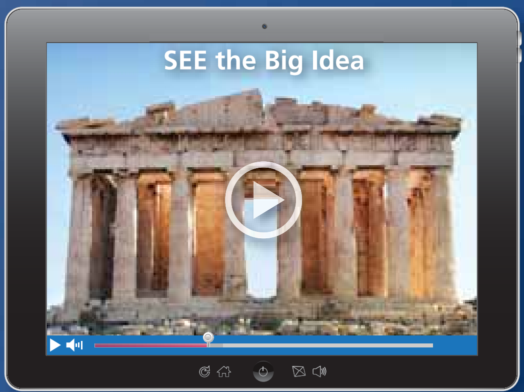
9 Solving Quadratic Equations

Dolphin *(p. 509)*

Half-pipe *(p. 501)*

Pond *(p. 489)*

Kicker *(p. 479)*



Parthenon *(p. 469)*

Mathematical Thinking: *Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.*

Maintaining Mathematical Proficiency

Factoring Perfect Square Trinomials (A.10.E)

Example 1 **Factor $x^2 + 14x + 49$.**

$$x^2 + 14x + 49 = x^2 + 2(x)(7) + 7^2 \qquad \text{Write as } a^2 + 2ab + b^2.$$

$$= (x + 7)^2 \qquad \text{Perfect square trinomial pattern}$$

Factor the trinomial.

1. $x^2 + 10x + 25$ **2.** $x^2 - 20x + 100$ **3.** $x^2 + 12x + 36$

4. $x^2 - 18x + 81$ **5.** $x^2 + 16x + 64$ **6.** $x^2 - 30x + 225$

Solving Systems of Linear Equations by Graphing (A.5.C)

Example 2 **Solve the system of linear equations by graphing.**

$$y = 2x + 1 \qquad \text{Equation 1}$$

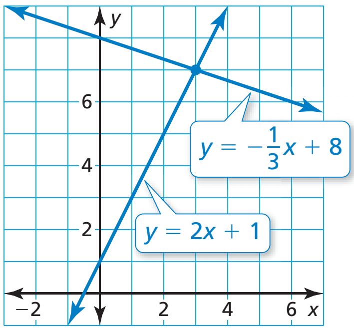
$$y = -\frac{1}{3}x + 8 \qquad \text{Equation 2}$$

Step 1 Graph each equation.

Step 2 Estimate the point of intersection. The graphs appear to intersect at (3, 7).

Step 3 Check your point from Step 2.

Equation 1	Equation 2
$y = 2x + 1$	$y = -\frac{1}{3}x + 8$
$7 \stackrel{?}{=} 2(3) + 1$	$7 \stackrel{?}{=} -\frac{1}{3}(3) + 8$
$7 = 7$ ✓	$7 = 7$ ✓



▶ The solution is (3, 7).

Solve the system of linear equations by graphing.

7. $y = -5x + 3$
$y = 2x - 4$

8. $y = \frac{3}{2}x - 2$
$y = -\frac{1}{4}x + 5$

9. $y = \frac{1}{2}x + 4$
$y = -3x - 3$

10. **ABSTRACT REASONING** What value of c makes $x^2 + bx + c$ a perfect square trinomial?

Mathematical Thinking

Mathematically proficient students use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. **(A.1.B)**

Problem-Solving Strategies

Core Concept

Guess, Check, and Revise

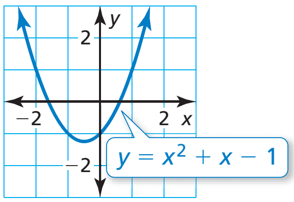
When solving a problem in mathematics, it is often helpful to estimate a solution and then observe how close that solution is to being correct. For instance, you can use the guess, check, and revise strategy to find a decimal approximation of the square root of 2.

	Guess	Check	How to revise
1.	1.4	$1.4^2 = 1.96$	Increase guess.
2.	1.41	$1.41^2 = 1.9881$	Increase guess.
3.	1.415	$1.415^2 = 2.002225$	Decrease guess.

By continuing this process, you can determine that the square root of 2 is approximately 1.4142.

EXAMPLE 1 Approximating a Solution of an Equation

The graph of $y = x^2 + x - 1$ is shown. Approximate the positive solution of the equation $x^2 + x - 1 = 0$ to the nearest thousandth.



SOLUTION

Using the graph, you can make an initial estimate of the positive solution to be $x = 0.65$.

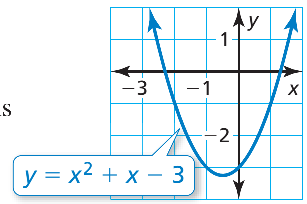
	Guess	Check	How to revise
1.	0.65	$0.65^2 + 0.65 - 1 = 0.0725$	Decrease guess.
2.	0.62	$0.62^2 + 0.62 - 1 = 0.0044$	Decrease guess.
3.	0.618	$0.618^2 + 0.618 - 1 = -0.000076$	Increase guess.
4.	0.6181	$0.6181^2 + 0.6181 - 1 \approx 0.00015$	The solution is between 0.618 and 0.6181.

▶ So, to the nearest thousandth, the positive solution of the equation is $x = 0.618$.

Monitoring Progress

1. Use the graph in Example 1 to approximate the negative solution of the equation $x^2 + x - 1 = 0$ to the nearest thousandth.

2. The graph of $y = x^2 + x - 3$ is shown. Approximate both solutions of the equation $x^2 + x - 3 = 0$ to the nearest thousandth.



9.1 Properties of Radicals

Essential Question

How can you multiply and divide square roots?

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
A.11.A

EXPLORATION 1 Operations with Square Roots

Work with a partner. For each operation with square roots, compare the results obtained using the two indicated orders of operations. What can you conclude?

a. Square Roots and Addition

Is $\sqrt{36} + \sqrt{64}$ equal to $\sqrt{36 + 64}$?

In general, is $\sqrt{a} + \sqrt{b}$ equal to $\sqrt{a + b}$? Explain your reasoning.

b. Square Roots and Multiplication

Is $\sqrt{4} \cdot \sqrt{9}$ equal to $\sqrt{4 \cdot 9}$?

In general, is $\sqrt{a} \cdot \sqrt{b}$ equal to $\sqrt{a \cdot b}$? Explain your reasoning.

c. Square Roots and Subtraction

Is $\sqrt{64} - \sqrt{36}$ equal to $\sqrt{64 - 36}$?

In general, is $\sqrt{a} - \sqrt{b}$ equal to $\sqrt{a - b}$? Explain your reasoning.

d. Square Roots and Division

Is $\dfrac{\sqrt{100}}{\sqrt{4}}$ equal to $\sqrt{\dfrac{100}{4}}$?

In general, is $\dfrac{\sqrt{a}}{\sqrt{b}}$ equal to $\sqrt{\dfrac{a}{b}}$? Explain your reasoning.

REASONING

To be proficient in math, you need to recognize and use counterexamples.

EXPLORATION 2 Writing Counterexamples

Work with a partner. A **counterexample** is an example that proves that a general statement is *not* true. For each general statement in Exploration 1 that is not true, write a counterexample different from the example given.

Communicate Your Answer

3. How can you multiply and divide square roots?

4. Give an example of multiplying square roots and an example of dividing square roots that are different from the examples in Exploration 1.

5. Write an algebraic rule for each operation.

a. the product of square roots

b. the quotient of square roots

9.1 Lesson

Core Vocabulary

counterexample, *p. 465*
radical expression, *p. 466*
simplest form, *p. 466*
rationalizing the denominator, *p. 468*
conjugates, *p. 468*
like radicals, *p. 470*

Previous
radicand
perfect cube

What You Will Learn

- Use properties of radicals to simplify expressions.
- Simplify expressions by rationalizing the denominator.
- Perform operations with radicals.

Using Properties of Radicals

A **radical expression** is an expression that contains a radical. A radical with index n is in **simplest form** when these three conditions are met.

- No radicands have perfect nth powers as factors other than 1.
- No radicands contain fractions.
- No radicals appear in the denominator of a fraction.

You can use the property below to simplify radical expressions involving square roots.

Core Concept

Product Property of Square Roots

Words The square root of a product equals the product of the square roots of the factors.

Numbers $\sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$

Algebra $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, where $a, b \geq 0$

STUDY TIP

There can be more than one way to factor a radicand. An efficient method is to find the greatest perfect square factor.

EXAMPLE 1 Using the Product Property of Square Roots

a. $\sqrt{108} = \sqrt{36 \cdot 3}$ Factor using the greatest perfect square factor.

$= \sqrt{36} \cdot \sqrt{3}$ Product Property of Square Roots

$= 6\sqrt{3}$ Simplify.

b. $\sqrt{9x^3} = \sqrt{9 \cdot x^2 \cdot x}$ Factor using the greatest perfect square factor.

$= \sqrt{9} \cdot \sqrt{x^2} \cdot \sqrt{x}$ Product Property of Square Roots

$= 3x\sqrt{x}$ Simplify.

STUDY TIP

In this course, whenever a variable appears in the radicand, assume that it has only *nonnegative* values.

Monitoring Progress

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Simplify the expression.

1. $\sqrt{24}$ **2.** $-\sqrt{80}$ **3.** $\sqrt{49x^3}$ **4.** $\sqrt{75n^5}$

Core Concept

Quotient Property of Square Roots

Words The square root of a quotient equals the quotient of the square roots of the numerator and denominator.

Numbers $\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$

Algebra $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, where $a \geq 0$ and $b > 0$

EXAMPLE 2 Using the Quotient Property of Square Roots

a. $\sqrt{\dfrac{15}{64}} = \dfrac{\sqrt{15}}{\sqrt{64}}$ — Quotient Property of Square Roots

$= \dfrac{\sqrt{15}}{8}$ — Simplify.

b. $\sqrt{\dfrac{81}{x^2}} = \dfrac{\sqrt{81}}{\sqrt{x^2}}$ — Quotient Property of Square Roots

$= \dfrac{9}{x}$ — Simplify.

You can extend the Product and Quotient Properties of Square Roots to other radicals, such as cube roots. When using these *properties of cube roots*, the radicands may contain negative numbers.

EXAMPLE 3 Using Properties of Cube Roots

STUDY TIP

To write a cube root in simplest form, find factors of the radicand that are perfect cubes.

a. $\sqrt[3]{-128} = \sqrt[3]{-64 \cdot 2}$ — Factor using the greatest perfect cube factor.

$= \sqrt[3]{-64} \cdot \sqrt[3]{2}$ — Product Property of Cube Roots

$= -4\sqrt[3]{2}$ — Simplify.

b. $\sqrt[3]{125x^7} = \sqrt[3]{125 \cdot x^6 \cdot x}$ — Factor using the greatest perfect cube factors.

$= \sqrt[3]{125} \cdot \sqrt[3]{x^6} \cdot \sqrt[3]{x}$ — Product Property of Cube Roots

$= 5x^2\sqrt[3]{x}$ — Simplify.

c. $\sqrt[3]{\dfrac{y}{216}} = \dfrac{\sqrt[3]{y}}{\sqrt[3]{216}}$ — Quotient Property of Cube Roots

$= \dfrac{\sqrt[3]{y}}{6}$ — Simplify.

d. $\sqrt[3]{\dfrac{8x^4}{27y^3}} = \dfrac{\sqrt[3]{8x^4}}{\sqrt[3]{27y^3}}$ — Quotient Property of Cube Roots

$= \dfrac{\sqrt[3]{8 \cdot x^3 \cdot x}}{\sqrt[3]{27 \cdot y^3}}$ — Factor using the greatest perfect cube factors.

$= \dfrac{\sqrt[3]{8} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x}}{\sqrt[3]{27} \cdot \sqrt[3]{y^3}}$ — Product Property of Cube Roots

$= \dfrac{2x\sqrt[3]{x}}{3y}$ — Simplify.

Monitoring Progress

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Simplify the expression.

5. $\sqrt{\dfrac{23}{9}}$ **6.** $-\sqrt{\dfrac{17}{100}}$ **7.** $\sqrt{\dfrac{36}{z^2}}$ **8.** $\sqrt{\dfrac{4x^2}{64}}$

9. $\sqrt[3]{54}$ **10.** $\sqrt[3]{16x^4}$ **11.** $\sqrt[3]{\dfrac{a}{-27}}$ **12.** $\sqrt[3]{\dfrac{25c^7d^3}{64}}$

Rationalizing the Denominator

When a radical is in the denominator of a fraction, you can multiply the fraction by an appropriate form of 1 to eliminate the radical from the denominator. This process is called **rationalizing the denominator**.

STUDY TIP

Rationalizing the denominator works because you multiply the numerator and denominator by the same nonzero number a, which is the same as multiplying by $\frac{a}{a}$, or 1.

EXAMPLE 4 Rationalizing the Denominator

a. $\dfrac{\sqrt{5}}{\sqrt{3n}} = \dfrac{\sqrt{5}}{\sqrt{3n}} \cdot \dfrac{\sqrt{3n}}{\sqrt{3n}}$ — Multiply by $\frac{\sqrt{3n}}{\sqrt{3n}}$.

$= \dfrac{\sqrt{15n}}{\sqrt{9n^2}}$ — Product Property of Square Roots

$= \dfrac{\sqrt{15n}}{\sqrt{9} \cdot \sqrt{n^2}}$ — Product Property of Square Roots

$= \dfrac{\sqrt{15n}}{3n}$ — Simplify.

b. $\dfrac{2}{\sqrt[3]{9}} = \dfrac{2}{\sqrt[3]{9}} \cdot \dfrac{\sqrt[3]{3}}{\sqrt[3]{3}}$ — Multiply by $\frac{\sqrt[3]{3}}{\sqrt[3]{3}}$.

$= \dfrac{2\sqrt[3]{3}}{\sqrt[3]{27}}$ — Product Property of Cube Roots

$= \dfrac{2\sqrt[3]{3}}{3}$ — Simplify.

The binomials $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$, where a, b, c, and d are rational numbers, are called **conjugates**. You can use conjugates to simplify radical expressions that contain a sum or difference involving square roots in the denominator.

ANALYZING MATHEMATICAL RELATIONSHIPS

Notice that the product of two conjugates $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$ does not contain a radical and is a *rational* number.

$$(a\sqrt{b} + c\sqrt{d})(a\sqrt{b} - c\sqrt{d})$$
$$= (a\sqrt{b})^2 - (c\sqrt{d})^2$$
$$= a^2b - c^2d$$

EXAMPLE 5 Rationalizing the Denominator Using Conjugates

Simplify $\dfrac{7}{2 - \sqrt{3}}$.

SOLUTION

$\dfrac{7}{2 - \sqrt{3}} = \dfrac{7}{2 - \sqrt{3}} \cdot \dfrac{2 + \sqrt{3}}{2 + \sqrt{3}}$ — The conjugate of $2 - \sqrt{3}$ is $2 + \sqrt{3}$.

$= \dfrac{7(2 + \sqrt{3})}{2^2 - (\sqrt{3})^2}$ — Sum and difference pattern

$= \dfrac{14 + 7\sqrt{3}}{1}$ — Simplify.

$= 14 + 7\sqrt{3}$ — Simplify.

Monitoring Progress

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Simplify the expression.

13. $\dfrac{1}{\sqrt{5}}$ **14.** $\dfrac{\sqrt{10}}{\sqrt{3}}$ **15.** $\dfrac{7}{\sqrt{2x}}$ **16.** $\sqrt{\dfrac{2y^2}{3}}$

17. $\dfrac{5}{\sqrt[3]{32}}$ **18.** $\dfrac{8}{1 + \sqrt{3}}$ **19.** $\dfrac{\sqrt{13}}{\sqrt{5} - 2}$ **20.** $\dfrac{12}{\sqrt{2} + \sqrt{7}}$

EXAMPLE 6 Solving a Real-Life Problem

The distance d (in miles) that you can see to the horizon with your eye level h feet above the water is given by $d = \sqrt{\frac{3h}{2}}$. How far can you see when your eye level is 5 feet above the water?

SOLUTION

$$d = \sqrt{\frac{3(5)}{2}} \qquad \text{Substitute 5 for } h.$$

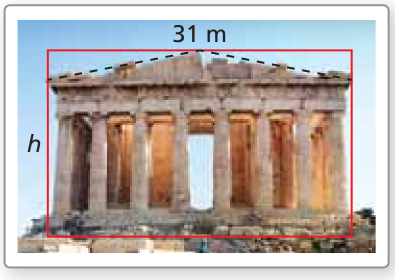
$$= \frac{\sqrt{15}}{\sqrt{2}} \qquad \text{Quotient Property of Square Roots}$$

$$= \frac{\sqrt{15}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \qquad \text{Multiply by } \frac{\sqrt{2}}{\sqrt{2}}.$$

$$= \frac{\sqrt{30}}{2} \qquad \text{Simplify.}$$

▶ You can see $\frac{\sqrt{30}}{2}$, or about 2.74 miles.

EXAMPLE 7 Modeling with Mathematics



The ratio of the length to the width of a *golden rectangle* is $(1 + \sqrt{5}) : 2$. The dimensions of the face of the Parthenon in Greece form a golden rectangle. What is the height h of the Parthenon?

SOLUTION

1. **Understand the Problem** Think of the length and height of the Parthenon as the length and width of a golden rectangle. The length of the rectangular face is 31 meters. You know the ratio of the length to the height. Find the height h.

2. **Make a Plan** Use the ratio $(1 + \sqrt{5}) : 2$ to write a proportion and solve for h.

3. **Solve the Problem**

$$\frac{1 + \sqrt{5}}{2} = \frac{31}{h} \qquad \text{Write a proportion.}$$

$$h(1 + \sqrt{5}) = 62 \qquad \text{Cross Products Property}$$

$$h = \frac{62}{1 + \sqrt{5}} \qquad \text{Divide each side by } 1 + \sqrt{5}.$$

$$h = \frac{62}{1 + \sqrt{5}} \cdot \frac{1 - \sqrt{5}}{1 - \sqrt{5}} \qquad \text{Multiply the numerator and denominator by the conjugate.}$$

$$h = \frac{62 - 62\sqrt{5}}{-4} \qquad \text{Simplify.}$$

$$h \approx 19.16 \qquad \text{Use a calculator.}$$

▶ The height is about 19 meters.

4. **Look Back** $\frac{1 + \sqrt{5}}{2} \approx 1.62$ and $\frac{31}{19.16} \approx 1.62$. So, your answer is reasonable.

Monitoring Progress

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21. WHAT IF? In Example 6, how far can you see when your eye level is 35 feet above the water?

22. The dimensions of a dance floor form a golden rectangle. The shorter side of the dance floor is 50 feet. What is the length of the longer side of the dance floor?

Performing Operations with Radicals

Radicals with the same index and radicand are called **like radicals**. You can add and subtract like radicals the same way you combine like terms by using the Distributive Property.

STUDY TIP

Do not assume that radicals with different radicands cannot be added or subtracted. Always check to see whether you can simplify the radicals. In some cases, the radicals will become like radicals.

EXAMPLE 8 Adding and Subtracting Radicals

a. $5\sqrt{7} + \sqrt{11} - 8\sqrt{7} = 5\sqrt{7} - 8\sqrt{7} + \sqrt{11}$ — Commutative Property of Addition

$= (5 - 8)\sqrt{7} + \sqrt{11}$ — Distributive Property

$= -3\sqrt{7} + \sqrt{11}$ — Subtract.

b. $10\sqrt{5} + \sqrt{20} = 10\sqrt{5} + \sqrt{4 \cdot 5}$ — Factor using the greatest perfect square factor.

$= 10\sqrt{5} + \sqrt{4} \cdot \sqrt{5}$ — Product Property of Square Roots

$= 10\sqrt{5} + 2\sqrt{5}$ — Simplify.

$= (10 + 2)\sqrt{5}$ — Distributive Property

$= 12\sqrt{5}$ — Add.

c. $6\sqrt[3]{x} + 2\sqrt[3]{x} = (6 + 2)\sqrt[3]{x}$ — Distributive Property

$= 8\sqrt[3]{x}$ — Add.

EXAMPLE 9 Multiplying Radicals

Simplify $\sqrt{5}\left(\sqrt{3} - \sqrt{75}\right)$.

SOLUTION

Method 1 $\sqrt{5}\left(\sqrt{3} - \sqrt{75}\right) = \sqrt{5} \cdot \sqrt{3} - \sqrt{5} \cdot \sqrt{75}$ — Distributive Property

$= \sqrt{15} - \sqrt{375}$ — Product Property of Square Roots

$= \sqrt{15} - 5\sqrt{15}$ — Simplify.

$= (1 - 5)\sqrt{15}$ — Distributive Property

$= -4\sqrt{15}$ — Subtract.

Method 2 $\sqrt{5}\left(\sqrt{3} - \sqrt{75}\right) = \sqrt{5}\left(\sqrt{3} - 5\sqrt{3}\right)$ — Simplify $\sqrt{75}$.

