,  $\overline{BC}$  is tangent to  $\odot P$  at point  $Y$ , and  $\overline{AC}$  is tangent to  $\odot P$  at point  $Z$ . Draw a diagram that illustrates this situation. Then classify  $\triangle ABC$  by its angles and sides. Justify your answer.

33. **PROVING A THEOREM** To prove the Tangent and Intersected Chord Theorem (Theorem 10.14), you must prove three cases.

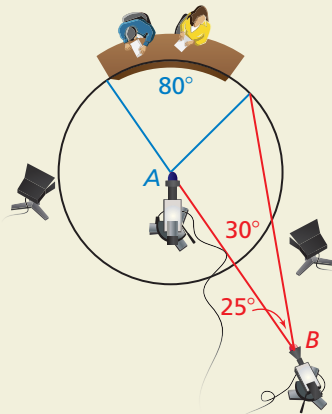
a. The diagram shows the case where  $\overline{AB}$  contains the center of the circle. Use the Tangent Line to Circle Theorem (Theorem 10.1) to write a paragraph proof for this case.



b. Draw a diagram and write a proof for the case where the center of the circle is in the interior of  $\angle CAB$ .

c. Draw a diagram and write a proof for the case where the center of the circle is in the exterior of  $\angle CAB$ .

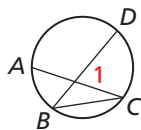
34. **HOW DO YOU SEE IT?** In the diagram, television cameras are positioned at  $A$  and  $B$  to record what happens on stage. The stage is an arc of  $\odot A$ . You would like the camera at  $B$  to have a  $30^\circ$  view of the stage. Should you move the camera closer or farther away? Explain your reasoning.



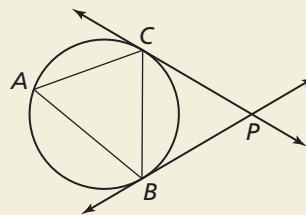
35. **PROVING A THEOREM** Write a proof of the Angles Inside the Circle Theorem (Theorem 10.15).

**Given** Chords  $\overline{AC}$  and  $\overline{BD}$  intersect inside a circle.

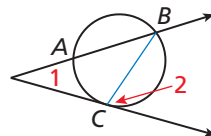
**Prove**  $m\angle 1 = \frac{1}{2}(m\widehat{DC} + m\widehat{AB})$



36. **THOUGHT PROVOKING** In the figure,  $\overleftrightarrow{BP}$  and  $\overleftrightarrow{CP}$  are tangent to the circle. Point  $A$  is any point on the major arc formed by the endpoints of the chord  $\overline{BC}$ . Label all congruent angles in the figure. Justify your reasoning.



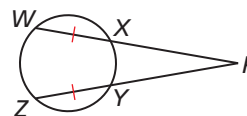
37. **PROVING A THEOREM** Use the diagram below to prove the Angles Outside the Circle Theorem (Theorem 10.16) for the case of a tangent and a secant. Then copy the diagrams for the other two cases on page 567 and draw appropriate auxiliary segments. Use your diagrams to prove each case.



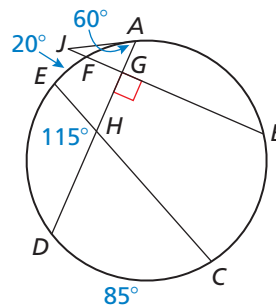
38. **PROVING A THEOREM** Prove that the Circumscribed Angle Theorem (Theorem 10.17) follows from the Angles Outside the Circle Theorem (Theorem 10.16).

In Exercises 39 and 40, find the indicated measure(s). Justify your answer.

39. Find  $m\angle P$  when  $m\widehat{WZY} = 200^\circ$ .



40. Find  $m\widehat{AB}$  and  $m\widehat{ED}$ .



## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. (*Skills Review Handbook*)

41.  $x^2 + x = 12$

42.  $x^2 = 12x + 35$

43.  $-3 = x^2 + 4x$

# 10.6 Segment Relationships in Circles



TEXAS ESSENTIAL  
KNOWLEDGE AND SKILLS  
G.5.A  
G.12.A

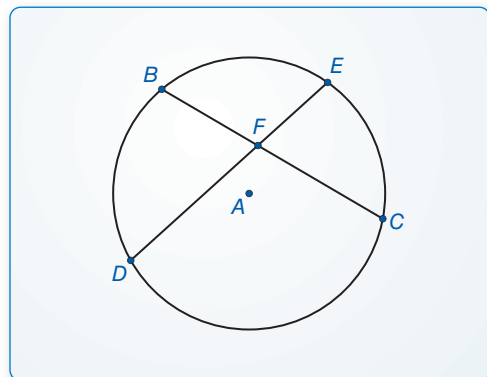
**Essential Question** What relationships exist among the segments formed by two intersecting chords or among segments of two secants that intersect outside a circle?

## EXPLORATION 1 Segments Formed by Two Intersecting Chords

**Work with a partner.** Use dynamic geometry software.

- Construct two chords  $\overline{BC}$  and  $\overline{DE}$  that intersect in the interior of a circle at a point  $F$ .
- Find the segment lengths  $BF$ ,  $CF$ ,  $DF$ , and  $EF$  and complete the table. What do you observe?

**Sample**



$BF$	$CF$	$BF \cdot CF$
$DF$	$EF$	$DF \cdot EF$

### REASONING

To be proficient in math, you need to make sense of quantities and their relationships in problem situations.

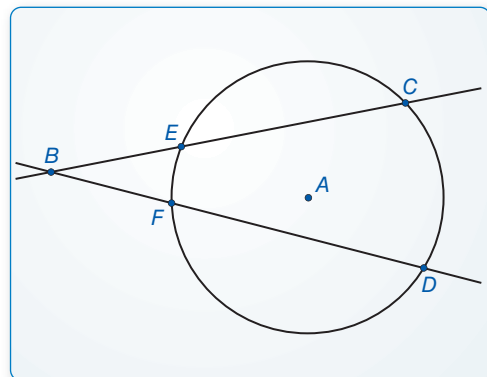
- Repeat parts (a) and (b) several times. Write a conjecture about your results.

## EXPLORATION 2 Secants Intersecting Outside a Circle

**Work with a partner.** Use dynamic geometry software.

- Construct two secants  $\overrightarrow{BC}$  and  $\overrightarrow{BD}$  that intersect at a point  $B$  outside a circle, as shown.
- Find the segment lengths  $BE$ ,  $BC$ ,  $BF$ , and  $BD$ , and complete the table. What do you observe?

**Sample**

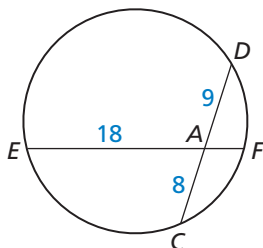


$BE$	$BC$	$BE \cdot BC$
$BF$	$BD$	$BF \cdot BD$

- Repeat parts (a) and (b) several times. Write a conjecture about your results.

### Communicate Your Answer

- What relationships exist among the segments formed by two intersecting chords or among segments of two secants that intersect outside a circle?
- Find the segment length  $AF$  in the figure at the left.



# 10.6 Lesson

## Core Vocabulary

segments of a chord, p. 574  
 tangent segment, p. 575  
 secant segment, p. 575  
 external segment, p. 575

## What You Will Learn

► Use segments of chords, tangents, and secants.

## Using Segments of Chords, Tangents, and Secants

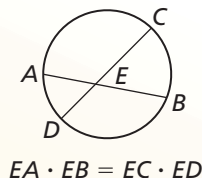
When two chords intersect in the interior of a circle, each chord is divided into two segments that are called **segments of the chord**.

## Theorem

### Theorem 10.18 Segments of Chords Theorem

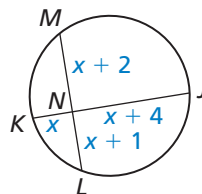
If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

*Proof* Ex. 19, p. 578



### EXAMPLE 1 Using Segments of Chords

Find  $ML$  and  $JK$ .



### SOLUTION

$$NK \cdot NJ = NL \cdot NM$$

$$x \cdot (x + 4) = (x + 1) \cdot (x + 2)$$

$$x^2 + 4x = x^2 + 3x + 2$$

$$4x = 3x + 2$$

$$x = 2$$

Segments of Chords Theorem

Substitute.

Simplify.

Subtract  $x^2$  from each side.

Subtract  $3x$  from each side.

Find  $ML$  and  $JK$  by substitution.

$$\begin{aligned} ML &= (x + 2) + (x + 1) \\ &= 2 + 2 + 2 + 1 \\ &= 7 \end{aligned}$$

$$\begin{aligned} JK &= x + (x + 4) \\ &= 2 + 2 + 4 \\ &= 8 \end{aligned}$$

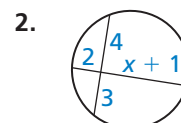
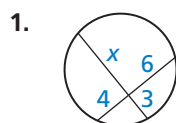
► So,  $ML = 7$  and  $JK = 8$ .

## Monitoring Progress



Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

Find the value of  $x$ .

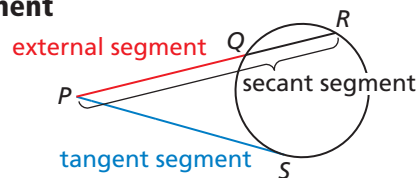


## Core Concept

### Tangent Segment and Secant Segment

A **tangent segment** is a segment that is tangent to a circle at an endpoint.

A **secant segment** is a segment that contains a chord of a circle and has exactly one endpoint outside the circle. The part of a secant segment that is outside the circle is called an **external segment**.



$\overline{PS}$  is a tangent segment.

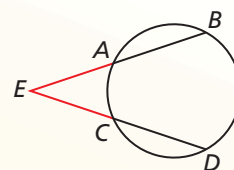
$\overline{PR}$  is a secant segment.

$\overline{PQ}$  is the external segment of  $\overline{PR}$ .

## Theorem

### Theorem 10.19 Segments of Secants Theorem

If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.



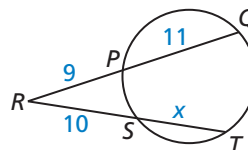
$$EA \cdot EB = EC \cdot ED$$

*Proof* Ex. 20, p. 578

### EXAMPLE 2

### Using Segments of Secants

Find the value of  $x$ .



### SOLUTION

$$RP \cdot RQ = RS \cdot RT$$

$$9 \cdot (11 + 9) = 10 \cdot (x + 10)$$

$$180 = 10x + 100$$

$$80 = 10x$$

$$8 = x$$

Segments of Secants Theorem

Substitute.

Simplify.

Subtract 100 from each side.

Divide each side by 10.

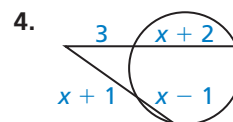
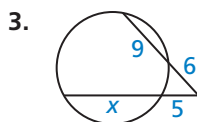
► The value of  $x$  is 8.

### Monitoring Progress



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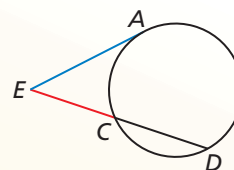
Find the value of  $x$ .



## Theorem

### Theorem 10.20 Segments of Secants and Tangents Theorem

If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.

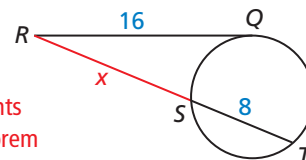


$$EA^2 = EC \cdot ED$$

*Proof* Exs. 21 and 22, p. 578

### EXAMPLE 3 Using Segments of Secants and Tangents

Find  $RS$ .



#### SOLUTION

$$RQ^2 = RS \cdot RT$$

$$16^2 = x \cdot (x + 8)$$

$$256 = x^2 + 8x$$

$$0 = x^2 + 8x - 256$$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(-256)}}{2(1)}$$

$$x = -4 \pm 4\sqrt{17}$$

Segments of Secants and Tangents Theorem

Substitute.

Simplify.

Write in standard form.

Use Quadratic Formula.

Simplify.

Use the positive solution because lengths cannot be negative.

▶ So,  $x = -4 + 4\sqrt{17} \approx 12.49$ , and  $RS \approx 12.49$ .

### EXAMPLE 4 Finding the Radius of a Circle

Find the radius of the aquarium tank.

#### SOLUTION

$$CB^2 = CE \cdot CD$$

$$20^2 = 8 \cdot (2r + 8)$$

$$400 = 16r + 64$$

$$336 = 16r$$

$$21 = r$$

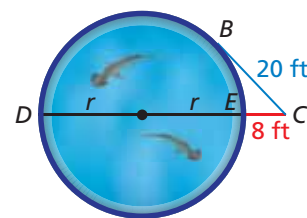
Segments of Secants and Tangents Theorem

Substitute.

Simplify.

Subtract 64 from each side.

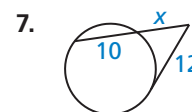
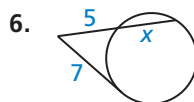
Divide each side by 16.



▶ So, the radius of the tank is 21 feet.

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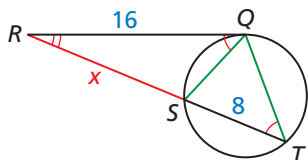
Find the value of  $x$ .



8. **WHAT IF?** In Example 4,  $CB = 35$  feet and  $CE = 14$  feet. Find the radius of the tank.

### ANOTHER WAY

In Example 3, you can draw segments  $\overline{QS}$  and  $\overline{QT}$ .



Because  $\angle RQS$  and  $\angle RTQ$  intercept the same arc, they are congruent. By the Reflexive Property of Congruence (Theorem 2.2),  $\angle QRS \cong \angle TRQ$ . So,  $\triangle RSQ \sim \triangle RQT$  by the AA Similarity Theorem (Theorem 8.3). You can use this fact to write and solve a proportion to find  $x$ .

# 10.6 Exercises

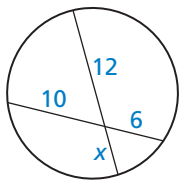
## Vocabulary and Core Concept Check

- VOCABULARY** The part of the secant segment that is outside the circle is called a(n) \_\_\_\_\_.
- WRITING** Explain the difference between a tangent segment and a secant segment.

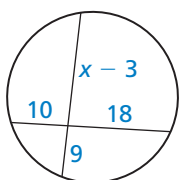
## Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the value of  $x$ . (See Example 1.)

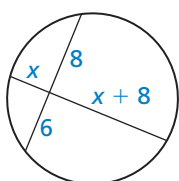
3.



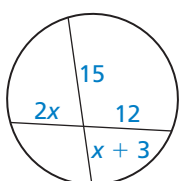
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5.

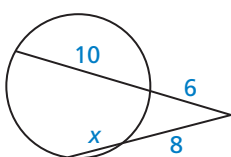


6.

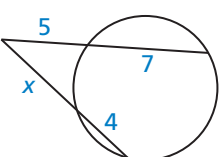


In Exercises 7–10, find the value of  $x$ . (See Example 2.)

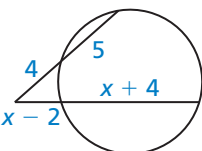
7.



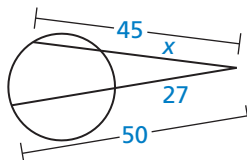
8.



9.

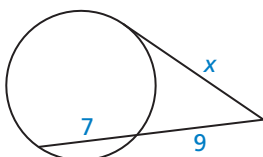


10.

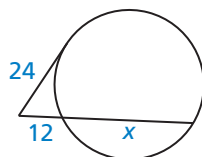


In Exercises 11–14, find the value of  $x$ . (See Example 3.)

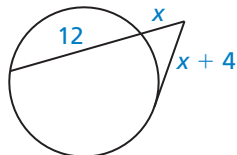
11.



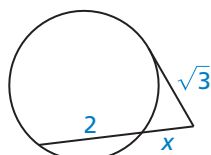
12.



13.



14.



15. **ERROR ANALYSIS** Describe and correct the error in finding  $CD$ .