$= \sqrt{5}\left[(1 - 5)\sqrt{3}\right]$ — Distributive Property

$= \sqrt{5}\left(-4\sqrt{3}\right)$ — Subtract.

$= -4\sqrt{15}$ — Product Property of Square Roots

Monitoring Progress

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Simplify the expression.

23. $3\sqrt{2} - \sqrt{6} + 10\sqrt{2}$

24. $4\sqrt{7} - 6\sqrt{63}$

25. $4\sqrt[3]{5x} - 11\sqrt[3]{5x}$

26. $\sqrt{3}\left(8\sqrt{2} + 7\sqrt{32}\right)$

27. $\left(2\sqrt{5} - 4\right)^2$

28. $\sqrt[3]{-4}\left(\sqrt[3]{2} - \sqrt[3]{16}\right)$

9.1 Exercises

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Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The process of eliminating a radical from the denominator of a radical expression is called ________________.

2. **VOCABULARY** What is the conjugate of the binomial $\sqrt{6} + 4$?

3. **WRITING** Are the expressions $\frac{1}{3}\sqrt{2x}$ and $\sqrt{\frac{2x}{9}}$ equivalent? Explain your reasoning.

4. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$-\frac{1}{3}\sqrt{6}$	$6\sqrt{3}$	$\frac{1}{6}\sqrt{3}$	$-3\sqrt{3}$

Monitoring Progress and Modeling with Mathematics

In Exercises 5–12, determine whether the expression is in simplest form. If the expression is not in simplest form, explain why.

5. $\sqrt{19}$

6. $\sqrt{\frac{1}{7}}$

7. $\sqrt{48}$

8. $\sqrt{34}$

9. $\frac{5}{\sqrt{2}}$

10. $\frac{3\sqrt{10}}{4}$

11. $\frac{1}{2 + \sqrt[3]{2}}$

12. $6 - \sqrt[3]{54}$

In Exercises 13–20, simplify the expression. *(See Example 1.)*

13. $\sqrt{20}$

14. $\sqrt{32}$

15. $\sqrt{128}$

16. $-\sqrt{72}$

17. $\sqrt{125b}$

18. $\sqrt{4x^2}$

19. $-\sqrt{81m^3}$

20. $\sqrt{48n^5}$

In Exercises 21–28, simplify the expression. *(See Example 2.)*

21. $\sqrt{\frac{4}{49}}$

22. $-\sqrt{\frac{7}{81}}$

23. $-\sqrt{\frac{23}{64}}$

24. $\sqrt{\frac{65}{121}}$

25. $\sqrt{\frac{a^3}{49}}$

26. $\sqrt{\frac{144}{k^2}}$

27. $\sqrt{\frac{100}{4x^2}}$

28. $\sqrt{\frac{25v^2}{36}}$

In Exercises 29–36, simplify the expression. *(See Example 3.)*

29. $\sqrt[3]{16}$

30. $\sqrt[3]{-108}$

31. $\sqrt[3]{-64x^5}$

32. $-\sqrt[3]{343n^2}$

33. $\sqrt[3]{\frac{6c}{-125}}$

34. $\sqrt[3]{\frac{8h^4}{27}}$

35. $-\sqrt[3]{\frac{81y^2}{1000x^3}}$

36. $\sqrt[3]{\frac{21}{-64a^3b^6}}$

ERROR ANALYSIS In Exercises 37 and 38, describe and correct the error in simplifying the expression.

37.
$$\begin{aligned}\sqrt{72} &= \sqrt{4 \cdot 18}\\ &= \sqrt{4} \cdot \sqrt{18}\\ &= 2\sqrt{18}\end{aligned}$$

38.
$$\begin{aligned}\sqrt[3]{\frac{128y^3}{125}} &= \frac{\sqrt[3]{128y^3}}{125}\\ &= \frac{\sqrt[3]{64 \cdot 2 \cdot y^3}}{125}\\ &= \frac{\sqrt[3]{64} \cdot \sqrt[3]{2} \cdot \sqrt[3]{y^3}}{125}\\ &= \frac{4y\sqrt[3]{2}}{125}\end{aligned}$$

In Exercises 39–44, write a factor that you can use to rationalize the denominator of the expression.

39. $\dfrac{4}{\sqrt{6}}$

40. $\dfrac{1}{\sqrt{13z}}$

41. $\dfrac{2}{\sqrt[3]{x^2}}$

42. $\dfrac{3m}{\sqrt[3]{4}}$

43. $\dfrac{\sqrt{2}}{\sqrt{5} - 8}$

44. $\dfrac{5}{\sqrt{3} + \sqrt{7}}$

In Exercises 45–54, simplify the expression.
(See Example 4.)

45. $\dfrac{2}{\sqrt{2}}$

46. $\dfrac{4}{\sqrt{3}}$

47. $\dfrac{\sqrt{5}}{\sqrt{48}}$

48. $\sqrt{\dfrac{4}{52}}$

49. $\dfrac{3}{\sqrt{a}}$

50. $\dfrac{1}{\sqrt{2x}}$

51. $\sqrt{\dfrac{3d^2}{5}}$

52. $\dfrac{\sqrt{8}}{\sqrt{3n^3}}$

53. $\dfrac{4}{\sqrt[3]{25}}$

54. $\sqrt[3]{\dfrac{1}{108y^2}}$

In Exercises 55–60, simplify the expression.
(See Example 5.)

55. $\dfrac{1}{\sqrt{7} + 1}$

56. $\dfrac{2}{5 - \sqrt{3}}$

57. $\dfrac{\sqrt{10}}{7 - \sqrt{2}}$

58. $\dfrac{\sqrt{5}}{6 + \sqrt{5}}$

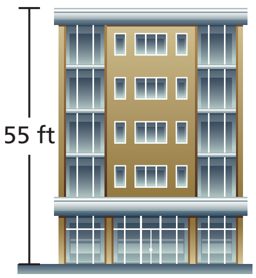
59. $\dfrac{3}{\sqrt{5} - \sqrt{2}}$

60. $\dfrac{\sqrt{3}}{\sqrt{7} + \sqrt{3}}$

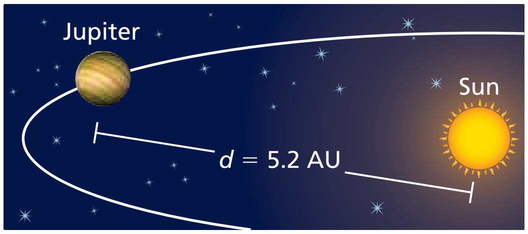
61. MODELING WITH MATHEMATICS The time t (in seconds) it takes an object to hit the ground is given by $t = \sqrt{\dfrac{h}{16}}$, where h is the height (in feet) from which the object was dropped. *(See Example 6.)*

a. How long does it take an earring to hit the ground when it falls from the roof of the building?

b. How much sooner does the earring hit the ground when it is dropped from two stories (22 feet) below the roof?



62. MODELING WITH MATHEMATICS The orbital period of a planet is the time it takes the planet to travel around the Sun. You can find the orbital period P (in Earth years) using the formula $P = \sqrt{d^3}$, where d is the average distance (in astronomical units, abbreviated AU) of the planet from the Sun.



a. Simplify the formula.

b. What is Jupiter's orbital period?

63. MODELING WITH MATHEMATICS The electric current I (in amperes) an appliance uses is given by the formula $I = \sqrt{\dfrac{P}{R}}$, where P is the power (in watts) and R is the resistance (in ohms). Find the current an appliance uses when the power is 147 watts and the resistance is 5 ohms.

64. MODELING WITH MATHEMATICS You can find the average annual interest rate r (in decimal form) of a savings account using the formula $r = \sqrt{\dfrac{V_2}{V_0}} - 1$, where V_0 is the initial investment and V_2 is the balance of the account after 2 years. Use the formula to compare the savings accounts. In which account would you invest money? Explain.

Account	Initial investment	Balance after 2 years
1	\$275	\$293
2	\$361	\$382
3	\$199	\$214
4	\$254	\$272
5	\$386	\$406

In Exercises 65–68, evaluate the function for the given value of x. Write your answer in simplest form and in decimal form rounded to the nearest hundredth.

65. $h(x) = \sqrt{5x};\ x = 10$

66. $g(x) = \sqrt{3x};\ x = 60$

67. $r(x) = \sqrt{\dfrac{3x}{3x^2 + 6}};\ x = 4$

68. $p(x) = \sqrt{\dfrac{x - 1}{5x}};\ x = 8$

In Exercises 69–72, evaluate the expression when $a = -2$, $b = 8$, and $c = \frac{1}{2}$. Write your answer in simplest form and in decimal form rounded to the nearest hundredth.

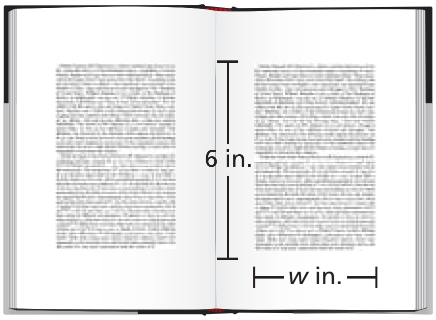
69. $\sqrt{a^2 + bc}$

70. $-\sqrt{4c - 6ab}$

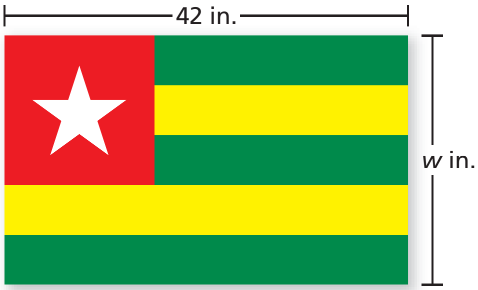
71. $-\sqrt{2a^2 + b^2}$

72. $\sqrt{b^2 - 4ac}$

73. **MODELING WITH MATHEMATICS** The text in the book shown forms a golden rectangle. What is the width w of the text? *(See Example 7.)*



74. **MODELING WITH MATHEMATICS** The flag of Togo is approximately the shape of a golden rectangle. What is the width w of the flag?



In Exercises 75–82, simplify the expression.
(See Example 8.)

75. $\sqrt{3} - 2\sqrt{2} + 6\sqrt{2}$

76. $\sqrt{5} - 5\sqrt{13} - 8\sqrt{5}$

77. $2\sqrt{6} - 5\sqrt{54}$

78. $9\sqrt{32} + \sqrt{2}$

79. $\sqrt{12} + 6\sqrt{3} + 2\sqrt{6}$

80. $3\sqrt{7} - 5\sqrt{14} + 2\sqrt{28}$

81. $\sqrt[3]{-81} + 4\sqrt[3]{3}$

82. $6\sqrt[3]{128t} - 2\sqrt[3]{2t}$

In Exercises 83–90, simplify the expression.
(See Example 9.)

83. $\sqrt{2}(\sqrt{45} + \sqrt{5})$

84. $\sqrt{3}(\sqrt{72} - 3\sqrt{2})$

85. $\sqrt{5}(2\sqrt{6x} - \sqrt{96x})$

86. $\sqrt{7y}(\sqrt{27y} + 5\sqrt{12y})$

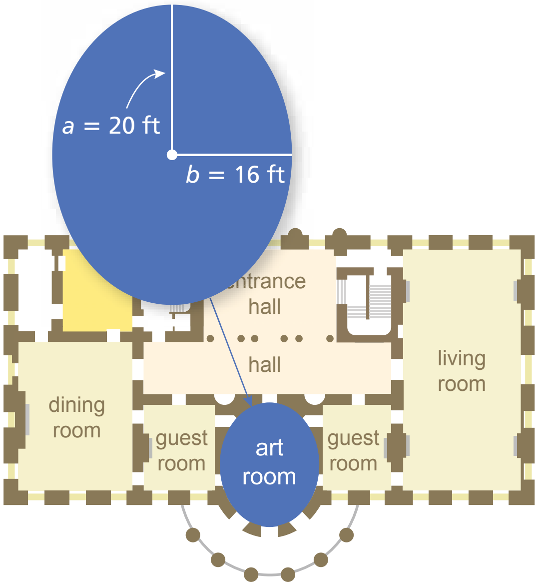
87. $(4\sqrt{2} - \sqrt{98})^2$

88. $(\sqrt{3} + \sqrt{48})(\sqrt{20} - \sqrt{5})$

89. $\sqrt[3]{3}(\sqrt[3]{4} + \sqrt[3]{32})$

90. $\sqrt[3]{2}(\sqrt[3]{135} - 4\sqrt[3]{5})$

91. **MODELING WITH MATHEMATICS** The circumference C of the art room in a mansion is approximated by the formula $C \approx 2\pi\sqrt{\dfrac{a^2 + b^2}{2}}$. Approximate the circumference of the room.



92. **CRITICAL THINKING** Determine whether each expression represents a *rational* or an *irrational* number. Justify your answer.

a. $4 + \sqrt{6}$

b. $\dfrac{\sqrt{48}}{\sqrt{3}}$

c. $\dfrac{8}{\sqrt{12}}$

d. $\sqrt{3} + \sqrt{7}$

e. $\dfrac{a}{\sqrt{10} - \sqrt{2}}$, where a is a positive integer

f. $\dfrac{2 + \sqrt{5}}{2b + \sqrt{5b^2}}$, where b is a positive integer

In Exercises 93–98, simplify the expression.

93. $\sqrt[5]{\dfrac{13}{5x^5}}$

94. $\sqrt[4]{\dfrac{10}{81}}$

95. $\sqrt[4]{256y}$

96. $\sqrt[5]{160x^6}$

97. $6\sqrt[4]{9} - \sqrt[5]{9} + 3\sqrt[4]{9}$

98. $\sqrt[5]{2}(\sqrt[4]{7} + \sqrt[5]{16})$

REASONING In Exercises 99 and 100, use the table shown.

	2	$\frac{1}{4}$	0	$\sqrt{3}$	$-\sqrt{3}$	π
2						
$\frac{1}{4}$						
0						
$\sqrt{3}$						
$-\sqrt{3}$						
π						

99. Copy and complete the table by (a) finding each sum $\left(2 + 2, 2 + \frac{1}{4}, \text{etc.}\right)$ and (b) finding each product $\left(2 \cdot 2, 2 \cdot \frac{1}{4}, \text{etc.}\right)$.

100. Use your answers in Exercise 99 to determine whether each statement is *always*, *sometimes*, or *never* true. Justify your answer.

a. The sum of a rational number and a rational number is rational.

b. The sum of a rational number and an irrational number is irrational.

c. The sum of an irrational number and an irrational number is irrational.

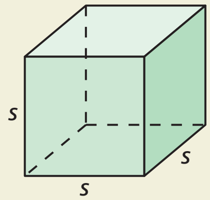
d. The product of a rational number and a rational number is rational.

e. The product of a nonzero rational number and an irrational number is irrational.

f. The product of an irrational number and an irrational number is irrational.

101. **REASONING** Let m be a positive integer. For what values of m will the simplified form of the expression $\sqrt{2^m}$ contain a radical? For what values will it *not* contain a radical? Explain.

102. **HOW DO YOU SEE IT?** The edge length s of a cube is an irrational number, the surface area is an irrational number, and the volume is a rational number. Give a possible value of s.



103. **REASONING** Let a and b be positive numbers. Explain why $\sqrt{ab}$ lies between a and b on a number line. (*Hint:* Let $a < b$ and multiply each side of $a < b$ by a. Then let $a < b$ and multiply each side by b.)

104. **MAKING AN ARGUMENT** Your friend says that you can rationalize the denominator of the expression $\dfrac{2}{4 + \sqrt[3]{5}}$ by multiplying the numerator and denominator by $4 - \sqrt[3]{5}$. Is your friend correct? Explain.

105. **PROBLEM SOLVING** The ratio of consecutive terms $\dfrac{a_n}{a_{n-1}}$ in the Fibonacci sequence gets closer and closer to the golden ratio $\dfrac{1 + \sqrt{5}}{2}$ as n increases. Find the term that precedes 610 in the sequence.

106. **THOUGHT PROVOKING** Use the golden ratio $\dfrac{1 + \sqrt{5}}{2}$ and the golden ratio conjugate $\dfrac{1 - \sqrt{5}}{2}$ for each of the following.

a. Show that the golden ratio and golden ratio conjugate are both solutions of $x^2 - x - 1 = 0$.

b. Construct a geometric diagram that has the golden ratio as the length of a part of the diagram.

107. **CRITICAL THINKING** Use the special product pattern $(a + b)(a^2 - ab + b^2) = a^3 + b^3$ to simplify the expression $\dfrac{2}{\sqrt[3]{x} + 1}$. Explain your reasoning.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Graph the linear equation. Identify the x-intercept. *(Section 3.5)*

108. $y = x - 4$ **109.** $y = -2x + 6$ **110.** $y = -\frac{1}{3}x - 1$ **111.** $y = \frac{3}{2}x + 6$

Solve the equation by graphing. Check your solution. *(Section 5.5)*

112. $-3x = -2x + 1$ **113.** $5x + 3 = \frac{3}{2}x - 4$ **114.** $4x - 3 = 8 - \frac{2}{5}x$ **115.** $6x - 1 = -4(x - 1)$

9.2 Solving Quadratic Equations by Graphing

Essential Question

How can you use a graph to solve a quadratic equation in one variable?

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS

A.7.A
A.8.B

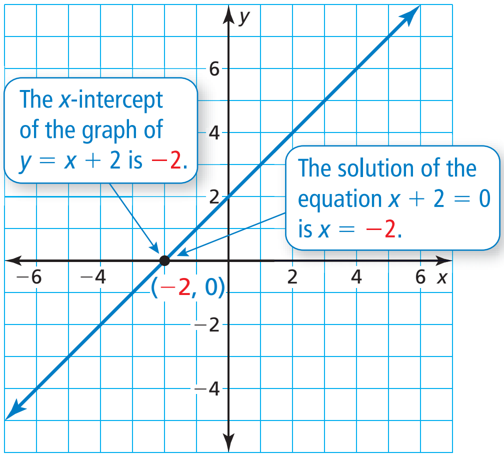
Based on what you learned about the x-intercepts of a graph in Section 3.4, it follows that the x-intercept of the graph of the linear equation

$y = ax + b$ 2 variables

is the same value as the solution of

$ax + b = 0.$ 1 variable

You can use similar reasoning to solve *quadratic equations*.



EXPLORATION 1 Solving a Quadratic Equation by Graphing

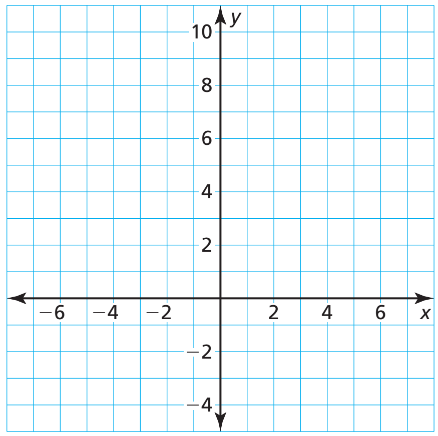
Work with a partner.

a. Sketch the graph of $y = x^2 - 2x$.

b. What is the definition of an x-intercept of a graph? How many x-intercepts does this graph have? What are they?

c. What is the definition of a solution of an equation in x? How many solutions does the equation $x^2 - 2x = 0$ have? What are they?

d. Explain how you can verify the solutions you found in part (c).



EXPLORATION 2 Solving Quadratic Equations by Graphing

Work with a partner. Solve each equation by graphing.

a. $x^2 - 4 = 0$

b. $x^2 + 3x = 0$

c. $-x^2 + 2x = 0$

d. $x^2 - 2x + 1 = 0$

e. $x^2 - 3x + 5 = 0$

f. $-x^2 + 3x - 6 = 0$

USING PROBLEM-SOLVING STRATEGIES

To be proficient in math, you need to check your answers to problems using a different method and continually ask yourself, "Does this make sense?"

Communicate Your Answer

3. How can you use a graph to solve a quadratic equation in one variable?

4. After you find a solution graphically, how can you check your result algebraically? Check your solutions for parts (a)–(d) in Exploration 2 algebraically.

5. How can you determine graphically that a quadratic equation has no solution?

9.2 Lesson

Core Vocabulary

quadratic equation, *p. 476*

Previous
x-intercept
root
zero of a function

What You Will Learn

- Solve quadratic equations by graphing.
- Use graphs to find and approximate the zeros of functions.
- Solve real-life problems using graphs of quadratic functions.

Solving Quadratic Equations by Graphing

A **quadratic equation** is a nonlinear equation that can be written in the standard form $ax^2 + bx + c = 0$, where $a \neq 0$.

In Chapter 7, you solved quadratic equations by factoring. You can also solve quadratic equations by graphing.