**X**

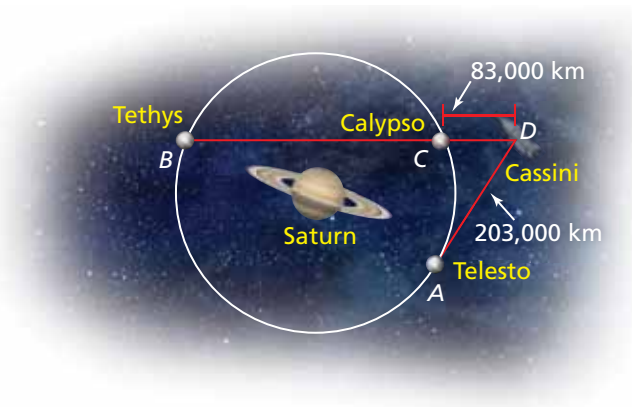
$$CD \cdot DF = AB \cdot AF$$

$$CD \cdot 4 = 5 \cdot 3$$

$$CD \cdot 4 = 15$$

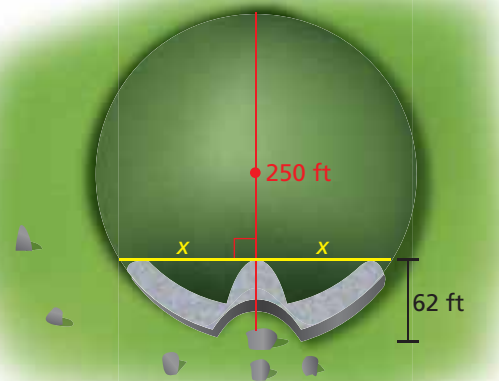
$$CD = 3.75$$

16. **MODELING WITH MATHEMATICS** The Cassini spacecraft is on a mission in orbit around Saturn until September 2017. Three of Saturn's moons, Tethys, Calypso, and Telesto, have nearly circular orbits of radius 295,000 kilometers. The diagram shows the positions of the moons and the spacecraft on one of Cassini's missions. Find the distance  $DB$  from Cassini to Tethys when  $AD$  is tangent to the circular orbit. (See Example 4.)

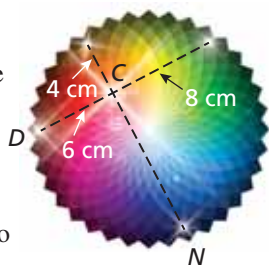




17. **MODELING WITH MATHEMATICS** The circular stone mound in Ireland called Newgrange has a diameter of 250 feet. A passage 62 feet long leads toward the center of the mound. Find the perpendicular distance  $x$  from the end of the passage to either side of the mound.



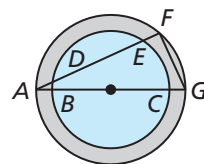
18. **MODELING WITH MATHEMATICS** You are designing an animated logo for your website. Sparkles leave point  $C$  and move to the outer circle along the segments shown so that all of the sparkles reach the outer circle at the same time. Sparkles travel from point  $C$  to point  $D$  at 2 centimeters per second. How fast should sparkles move from point  $C$  to point  $N$ ? Explain.



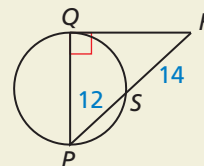
19. **PROVING A THEOREM** Write a two-column proof of the Segments of Chords Theorem (Theorem 10.18).

**Plan for Proof** Use the diagram from page 574. Draw  $AC$  and  $DB$ . Show that  $\triangle EAC$  and  $\triangle EDB$  are similar. Use the fact that corresponding side lengths in similar triangles are proportional.

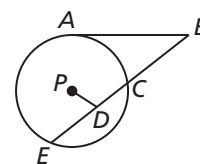
20. **PROVING A THEOREM** Prove the Segments of Secants Theorem (Theorem 10.19). (*Hint*: Draw a diagram and add auxiliary line segments to form similar triangles.)
21. **PROVING A THEOREM** Use the Tangent Line to Circle Theorem (Theorem 10.1) to prove the Segments of Secants and Tangents Theorem (Theorem 10.20) for the special case when the secant segment contains the center of the circle.
22. **PROVING A THEOREM** Prove the Segments of Secants and Tangents Theorem (Theorem 10.20). (*Hint*: Draw a diagram and add auxiliary line segments to form similar triangles.)
23. **WRITING EQUATIONS** In the diagram of the water well,  $AB$ ,  $AD$ , and  $DE$  are known. Write an equation for  $BC$  using these three measurements.



24. **HOW DO YOU SEE IT?** Which two theorems would you need to use to find  $PQ$ ? Explain your reasoning.



25. **CRITICAL THINKING** In the figure,  $AB = 12$ ,  $BC = 8$ ,  $DE = 6$ ,  $PD = 4$ , and  $A$  is a point of tangency. Find the radius of  $\odot P$ .



26. **THOUGHT PROVOKING** Circumscribe a triangle about a circle. Then, using the points of tangency, inscribe a triangle in the circle. Must it be true that the two triangles are similar? Explain your reasoning.

## Maintaining Mathematical Proficiency Reviewing what you learned in previous grades and lessons

Solve the equation by completing the square. (*Skills Review Handbook*)

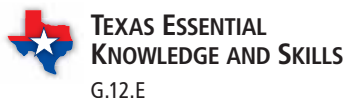
27.  $x^2 + 4x = 45$

28.  $x^2 - 2x - 1 = 8$

29.  $2x^2 + 12x + 20 = 34$

30.  $-4x^2 + 8x + 44 = 16$

# 10.7 Circles in the Coordinate Plane



**Essential Question** What is the equation of a circle with center  $(h, k)$  and radius  $r$  in the coordinate plane?

## EXPLORATION 1

### The Equation of a Circle with Center at the Origin

**Work with a partner.** Use dynamic geometry software to construct and determine the equations of circles centered at  $(0, 0)$  in the coordinate plane, as described below.

- Complete the first two rows of the table for circles with the given radii. Complete the other rows for circles with radii of your choice.
- Write an equation of a circle with center  $(0, 0)$  and radius  $r$ .

Radius	Equation of circle
1	
2	

## EXPLORATION 2

### The Equation of a Circle with Center $(h, k)$

**Work with a partner.** Use dynamic geometry software to construct and determine the equations of circles of radius 2 in the coordinate plane, as described below.

- Complete the first two rows of the table for circles with the given centers. Complete the other rows for circles with centers of your choice.
- Write an equation of a circle with center  $(h, k)$  and radius 2.
- Write an equation of a circle with center  $(h, k)$  and radius  $r$ .

Center	Equation of circle
$(0, 0)$	
$(2, 0)$	

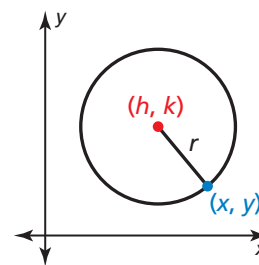
## EXPLORATION 3

### Deriving the Standard Equation of a Circle

**Work with a partner.** Consider a circle with radius  $r$  and center  $(h, k)$ .

Write the Distance Formula to represent the distance  $d$  between a point  $(x, y)$  on the circle and the center  $(h, k)$  of the circle. Then square each side of the Distance Formula equation.

How does your result compare with the equation you wrote in part (c) of Exploration 2?



## USING PROBLEM-SOLVING STRATEGIES

To be proficient in math, you need to explain correspondences between equations and graphs.

## Communicate Your Answer

- What is the equation of a circle with center  $(h, k)$  and radius  $r$  in the coordinate plane?
- Write an equation of the circle with center  $(4, -1)$  and radius 3.

# 10.7 Lesson

## Core Vocabulary

standard equation of a circle,  
p. 580

**Previous**  
completing the square

## What You Will Learn

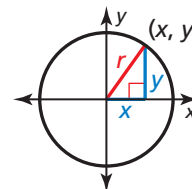
- ▶ Write and graph equations of circles.
- ▶ Write coordinate proofs involving circles.
- ▶ Solve real-life problems using graphs of circles.

## Writing and Graphing Equations of Circles

Let  $(x, y)$  represent any point on a circle with center at the origin and radius  $r$ . By the Pythagorean Theorem (Theorem 9.1),

$$x^2 + y^2 = r^2.$$

This is the equation of a circle with center at the origin and radius  $r$ .



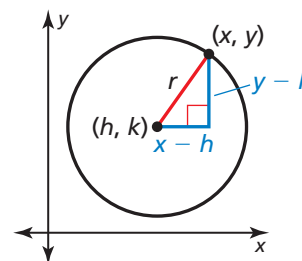
## Core Concept

### Standard Equation of a Circle

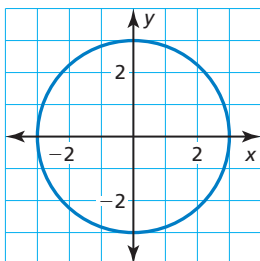
Let  $(x, y)$  represent any point on a circle with center  $(h, k)$  and radius  $r$ . By the Pythagorean Theorem (Theorem 9.1),

$$(x - h)^2 + (y - k)^2 = r^2.$$

This is the **standard equation of a circle** with center  $(h, k)$  and radius  $r$ .