Core Concept

Solving Quadratic Equations by Graphing

Step 1 Write the equation in standard form, $ax^2 + bx + c = 0$.

Step 2 Graph the related function $y = ax^2 + bx + c$.

Step 3 Find the x-intercepts, if any.

The solutions, or *roots*, of $ax^2 + bx + c = 0$ are the x-intercepts of the graph.

EXAMPLE 1 Solving a Quadratic Equation: Two Real Solutions

Solve $x^2 + 2x = 3$ by graphing.

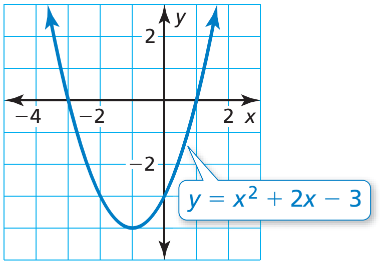
SOLUTION

Step 1 Write the equation in standard form.

$$x^2 + 2x = 3 \quad \text{Write original equation.}$$

$$x^2 + 2x - 3 = 0 \quad \text{Subtract 3 from each side.}$$

Step 2 Graph the related function $y = x^2 + 2x - 3$.



Step 3 Find the x-intercepts. The x-intercepts are -3 and 1.

- So, the solutions are $x = -3$ and $x = 1$.

Check

$x^2 + 2x = 3$	Original equation	$x^2 + 2x = 3$
$(-3)^2 + 2(-3) \stackrel{?}{=} 3$	Substitute.	$1^2 + 2(1) \stackrel{?}{=} 3$
$3 = 3$ ✓	Simplify.	$3 = 3$ ✓

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Solve the equation by graphing. Check your solutions.

1. $x^2 - x - 2 = 0$ **2.** $x^2 + 7x = -10$ **3.** $x^2 + x = 12$

EXAMPLE 2 Solving a Quadratic Equation: One Real Solution

Solve $x^2 - 8x = -16$ by graphing.

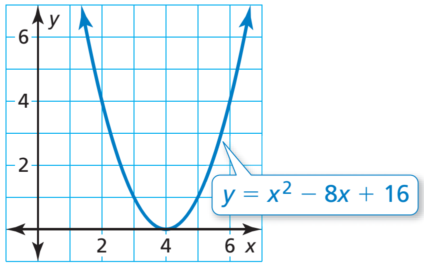
SOLUTION

Step 1 Write the equation in standard form.

$$x^2 - 8x = -16 \qquad \text{Write original equation.}$$

$$x^2 - 8x + 16 = 0 \qquad \text{Add 16 to each side.}$$

Step 2 Graph the related function $y = x^2 - 8x + 16$.



Step 3 Find the x-intercept. The only x-intercept is at the vertex, (4, 0).

▶ So, the solution is $x = 4$.

ANOTHER WAY

You can also solve the equation in Example 2 by factoring.

$$x^2 - 8x + 16 = 0$$

$$(x - 4)(x - 4) = 0$$

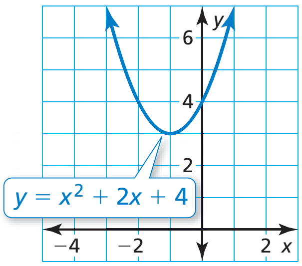
So, $x = 4$.

EXAMPLE 3 Solving a Quadratic Equation: No Real Solutions

Solve $-x^2 = 2x + 4$ by graphing.

SOLUTION

Method 1 Write the equation in standard form, $x^2 + 2x + 4 = 0$. Then graph the related function $y = x^2 + 2x + 4$, as shown at the left.

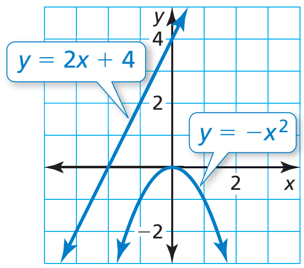


▶ There are no x-intercepts. So, $-x^2 = 2x + 4$ has no real solutions.

Method 2 Graph each side of the equation.

$$y = -x^2 \qquad \text{Left side}$$

$$y = 2x + 4 \qquad \text{Right side}$$



▶ The graphs do not intersect.
So, $-x^2 = 2x + 4$ has no real solutions.

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Solve the equation by graphing.

4. $x^2 + 36 = 12x$ **5.** $x^2 + 4x = 0$ **6.** $x^2 + 10x = -25$

7. $x^2 = 3x - 3$ **8.** $x^2 + 7x = -6$ **9.** $2x + 5 = -x^2$

Concept Summary

Number of Solutions of a Quadratic Equation

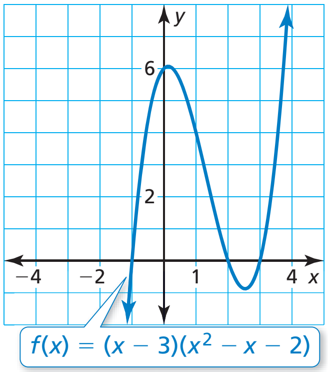
A quadratic equation has:

- two real solutions when the graph of its related function has two x-intercepts.
- one real solution when the graph of its related function has one x-intercept.
- no real solutions when the graph of its related function has no x-intercepts.

Finding Zeros of Functions

Recall that a zero of a function is an x-intercept of the graph of the function.

EXAMPLE 4 Finding the Zeros of a Function



The graph of $f(x) = (x - 3)(x^2 - x - 2)$ is shown. Find the zeros of f.

SOLUTION

The x-intercepts are -1, 2, and 3.

▶ So, the zeros of f are -1, 2, and 3.

Check

$f(-1) = (-1 - 3)[(-1)^2 - (-1) - 2] = 0$ ✓

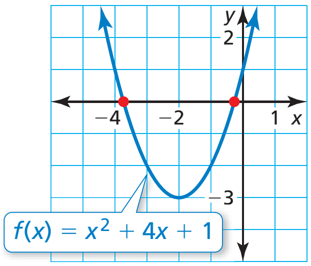
$f(2) = (2 - 3)(2^2 - 2 - 2) = 0$ ✓

$f(3) = (3 - 3)(3^2 - 3 - 2) = 0$ ✓

The zeros of a function are not necessarily integers. To approximate zeros, analyze the signs of function values. When two function values have different signs, a zero lies between the x-values that correspond to the function values.

EXAMPLE 5 Approximating the Zeros of a Function

The graph of $f(x) = x^2 + 4x + 1$ is shown. Approximate the zeros of f to the nearest tenth.



SOLUTION

There are two x-intercepts: one between -4 and -3, and another between -1 and 0.

Make tables using x-values between -4 and -3, and between -1 and 0. Use an increment of 0.1. Look for a change in the signs of the function values.

x	-3.9	-3.8	-3.7	-3.6	-3.5	-3.4	-3.3	-3.2	-3.1
$f(x)$	0.61	0.24	-0.11	-0.44	-0.75	-1.04	-1.31	-1.56	-1.79

change in signs

x	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1
$f(x)$	-1.79	-1.56	-1.31	-1.04	-0.75	-0.44	-0.11	0.24	0.61

change in signs

The function values that are closest to 0 correspond to x-values that best approximate the zeros of the function.

ANOTHER WAY

You could approximate one zero using a table and then use the axis of symmetry to find the other zero.

▶ In each table, the function value closest to 0 is -0.11. So, the zeros of f are about -3.7 and -0.3.

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10. Graph $f(x) = x^2 + x - 6$. Find the zeros of f.

11. Graph $f(x) = -x^2 + 2x + 2$. Approximate the zeros of f to the nearest tenth.

Solving Real-Life Problems

Seconds, t	Height, h
0	2
1	61
2	88
3	83
4	46
5	−23

EXAMPLE 6 **Real-Life Application**

A football player kicks a football 2 feet above the ground with an initial vertical velocity of 75 feet per second. The function $h = -16t^2 + 75t + 2$ represents the height h (in feet) of the football after t seconds. (a) Find the height of the football each second after it is kicked. (b) Use the results of part (a) to estimate when the height of the football is 50 feet. (c) Using a graph, after how many seconds is the football 50 feet above the ground?

SOLUTION

a. Make a table of values starting with $t = 0$ seconds using an increment of 1. Continue the table until a function value is negative.

▶ The height of the football is 61 feet after 1 second, 88 feet after 2 seconds, 83 feet after 3 seconds, and 46 feet after 4 seconds.

b. From part (a), you can estimate that the height of the football is 50 feet between 0 and 1 second and between 3 and 4 seconds.

▶ Based on the function values, it is reasonable to estimate that the height of the football is 50 feet slightly less than 1 second and slightly less than 4 seconds after it is kicked.

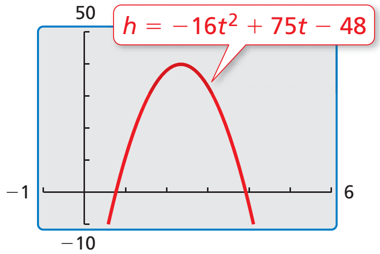
c. To determine when the football is 50 feet above the ground, find the t-values for which $h = 50$. So, solve the equation $-16t^2 + 75t + 2 = 50$ by graphing.

Step 1 Write the equation in standard form.

$$-16t^2 + 75t + 2 = 50 \qquad \text{Write the equation.}$$

$$-16t^2 + 75t - 48 = 0 \qquad \text{Subtract 50 from each side.}$$

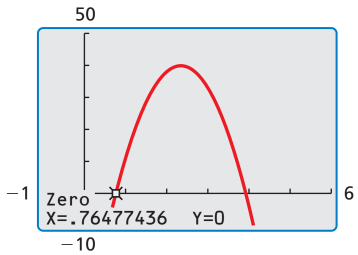
Step 2 Use a graphing calculator to graph the related function $h = -16t^2 + 75t - 48$.

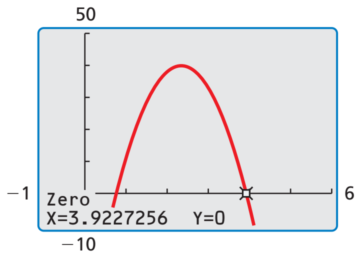


REMEMBER

Equations have *solutions*, or *roots*. Graphs have *x-intercepts*. Functions have *zeros*.

Step 3 Use the *zero* feature to find the zeros of the function.





▶ The football is 50 feet above the ground after about 0.8 second and about 3.9 seconds, which supports the estimates in part (b).

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12. WHAT IF? After how many seconds is the football 65 feet above the ground?

In Section 4.6, you used a graphing calculator to perform linear regression on a set of data to find a linear model for the data. You can also perform *quadratic regression*.

EXAMPLE 7 Finding a Quadratic Model Using Technology

Time	Temperature (°F)
6 A.M.	58
8 A.M.	68
10 A.M.	76
12 P.M.	82
2 P.M.	84
4 P.M.	81
6 P.M.	75

The table shows the recorded temperatures (in degrees Fahrenheit) for a portion of a day. (a) Use a graphing calculator to find a quadratic model for the data. Then determine whether the model is a good fit. (b) At what time(s) during the day is the temperature 77°F?

SOLUTION

a. Step 1 Enter the data from the table into two lists. Let x represent the number of hours after midnight.

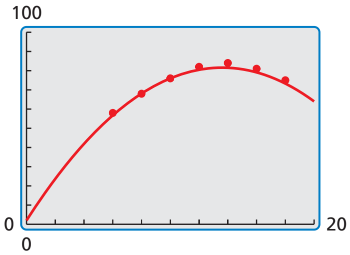
```
L1    L2    L3    1
6     58    ------
8     68
10    76
12    82
14    84
16    81
18    75
L1(1)=6
```

Step 2 Use the *quadratic regression* feature. The values in the equation can be rounded to obtain $y = -0.43x^2 + 11.7x + 2$.

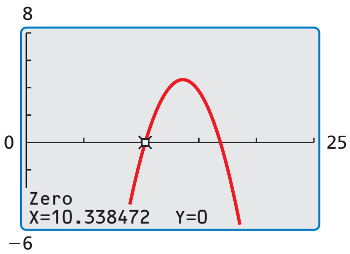
```
QuadReg
 y=ax²+bx+c
 a=-.4255952381
 b=11.73214286
 c=2.166666667
 R²=.9933031503
```

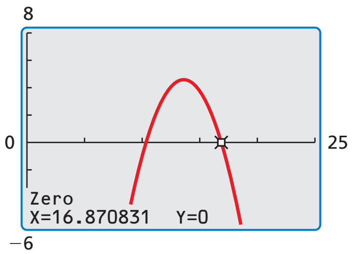
Step 3 Enter the equation $y = -0.43x^2 + 11.7x + 2$ into the calculator. Then plot the data and graph the equation in the same viewing window.

▶ The graph of the equation passes through or is close to all of the data points. So, the model is a good fit.



b. Find the x-values for which $y = 77$ by writing $-0.43x^2 + 11.7x + 2 = 77$ in standard form, graphing the related function $y = -0.43x^2 + 11.7x - 75$, and finding its zeros.





▶ The temperature is 77°F at about 10.3, or 10:18 A.M., and at about 16.9, or 4:54 P.M.

STUDY TIP

Notice that the graphing calculator does not calculate the correlation coefficient r, but it does calculate R^2, which is called the *coefficient of determination*. An R^2 value that is close to 1 also indicates that the model is a good fit for the data.

JUSTIFYING THE SOLUTION

From the table, you can estimate that the temperature is 77°F between 10 A.M. and 12 P.M. and between 4 P.M. and 6 P.M. So, your answers are reasonable.

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13. After a break, two students come to school with the flu. The table shows the total numbers of students infected with the flu x days after the break. (a) Use a graphing calculator to find a quadratic model for the data. Then determine whether the model is a good fit. (b) How many days after the break are 26 students infected?

Days after break	0	7	14	21	28	35	42	49
Students with flu	2	3	5	8	18	30	42	55

Vocabulary and Core Concept Check

1. **VOCABULARY** What is a quadratic equation?

2. **WHICH ONE DOESN'T BELONG?** Which equation does *not* belong with the other three? Explain your reasoning.

$x^2 + 5x = 20$	$x^2 + x - 4 = 0$	$x^2 - 6 = 4x$	$7x + 12 = x^2$

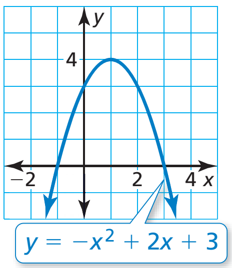
3. **WRITING** How can you use a graph to find the number of solutions of a quadratic equation?

4. **WRITING** How are solutions, roots, x-intercepts, and zeros related?

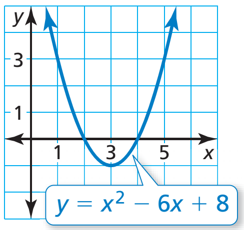
Monitoring Progress and Modeling with Mathematics

In Exercises 5–8, use the graph to solve the equation.

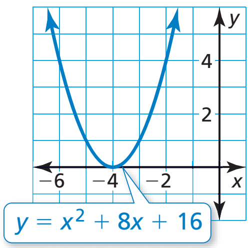
5. $-x^2 + 2x + 3 = 0$



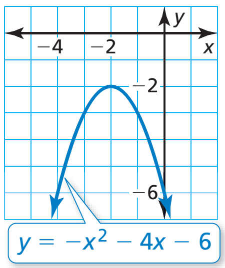
6. $x^2 - 6x + 8 = 0$



7. $x^2 + 8x + 16 = 0$



8. $-x^2 - 4x - 6 = 0$



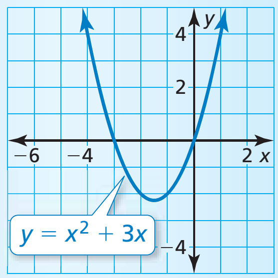
In Exercises 9–12, write the equation in standard form.

9. $4x^2 = 12$
10. $-x^2 = 15$
11. $2x - x^2 = 1$
12. $5 + x = 3x^2$

In Exercises 13–24, solve the equation by graphing. *(See Examples 1, 2, and 3.)*

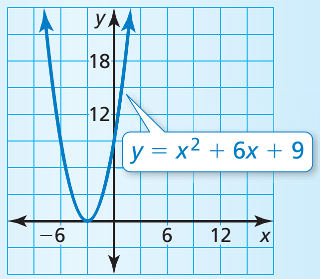
13. $x^2 - 5x = 0$
14. $x^2 - 4x + 4 = 0$
15. $x^2 - 2x + 5 = 0$
16. $x^2 - 6x - 7 = 0$
17. $x^2 = 6x - 9$
18. $-x^2 = 8x + 20$
19. $x^2 = -1 - 2x$
20. $x^2 = -x - 3$
21. $4x - 12 = -x^2$
22. $5x - 6 = x^2$
23. $x^2 - 2 = -x$
24. $16 - x^2 = -8x$

25. **ERROR ANALYSIS** Describe and correct the error in solving $x^2 + 3x = 18$ by graphing.



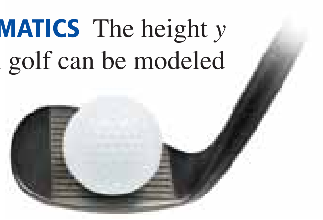
The solutions of the equation $x^2 + 3x = 18$ are $x = -3$ and $x = 0$.

26. **ERROR ANALYSIS** Describe and correct the error in solving $x^2 + 6x + 9 = 0$ by graphing.



The solution of the equation $x^2 + 6x + 9 = 0$ is $x = 9$.

27. **MODELING WITH MATHEMATICS** The height y (in yards) of a flop shot in golf can be modeled by $y = -x^2 + 5x$, where x is the horizontal distance (in yards).



a. Interpret the x-intercepts of the graph of the equation.

b. How far away does the golf ball land?

28. **MODELING WITH MATHEMATICS** The height h (in feet) of an underhand volleyball serve can be modeled by $h = -16t^2 + 30t + 4$, where t is the time (in seconds).

a. Do both t-intercepts of the graph of the function have meaning in this situation? Explain.

b. No one receives the serve. After how many seconds does the volleyball hit the ground?

In Exercises 29–36, solve the equation by using Method 2 from Example 3.

29. $x^2 = 10 - 3x$

30. $2x - 3 = x^2$

31. $5x - 7 = x^2$

32. $x^2 = 6x - 5$

33. $x^2 + 12x = -20$

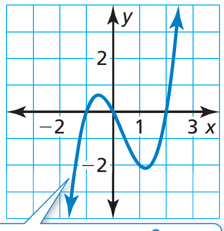
34. $x^2 + 8x = 9$

35. $-x^2 - 5 = -2x$

36. $-x^2 - 4 = -4x$

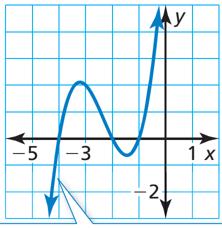
In Exercises 37–42, find the zero(s) of f. *(See Example 4.)*

37.



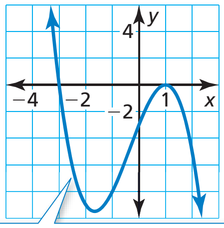
$f(x) = (x - 2)(x^2 + x)$

38.



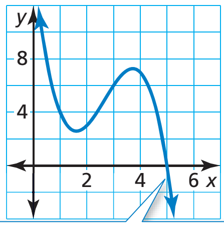
$f(x) = (x + 1)(x^2 + 6x + 8)$

39.



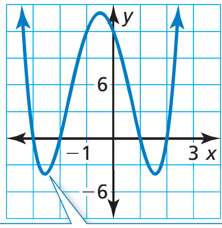
$f(x) = (x + 3)(-x^2 + 2x - 1)$

40.



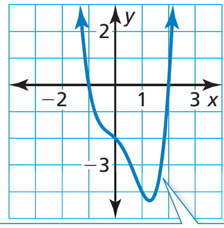
$f(x) = (x - 5)(-x^2 + 3x - 3)$

41.



$f(x) = (x^2 - 4)(x^2 + 2x - 3)$

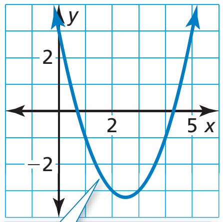
42.



$f(x) = (x^2 + 1)(x^2 - x - 2)$

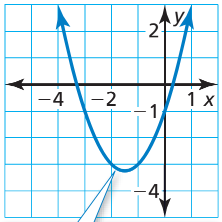
In Exercises 43–46, approximate the zeros of f to the nearest tenth. *(See Example 5.)*

43.



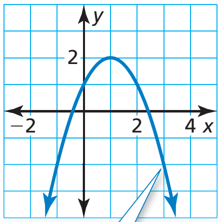
$f(x) = x^2 - 5x + 3$

44.



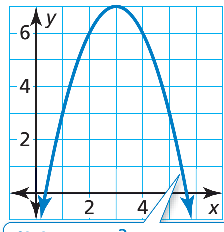
$f(x) = x^2 + 3x - 1$

45.



$f(x) = -x^2 + 2x + 1$

46.



$f(x) = -x^2 + 6x - 2$

In Exercises 47–52, graph the function. Approximate the zeros of the function to the nearest tenth, if necessary.

47. $f(x) = x^2 + 6x + 1$

48. $f(x) = x^2 - 3x + 2$

49. $y = -x^2 + 4x - 2$

50. $y = -x^2 + 9x - 6$

51. $f(x) = \frac{1}{2}x^2 + 2x - 5$

52. $f(x) = -3x^2 + 4x + 3$

53. **MODELING WITH MATHEMATICS** At a Civil War reenactment, a cannonball is fired into the air with an initial vertical velocity of 128 feet per second. The release point is 6 feet above the ground. The function $h = -16t^2 + 128t + 6$ represents the height h (in feet) of the cannonball after t seconds. *(See Example 6.)*

a. Find the height of the cannonball each second after it is fired.

b. Use the results of part (a) to estimate when the height of the cannonball is 150 feet.

c. Using a graph, after how many seconds is the cannonball 150 feet above the ground?

54. **MODELING WITH MATHEMATICS** You throw a softball straight up into the air with an initial vertical velocity of 40 feet per second. The release point is 5 feet above the ground. The function $h = -16t^2 + 40t + 5$ represents the height h (in feet) of the softball after t seconds.

a. Find the height of the softball each second after it is released.

b. Use the results of part (a) to estimate when the height of the softball is 15 feet.

c. Using a graph, after how many seconds is the softball 15 feet above the ground?

55. **MODELING WITH MATHEMATICS** The table shows the temperatures (in degrees Fahrenheit) of a cup of hot chocolate over time. (*See Example 7.*)

Time (minutes)	Temperature (°F)
0	200
10	157
20	128
30	109
40	99
50	92
60	90

a. Use a graphing calculator to find a quadratic model for the data. Then determine whether the model is a good fit.

b. After how many minutes is the temperature of the hot chocolate 120°F? Round your answer to the nearest tenth.

c. Should you use the quadratic model you found in part (a) to predict the temperature of the hot chocolate after 60 minutes? Explain.

56. **MODELING WITH MATHEMATICS** The table shows the values (in dollars) of a car over time.

Age (years)	0	3	6	9	12
Value (dollars)	18,900	12,275	7972	5178	3363

a. Use a graphing calculator to find a quadratic model for the data. Then determine whether the model is a good fit.

b. After how many years is the value of the car \$10,000? Round your answer to the nearest tenth.

c. Should you use the quadratic model you found in part (a) to predict the value of the car after it is 12 years old? Explain your reasoning.

57. **MATHEMATICAL CONNECTIONS** The table shows the numbers of line segments that you can draw whose endpoints are chosen from x points, no three of which are collinear.

Number of points, x	2	3	4	5	6
Number of line segments, y	1	3			

a. Copy and complete the table. Use diagrams to support your answers.

b. Use a graphing calculator to find a quadratic model for the data. Then determine whether the model is a good fit.

c. Predict the number of line segments that you can draw whose endpoints are chosen from 9 points.

d. How many points are chosen when you can draw 66 line segments? Explain how you found your answer.

58. **MODELING WITH MATHEMATICS** The table shows the numbers of cellular telephone sites (in thousands) in the U.S. for selected years from 1990 to 2012.

Year	Cellular sites (thousands)
1992	10.3
1996	30.0
2000	104.3
2004	175.7
2008	242.1
2012	301.8

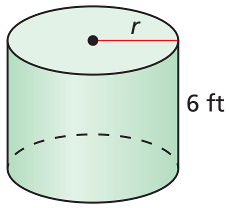
a. Use a graphing calculator to find a linear model and a quadratic model for the data. Let $x = 0$ represent 1990. Is either model a better fit for the data? Explain.

b. Use each model in part (a) to determine in what year the number of cellular sites reached 200,000. Do you get the same result? Justify your answer.

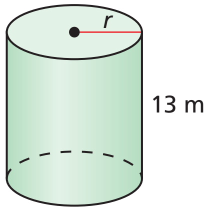
c. Use each model in part (a) to predict in what year the number of cellular sites will reach 500,000. Do you get the same result? Justify your answer.

MATHEMATICAL CONNECTIONS **In Exercises 59 and 60, use the given surface area S of the cylinder to find the radius r to the nearest tenth.**

59. $S = 225 \text{ ft}^2$

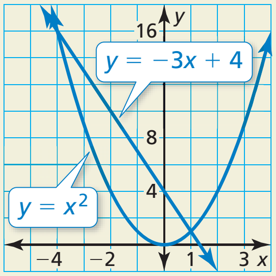


60. $S = 750 \text{ m}^2$



61. **WRITING** Explain how to approximate zeros of a function when the zeros are not integers.

62. **HOW DO YOU SEE IT?** Consider the graph shown.



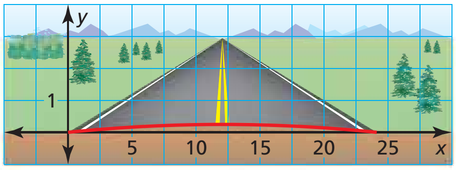
a. How many solutions does the quadratic equation $x^2 = -3x + 4$ have? Explain.

b. Without graphing, describe what you know about the graph of $y = x^2 + 3x - 4$.

63. **COMPARING METHODS** Example 3 shows two methods for solving a quadratic equation. Which method do you prefer? Explain your reasoning.

64. **THOUGHT PROVOKING** How many different parabolas have -2 and 2 as x-intercepts? Sketch examples of parabolas that have these two x-intercepts.

65. **MODELING WITH MATHEMATICS** To keep water off a road, the surface of the road is shaped like a parabola. A cross section of the road is shown in the diagram. The surface of the road can be modeled by $y = -0.0017x^2 + 0.041x$, where x and y are measured in feet. Find the width of the road to the nearest tenth of a foot.



66. **MAKING AN ARGUMENT** A stream of water from a fire hose can be modeled by $y = -0.003x^2 + 0.58x + 3$, where x and y are measured in feet. A firefighter is standing 57 feet from a building and is holding the hose 3 feet above the ground. The bottom of a window of the building is 26 feet above the ground. Your friend claims the stream of water will pass through the window. Is your friend correct? Explain.

REASONING **In Exercises 67–69, determine whether the statement is *always*, *sometimes*, or *never* true. Justify your answer.**

67. The graph of $y = ax^2 + c$ has two x-intercepts when a is negative.

68. The graph of $y = ax^2 + c$ has no x-intercepts when a and c have the same sign.

69. The graph of $y = ax^2 + bx + c$ has more than two x-intercepts when $a \neq 0$.

70. **WRITING** You want to find a model for a set of data. How do you determine whether to perform linear regression or quadratic regression on the set of data?

71. **REASONING** Show how you can use a system of equations to solve the problem in Example 7(b).

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Determine whether the table represents an *exponential growth function*, an *exponential decay function*, or *neither*. Explain. *(Section 6.4)*

72.

x	-1	0	1	2
y	18	3	$\frac{1}{2}$	$\frac{1}{12}$

73.

x	0	1	2	3
y	2	8	32	128

Simplify the expression. *(Section 9.1)*

74. $\dfrac{5}{\sqrt{13}}$

75. $\dfrac{\sqrt{10}}{\sqrt{6x}}$

76. $\dfrac{8}{7 + \sqrt{5}}$

77. $\dfrac{3}{\sqrt{3} - 2}$

9.3 Solving Quadratic Equations Using Square Roots

Essential Question

How can you determine the number of solutions of a quadratic equation of the form $ax^2 + c = 0$?

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
A.8.A

EXPLORATION 1 The Number of Solutions of $ax^2 + c = 0$

Work with a partner. Solve each equation by graphing. Explain how the number of solutions of $ax^2 + c = 0$ relates to the graph of $y = ax^2 + c$.

a. $x^2 - 4 = 0$

b. $2x^2 + 5 = 0$

c. $x^2 = 0$

d. $x^2 - 5 = 0$

EXPLORATION 2 Estimating Solutions

Work with a partner. Complete each table. Use the completed tables to estimate the solutions of $x^2 - 5 = 0$. Explain your reasoning.

a.

x	$x^2 - 5$
2.21	
2.22	
2.23	
2.24	
2.25	
2.26	

b.

x	$x^2 - 5$
−2.21	
−2.22	
−2.23	
−2.24	
−2.25	
−2.26	

USING PRECISE MATHEMATICAL LANGUAGE

To be proficient in math, you need to calculate accurately and express numerical answers with a level of precision appropriate for the problem's context.

EXPLORATION 3 Using Technology to Estimate Solutions

Work with a partner. Two equations are equivalent when they have the same solutions.

a. Are the equations $x^2 - 5 = 0$ and $x^2 = 5$ equivalent? Explain your reasoning.

b. Use the square root key on a calculator to estimate the solutions of $x^2 - 5 = 0$. Describe the accuracy of your estimates in Exploration 2.

c. Write the exact solutions of $x^2 - 5 = 0$.

Communicate Your Answer

4. How can you determine the number of solutions of a quadratic equation of the form $ax^2 + c = 0$?

5. Write the exact solutions of each equation. Then use a calculator to estimate the solutions.

a. $x^2 - 2 = 0$

b. $3x^2 - 18 = 0$

c. $x^2 = 8$

9.3 Lesson

Core Vocabulary

Previous
square root
zero of a function

What You Will Learn

- Solve quadratic equations using square roots.
- Approximate the solutions of quadratic equations.

Solving Quadratic Equations Using Square Roots

Earlier in this chapter, you studied properties of square roots. Now you will use square roots to solve quadratic equations of the form $ax^2 + c = 0$. First isolate x^2 on one side of the equation to obtain $x^2 = d$. Then solve by taking the square root of each side.

Core Concept

Solutions of $x^2 = d$

- When $d > 0$, $x^2 = d$ has two real solutions, $x = \pm\sqrt{d}$.
- When $d = 0$, $x^2 = d$ has one real solution, $x = 0$.
- When $d < 0$, $x^2 = d$ has no real solutions.

ANOTHER WAY

You can also solve $3x^2 - 27 = 0$ by factoring.

$$3(x^2 - 9) = 0$$
$$3(x - 3)(x + 3) = 0$$
$$x = 3 \text{ or } x = -3$$

EXAMPLE 1 Solving Quadratic Equations Using Square Roots

a. Solve $3x^2 - 27 = 0$ using square roots.

$3x^2 - 27 = 0$	Write the equation.
$3x^2 = 27$	Add 27 to each side.
$x^2 = 9$	Divide each side by 3.
$x = \pm\sqrt{9}$	Take the square root of each side.
$x = \pm 3$	Simplify.

▶ The solutions are $x = 3$ and $x = -3$.

b. Solve $x^2 - 10 = -10$ using square roots.

$x^2 - 10 = -10$	Write the equation.
$x^2 = 0$	Add 10 to each side.
$x = 0$	Take the square root of each side.

▶ The only solution is $x = 0$.

c. Solve $-5x^2 + 11 = 16$ using square roots.

$-5x^2 + 11 = 16$	Write the equation.
$-5x^2 = 5$	Subtract 11 from each side.
$x^2 = -1$	Divide each side by -5.

▶ The square of a real number cannot be negative. So, the equation has no real solutions.

STUDY TIP

Each side of the equation $(x - 1)^2 = 25$ is a square. So, you can still solve by taking the square root of each side.

EXAMPLE 2 Solving a Quadratic Equation Using Square Roots

Solve $(x - 1)^2 = 25$ using square roots.

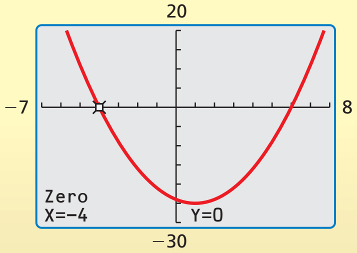
SOLUTION

$(x - 1)^2 = 25$	Write the equation.
$x - 1 = \pm 5$	Take the square root of each side.
$x = 1 \pm 5$	Add 1 to each side.

▶ So, the solutions are $x = 1 + 5 = 6$ and $x = 1 - 5 = -4$.

Check

Use a graphing calculator to check your answer. Rewrite the equation as $(x - 1)^2 - 25 = 0$. Graph the related function $f(x) = (x - 1)^2 - 25$ and find the zeros of the function. The zeros are -4 and 6.



Monitoring Progress

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Solve the equation using square roots.

1. $-3x^2 = -75$ **2.** $x^2 + 12 = 10$ **3.** $4x^2 - 15 = -15$

4. $(x + 7)^2 = 0$ **5.** $4(x - 3)^2 = 9$ **6.** $(2x + 1)^2 = 36$

Approximating Solutions of Quadratic Equations

EXAMPLE 3 Approximating Solutions of a Quadratic Equation

Solve $4x^2 - 13 = 15$ using square roots. Round the solutions to the nearest hundredth.

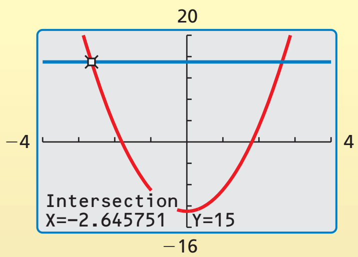
SOLUTION

$4x^2 - 13 = 15$	Write the equation.
$4x^2 = 28$	Add 13 to each side.
$x^2 = 7$	Divide each side by 4.
$x = \pm\sqrt{7}$	Take the square root of each side.
$x \approx \pm 2.65$	Use a calculator.

▶ The solutions are $x \approx -2.65$ and $x \approx 2.65$.

Check

Graph each side of the equation and find the points of intersection. The x-values of the points of intersection are about -2.65 and 2.65.



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Solve the equation using square roots. Round your solutions to the nearest hundredth.

7. $x^2 + 8 = 19$ **8.** $5x^2 - 2 = 0$ **9.** $3x^2 - 30 = 4$

EXAMPLE 4 Solving a Real-Life Problem

A touch tank has a height of 3 feet. Its length is three times its width. The volume of the tank is 270 cubic feet. Find the length and width of the tank.

SOLUTION

The length ℓ is three times the width w, so $\ell = 3w$. Write an equation using the formula for the volume of a rectangular prism.

$$V = \ell wh \qquad \text{Write the formula.}$$

$$270 = 3w(w)(3) \qquad \text{Substitute 270 for } V\text{, } 3w \text{ for } \ell\text{, and 3 for } h.$$

$$270 = 9w^2 \qquad \text{Multiply.}$$

$$30 = w^2 \qquad \text{Divide each side by 9.}$$

$$\pm\sqrt{30} = w \qquad \text{Take the square root of each side.}$$

The solutions are $\sqrt{30}$ and $-\sqrt{30}$. Use the positive solution.

▶ So, the width is $\sqrt{30} \approx 5.5$ feet and the length is $3\sqrt{30} \approx 16.4$ feet.

EXPLAINING MATHEMATICAL IDEAS

Use the positive square root because negative solutions do not make sense in this context. Length and width cannot be negative.

EXAMPLE 5 Rearranging and Evaluating a Formula

The area A of an equilateral triangle with side length s is given by the formula $A = \frac{\sqrt{3}}{4}s^2$. Solve the formula for s. Then approximate the side length of the traffic sign that has an area of 390 square inches.

SOLUTION

Step 1 Solve the formula for s.

$$A = \frac{\sqrt{3}}{4}s^2 \qquad \text{Write the formula.}$$

$$\frac{4A}{\sqrt{3}} = s^2 \qquad \text{Multiply each side by } \frac{4}{\sqrt{3}}.$$

$$\sqrt{\frac{4A}{\sqrt{3}}} = s \qquad \text{Take the positive square root of each side.}$$

Step 2 Substitute 390 for A in the new formula and evaluate.

$$s = \sqrt{\frac{4A}{\sqrt{3}}} = \sqrt{\frac{4(390)}{\sqrt{3}}} = \sqrt{\frac{1560}{\sqrt{3}}} \approx 30 \qquad \text{Use a calculator.}$$

▶ The side length of the traffic sign is about 30 inches.

ANOTHER WAY

Notice that you can rewrite the formula as $s = \frac{2}{3^{1/4}}\sqrt{A}$, or $s \approx 1.52\sqrt{A}$. This can help you efficiently find the value of s for various values of A.

Monitoring Progress

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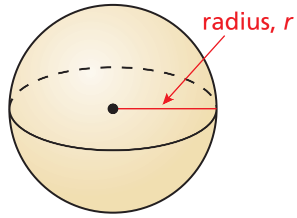
10. **WHAT IF?** In Example 4, the volume of the tank is 315 cubic feet. Find the length and width of the tank.

11. The surface area S of a sphere with radius r is given by the formula $S = 4\pi r^2$. Solve the formula for r. Then find the radius of a globe with a surface area of 804 square inches.



9.3 Exercises

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Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The equation $x^2 = d$ has ____ real solutions when $d > 0$.

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

Solve $x^2 = 144$ using square roots.	Solve $x^2 - 144 = 0$ using square roots.
Solve $x^2 + 146 = 2$ using square roots.	Solve $x^2 + 2 = 146$ using square roots.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, determine the number of real solutions of the equation. Then solve the equation using square roots.

3. $x^2 = 25$
4. $x^2 = -36$
5. $x^2 = -21$
6. $x^2 = 400$
7. $x^2 = 0$
8. $x^2 = 169$

In Exercises 9–18, solve the equation using square roots. *(See Example 1.)*

9. $x^2 - 16 = 0$
10. $x^2 + 6 = 0$
11. $3x^2 + 12 = 0$
12. $x^2 - 55 = 26$
13. $2x^2 - 98 = 0$
14. $-x^2 + 9 = 9$
15. $-3x^2 - 5 = -5$
16. $4x^2 - 371 = 29$
17. $4x^2 + 10 = 11$
18. $9x^2 - 35 = 14$

In Exercises 19–24, solve the equation using square roots. *(See Example 2.)*

19. $(x + 3)^2 = 0$
20. $(x - 1)^2 = 4$
21. $(2x - 1)^2 = 81$
22. $(4x + 5)^2 = 9$
23. $9(x + 1)^2 = 16$
24. $4(x - 2)^2 = 25$

In Exercises 25–30, solve the equation using square roots. Round your solutions to the nearest hundredth. *(See Example 3.)*

25. $x^2 + 6 = 13$
26. $x^2 + 11 = 24$
27. $2x^2 - 9 = 11$
28. $5x^2 + 2 = 6$
29. $-21 = 15 - 2x^2$
30. $2 = 4x^2 - 5$

31. **ERROR ANALYSIS** Describe and correct the error in solving the equation $2x^2 - 33 = 39$ using square roots.

$$2x^2 - 33 = 39$$
$$2x^2 = 72$$
$$x^2 = 36$$
$$x = 6$$

The solution is $x = 6$.

32. **MODELING WITH MATHEMATICS** An in-ground pond has the shape of a rectangular prism. The pond has a depth of 24 inches and a volume of 72,000 cubic inches. The length of the pond is two times its width. Find the length and width of the pond. *(See Example 4.)*

33. **MODELING WITH MATHEMATICS** A person sitting in the top row of the bleachers at a sporting event drops a pair of sunglasses from a height of 24 feet. The function $h = -16x^2 + 24$ represents the height h (in feet) of the sunglasses after x seconds. How long does it take the sunglasses to hit the ground?

34. MAKING AN ARGUMENT Your friend says that the solution of the equation $x^2 + 4 = 0$ is $x = 0$. Your cousin says that the equation has no real solutions. Who is correct? Explain your reasoning.

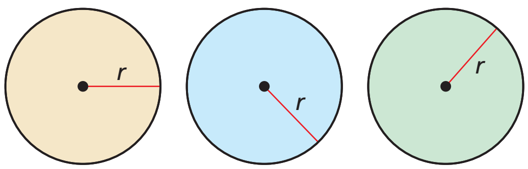
35. MODELING WITH MATHEMATICS The design of a square rug for your living room is shown. You want the area of the inner square to be 25% of the total area of the rug. Find the side length x of the inner square.

6 ft

36. MATHEMATICAL CONNECTIONS The area A of a circle with radius r is given by the formula $A = \pi r^2$. *(See Example 5.)*

a. Solve the formula for r.

b. Use the formula from part (a) to find the radius of each circle.



$A = 113\text{ ft}^2$ $\quad A = 1810\text{ in.}^2$ $\quad A = 531\text{ m}^2$

c. Explain why it is beneficial to solve the formula for r before finding the radius.

37. WRITING How can you approximate the roots of a quadratic equation when the roots are not integers?

38. WRITING Given the equation $ax^2 + c = 0$, describe the values of a and c so the equation has the following number of solutions.