### EXAMPLE 1 Writing the Standard Equation of a Circle



Write the standard equation of each circle.

- a. the circle shown at the left
- b. a circle with center  $(0, -9)$  and radius 4.2

### SOLUTION

- a. The radius is 3, and the center is at the origin.

$$(x - h)^2 + (y - k)^2 = r^2 \quad \text{Standard equation of a circle}$$

$$(x - 0)^2 + (y - 0)^2 = 3^2 \quad \text{Substitute.}$$

$$x^2 + y^2 = 9 \quad \text{Simplify.}$$

- ▶ The standard equation of the circle is  $x^2 + y^2 = 9$ .

- b. The radius is 4.2, and the center is at  $(0, -9)$ .

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 0)^2 + [y - (-9)]^2 = 4.2^2$$

$$x^2 + (y + 9)^2 = 17.64$$

- ▶ The standard equation of the circle is  $x^2 + (y + 9)^2 = 17.64$ .

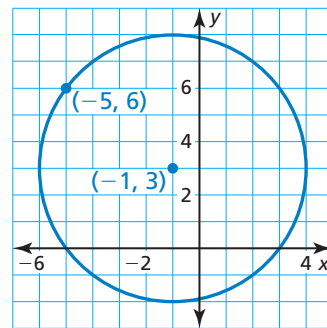
## Monitoring Progress Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

Write the standard equation of the circle with the given center and radius.

1. center:  $(0, 0)$ , radius: 2.5
2. center:  $(-2, 5)$ , radius: 7

**EXAMPLE 2****Writing the Standard Equation of a Circle**

The point  $(-5, 6)$  is on a circle with center  $(-1, 3)$ . Write the standard equation of the circle.

**SOLUTION**

To write the standard equation, you need to know the values of  $h$ ,  $k$ , and  $r$ . To find  $r$ , find the distance between the center and the point  $(-5, 6)$  on the circle.

$$\begin{aligned} r &= \sqrt{[-5 - (-1)]^2 + (6 - 3)^2} \\ &= \sqrt{(-4)^2 + 3^2} \\ &= 5 \end{aligned}$$

Distance Formula

Simplify.

Simplify.

Substitute the values for the center and the radius into the standard equation of a circle.

$$\begin{aligned} (x - h)^2 + (y - k)^2 &= r^2 \\ [x - (-1)]^2 + (y - 3)^2 &= 5^2 \\ (x + 1)^2 + (y - 3)^2 &= 25 \end{aligned}$$

Standard equation of a circle

Substitute  $(h, k) = (-1, 3)$  and  $r = 5$ .

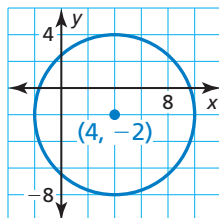
Simplify.

► The standard equation of the circle is  $(x + 1)^2 + (y - 3)^2 = 25$ .

**REMEMBER**

To complete the square for the expression  $x^2 + bx$ , add the square of half the coefficient of the term  $bx$ .

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

**EXAMPLE 3****Graphing a Circle**

The equation of a circle is  $x^2 + y^2 - 8x + 4y - 16 = 0$ . Find the center and the radius of the circle. Then graph the circle.

**SOLUTION**

You can write the equation in standard form by completing the square on the  $x$ -terms and the  $y$ -terms.

$$\begin{aligned} x^2 + y^2 - 8x + 4y - 16 &= 0 \\ x^2 - 8x + y^2 + 4y &= 16 \\ x^2 - 8x + 16 + y^2 + 4y + 4 &= 16 + 16 + 4 \\ (x - 4)^2 + (y + 2)^2 &= 36 \\ (x - 4)^2 + [y - (-2)]^2 &= 6^2 \end{aligned}$$

Equation of circle

Isolate constant. Group terms.

Complete the square twice.

Factor left side. Simplify right side.

Rewrite the equation to find the center and the radius.

► The center is  $(4, -2)$ , and the radius is 6. Use a compass to graph the circle.

**Monitoring Progress**

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- The point  $(3, 4)$  is on a circle with center  $(1, 4)$ . Write the standard equation of the circle.
- The equation of a circle is  $x^2 + y^2 - 8x + 6y + 9 = 0$ . Find the center and the radius of the circle. Then graph the circle.

## Writing Coordinate Proofs Involving Circles

### EXAMPLE 4 Writing a Coordinate Proof Involving a Circle

Prove or disprove that the point  $(\sqrt{2}, \sqrt{2})$  lies on the circle centered at the origin and containing the point  $(2, 0)$ .

#### SOLUTION

The circle centered at the origin and containing the point  $(2, 0)$  has the following radius.

$$r = \sqrt{(x - h)^2 + (y - k)^2} = \sqrt{(2 - 0)^2 + (0 - 0)^2} = 2$$

So, a point lies on the circle if and only if the distance from that point to the origin is 2. The distance from  $(\sqrt{2}, \sqrt{2})$  to  $(0, 0)$  is

$$d = \sqrt{(\sqrt{2} - 0)^2 + (\sqrt{2} - 0)^2} = 2.$$

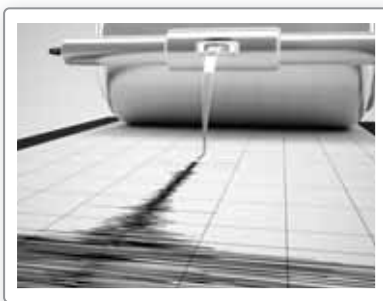
► So, the point  $(\sqrt{2}, \sqrt{2})$  lies on the circle centered at the origin and containing the point  $(2, 0)$ .

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5. Prove or disprove that the point  $(1, \sqrt{5})$  lies on the circle centered at the origin and containing the point  $(0, 1)$ .

## Solving Real-Life Problems

### EXAMPLE 5 Using Graphs of Circles



The epicenter of an earthquake is the point on Earth's surface directly above the earthquake's origin. A seismograph can be used to determine the distance to the epicenter of an earthquake. Seismographs are needed in three different places to locate an earthquake's epicenter.

Use the seismograph readings from locations  $A$ ,  $B$ , and  $C$  to find the epicenter of an earthquake.

- The epicenter is 7 miles away from  $A(-2, 2.5)$ .
- The epicenter is 4 miles away from  $B(4, 6)$ .
- The epicenter is 5 miles away from  $C(3, -2.5)$ .

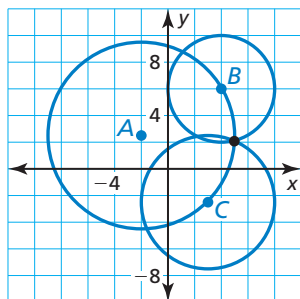
#### SOLUTION

The set of all points equidistant from a given point is a circle, so the epicenter is located on each of the following circles.

- $\odot A$  with center  $(-2, 2.5)$  and radius 7
- $\odot B$  with center  $(4, 6)$  and radius 4
- $\odot C$  with center  $(3, -2.5)$  and radius 5

To find the epicenter, graph the circles on a coordinate plane where each unit corresponds to one mile. Find the point of intersection of the three circles.

► The epicenter is at about  $(5, 2)$ .



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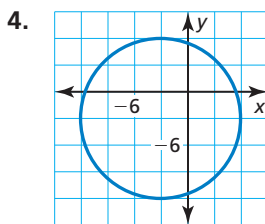
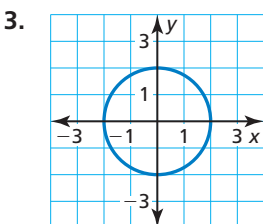
6. Why are three seismographs needed to locate an earthquake's epicenter?

## Vocabulary and Core Concept Check

- VOCABULARY** What is the standard equation of a circle?
- WRITING** Explain why knowing the location of the center and one point on a circle is enough to graph the circle.

## Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, write the standard equation of the circle. (See Example 1.)



- a circle with center  $(0, 0)$  and radius 7
- a circle with center  $(4, 1)$  and radius 5
- a circle with center  $(-3, 4)$  and radius 1
- a circle with center  $(3, -5)$  and radius 7

In Exercises 9–11, use the given information to write the standard equation of the circle. (See Example 2.)

- The center is  $(0, 0)$ , and a point on the circle is  $(0, 6)$ .
- The center is  $(1, 2)$ , and a point on the circle is  $(4, 2)$ .
- The center is  $(0, 0)$ , and a point on the circle is  $(3, -7)$ .
- ERROR ANALYSIS** Describe and correct the error in writing the standard equation of a circle.