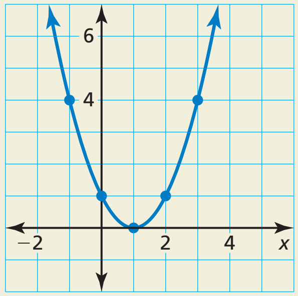
a. two real solutions

b. one real solution

c. no real solutions

39. REASONING Without graphing, where do the graphs of $y = x^2$ and $y = 9$ intersect? Explain.

40. HOW DO YOU SEE IT? The graph represents the function $f(x) = (x - 1)^2$. How many solutions does the equation $(x - 1)^2 = 0$ have? Explain.



41. REASONING Solve $x^2 = 1.44$ without using a calculator. Explain your reasoning.

42. THOUGHT PROVOKING The quadratic equation

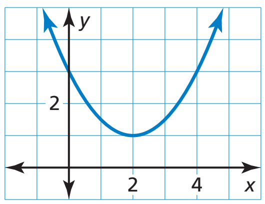
$$ax^2 + bx + c = 0$$

can be rewritten in the following form.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Use this form to write the solutions of the equation.

43. REASONING An equation of the graph shown is $y = \frac{1}{2}(x - 2)^2 + 1$. Two points on the parabola have y-coordinates of 9. Find the x-coordinates of these points.



44. CRITICAL THINKING Solve each equation without graphing.

a. $x^2 - 12x + 36 = 64$

b. $x^2 + 14x + 49 = 16$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Factor the polynomial. *(Section 7.8)*

45. $x^2 + 8x + 16$ **46.** $x^2 - 4x + 4$ **47.** $x^2 - 14x + 49$

48. $x^2 + 18x + 81$ **49.** $x^2 + 12x + 36$ **50.** $x^2 - 22x + 121$

9.1–9.3 What Did You Learn?

Core Vocabulary

counterexample, *p. 465*
radical expression, *p. 466*
simplest form, *p. 466*
rationalizing the denominator, *p. 468*
conjugates, *p. 468*
like radicals, *p. 470*
quadratic equation, *p. 476*

Core Concepts

Section 9.1

Product Property of Square Roots, *p. 466*
Quotient Property of Square Roots, *p. 466*
Rationalizing the Denominator, *p. 468*
Performing Operations with Radicals, *p. 470*

Section 9.2

Solving Quadratic Equations by Graphing, *p. 476*
Number of Solutions of a Quadratic Equation, *p. 477*
Finding Zeros of Functions, *p. 478*

Section 9.3

Solutions of $x^2 = d$, *p. 486*
Approximating Solutions of Quadratic Equations, *p. 487*

Mathematical Thinking

1. For each part of Exercise 100 on page 474 that is *sometimes* true, list all examples and counterexamples from the table that represent the sum or product being described.
2. Which Examples can you use to help you solve Exercise 54 on page 483?
3. Describe how solving a simpler equation can help you solve the equation in Exercise 41 on page 490.

Study Skills

Keeping a Positive Attitude

Do you ever feel frustrated or overwhelmed by math? You're not alone. Just take a deep breath and assess the situation. Try to find a productive study environment, review your notes and the examples in the textbook, and ask your teacher or friends for help.

9.1–9.3 Quiz

Simplify the expression. *(Section 9.1)*

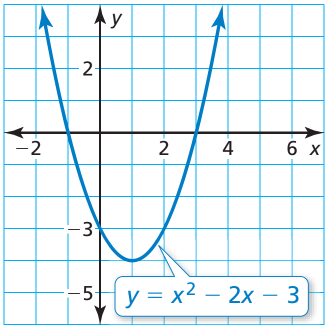
1. $\sqrt{112x^3}$ **2.** $\sqrt{\dfrac{18}{81}}$ **3.** $\sqrt[3]{-625}$ **4.** $\dfrac{12}{\sqrt{32}}$

5. $\dfrac{4}{\sqrt{11}}$ **6.** $\sqrt{\dfrac{144}{13}}$ **7.** $\sqrt[3]{\dfrac{54x^4}{343y^6}}$ **8.** $\sqrt{\dfrac{4x^2}{28y^4z^5}}$

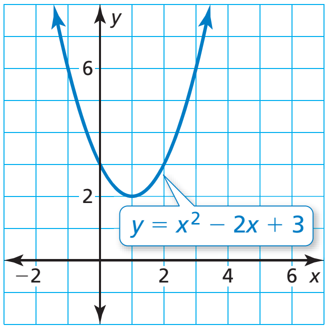
9. $\dfrac{6}{5+\sqrt{3}}$ **10.** $2\sqrt{5} + 7\sqrt{10} - 3\sqrt{20}$ **11.** $\dfrac{10}{\sqrt{8}-\sqrt{10}}$ **12.** $\sqrt{6}\left(7\sqrt{12} - 4\sqrt{3}\right)$

Use the graph to solve the equation. *(Section 9.2)*

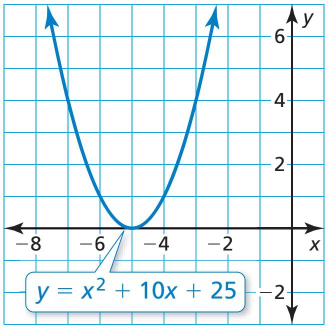
13. $x^2 - 2x - 3 = 0$



14. $x^2 - 2x + 3 = 0$



15. $x^2 + 10x + 25 = 0$



Solve the equation by graphing. *(Section 9.2)*

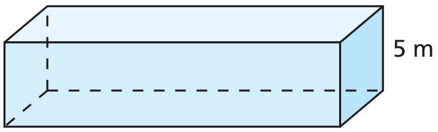
16. $x^2 + 9x + 14 = 0$ **17.** $x^2 - 7x = 8$ **18.** $x + 4 = -x^2$

Solve the equation using square roots. *(Section 9.3)*

19. $4x^2 = 64$ **20.** $-3x^2 + 6 = 10$ **21.** $(x-8)^2 = 1$

22. Explain how to determine the number of real solutions of $x^2 = 100$ without solving. *(Section 9.3)*

23. The length of a rectangular prism is four times its width. The volume of the prism is 380 cubic meters. Find the length and width of the prism. *(Section 9.3)*



24. You cast a fishing lure into the water from a height of 4 feet above the water. The height h (in feet) of the fishing lure after t seconds can be modeled by the equation $h = -16t^2 + 24t + 4$. *(Section 9.2)*

a. After how many seconds does the fishing lure reach a height of 12 feet?

b. After how many seconds does the fishing lure hit the water?

9.4 Solving Quadratic Equations by Completing the Square

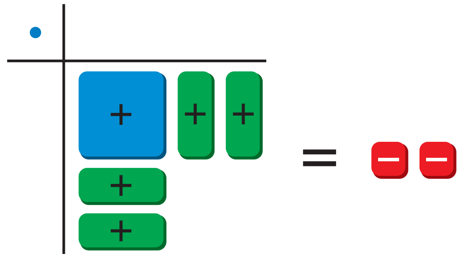
Essential Question How can you use “completing the square” to solve a quadratic equation?

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
A.8.A

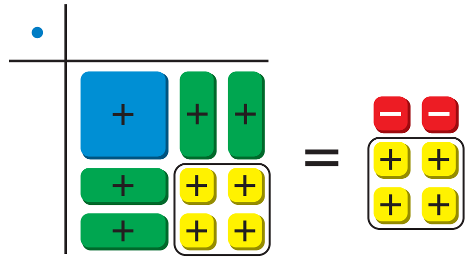
EXPLORATION 1 Solving by Completing the Square

Work with a partner.

a. Write the equation modeled by the algebra tiles. This is the equation to be solved.

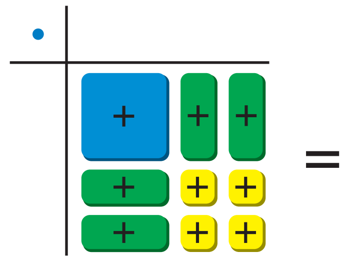


b. Four algebra tiles are added to the left side to “complete the square.” Why are four algebra tiles also added to the right side?



c. Use algebra tiles to label the dimensions of the square on the left side and simplify on the right side.

d. Write the equation modeled by the algebra tiles so that the left side is the square of a binomial. Solve the equation using square roots.



USING PROBLEM-SOLVING STRATEGIES

To be proficient in math, you need to explain to yourself the meaning of a problem. After that, you need to look for entry points to its solution.

EXPLORATION 2 Solving by Completing the Square

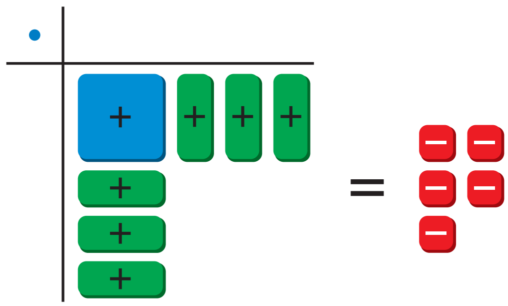
Work with a partner.

a. Write the equation modeled by the algebra tiles.

b. Use algebra tiles to “complete the square.”

c. Write the solutions of the equation.

d. Check each solution in the original equation.



Communicate Your Answer

3. How can you use “completing the square” to solve a quadratic equation?

4. Solve each quadratic equation by completing the square.

a. $x^2 - 2x = 1$ **b.** $x^2 - 4x = -1$ **c.** $x^2 + 4x = -3$

9.4 Lesson

Core Vocabulary

completing the square, *p. 494*

Previous
perfect square trinomial
coefficient
maximum value
minimum value
vertex form of a quadratic function

What You Will Learn

- Complete the square for expressions of the form $x^2 + bx$.
- Solve quadratic equations by completing the square.
- Find and use maximum and minimum values.
- Solve real-life problems by completing the square.

Completing the Square

For an expression of the form $x^2 + bx$, you can add a constant c to the expression so that $x^2 + bx + c$ is a perfect square trinomial. This process is called **completing the square**.

Core Concept

Completing the Square

Words To complete the square for an expression of the form $x^2 + bx$, follow these steps.

Step 1 Find one-half of b, the coefficient of x.

Step 2 Square the result from Step 1.

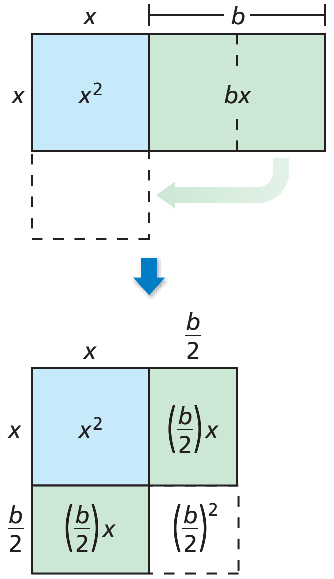
Step 3 Add the result from Step 2 to $x^2 + bx$.

Factor the resulting expression as the square of a binomial.

Algebra $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

JUSTIFYING STEPS

In each diagram below, the combined area of the shaded regions is $x^2 + bx$. Adding $\left(\frac{b}{2}\right)^2$ completes the square in the second diagram.



EXAMPLE 1 Completing the Square

Complete the square for each expression. Then factor the trinomial.

a. $x^2 + 6x$ **b.** $x^2 - 9x$

SOLUTION

a. Step 1 Find one-half of b. $\frac{b}{2} = \frac{6}{2} = 3$

Step 2 Square the result from Step 1. $3^2 = 9$

Step 3 Add the result from Step 2 to $x^2 + bx$. $x^2 + 6x + 9$

▶ $x^2 + 6x + 9 = (x + 3)^2$

b. Step 1 Find one-half of b. $\frac{b}{2} = \frac{-9}{2}$

Step 2 Square the result from Step 1. $\left(\frac{-9}{2}\right)^2 = \frac{81}{4}$

Step 3 Add the result from Step 2 to $x^2 + bx$. $x^2 - 9x + \frac{81}{4}$

▶ $x^2 - 9x + \frac{81}{4} = \left(x - \frac{9}{2}\right)^2$

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Complete the square for the expression. Then factor the trinomial.

1. $x^2 + 10x$ **2.** $x^2 - 4x$ **3.** $x^2 + 7x$

Solving Quadratic Equations by Completing the Square

The method of completing the square can be used to solve any quadratic equation. To solve a quadratic equation by completing the square, you must write the equation in the form $x^2 + bx = d$.

COMMON ERROR

When completing the square to solve an equation, be sure to add $\left(\frac{b}{2}\right)^2$ to each side of the equation.

EXAMPLE 2 Solving a Quadratic Equation: $x^2 + bx = d$

Solve $x^2 - 16x = -15$ by completing the square.

SOLUTION

$x^2 - 16x = -15$	Write the equation.
$x^2 - 16x + (-8)^2 = -15 + (-8)^2$	Complete the square by adding $\left(\frac{-16}{2}\right)^2$, or $(-8)^2$, to each side.
$(x - 8)^2 = 49$	Write the left side as the square of a binomial.
$x - 8 = \pm 7$	Take the square root of each side.
$x = 8 \pm 7$	Add 8 to each side.

▶ The solutions are $x = 8 + 7 = 15$ and $x = 8 - 7 = 1$.

Check

$x^2 - 16x = -15$	Original equation	$x^2 - 16x = -15$
$15^2 - 16(15) \stackrel{?}{=} -15$	Substitute.	$1^2 - 16(1) \stackrel{?}{=} -15$
$-15 = -15$ ✓	Simplify.	$-15 = -15$ ✓

EXAMPLE 3 Solving a Quadratic Equation: $ax^2 + bx + c = 0$

Solve $2x^2 + 20x - 8 = 0$ by completing the square.

COMMON ERROR

Before you complete the square, be sure that the coefficient of the x^2-term is 1.

SOLUTION

$2x^2 + 20x - 8 = 0$	Write the equation.
$2x^2 + 20x = 8$	Add 8 to each side.
$x^2 + 10x = 4$	Divide each side by 2.
$x^2 + 10x + 5^2 = 4 + 5^2$	Complete the square by adding $\left(\frac{10}{2}\right)^2$, or 5^2, to each side.
$(x + 5)^2 = 29$	Write the left side as the square of a binomial.
$x + 5 = \pm\sqrt{29}$	Take the square root of each side.
$x = -5 \pm \sqrt{29}$	Subtract 5 from each side.

▶ The solutions are $x = -5 + \sqrt{29} \approx 0.39$ and $x = -5 - \sqrt{29} \approx -10.39$.

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Solve the equation by completing the square. Round your solutions to the nearest hundredth, if necessary.

4. $x^2 - 2x = 3$ **5.** $m^2 + 12m = -8$ **6.** $3g^2 - 24g + 27 = 0$

Finding and Using Maximum and Minimum Values

One way to find the maximum or minimum value of a quadratic function is to write the function in vertex form by completing the square. Recall that the vertex form of a quadratic function is $y = a(x - h)^2 + k$, where $a \neq 0$. The vertex of the graph is (h, k).

EXAMPLE 4 Finding a Minimum Value

Find the minimum value of $y = x^2 + 4x - 1$.

SOLUTION

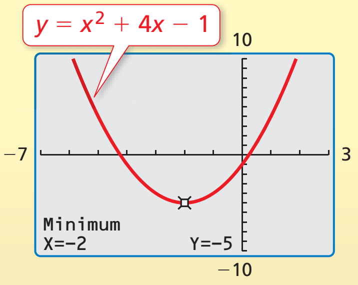
Write the function in vertex form.

$y = x^2 + 4x - 1$	Write the function.
$y + 1 = x^2 + 4x$	Add 1 to each side.
$y + 1 + 4 = x^2 + 4x + 4$	Complete the square for $x^2 + 4x$.
$y + 5 = x^2 + 4x + 4$	Simplify the left side.
$y + 5 = (x + 2)^2$	Write the right side as the square of a binomial.
$y = (x + 2)^2 - 5$	Write in vertex form.

The vertex is $(-2, -5)$. Because a is positive $(a = 1)$, the parabola opens up and the y-coordinate of the vertex is the minimum value.

▶ So, the function has a minimum value of -5.

Check



EXAMPLE 5 Finding a Maximum Value

Find the maximum value of $y = -x^2 + 2x + 7$.

SOLUTION

Write the function in vertex form.

$y = -x^2 + 2x + 7$	Write the function.
$y - 7 = -x^2 + 2x$	Subtract 7 from each side.
$y - 7 = -(x^2 - 2x)$	Factor out -1.
$y - 7 - 1 = -(x^2 - 2x + 1)$	Complete the square for $x^2 - 2x$.
$y - 8 = -(x^2 - 2x + 1)$	Simplify the left side.
$y - 8 = -(x - 1)^2$	Write $x^2 - 2x + 1$ as the square of a binomial.
$y = -(x - 1)^2 + 8$	Write in vertex form.

STUDY TIP

Adding 1 inside the parentheses results in subtracting 1 from the right side of the equation.

The vertex is $(1, 8)$. Because a is negative $(a = -1)$, the parabola opens down and the y-coordinate of the vertex is the maximum value.

▶ So, the function has a maximum value of 8.

Monitoring Progress

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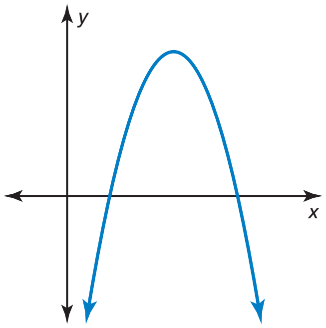
Determine whether the quadratic function has a maximum or minimum value. Then find the value.

7. $y = -x^2 - 4x + 4$ **8.** $y = x^2 + 12x + 40$ **9.** $y = x^2 - 2x - 2$

EXAMPLE 6 Interpreting Forms of Quadratic Functions

Which of the functions could be represented by the graph? Explain.



$f(x) = -\frac{1}{2}(x + 4)^2 + 8$

$g(x) = -(x - 5)^2 + 9$

$m(x) = (x - 3)(x - 12)$

$p(x) = -(x - 2)(x - 8)$

SOLUTION

You do not know the scale of either axis. To eliminate functions, consider the characteristics of the graph and information provided by the form of each function. The graph appears to be a parabola that opens down, which means the function has a maximum value. The vertex of the graph is in the first quadrant. Both x-intercepts are positive.

- The graph of f opens down because $a < 0$, which means f has a maximum value. However, the vertex $(-4, 8)$ of the graph of f is in the second quadrant. So, the graph does not represent f.
- The graph of g opens down because $a < 0$, which means g has a maximum value. The vertex $(5, 9)$ of the graph of g is in the first quadrant. By solving $0 = -(x - 5)^2 + 9$, you see that the x-intercepts of the graph of g are 2 and 8. So, the graph could represent g.
- The graph of m has two positive x-intercepts. However, its graph opens up because $a > 0$, which means m has a minimum value. So, the graph does not represent m.
- The graph of p has two positive x-intercepts, and its graph opens down because $a < 0$. This means that p has a maximum value and the vertex must be in the first quadrant. So, the graph could represent p.

▶ The graph could represent function g or function p.

EXAMPLE 7 Real-Life Application

The function $y = -16x^2 + 96x$ represents the height y (in feet) of a model rocket x seconds after it is launched. (a) Find the maximum height of the rocket. (b) Find and interpret the axis of symmetry.

SOLUTION

a. To find the maximum height, identify the maximum value of the function.

$y = -16x^2 + 96x$	Write the function.
$y = -16(x^2 - 6x)$	Factor out -16.
$y - 144 = -16(x^2 - 6x + 9)$	Complete the square for $x^2 - 6x$.
$y = -16(x - 3)^2 + 144$	Write in vertex form.

STUDY TIP

Adding 9 inside the parentheses results in subtracting 144 from the right side of the equation.

▶ Because the maximum value is 144, the model rocket reaches a maximum height of 144 feet.

b. The vertex is $(3, 144)$. So, the axis of symmetry is $x = 3$. On the left side of $x = 3$, the height increases as time increases. On the right side of $x = 3$, the height decreases as time increases.

Monitoring Progress

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Determine whether the function could be represented by the graph in Example 6. Explain.

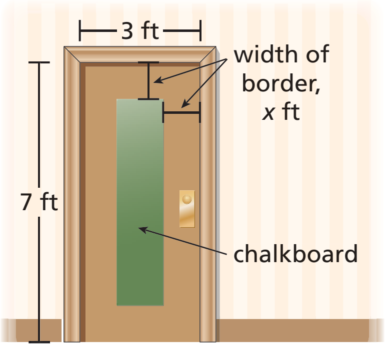
10. $h(x) = (x - 8)^2 + 10$

11. $n(x) = -2(x - 5)(x - 20)$

12. WHAT IF? Repeat Example 7 when the function is $y = -16x^2 + 128x$.

Solving Real-Life Problems

EXAMPLE 8 **Modeling with Mathematics**



You decide to use chalkboard paint to create a chalkboard on a door. You want the chalkboard to cover 6 square feet and to have a uniform border, as shown. Find the width of the border to the nearest inch.

SOLUTION

1. **Understand the Problem** You know the dimensions (in feet) of the door from the diagram. You also know the area (in square feet) of the chalkboard and that it will have a uniform border. You are asked to find the width of the border to the nearest inch.
2. **Make a Plan** Use a verbal model to write an equation that represents the area of the chalkboard. Then solve the equation.
3. **Solve the Problem**

Let x be the width (in feet) of the border, as shown in the diagram.

Area of chalkboard (square feet)	=	Length of chalkboard (feet)	$\cdot$	Width of chalkboard (feet)
6	=	$(7 - 2x)$	$\cdot$	$(3 - 2x)$

$$6 = (7 - 2x)(3 - 2x) \qquad \text{Write the equation.}$$
$$6 = 21 - 20x + 4x^2 \qquad \text{Multiply the binomials.}$$
$$-15 = 4x^2 - 20x \qquad \text{Subtract 21 from each side.}$$
$$-\frac{15}{4} = x^2 - 5x \qquad \text{Divide each side by 4.}$$
$$-\frac{15}{4} + \frac{25}{4} = x^2 - 5x + \frac{25}{4} \qquad \text{Complete the square for } x^2 - 5x.$$
$$\frac{5}{2} = x^2 - 5x + \frac{25}{4} \qquad \text{Simplify the left side.}$$
$$\frac{5}{2} = \left(x - \frac{5}{2}\right)^2 \qquad \text{Write the right side as the square of a binomial.}$$
$$\pm\sqrt{\frac{5}{2}} = x - \frac{5}{2} \qquad \text{Take the square root of each side.}$$
$$\frac{5}{2} \pm \sqrt{\frac{5}{2}} = x \qquad \text{Add } \frac{5}{2} \text{ to each side.}$$

The solutions of the equation are $x = \frac{5}{2} + \sqrt{\frac{5}{2}} \approx 4.08$ and $x = \frac{5}{2} - \sqrt{\frac{5}{2}} \approx 0.92$.

It is not possible for the width of the border to be 4.08 feet because the width of the door is 3 feet. So, the width of the border is about 0.92 foot.

$$0.92\,\cancel{\text{ft}} \cdot \frac{12 \text{ in.}}{1\,\cancel{\text{ft}}} = 11.04 \text{ in.} \qquad \text{Convert 0.92 foot to inches.}$$

▶ The width of the border should be about 11 inches.

4. **Look Back** When the width of the border is slightly less than 1 foot, the length of the chalkboard is slightly more than 5 feet and the width of the chalkboard is slightly more than 1 foot. Multiplying these dimensions gives an area close to 6 square feet. So, an 11-inch border is reasonable.

Monitoring Progress

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13. **WHAT IF?** You want the chalkboard to cover 4 square feet. Find the width of the border to the nearest inch.

9.4 Exercises

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Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The process of adding a constant c to the expression $x^2 + bx$ so that $x^2 + bx + c$ is a perfect square trinomial is called ________________.

2. **VOCABULARY** Explain how to complete the square for an expression of the form $x^2 + bx$.

3. **WRITING** Is it more convenient to complete the square for $x^2 + bx$ when b is odd or when b is even? Explain.

4. **WRITING** Describe how you can use the process of completing the square to find the maximum or minimum value of a quadratic function.

Monitoring Progress and Modeling with Mathematics

In Exercises 5–10, find the value of c that completes the square.

5. $x^2 - 8x + c$
6. $x^2 - 2x + c$
7. $x^2 + 4x + c$
8. $x^2 + 12x + c$
9. $x^2 - 15x + c$
10. $x^2 + 9x + c$

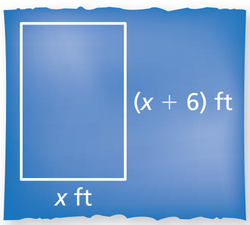
In Exercises 11–16, complete the square for the expression. Then factor the trinomial. *(See Example 1.)*

11. $x^2 - 10x$
12. $x^2 - 40x$
13. $x^2 + 16x$
14. $x^2 + 22x$
15. $x^2 + 5x$
16. $x^2 - 3x$

In Exercises 17–22, solve the equation by completing the square. Round your solutions to the nearest hundredth, if necessary. *(See Example 2.)*

17. $x^2 + 14x = 15$
18. $x^2 - 6x = 16$
19. $x^2 - 4x = -2$
20. $x^2 + 2x = 5$
21. $x^2 - 5x = 8$
22. $x^2 + 11x = -10$

23. **MODELING WITH MATHEMATICS** The area of the patio is 216 square feet.



   a. Write an equation that represents the area of the patio.

   b. Find the dimensions of the patio by completing the square.

24. **MODELING WITH MATHEMATICS** Some sand art contains sand and water sealed in a glass case, similar to the one shown. When the art is turned upside down, the sand and water fall to create a new picture. The glass case has a depth of 1 centimeter and a volume of 768 cubic centimeters.

   a. Write an equation that represents the volume of the glass case.

   b. Find the dimensions of the glass case by completing the square.

In Exercises 25–32, solve the equation by completing the square. Round your solutions to the nearest hundredth, if necessary. *(See Example 3.)*

25. $x^2 - 8x + 15 = 0$
26. $x^2 + 4x - 21 = 0$
27. $2x^2 + 20x + 44 = 0$
28. $3x^2 - 18x + 12 = 0$
29. $-3x^2 - 24x + 17 = -40$
30. $-5x^2 - 20x + 35 = 30$
31. $2x^2 - 14x + 10 = 26$
32. $4x^2 + 12x - 15 = 5$

33. **ERROR ANALYSIS** Describe and correct the error in solving $x^2 + 8x = 10$ by completing the square.

$$x^2 + 8x = 10$$
$$x^2 + 8x + 16 = 10$$
$$(x + 4)^2 = 10$$
$$x + 4 = \pm\sqrt{10}$$
$$x = -4 \pm \sqrt{10}$$

34. **ERROR ANALYSIS** Describe and correct the error in the first two steps of solving $2x^2 - 2x - 4 = 0$ by completing the square.

$$2x^2 - 2x - 4 = 0$$
$$2x^2 - 2x = 4$$
$$2x^2 - 2x + 1 = 4 + 1$$

35. **NUMBER SENSE** Find all values of b for which $x^2 + bx + 25$ is a perfect square trinomial. Explain how you found your answer.

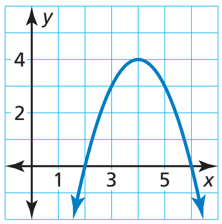
36. **REASONING** You are completing the square to solve $3x^2 + 6x = 12$. What is the first step?

In Exercises 37–40, write the function in vertex form by completing the square. Then match the function with its graph.

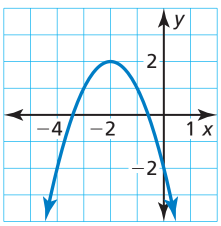
37. $y = x^2 + 6x + 3$ **38.** $y = -x^2 + 8x - 12$

39. $y = -x^2 - 4x - 2$ **40.** $y = x^2 - 2x + 4$

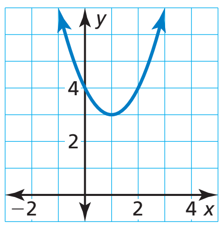
A.



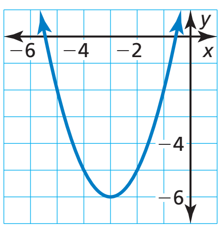
B.



C.



D.



In Exercises 41–46, determine whether the quadratic function has a maximum or minimum value. Then find the value. *(See Examples 4 and 5.)*

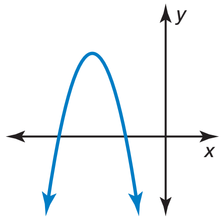
41. $y = x^2 - 4x - 2$ **42.** $y = x^2 + 6x + 10$

43. $y = -x^2 - 10x - 30$ **44.** $y = -x^2 + 14x - 34$

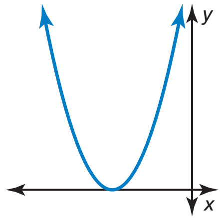
45. $f(x) = -3x^2 - 6x - 9$ **46.** $f(x) = 4x^2 - 28x + 32$

In Exercises 47–50, determine whether the graph could represent the function. Explain.

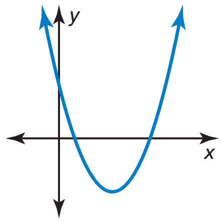
47. $y = -(x + 8)(x + 3)$



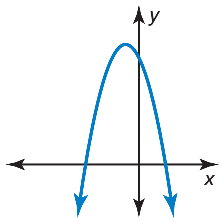
48. $y = (x - 5)^2$



49. $y = \frac{1}{4}(x + 2)^2 - 4$

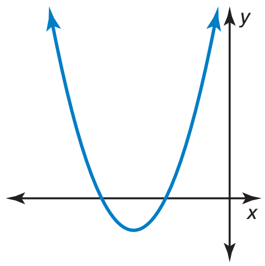


50. $y = -2(x - 1)(x + 2)$



In Exercises 51 and 52, determine which of the functions could be represented by the graph. Explain. *(See Example 6.)*

51.



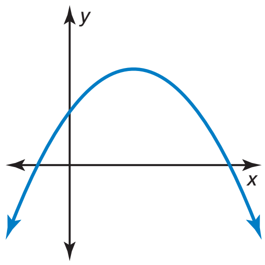
$h(x) = (x + 2)^2 + 3$

$f(x) = 2(x + 3)^2 - 2$

$g(x) = -\frac{1}{2}(x - 8)(x - 4)$

$m(x) = (x + 2)(x + 4)$

52.



$r(x) = -\frac{1}{3}(x - 5)(x + 1)$

$p(x) = -2(x - 2)(x - 6)$

$q(x) = (x + 1)^2 + 4$

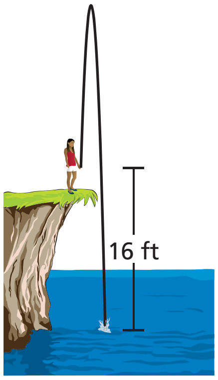
$n(x) = -(x - 2)^2 + 9$

53. **MODELING WITH MATHEMATICS** The function $h = -16t^2 + 48t$ represents the height h (in feet) of a kickball t seconds after it is kicked from the ground. *(See Example 7.)*

a. Find the maximum height of the kickball.

b. Find and interpret the axis of symmetry.

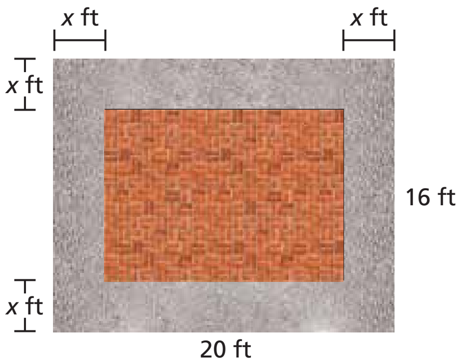
54. MODELING WITH MATHEMATICS You throw a stone from a height of 16 feet with an initial vertical velocity of 32 feet per second. The function $h = -16t^2 + 32t + 16$ represents the height h (in feet) of the stone after t seconds.



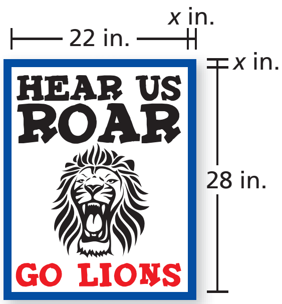
a. Find the maximum height of the stone.

b. Find and interpret the axis of symmetry.

55. MODELING WITH MATHEMATICS You are building a rectangular brick patio surrounded by a crushed stone border with a uniform width, as shown. You purchase patio bricks to cover 140 square feet. Find the width of the border. *(See Example 8.)*

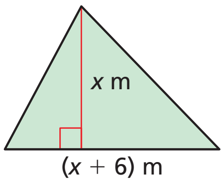


56. MODELING WITH MATHEMATICS You are making a poster that will have a uniform border, as shown. The total area of the poster is 722 square inches. Find the width of the border to the nearest inch.

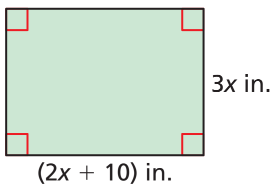


MATHEMATICAL CONNECTIONS **In Exercises 57 and 58, find the value of x. Round your answer to the nearest hundredth, if necessary.**

57. $A = 108\text{ m}^2$



58. $A = 288\text{ in.}^2$



In Exercises 59–62, solve the equation by completing the square. Round your solutions to the nearest hundredth, if necessary.

59. $0.5x^2 + x - 2 = 0$

60. $0.75x^2 + 1.5x = 4$

61. $\frac{8}{3}x - \frac{2}{3}x^2 = -\frac{5}{6}$

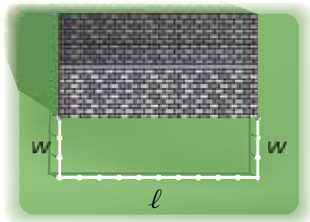
62. $\frac{1}{4}x^2 + \frac{1}{2}x - \frac{5}{4} = 0$

63. PROBLEM SOLVING The distance d (in feet) that it takes a car to come to a complete stop can be modeled by $d = 0.05s^2 + 2.2s$, where s is the speed of the car (in miles per hour). A car has 168 feet to come to a complete stop. Find the maximum speed at which the car can travel.

64. PROBLEM SOLVING During a "big air" competition, snowboarders launch themselves from a half-pipe, perform tricks in the air, and land back in the half-pipe. The height h (in feet) of a snowboarder above the bottom of the half-pipe can be modeled by $h = -16t^2 + 24t + 16.4$, where t is the time (in seconds) after the snowboarder launches into the air. The snowboarder lands 3.2 feet lower than the height of the launch. How long is the snowboarder in the air? Round your answer to the nearest tenth of a second.

Cross section of a half-pipe

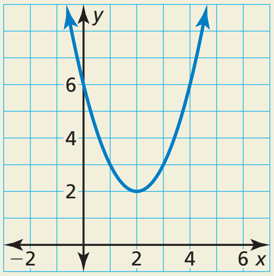
65. PROBLEM SOLVING You have 80 feet of fencing to make a rectangular horse pasture that covers 750 square feet. A barn will be used as one side of the pasture, as shown.



a. Write equations for the amount of fencing to be used and the area enclosed by the fencing.

b. Use substitution to solve the system of equations from part (a). What are the possible dimensions of the pasture?

66. **HOW DO YOU SEE IT?** The graph represents the quadratic function $y = x^2 - 4x + 6$.



a. Use the graph to estimate the x-values for which $y = 3$.

b. Explain how you can use the method of completing the square to check your estimates in part (a).

67. **COMPARING METHODS** Consider the quadratic equation $x^2 + 12x + 2 = 12$.

a. Solve the equation by graphing.

b. Solve the equation by completing the square.

c. Compare the two methods. Which do you prefer? Explain.

68. **THOUGHT PROVOKING** Sketch the graph of the equation $x^2 - 2xy + y^2 - x - y = 0$. Identify the graph.

69. **REASONING** The product of two consecutive even integers that are positive is 48. Write and solve an equation to find the integers.

70. **REASONING** The product of two consecutive odd integers that are negative is 195. Write and solve an equation to find the integers.

71. **MAKING AN ARGUMENT** You purchase stock for \$16 per share. You sell the stock 30 days later for \$23.50 per share. The price y (in dollars) of a share during the 30-day period can be modeled by $y = -0.025x^2 + x + 16$, where x is the number of days after the stock is purchased. Your friend says you could have sold the stock earlier for \$23.50 per share. Is your friend correct? Explain.

72. **REASONING** You are solving the equation $x^2 + 9x = 18$. What are the advantages of solving the equation by completing the square instead of using other methods you have learned?

73. **PROBLEM SOLVING** You are knitting a rectangular scarf. The pattern results in a scarf that is 60 inches long and 4 inches wide. However, you have enough yarn to knit 396 square inches. You decide to increase the dimensions of the scarf so that you will use all your yarn. The increase in the length is three times the increase in the width. What are the dimensions of your scarf?

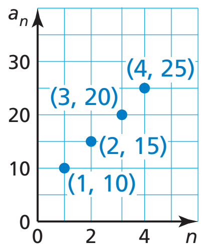
74. **WRITING** How many solutions does $x^2 + bx = c$ have when $c < -\left(\frac{b}{2}\right)^2$? Explain.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write a recursive rule for the sequence. *(Section 6.6)*

75.



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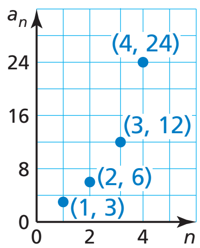

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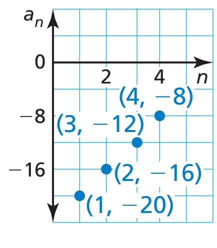

Simplify the expression $\sqrt{b^2 - 4ac}$ for the given values. *(Section 9.1)*

78. $a = 3, b = -6, c = 2$

79. $a = -2, b = 4, c = 7$

80. $a = 1, b = 6, c = 4$

9.5 Solving Quadratic Equations Using the Quadratic Formula

TEXAS ESSENTIAL KNOWLEDGE AND SKILLS
A.8.A

Essential Question How can you derive a formula that can be used to write the solutions of any quadratic equation in standard form?

EXPLORATION 1 Deriving the Quadratic Formula

Work with a partner. The following steps show a method of solving $ax^2 + bx + c = 0$. Explain what was done in each step.

$$ax^2 + bx + c = 0$$ ← 1. Write the equation.

$$4a^2x^2 + 4abx + 4ac = 0$$ ← 2. What was done?

$$4a^2x^2 + 4abx + 4ac + b^2 = b^2$$ ← 3. What was done?

$$4a^2x^2 + 4abx + b^2 = b^2 - 4ac$$ ← 4. What was done?

$$(2ax + b)^2 = b^2 - 4ac$$ ← 5. What was done?

$$2ax + b = \pm\sqrt{b^2 - 4ac}$$ ← 6. What was done?

$$2ax = -b \pm \sqrt{b^2 - 4ac}$$ ← 7. What was done?

Quadratic Formula: $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ ← 8. What was done?

EXPLORATION 2 Deriving the Quadratic Formula by Completing the Square

Work with a partner.

a. Solve $ax^2 + bx + c = 0$ by completing the square. (*Hint:* Subtract c from each side, divide each side by a, and then proceed by completing the square.)

b. Compare this method with the method in Exploration 1. Explain why you think $4a$ and b^2 were chosen in Steps 2 and 3 of Exploration 1.

Communicate Your Answer

SELECTING TOOLS
To be proficient in math, you need to identify relevant external mathematical resources.

3. How can you derive a formula that can be used to write the solutions of any quadratic equation in standard form?

4. Use the Quadratic Formula to solve each quadratic equation.

a. $x^2 + 2x - 3 = 0$ **b.** $x^2 - 4x + 4 = 0$ **c.** $x^2 + 4x + 5 = 0$

5. Use the Internet to research *imaginary numbers*. How are they related to quadratic equations?

9.5 Lesson

Core Vocabulary

Quadratic Formula, *p. 504*
discriminant, *p. 506*

What You Will Learn

- Solve quadratic equations using the Quadratic Formula.
- Interpret the discriminant.
- Choose efficient methods for solving quadratic equations.

Using the Quadratic Formula

By completing the square for the quadratic equation $ax^2 + bx + c = 0$, you can develop a formula that gives the solutions of any quadratic equation in standard form. This formula is called the **Quadratic Formula**.

Core Concept

Quadratic Formula

The real solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Quadratic Formula}$$

where $a \neq 0$ and $b^2 - 4ac \geq 0$.

EXAMPLE 1 Using the Quadratic Formula

Solve $2x^2 - 5x + 3 = 0$ using the Quadratic Formula.

SOLUTION

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Quadratic Formula}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(3)}}{2(2)} \qquad \text{Substitute 2 for } a, -5 \text{ for } b, \text{ and 3 for } c.$$

$$= \frac{5 \pm \sqrt{1}}{4} \qquad \text{Simplify.}$$

$$= \frac{5 \pm 1}{4} \qquad \text{Evaluate the square root.}$$

So, the solutions are $x = \frac{5 + 1}{4} = \frac{3}{2}$ and $x = \frac{5 - 1}{4} = 1$.

STUDY TIP

You can use the roots of a quadratic equation to factor the related expression. In Example 1, you can use 1 and $\frac{3}{2}$ to factor $2x^2 - 5x + 3$ as $(x - 1)(2x - 3)$.