The standard equation of a circle with center  $(-3, -5)$  and radius 3 is  $(x - 3)^2 + (y - 5)^2 = 9$ .

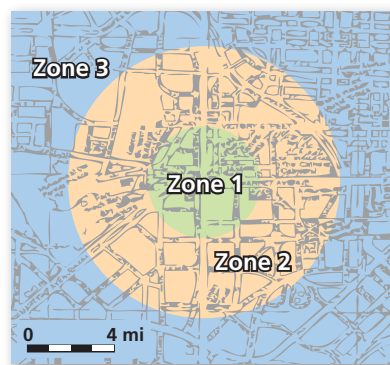
In Exercises 13–18, find the center and radius of the circle. Then graph the circle. (See Example 3.)

- $x^2 + y^2 = 49$
- $(x + 5)^2 + (y - 3)^2 = 9$
- $x^2 + y^2 - 6x = 7$
- $x^2 + y^2 + 4y = 32$
- $x^2 + y^2 - 8x - 2y = -16$

18.  $x^2 + y^2 + 4x + 12y = -15$

In Exercises 19–22, prove or disprove the statement. (See Example 4.)

- The point  $(2, 3)$  lies on the circle centered at the origin with radius 8.
- The point  $(4, \sqrt{5})$  lies on the circle centered at the origin with radius 3.
- The point  $(\sqrt{6}, 2)$  lies on the circle centered at the origin and containing the point  $(3, -1)$ .
- The point  $(\sqrt{7}, 5)$  lies on the circle centered at the origin and containing the point  $(5, 2)$ .
- MODELING WITH MATHEMATICS** A city's commuter system has three zones. Zone 1 serves people living within 3 miles of the city's center. Zone 2 serves those between 3 and 7 miles from the center. Zone 3 serves those over 7 miles from the center. (See Example 5.)



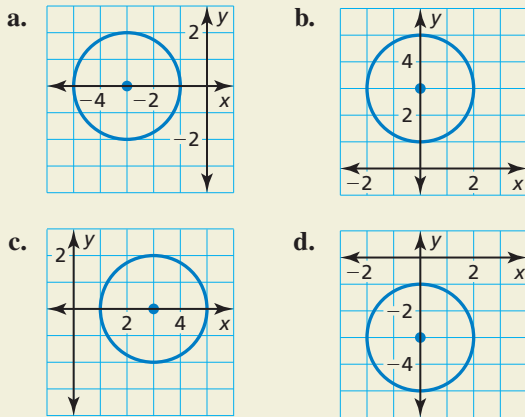
- Graph this situation on a coordinate plane where each unit corresponds to 1 mile. Locate the city's center at the origin.
- Determine which zone serves people whose homes are represented by the points  $(3, 4)$ ,  $(6, 5)$ ,  $(1, 2)$ ,  $(0, 3)$ , and  $(1, 6)$ .

**24. MODELING WITH MATHEMATICS** Telecommunication towers can be used to transmit cellular phone calls. A graph with units measured in kilometers shows towers at points (0, 0), (0, 5), and (6, 3). These towers have a range of about 3 kilometers.

- Sketch a graph and locate the towers. Are there any locations that may receive calls from more than one tower? Explain your reasoning.
- The center of City A is located at (-2, 2.5), and the center of City B is located at (5, 4). Each city has a radius of 1.5 kilometers. Which city seems to have better cell phone coverage? Explain your reasoning.

**25. REASONING** Sketch the graph of the circle whose equation is  $x^2 + y^2 = 16$ . Then sketch the graph of the circle after the translation  $(x, y) \rightarrow (x - 2, y - 4)$ . What is the equation of the image? Make a conjecture about the equation of the image of a circle centered at the origin after a translation  $m$  units to the left and  $n$  units down.

**26. HOW DO YOU SEE IT?** Match each graph with its equation.



- |                                 |                                 |
|---------------------------------|---------------------------------|
| <b>A.</b> $x^2 + (y + 3)^2 = 4$ | <b>B.</b> $(x - 3)^2 + y^2 = 4$ |
| <b>C.</b> $(x + 3)^2 + y^2 = 4$ | <b>D.</b> $x^2 + (y - 3)^2 = 4$ |

**27. USING STRUCTURE** The vertices of  $\triangle XYZ$  are  $X(4, 5)$ ,  $Y(4, 13)$ , and  $Z(8, 9)$ . Find the equation of the circle circumscribed about  $\triangle XYZ$ . Justify your answer.

**28. THOUGHT PROVOKING** A circle has center  $(h, k)$  and contains point  $(a, b)$ . Write the equation of the line tangent to the circle at point  $(a, b)$ .

**MATHEMATICAL CONNECTIONS** In Exercises 29–32, use the equations to determine whether the line is a *tangent*, a *secant*, a *secant that contains the diameter*, or *none of these*. Explain your reasoning.

**29. Circle:**  $(x - 4)^2 + (y - 3)^2 = 9$   
**Line:**  $y = 6$

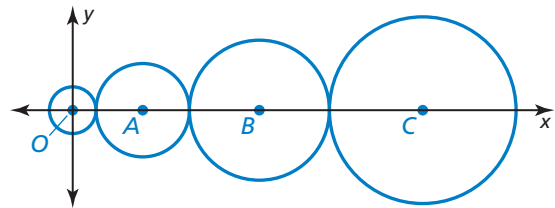
**30. Circle:**  $(x + 2)^2 + (y - 2)^2 = 16$   
**Line:**  $y = 2x - 4$

**31. Circle:**  $(x - 5)^2 + (y + 1)^2 = 4$   
**Line:**  $y = \frac{1}{5}x - 3$

**32. Circle:**  $(x + 3)^2 + (y - 6)^2 = 25$   
**Line:**  $y = -\frac{4}{3}x + 2$

**33. MAKING AN ARGUMENT** Your friend claims that the equation of a circle passing through the points  $(-1, 0)$  and  $(1, 0)$  is  $x^2 - 2yk + y^2 = 1$  with center  $(0, k)$ . Is your friend correct? Explain your reasoning.

**34. REASONING** Four tangent circles are centered on the  $x$ -axis. The radius of  $\odot A$  is twice the radius of  $\odot O$ . The radius of  $\odot B$  is three times the radius of  $\odot O$ . The radius of  $\odot C$  is four times the radius of  $\odot O$ . All circles have integer radii, and the point  $(63, 16)$  is on  $\odot C$ . What is the equation of  $\odot A$ ? Explain your reasoning.

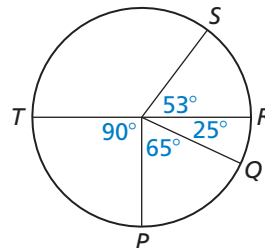


## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Identify the arc as a *major arc*, *minor arc*, or *semicircle*. Then find the measure of the arc. (Section 10.2)

- |                     |                    |
|---------------------|--------------------|
| 35. $\widehat{RS}$  | 36. $\widehat{PR}$ |
| 37. $\widehat{PRT}$ | 38. $\widehat{ST}$ |
| 39. $\widehat{RST}$ | 40. $\widehat{QS}$ |



# 10.4–10.7 What Did You Learn?

## Core Vocabulary

inscribed angle, *p.* 558  
intercepted arc, *p.* 558  
subtend, *p.* 558  
inscribed polygon, *p.* 560

circumscribed circle, *p.* 560  
circumscribed angle, *p.* 568  
segments of a chord, *p.* 574  
tangent segment, *p.* 575

secant segment, *p.* 575  
external segment, *p.* 575  
standard equation of a circle, *p.* 580

## Core Concepts

### Section 10.4

Inscribed Angle and Intercepted Arc, *p.* 558  
Theorem 10.10 Measure of an Inscribed Angle  
Theorem, *p.* 558  
Theorem 10.11 Inscribed Angles of a Circle Theorem,  
*p.* 559

Inscribed Polygon, *p.* 560  
Theorem 10.12 Inscribed Right Triangle Theorem,  
*p.* 560  
Theorem 10.13 Inscribed Quadrilateral Theorem,  
*p.* 560

### Section 10.5

Theorem 10.14 Tangent and Intersected Chord  
Theorem, *p.* 566  
Intersecting Lines and Circles, *p.* 566  
Theorem 10.15 Angles Inside the Circle Theorem,  
*p.* 567