Check

$2x^2 - 5x + 3 = 0$	Original equation	$2x^2 - 5x + 3 = 0$
$2\left(\frac{3}{2}\right)^2 - 5\left(\frac{3}{2}\right) + 3 \stackrel{?}{=} 0$	Substitute.	$2(1)^2 - 5(1) + 3 \stackrel{?}{=} 0$
$\frac{9}{2} - \frac{15}{2} + 3 \stackrel{?}{=} 0$	Simplify.	$2 - 5 + 3 \stackrel{?}{=} 0$
$0 = 0$ ✓	Simplify.	$0 = 0$ ✓

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Solve the equation using the Quadratic Formula. Round your solutions to the nearest tenth, if necessary.

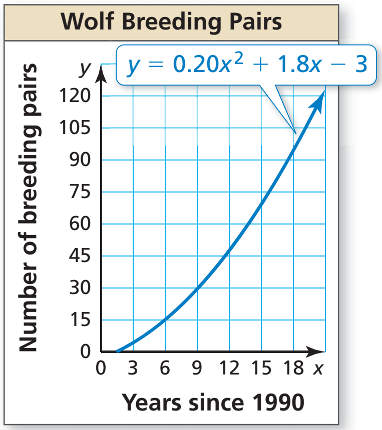
1. $x^2 - 6x + 5 = 0$

2. $\frac{1}{2}x^2 + x - 10 = 0$

3. $-3x^2 + 2x + 7 = 0$

4. $4x^2 - 4x = -1$

EXAMPLE 2 Modeling With Mathematics



The number y of Northern Rocky Mountain wolf breeding pairs x years since 1990 can be modeled by the function $y = 0.20x^2 + 1.8x - 3$. When were there about 35 breeding pairs?

SOLUTION

1. **Understand the Problem** You are given a quadratic function that represents the number of wolf breeding pairs for years after 1990. You need to use the model to determine when there were 35 wolf breeding pairs.

2. **Make a Plan** To determine when there were 35 wolf breeding pairs, find the x-values for which $y = 35$. So, solve the equation $35 = 0.20x^2 + 1.8x - 3$.

3. **Solve the Problem**

$$35 = 0.20x^2 + 1.8x - 3 \quad \text{Write the equation.}$$

$$0 = 0.20x^2 + 1.8x - 38 \quad \text{Write in standard form.}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$

$$= \frac{-1.8 \pm \sqrt{1.8^2 - 4(0.2)(-38)}}{2(0.2)} \quad \text{Substitute 0.2 for } a\text{, 1.8 for } b\text{, and } -38 \text{ for } c.$$

$$= \frac{-1.8 \pm \sqrt{33.64}}{0.4} \quad \text{Simplify.}$$

$$= \frac{-1.8 \pm 5.8}{0.4} \quad \text{Simplify.}$$

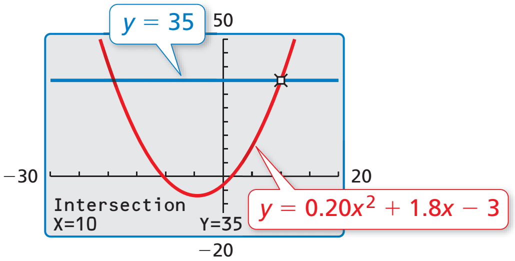
The solutions are $x = \dfrac{-1.8 + 5.8}{0.4} = 10$ and $x = \dfrac{-1.8 - 5.8}{0.4} = -19$.

EXPLAINING MATHEMATICAL IDEAS

You can ignore the solution $x = -19$ because -19 represents the year 1971, which is not in the given time period.

▶ Because x represents the number of years since 1990, x is greater than or equal to zero. So, there were about 35 breeding pairs 10 years after 1990, in 2000.

4. **Look Back** Use a graphing calculator to graph the equations $y = 0.20x^2 + 1.8x - 3$ and $y = 35$. Then use the *intersect* feature to find the point of intersection. The graphs intersect at (10, 35).



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5. **WHAT IF?** When were there about 60 wolf breeding pairs?

6. The number y of bald eagle nesting pairs in a state x years since 2000 can be modeled by the function $y = 0.34x^2 + 13.1x + 51$.

   a. When were there about 160 bald eagle nesting pairs?

   b. How many bald eagle nesting pairs were there in 2000?

Interpreting the Discriminant

The expression $b^2 - 4ac$ in the Quadratic Formula is called the **discriminant**.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ discriminant

Because the discriminant is under the radical symbol, you can use the value of the discriminant to determine the number of real solutions of a quadratic equation and the number of x-intercepts of the graph of the related function.

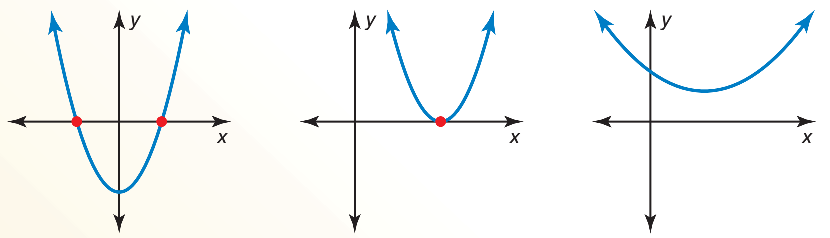
STUDY TIP

The solutions of a quadratic equation may be real numbers or *imaginary numbers*. You will study imaginary numbers in a future course.

Core Concept

Interpreting the Discriminant

$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$



• two real solutions • two x-intercepts	• one real solution • one x-intercept	• no real solutions • no x-intercepts

EXAMPLE 3 Determining the Number of Real Solutions

a. Determine the number of real solutions of $x^2 + 8x - 3 = 0$.

$b^2 - 4ac = 8^2 - 4(1)(-3)$ Substitute 1 for *a*, 8 for *b*, and −3 for *c*.

$= 64 + 12$ Simplify.

$= 76$ Add.

▶ The discriminant is greater than 0. So, the equation has two real solutions.

b. Determine the number of real solutions of $9x^2 + 1 = 6x$.

Write the equation in standard form: $9x^2 - 6x + 1 = 0$.

$b^2 - 4ac = (-6)^2 - 4(9)(1)$ Substitute 9 for *a*, −6 for *b*, and 1 for *c*.

$= 36 - 36$ Simplify.

$= 0$ Subtract.

▶ The discriminant is 0. So, the equation has one real solution.

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Determine the number of real solutions of the equation.

7. $-x^2 + 4x - 4 = 0$

8. $6x^2 + 2x = -1$

9. $\frac{1}{2}x^2 = 7x - 1$

EXAMPLE 4 Finding the Number of x-Intercepts of a Parabola

Find the number of x-intercepts of the graph of $y = 2x^2 + 3x + 9$.

SOLUTION

Determine the number of real solutions of $0 = 2x^2 + 3x + 9$.

$$b^2 - 4ac = 3^2 - 4(2)(9)$$ Substitute 2 for a, 3 for b, and 9 for c.

$$= 9 - 72$$ Simplify.

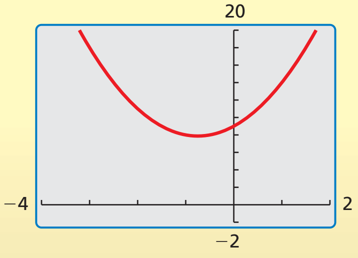
$$= -63$$ Subtract.

Because the discriminant is less than 0, the equation has no real solutions.

▶ So, the graph of $y = 2x^2 + 3x + 9$ has no x-intercepts.

Check

Use a graphing calculator to check your answer. Notice that the graph of $y = 2x^2 + 3x + 9$ has no x-intercepts.



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Find the number of x-intercepts of the graph of the function.

10. $y = -x^2 + x - 6$ **11.** $y = x^2 - x$ **12.** $f(x) = x^2 + 12x + 36$

Choosing an Efficient Method

The table shows five methods for solving quadratic equations. For a given equation, it may be more efficient to use one method instead of another. Some advantages and disadvantages of each method are shown.

Core Concept

Methods for Solving Quadratic Equations

Method	Advantages	Disadvantages
Factoring *(Lessons 7.6–7.9)*	• Straightforward when the equation can be factored easily	• Some equations are not factorable.
Graphing *(Lesson 9.2)*	• Can easily see the number of solutions • Use when approximate solutions are sufficient. • Can use a graphing calculator	• May not give exact solutions
Using Square Roots *(Lesson 9.3)*	• Use to solve equations of the form $x^2 = d$.	• Can only be used for certain equations
Completing the Square *(Lesson 9.4)*	• Best used when $a = 1$ and b is even	• May involve difficult calculations
Quadratic Formula *(Lesson 9.5)*	• Can be used for any quadratic equation • Gives exact solutions	• Takes time to do calculations

EXAMPLE 5 Choosing a Method

Solve the equation using any method. Explain your choice of method.

a. $x^2 - 10x = 1$ **b.** $2x^2 - 13x - 24 = 0$ **c.** $x^2 + 8x + 12 = 0$

SOLUTION

a. The coefficient of the x^2-term is 1, and the coefficient of the x-term is an even number. So, solve by completing the square.

$x^2 - 10x = 1$	Write the equation.
$x^2 - 10x + 25 = 1 + 25$	Complete the square for $x^2 - 10x$.
$(x - 5)^2 = 26$	Write the left side as the square of a binomial.
$x - 5 = \pm\sqrt{26}$	Take the square root of each side.
$x = 5 \pm \sqrt{26}$	Add 5 to each side.

▶ So, the solutions are $x = 5 + \sqrt{26} \approx 10.1$ and $x = 5 - \sqrt{26} \approx -0.1$.

b. The equation is not easily factorable, and the numbers are somewhat large. So, solve using the Quadratic Formula.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic Formula
$= \dfrac{-(-13) \pm \sqrt{(-13)^2 - 4(2)(-24)}}{2(2)}$	Substitute 2 for a, -13 for b, and -24 for c.
$= \dfrac{13 \pm \sqrt{361}}{4}$	Simplify.
$= \dfrac{13 \pm 19}{4}$	Evaluate the square root.

▶ So, the solutions are $x = \dfrac{13 + 19}{4} = 8$ and $x = \dfrac{13 - 19}{4} = -\dfrac{3}{2}$.

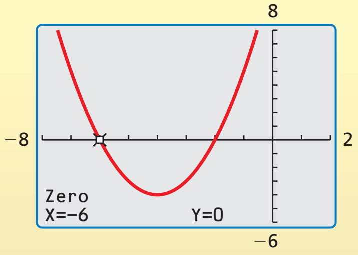
c. The equation is easily factorable. So, solve by factoring.

$x^2 + 8x + 12 = 0$	Write the equation.
$(x + 2)(x + 6) = 0$	Factor the polynomial.
$x + 2 = 0$ or $x + 6 = 0$	Zero-Product Property
$x = -2$ or $x = -6$	Solve for x.

▶ The solutions are $x = -2$ and $x = -6$.

Check

Graph the related function $f(x) = x^2 + 8x + 12$ and find the zeros. The zeros are -6 and -2.



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Solve the equation using any method. Explain your choice of method.

13. $x^2 + 11x - 12 = 0$

14. $9x^2 - 5 = 4$

15. $5x^2 - x - 1 = 0$

16. $x^2 = 2x - 5$

9.5 Exercises

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Vocabulary and Core Concept Check

1. **VOCABULARY** What formula can you use to solve any quadratic equation? Write the formula.

2. **VOCABULARY** In the Quadratic Formula, what is the discriminant? What does the value of the discriminant determine?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, write the equation in standard form. Then identify the values of a, b, and c that you would use to solve the equation using the Quadratic Formula.

3. $x^2 = 7x$
4. $x^2 - 4x = -12$
5. $-2x^2 + 1 = 5x$
6. $3x + 2 = 4x^2$
7. $4 - 3x = -x^2 + 3x$
8. $-8x - 1 = 3x^2 + 2$

In Exercises 9–22, solve the equation using the Quadratic Formula. Round your solutions to the nearest tenth, if necessary. *(See Example 1.)*

9. $x^2 - 12x + 36 = 0$
10. $x^2 + 7x + 16 = 0$
11. $x^2 - 10x - 11 = 0$
12. $2x^2 - x - 1 = 0$
13. $2x^2 - 6x + 5 = 0$
14. $9x^2 - 6x + 1 = 0$
15. $6x^2 - 13x = -6$
16. $-3x^2 + 6x = 4$
17. $1 - 8x = -16x^2$
18. $x^2 - 5x + 3 = 0$
19. $x^2 + 2x = 9$
20. $5x^2 - 2 = 4x$
21. $2x^2 + 9x + 7 = 3$
22. $8x^2 + 8 = 6 - 9x$

23. **MODELING WITH MATHEMATICS** A dolphin jumps out of the water, as shown in the diagram. The function $h = -16t^2 + 26t$ models the height h (in feet) of the dolphin after t seconds. After how many seconds is the dolphin at a height of 5 feet? *(See Example 2.)*

24. **MODELING WITH MATHEMATICS** The amount of trout y (in tons) caught in a lake from 1995 to 2014 can be modeled by the equation $y = -0.08x^2 + 1.6x + 10$, where x is the number of years since 1995.

    a. When were about 15 tons of trout caught in the lake?

    b. Do you think this model can be used to determine the amounts of trout caught in future years? Explain your reasoning.

In Exercises 25–30, determine the number of real solutions of the equation. *(See Example 3.)*

25. $x^2 - 6x + 10 = 0$
26. $x^2 - 5x - 3 = 0$
27. $2x^2 - 12x = -18$
28. $4x^2 = 4x - 1$
29. $-\frac{1}{4}x^2 + 4x = -2$
30. $-5x^2 + 8x = 9$

In Exercises 31–36, find the number of x-intercepts of the graph of the function. *(See Example 4.)*

31. $y = x^2 + 5x - 1$
32. $y = 4x^2 + 4x + 1$
33. $y = -6x^2 + 3x - 4$
34. $y = -x^2 + 5x + 13$
35. $f(x) = 4x^2 + 3x - 6$
36. $f(x) = 2x^2 + 8x + 8$

In Exercises 37–44, solve the equation using any method. Explain your choice of method. *(See Example 5.)*

37. $-10x^2 + 13x = 4$
38. $x^2 - 3x - 40 = 0$
39. $x^2 + 6x = 5$
40. $-5x^2 = -25$
41. $x^2 + x - 12 = 0$
42. $x^2 - 4x + 1 = 0$
43. $4x^2 - x = 17$
44. $x^2 + 6x + 9 = 16$

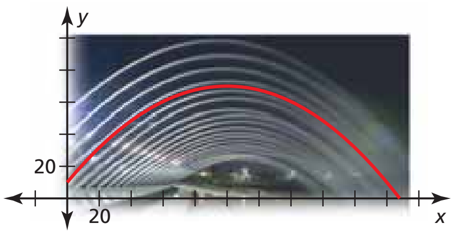
45. **ERROR ANALYSIS** Describe and correct the error in solving the equation $3x^2 - 7x - 6 = 0$ using the Quadratic Formula.

$$x = \frac{-7 \pm \sqrt{(-7)^2 - 4(3)(-6)}}{2(3)}$$
$$= \frac{-7 \pm \sqrt{121}}{6}$$
$$x = \frac{2}{3} \text{ and } x = -3$$

46. **ERROR ANALYSIS** Describe and correct the error in solving the equation $-2x^2 + 9x = 4$ using the Quadratic Formula.

$$x = \frac{-9 \pm \sqrt{9^2 - 4(-2)(4)}}{2(-2)}$$
$$= \frac{-9 \pm \sqrt{113}}{-4}$$
$$x \approx -0.41 \text{ and } x \approx 4.91$$

47. **MODELING WITH MATHEMATICS** A fountain shoots a water arc that can be modeled by the graph of the equation $y = -0.006x^2 + 1.2x + 10$, where x is the horizontal distance (in feet) from the river's north shore and y is the height (in feet) above the river. Does the water arc reach a height of 50 feet? If so, about how far from the north shore is the water arc 50 feet above the water?



48. **MODELING WITH MATHEMATICS** Between the months of April and September, the number y of hours of daylight per day in Seattle, Washington, can be modeled by $y = -0.00046x^2 + 0.076x + 13$, where x is the number of days since April 1.

a. Do any of the days between April and September in Seattle have 17 hours of daylight? If so, how many?

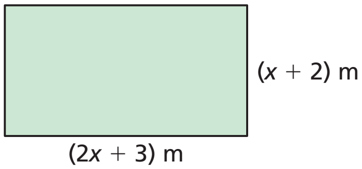
b. Do any of the days between April and September in Seattle have 14 hours of daylight? If so, how many?

49. **MAKING AN ARGUMENT** Your friend uses the discriminant of the equation $2x^2 - 5x - 2 = -11$ and determines that the equation has two real solutions. Is your friend correct? Explain your reasoning.

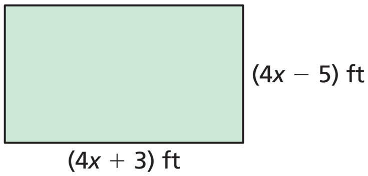
50. **MODELING WITH MATHEMATICS** The frame of the tent shown is defined by a rectangular base and two parabolic arches that connect the opposite corners of the base. The graph of $y = -0.18x^2 + 1.6x$ models the height y (in feet) of one of the arches x feet along the diagonal of the base. Can a child who is 4 feet tall walk under one of the arches without having to bend over? Explain.

MATHEMATICAL CONNECTIONS **In Exercises 51 and 52, use the given area A of the rectangle to find the value of x. Then give the dimensions of the rectangle.**

51. $A = 91 \text{ m}^2$



52. $A = 209 \text{ ft}^2$



COMPARING METHODS **In Exercises 53 and 54, solve the equation by (a) graphing, (b) factoring, and (c) using the Quadratic Formula. Which method do you prefer? Explain your reasoning.**

53. $x^2 + 4x + 4 = 0$

54. $3x^2 + 11x + 6 = 0$

55. **REASONING** How many solutions does the equation $ax^2 + bx + c = 0$ have when a and c have different signs? Explain your reasoning.

56. **REASONING** When the discriminant is a perfect square, are the solutions of $ax^2 + bx + c = 0$ rational or irrational? (Assume a, b, and c are integers.) Explain your reasoning.

REASONING **In Exercises 57–59, give a value of c for which the equation has (a) two solutions, (b) one solution, and (c) no solutions.**

57. $x^2 - 2x + c = 0$

58. $x^2 - 8x + c = 0$

59. $4x^2 + 12x + c = 0$

60. **REPEATED REASONING** You use the Quadratic Formula to solve an equation.

a. You obtain solutions that are integers. Could you have used factoring to solve the equation? Explain your reasoning.

b. You obtain solutions that are fractions. Could you have used factoring to solve the equation? Explain your reasoning.

c. Make a generalization about quadratic equations with rational solutions.

61. **MODELING WITH MATHEMATICS** The fuel economy y (in miles per gallon) of a car can be modeled by the equation $y = -0.013x^2 + 1.25x + 5.6$, where $5 \le x \le 75$ and x is the speed (in miles per hour) of the car. Find the speed(s) at which you can travel and have a fuel economy of 32 miles per gallon.

62. **MODELING WITH MATHEMATICS** The depth d (in feet) of a river can be modeled by the equation $d = -0.25t^2 + 1.7t + 3.5$, where $0 \le t \le 7$ and t is the time (in hours) after a heavy rain begins. When is the river 6 feet deep?

ANALYZING EQUATIONS **In Exercises 63–68, tell whether the vertex of the graph of the function lies above, below, or on the x-axis. Explain your reasoning without using a graph.**

63. $y = x^2 - 3x + 2$

64. $y = 3x^2 - 6x + 3$

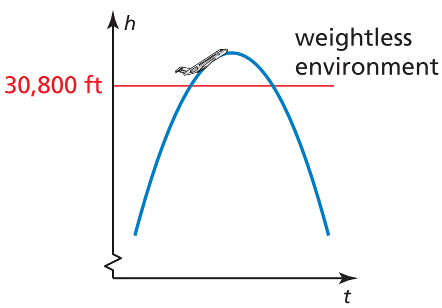
65. $y = 6x^2 - 2x + 4$

66. $y = -15x^2 + 10x - 25$

67. $f(x) = -3x^2 - 4x + 8$

68. $f(x) = 9x^2 - 24x + 16$

69. **REASONING** NASA creates a weightless environment by flying a plane in a series of parabolic paths. The height h (in feet) of a plane after t seconds in a parabolic flight path can be modeled by $h = -11t^2 + 700t + 21{,}000$. The passengers experience a weightless environment when the height of the plane is greater than or equal to 30,800 feet. For approximately how many seconds do passengers experience weightlessness on such a flight? Explain.