Theorem 10.16 Angles Outside the Circle Theorem,  
*p.* 567  
Circumscribed Angle, *p.* 568  
Theorem 10.17 Circumscribed Angle Theorem, *p.* 568

### Section 10.6

Theorem 10.18 Segments of Chords Theorem, *p.* 574  
Tangent Segment and Secant Segment, *p.* 575  
Theorem 10.19 Segments of Secants Theorem, *p.* 575

Theorem 10.20 Segments of Secants and Tangents  
Theorem, *p.* 576

### Section 10.7

Standard Equation of a Circle, *p.* 580

Writing Coordinate Proofs Involving Circles, *p.* 582

## Mathematical Thinking

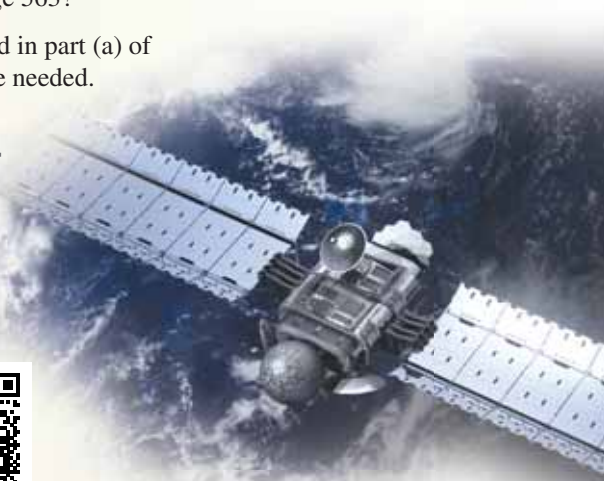
1. What other tools could you use to complete the task in Exercise 18 on page 563?
2. You have a classmate who is confused about why two diagrams are needed in part (a) of Exercise 31 on page 571. Explain to your classmate why two diagrams are needed.

### Performance Task

## Circular Motion

What do the properties of tangents tell us about the forces acting on a satellite orbiting around Earth? How would the path of the satellite change if the force of gravity were removed?

To explore the answers to these questions and more, go to [BigIdeasMath.com](http://BigIdeasMath.com).





# 10 Chapter Review

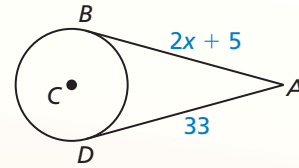
## 10.1 Lines and Segments That Intersect Circles (pp. 533–540)

In the diagram,  $\overline{AB}$  is tangent to  $\odot C$  at  $B$  and  $\overline{AD}$  is tangent to  $\odot C$  at  $D$ . Find the value of  $x$ .

$$AB = AD \quad \text{External Tangent Congruence Theorem (Theorem 10.2)}$$

$$2x + 5 = 33 \quad \text{Substitute.}$$

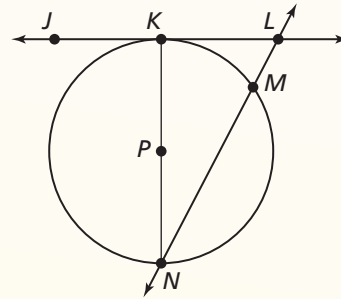
$$x = 14 \quad \text{Solve for } x.$$



► The value of  $x$  is 14.

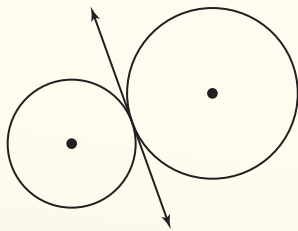
Tell whether the line, ray, or segment is best described as a *radius*, *chord*, *diameter*, *secant*, or *tangent* of  $\odot P$ .

- |                          |                    |
|--------------------------|--------------------|
| 1. $\overline{PK}$       | 2. $\overline{NM}$ |
| 3. $\overrightarrow{JL}$ | 4. $\overline{KN}$ |
| 5. $\overrightarrow{NL}$ | 6. $\overline{PN}$ |

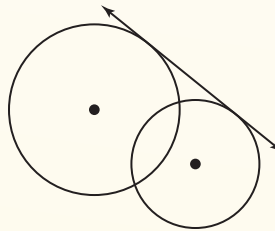


Tell whether the common tangent is *internal* or *external*.

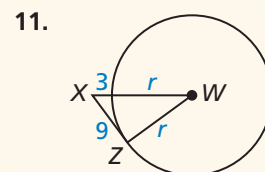
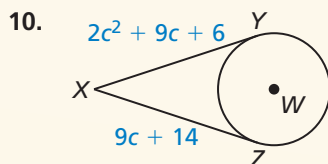
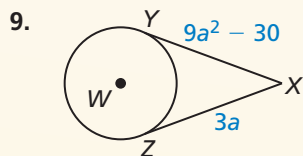
7.



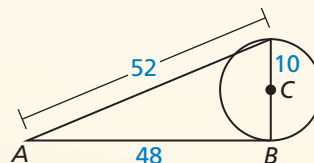
8.



Points  $Y$  and  $Z$  are points of tangency. Find the value of the variable.



12. Tell whether  $\overline{AB}$  is tangent to  $\odot C$ . Explain.



### 10.2 Finding Arc Measures (pp. 541–548)

Find the measure of each arc of  $\odot P$ , where  $\overline{LN}$  is a diameter.

a.  $\widehat{MN}$

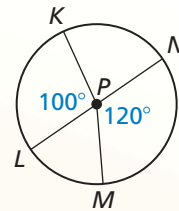
▶  $\widehat{MN}$  is a minor arc, so  $m\widehat{MN} = m\angle MPN = 120^\circ$ .

b.  $\widehat{NLM}$

▶  $\widehat{NLM}$  is a major arc, so  $m\widehat{NLM} = 360^\circ - 120^\circ = 240^\circ$ .

c.  $\widehat{NML}$

▶  $\overline{NL}$  is a diameter, so  $\widehat{NML}$  is a semicircle, and  $m\widehat{NML} = 180^\circ$ .



Use the diagram above to find the measure of the indicated arc.

13.  $\widehat{KL}$

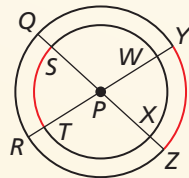
14.  $\widehat{LM}$

15.  $\widehat{KM}$

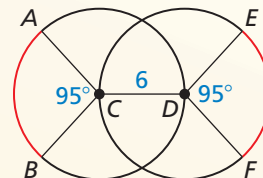
16.  $\widehat{KN}$

Tell whether the red arcs are congruent. Explain why or why not.

17.



18.

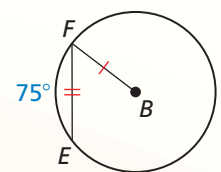
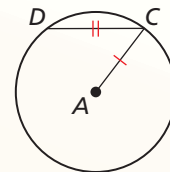


### 10.3 Using Chords (pp. 549–554)

In the diagram,  $\odot A \cong \odot B$ ,  $\overline{CD} \cong \overline{FE}$ , and  $m\widehat{FE} = 75^\circ$ . Find  $m\widehat{CD}$ .

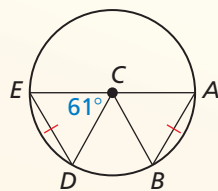
Because  $\overline{CD}$  and  $\overline{FE}$  are congruent chords in congruent circles, the corresponding minor arcs  $\widehat{CD}$  and  $\widehat{FE}$  are congruent by the Congruent Corresponding Chords Theorem (Theorem 10.6).

▶ So,  $m\widehat{CD} = m\widehat{FE} = 75^\circ$ .

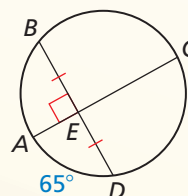


Find the measure of  $\widehat{AB}$ .

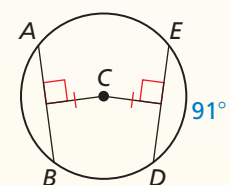
19.



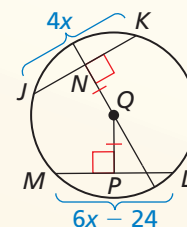
20.



21.



22. In the diagram,  $QN = QP = 10$ ,  $JK = 4x$ , and  $LM = 6x - 24$ . Find the radius of  $\odot Q$ .

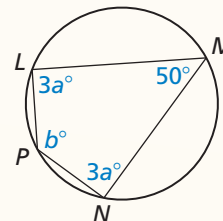


## 10.4 Inscribed Angles and Polygons (pp. 557–564)

Find the value of each variable.

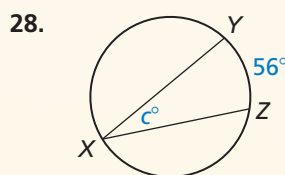
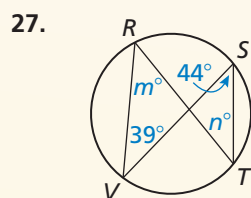
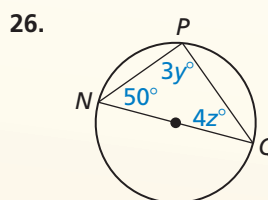
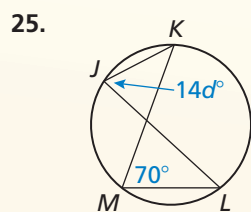
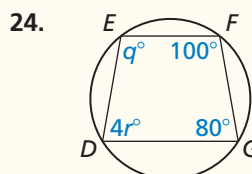
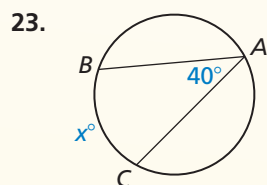
$LMNP$  is inscribed in a circle, so opposite angles are supplementary by the Inscribed Quadrilateral Theorem (Theorem 10.13).

$$\begin{aligned} m\angle L + m\angle N &= 180^\circ & m\angle P + m\angle M &= 180^\circ \\ 3a^\circ + 3a^\circ &= 180^\circ & b^\circ + 50^\circ &= 180^\circ \\ 6a &= 180 & b &= 130 \\ a &= 30 \end{aligned}$$



▶ The value of  $a$  is 30, and the value of  $b$  is 130.

Find the value(s) of the variable(s).

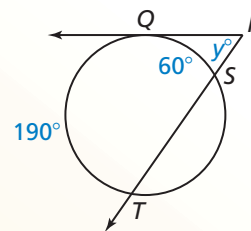


## 10.5 Angle Relationships in Circles (pp. 565–572)

Find the value of  $y$ .

The tangent  $\overrightarrow{RQ}$  and secant  $\overrightarrow{RT}$  intersect outside the circle, so you can use the Angles Outside the Circle Theorem (Theorem 10.16).

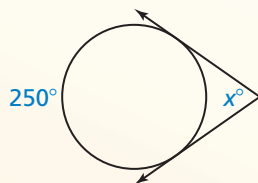
$$\begin{aligned} y^\circ &= \frac{1}{2}(m\widehat{QT} - m\widehat{SQ}) && \text{Angles Outside the Circle Theorem} \\ y^\circ &= \frac{1}{2}(190^\circ - 60^\circ) && \text{Substitute.} \\ y &= 65 && \text{Simplify.} \end{aligned}$$



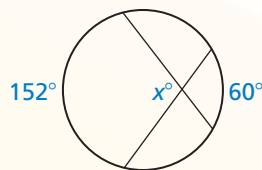
▶ The value of  $y$  is 65.

Find the value of  $x$ .

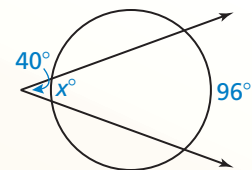
29.



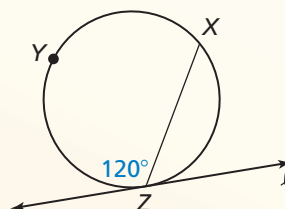
30.



31.



32. Line  $\ell$  is tangent to the circle. Find  $m\widehat{XYZ}$ .



## 10.6 Segment Relationships in Circles (pp. 573–578)

Find the value of  $x$ .

The chords  $\overline{EG}$  and  $\overline{FH}$  intersect inside the circle, so you can use the Segments of Chords Theorem (Theorem 10.18).

$$EP \cdot PG = FP \cdot PH$$

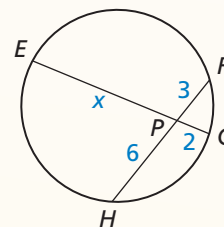
Segments of Chords Theorem

$$x \cdot 2 = 3 \cdot 6$$

Substitute.

$$x = 9$$

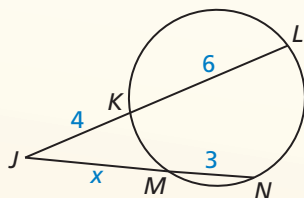
Simplify.



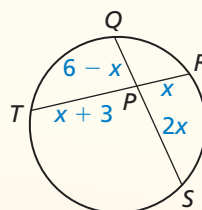
► The value of  $x$  is 9.

Find the value of  $x$ .

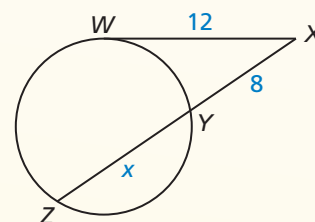
33.



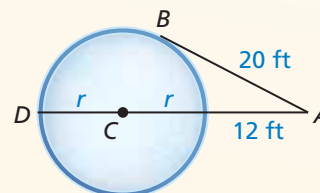
34.



35.

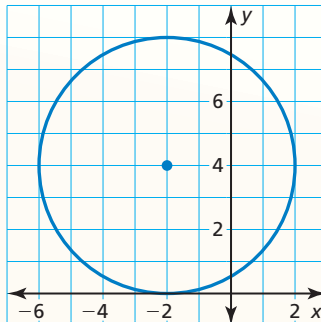


36. A local park has a circular ice skating rink. You are standing at point A, about 12 feet from the edge of the rink. The distance from you to a point of tangency on the rink is about 20 feet. Estimate the radius of the rink.



## 10.7 Circles in the Coordinate Plane (pp. 579–584)

Write the standard equation of the circle shown.



The radius is 4, and the center is  $(-2, 4)$ .

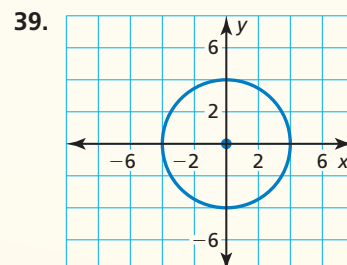
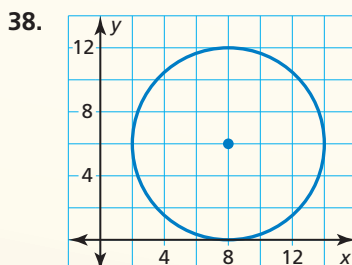
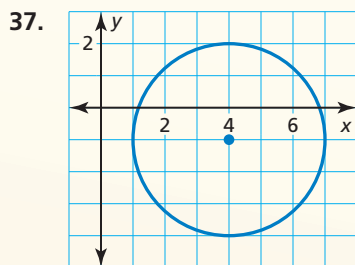
$$(x - h)^2 + (y - k)^2 = r^2 \quad \text{Standard equation of a circle}$$

$$[x - (-2)]^2 + (y - 4)^2 = 4^2 \quad \text{Substitute.}$$

$$(x + 2)^2 + (y - 4)^2 = 16 \quad \text{Simplify.}$$

► The standard equation of the circle is  $(x + 2)^2 + (y - 4)^2 = 16$ .

Write the standard equation of the circle shown.



Write the standard equation of the circle with the given center and radius.

40. center:  $(0, 0)$ , radius: 9

41. center:  $(-5, 2)$ , radius: 1.3

42. center:  $(6, 21)$ , radius: 4

43. center:  $(-3, 2)$ , radius: 16

44. center:  $(10, 7)$ , radius: 3.5

45. center:  $(0, 0)$ , radius: 5.2

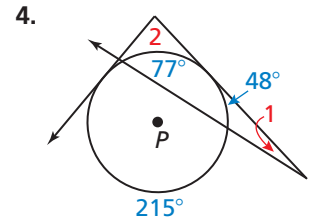
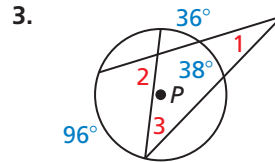
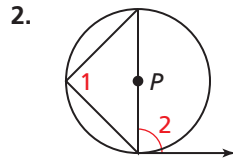
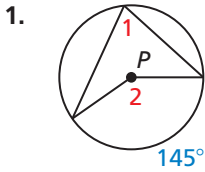
46. The point  $(-7, 1)$  is on a circle with center  $(-7, 6)$ . Write the standard equation of the circle.

47. The equation of a circle is  $x^2 + y^2 - 12x + 8y + 48 = 0$ . Find the center and the radius of the circle. Then graph the circle.

48. Prove or disprove that the point  $(4, -3)$  lies on the circle centered at the origin and containing the point  $(-5, 0)$ .

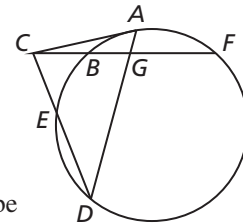
# 10 Chapter Test

Find the measure of each numbered angle in  $\odot P$ . Justify your answer.

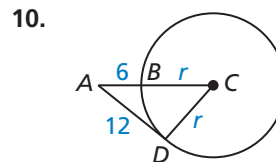
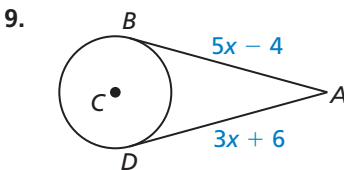


Use the diagram.

5.  $AG = 2$ ,  $GD = 9$ , and  $BG = 3$ . Find  $GF$ .
6.  $CF = 12$ ,  $CB = 3$ , and  $CD = 9$ . Find  $CE$ .
7.  $BF = 9$  and  $CB = 3$ . Find  $CA$ .
8. Sketch a pentagon inscribed in a circle. Label the pentagon  $ABCDE$ . Describe the relationship between each pair of angles. Explain your reasoning.
- a.  $\angle CDE$  and  $\angle CAE$                       b.  $\angle CBE$  and  $\angle CAE$

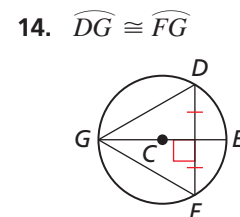
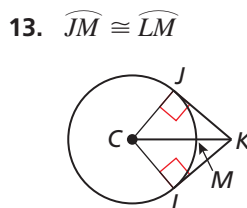
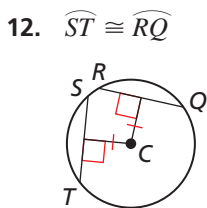


Find the value of the variable. Justify your answer.



11. Prove or disprove that the point  $(2\sqrt{2}, -1)$  lies on the circle centered at  $(0, 2)$  and containing the point  $(-1, 4)$ .

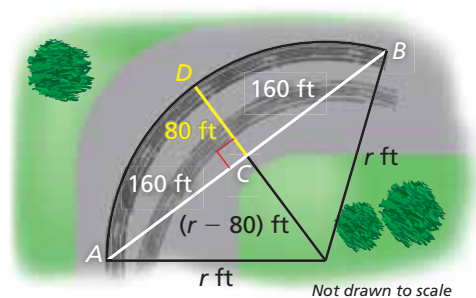
Prove the given statement.



15. A bank of lighting hangs over a stage. Each light illuminates a circular region on the stage. A coordinate plane is used to arrange the lights, using a corner of the stage as the origin. The equation  $(x - 13)^2 + (y - 4)^2 = 16$  represents the boundary of the region illuminated by one of the lights. Three actors stand at the points  $A(11, 4)$ ,  $B(8, 5)$ , and  $C(15, 5)$ . Graph the given equation. Then determine which actors are illuminated by the light.

16. If a car goes around a turn too quickly, it can leave tracks that form an arc of a circle. By finding the radius of the circle, accident investigators can estimate the speed of the car.

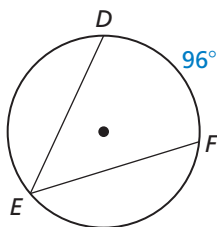
- a. To find the radius, accident investigators choose points  $A$  and  $B$  on the tire marks. Then the investigators find the midpoint  $C$  of  $\overline{AB}$ . Use the diagram to find the radius  $r$  of the circle. Explain why this method works.
- b. The formula  $S = 3.87\sqrt{fr}$  can be used to estimate a car's speed in miles per hour, where  $f$  is the *coefficient of friction* and  $r$  is the radius of the circle in feet. If  $f = 0.7$ , estimate the car's speed in part (a).



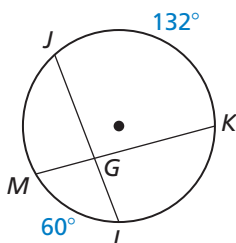
# 10 Standards Assessment

1. Which angle has a measure of  $96^\circ$ ? (TEKS G.12.A)

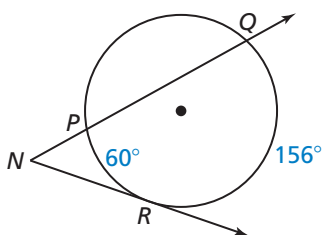
(A)  $\angle DEF$



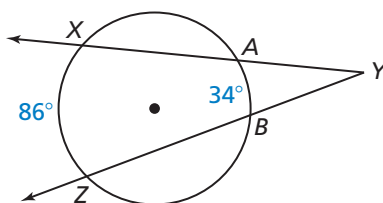
(B)  $\angle JGK$



(C)  $\angle QNR$



(D)  $\angle XYZ$



2. Which conditional statement is the inverse of the statement below? (TEKS G.4.B)

*“If you are a soccer player, then you are an athlete.”*

(F) If you are not a soccer player, then you are not an athlete.

(G) If you are an athlete, then you are a soccer player.

(H) You are a soccer player if and only if you are an athlete.

(J) If you are not an athlete, then you are not a soccer player.

3. The equation of a circle is  $x^2 + y^2 + 14x - 16y + 77 = 0$ . What are the center and radius of the circle? (TEKS G.12.E)

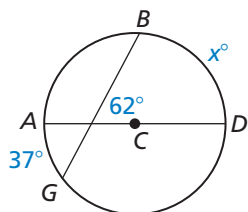
(A) center:  $(14, -16)$ , radius: 8.8

(B) center:  $(-7, 8)$ , radius: 6

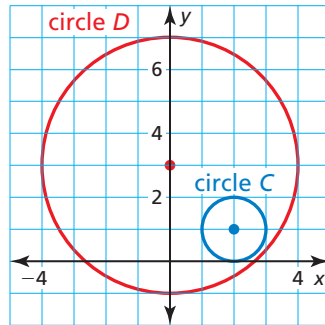
(C) center:  $(-14, 16)$ , radius: 8.8

(D) center:  $(7, -8)$ , radius: 5.2

4. **GRIDDED ANSWER** Find the value of  $x$ . (TEKS G.12.A)

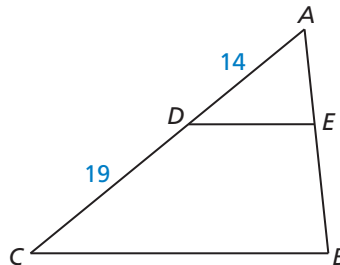


5. Which composition of transformations maps circle  $C$  to circle  $D$ ? (TEKS G.3.C)



- (F) Translation:  $(x, y) \rightarrow (x + 2, y + 2)$     (G) Translation:  $(x, y) \rightarrow (x + 2, y - 2)$   
 Dilation: center  $(0, 3)$  and  $k = 4$     Dilation: center  $(0, 3)$  and  $k = \frac{1}{4}$
- (H) Translation:  $(x, y) \rightarrow (x - 2, y + 2)$     (J) Translation:  $(x, y) \rightarrow (x - 2, y + 2)$   
 Dilation: center  $(0, 0)$  and  $k = 4$     Dilation: center  $(0, 3)$  and  $k = 4$
6. In  $\triangle ABC$ ,  $\overline{DE} \parallel \overline{CB}$ . What is the approximate length of  $\overline{AE}$  when  $AB = 21$ ? (TEKS G.8.A)

- (A) 8.9  
 (B) 12.1  
 (C) 15.5  
 (D) 49.5



7. The coordinates of the vertices of a quadrilateral are  $W(-7, -6)$ ,  $X(1, -2)$ ,  $Y(3, -6)$ , and  $Z(-5, -10)$ . What is the most specific name for quadrilateral  $WXYZ$ ? (TEKS G.6.E)
- (F) rectangle    (G) rhombus  
 (H) square    (J) trapezoid
8. Which quadrilaterals could be inscribed in a circle? (TEKS G.12.A)

- I. square  
 II. rectangle  
 III. parallelogram  
 IV. isosceles trapezoid
- (A) I and II only    (B) III and IV only  
 (C) I, II, and III only    (D) I, II, and IV only