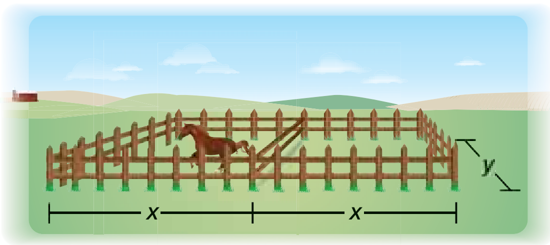


70. **WRITING EQUATIONS** Use the numbers to create a quadratic equation with the solutions $x = -1$ and $x = -\frac{1}{4}$.

$$\_\_\_x^2 + \_\_\_x + \_\_\_ = 0$$

−5	−4	−3	−2	−1
1	2	3	4	5

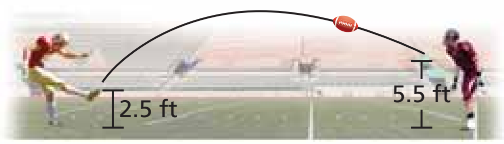
71. **PROBLEM SOLVING** A rancher constructs two rectangular horse pastures that share a side, as shown. The pastures are enclosed by 1050 feet of fencing. Each pasture has an area of 15,000 square feet.



a. Show that $y = 350 - \frac{4}{3}x$.

b. Find the possible lengths and widths of each pasture.

72. **PROBLEM SOLVING** A kicker punts a football from a height of 2.5 feet above the ground with an initial vertical velocity of 45 feet per second.



Not drawn to scale

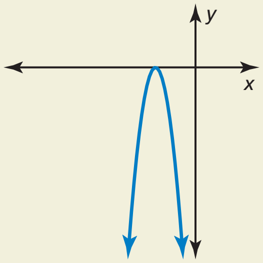
a. Write an equation that models this situation using the function $h = -16t^2 + v_0t + s_0$, where h is the height (in feet) of the football, t is the time (in seconds) after the football is punted, v_0 is the initial vertical velocity (in feet per second), and s_0 is the initial height (in feet).

b. The football is caught 5.5 feet above the ground, as shown in the diagram. Find the amount of time that the football is in the air.

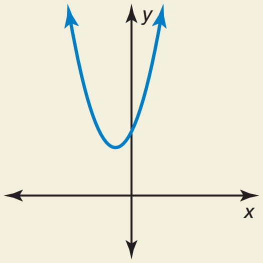
73. **CRITICAL THINKING** The solutions of the quadratic equation $ax^2 + bx + c = 0$ are $x = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$. Find the mean of the solutions. How is the mean of the solutions related to the graph of $y = ax^2 + bx + c$? Explain.

74. HOW DO YOU SEE IT? Match each graph with its discriminant. Explain your reasoning.

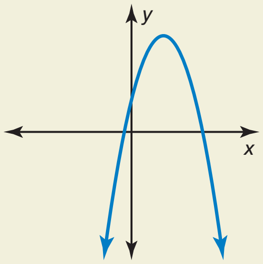
A.



B.



C.



a. $b^2 - 4ac > 0$

b. $b^2 - 4ac = 0$

c. $b^2 - 4ac < 0$

75. CRITICAL THINKING You are trying to hang a tire swing. To get the rope over a tree branch that is 15 feet high, you tie the rope to a weight and throw it over the branch. You release the weight at a height s_0 of 5.5 feet. What is the minimum initial vertical velocity v_0 needed to reach the branch? (*Hint*: Use the equation $h = -16t^2 + v_0t + s_0$.)

76. THOUGHT PROVOKING Consider the graph of the standard form of a quadratic function $y = ax^2 + bx + c$. Then consider the Quadratic Formula as given by

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}.$$

Write a graphical interpretation of the two parts of this formula.

77. ANALYZING RELATIONSHIPS Find the sum and product of $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$. Then write a quadratic equation whose solutions have a sum of 2 and a product of $\frac{1}{2}$.

78. WRITING A FORMULA Derive a formula that can be used to find solutions of equations that have the form $ax^2 + x + c = 0$. Use your formula to solve $-2x^2 + x + 8 = 0$.

79. MULTIPLE REPRESENTATIONS If p is a solution of a quadratic equation $ax^2 + bx + c = 0$, then $(x - p)$ is a factor of $ax^2 + bx + c$.

a. Copy and complete the table for each pair of solutions.

Solutions	Factors	Quadratic equation
3, 4	$(x - 3), (x - 4)$	$x^2 - 7x + 12 = 0$
$-1, 6$		
0, 2		
$-\frac{1}{2}, 5$		

b. Graph the related function for each equation. Identify the zeros of the function.

CRITICAL THINKING **In Exercises 80–82, find all values of k for which the equation has (a) two solutions, (b) one solution, and (c) no solutions.**

80. $2x^2 + x + 3k = 0$

81. $x^2 - 4kx + 36 = 0$

82. $kx^2 + 5x - 16 = 0$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the system of linear equations using any method. Explain why you chose the method. *(Section 5.1, Section 5.2, and Section 5.3)*

83. $y = -x + 4$
$y = 2x - 8$

84. $x = 16 - 4y$
$3x + 4y = 8$

85. $2x - y = 7$
$2x + 7y = 31$

86. $3x - 2y = -20$
$x + 1.2y = 6.4$

9.4–9.5 What Did You Learn?

Core Vocabulary

completing the square, *p. 494*
Quadratic Formula, *p. 504*
discriminant, *p. 506*

Core Concepts

Section 9.4

Completing the Square, *p. 494*

Section 9.5

Quadratic Formula, *p. 504*
Interpreting the Discriminant, *p. 506*

Mathematical Thinking

1. How does your answer to Exercise 74 on page 502 help create a shortcut when solving some quadratic equations by completing the square?
2. What logical progression led you to your answer in Exercise 55 on page 510?

Performance Task

Form Matters

Each form of a quadratic function has its pros and cons. Using one form, you can easily find the vertex, but the zeros are more difficult to find. Using another form, you can easily find the y-intercept, but the vertex is more difficult to find. Which form would you use in different situations? How can you convert one form into another?

To explore the answers to these questions and more, go to ***BigIdeasMath.com***.

9 Chapter Review

9.1 Properties of Radicals (pp. 465–474)

a. Simplify $\sqrt{\dfrac{19}{169}}$.

$$\sqrt{\frac{19}{169}} = \frac{\sqrt{19}}{\sqrt{169}}$$ Quotient Property of Square Roots

$$= \frac{\sqrt{19}}{13}$$ Simplify.

b. Simplify $\sqrt[3]{27x^{10}}$.

$$\sqrt[3]{27x^{10}} = \sqrt[3]{27 \cdot x^9 \cdot x}$$ Factor using the greatest perfect cube factors.

$$= \sqrt[3]{27} \cdot \sqrt[3]{x^9} \cdot \sqrt[3]{x}$$ Product Property of Cube Roots

$$= 3x^3\sqrt[3]{x}$$ Simplify.

c. Simplify $\dfrac{12}{3 + \sqrt{5}}$.

$$\frac{12}{3+\sqrt{5}} = \frac{12}{3+\sqrt{5}} \cdot \frac{3-\sqrt{5}}{3-\sqrt{5}}$$ The conjugate of $3 + \sqrt{5}$ is $3 - \sqrt{5}$.

$$= \frac{12(3-\sqrt{5})}{3^2 - (\sqrt{5})^2}$$ Sum and difference pattern

$$= \frac{36 - 12\sqrt{5}}{4}$$ Simplify.

$$= 9 - 3\sqrt{5}$$ Simplify.

d. Simplify $9\sqrt{6} + \sqrt{10} + 7\sqrt{6}$.

$$9\sqrt{6} + \sqrt{10} + 7\sqrt{6} = 9\sqrt{6} + 7\sqrt{6} + \sqrt{10}$$ Commutative Property of Addition

$$= (9 + 7)\sqrt{6} + \sqrt{10}$$ Distributive Property

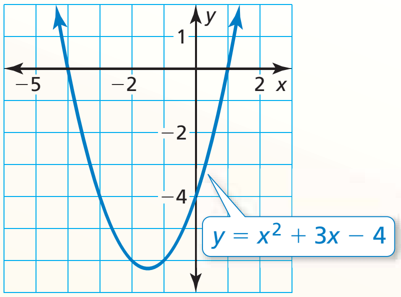
$$= 16\sqrt{6} + \sqrt{10}$$ Add.

Simplify the expression.

1. $\sqrt{124}$
2. $\sqrt{72p^7}$
3. $\sqrt{\dfrac{45}{7y}}$
4. $\sqrt[3]{\dfrac{125x^{11}}{4}}$
5. $\dfrac{\sqrt{15}}{\sqrt{10}}$
6. $\dfrac{4}{\sqrt{6x}}$
7. $\dfrac{8}{\sqrt{6} + 2}$
8. $4\sqrt{3} + 5\sqrt{12}$
9. $\sqrt{5} + 2\sqrt{7} - 2\sqrt{5}$
10. $15\sqrt[3]{2} - 2\sqrt[3]{54}$
11. $(3\sqrt{7} + 5)^2$
12. $\sqrt{6}(\sqrt{18} + \sqrt{8})$

9.2 Solving Quadratic Equations by Graphing (pp. 475–484)

Solve $x^2 + 3x = 4$ by graphing.



Step 1 Write the equation in standard form.

$x^2 + 3x = 4$ Write original equation.

$x^2 + 3x - 4 = 0$ Subtract 4 from each side.

Step 2 Graph the related function $y = x^2 + 3x - 4$.

Step 3 Find the x-intercepts. The x-intercepts are -4 and 1.

▶ So, the solutions are $x = -4$ and $x = 1$.

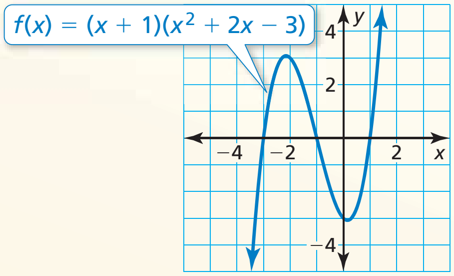
Solve the equation by graphing.

13. $x^2 - 9x + 18 = 0$

14. $x^2 - 2x = -4$

15. $-8x - 16 = x^2$

16. The graph of $f(x) = (x + 1)(x^2 + 2x - 3)$ is shown. Find the zeros of f.

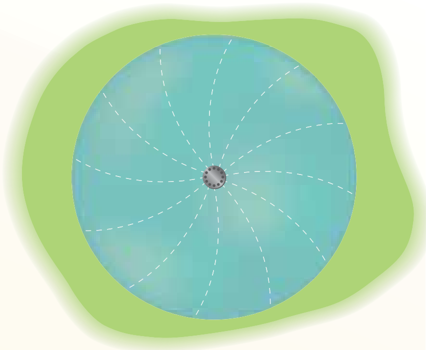


17. Graph $f(x) = x^2 + 2x - 5$. Approximate the zeros of f to the nearest tenth.

9.3 Solving Quadratic Equations Using Square Roots (pp. 485–490)

A sprinkler sprays water that covers a circular region of 90π square feet. Find the diameter of the circle.

Write an equation using the formula for the area of a circle.



$A = \pi r^2$ Write the formula.

$90\pi = \pi r^2$ Substitute 90π for A.

$90 = r^2$ Divide each side by π.

$\pm\sqrt{90} = r$ Take the square root of each side.

$\pm 3\sqrt{10} = r$ Simplify.

A diameter cannot be negative, so use the positive square root. The diameter is twice the radius. So, the diameter is $6\sqrt{10}$.

▶ The diameter of the circle is $6\sqrt{10} \approx 19$ feet.

Solve the equation using square roots. Round your solutions to the nearest hundredth, if necessary.

18. $x^2 + 5 = 17$

19. $x^2 - 14 = -14$

20. $(x + 2)^2 = 64$

21. $4x^2 + 25 = -75$

22. $(x - 1)^2 = 0$

23. $19 = 30 - 5x^2$

9.4 Solving Quadratic Equations by Completing the Square *(pp. 493–502)*

Solve $x^2 - 6x + 4 = 11$ by completing the square.

$x^2 - 6x + 4 = 11$	Write the equation.
$x^2 - 6x = 7$	Subtract 4 from each side.
$x^2 - 6x + (-3)^2 = 7 + (-3)^2$	Complete the square by adding $\left(\frac{-6}{2}\right)^2$, or $(-3)^2$, to each side.
$(x - 3)^2 = 16$	Write the left side as the square of a binomial.
$x - 3 = \pm 4$	Take the square root of each side.
$x = 3 \pm 4$	Add 3 to each side.

▶ The solutions are $x = 3 + 4 = 7$ and $x = 3 - 4 = -1$.

Solve the equation by completing the square. Round your solutions to the nearest hundredth, if necessary.

24. $x^2 + 6x - 40 = 0$ **25.** $x^2 + 2x + 5 = 4$ **26.** $2x^2 - 4x = 10$

Determine whether the quadratic function has a maximum or minimum value. Then find the value.

27. $y = -x^2 + 6x - 1$ **28.** $f(x) = x^2 + 4x + 11$ **29.** $y = 3x^2 - 24x + 15$

30. The width w of a credit card is 3 centimeters shorter than the length ℓ. The area is 46.75 square centimeters. Find the perimeter.

9.5 Solving Quadratic Equations Using the Quadratic Formula *(pp. 503–512)*

Solve $-3x^2 + x = -8$ using the Quadratic Formula.

$-3x^2 + x = -8$	Write the equation.
$-3x^2 + x + 8 = 0$	Write in standard form.
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic Formula
$x = \dfrac{-1 \pm \sqrt{1^2 - 4(-3)(8)}}{2(-3)}$	Substitute -3 for a, 1 for b, and 8 for c.
$x = \dfrac{-1 \pm \sqrt{97}}{-6}$	Simplify.

▶ So, the solutions are $x = \dfrac{-1 + \sqrt{97}}{-6} \approx -1.5$ and $x = \dfrac{-1 - \sqrt{97}}{-6} \approx 1.8$.

Solve the equation using the Quadratic Formula. Round your solutions to the nearest tenth, if necessary.

31. $x^2 + 2x - 15 = 0$ **32.** $2x^2 - x + 8 = 16$ **33.** $-5x^2 + 10x = 5$

Find the number of x-intercepts of the graph of the function.

34. $y = -x^2 + 6x - 9$ **35.** $y = 2x^2 + 4x + 8$ **36.** $y = -\frac{1}{2}x^2 + 2x$

9 Chapter Test

Simplify the expression.

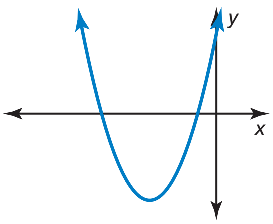
1. $-\sqrt{117}$
2. $\sqrt{98b^5}$
3. $\sqrt[3]{40w^3}$
4. $\sqrt[3]{\dfrac{27y^4}{1000z^3}}$
5. $\dfrac{10}{\sqrt{7}}$
6. $\dfrac{13}{\sqrt{3}-4}$
7. $(2\sqrt{5}-1)^2$
8. $12\sqrt{8}+3\sqrt{18}$

Solve the equation using any method. Explain your choice of method.

9. $x^2 - 121 = 0$
10. $x^2 - 6x = 10$
11. $-2x^2 + 3x + 7 = 0$
12. $x^2 - 7x + 12 = 0$
13. $5x^2 + x - 4 = 0$
14. $(4x+3)^2 = 16$

15. Explain how you can determine, without graphing, whether the quadratic function $y = -x^2 - 6x - 1$ has a maximum or minimum value. Then find the value.

16. Describe how you can use the method of completing the square to determine whether the function $f(x) = 2x^2 + 4x - 6$ can be represented by the graph shown.



17. The dimensions of a park form a golden rectangle. The shorter side of the park is 84 meters. What is the length of the longer side of the park?

18. A skier leaves an 8-foot-tall ramp with an initial vertical velocity of 28 feet per second. The function $h = -16t^2 + 28t + 8$ represents the height h (in feet) of the skier after t seconds. The skier has a perfect landing. How many points does the skier earn?

Criteria	Scoring
Maximum height	1 point per foot
Time in air	5 points per second
Perfect landing	25 points

19. An amusement park ride lifts seated riders 265 feet above the ground. The riders are then dropped and experience free fall until the brakes are activated 105 feet above the ground. The function $h = -16t^2 + 265$ represents the height h (in feet) of the riders t seconds after they are dropped. How long do the riders experience free fall? Round your solution to the nearest hundredth.

20. Write an expression in simplest form that represents the area of the painting shown.

$\dfrac{36}{\sqrt{3}}$ in.

$\sqrt{30x^7}$ in.

21. Explain how you can determine the number of times the graph of $y = 5x^2 - 10x + 5$ intersects the x-axis without graphing or solving an equation.

22. Consider the quadratic equation $ax^2 + bx = c$. Find values of a, b, and c so that the graph of its related function has (a) two x-intercepts, (b) one x-intercept, and (c) no x-intercepts.

9 Standards Assessment

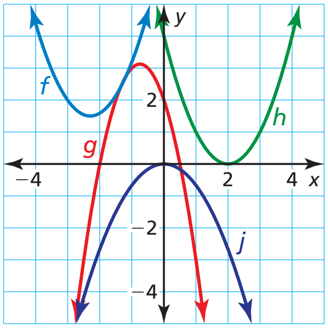
1. The graphs of four quadratic functions are shown. Which equation has a negative discriminant? *(TEKS A.7.A)*

Ⓐ $f(x) = 0$

Ⓑ $g(x) = 0$

Ⓒ $h(x) = 0$

Ⓓ $j(x) = 0$



2. The population p (in thousands) of Phoenix, Arizona, can be modeled by the function $p(t) = 0.08t^2 + 17.4t + 421$, where $t = 0$ represents the year 1960. Using the model, what will the population of Phoenix be in the year 2030? *(TEKS A.8.B)*

Ⓕ 1015

Ⓖ 2031

Ⓗ 1,015,000

Ⓙ 2,031,000

3. The table represents the numbers of bottles of sports drink sold at a concession stand on days with different average temperatures. You find an equation of the line of best fit. Which of the following is a reasonable correlation coefficient? *(TEKS A.4.A)*

Temperature (°F), *x*	14	27	32	41	48	62	73
Bottles of sports drink, *y*	8	12	13	16	19	27	29

Ⓐ −0.99

Ⓑ −0.12

Ⓒ 0.12

Ⓓ 0.99

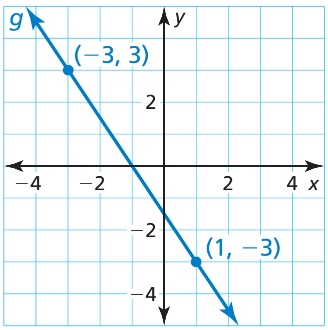
4. Which equation represents the line that passes through $(6, -1)$ and is perpendicular to line g? *(TEKS A.2.F)*

Ⓕ $y = -\frac{3}{2}x + \frac{9}{2}$

Ⓖ $y = -\frac{3}{2}x + 8$

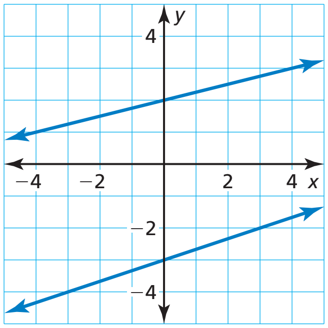
Ⓗ $y = \frac{2}{3}x - 5$

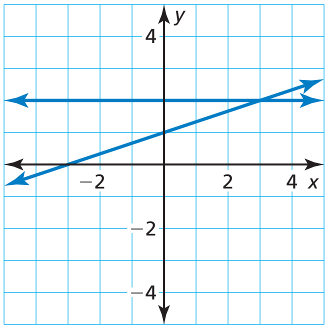
Ⓙ $y = \frac{2}{3}x + \frac{20}{3}$

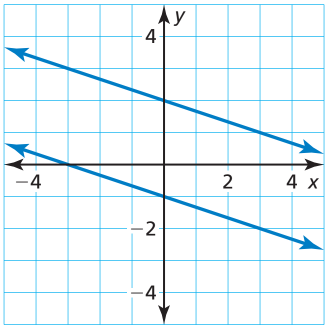


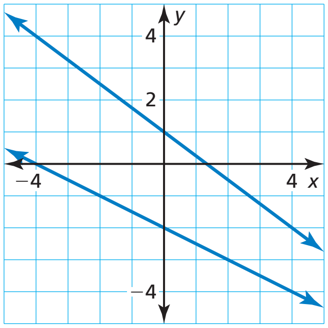
5. **GRIDDED ANSWER** What is the radius (in centimeters) of a cylinder with a height of 12 centimeters and a surface area of 320π square centimeters? *(TEKS A.8.A)*

6. Which graph represents a system of equations that has no solution? *(TEKS A.3.F)*

Ⓐ 

Ⓑ 

Ⓒ 

Ⓓ 

7. Which of the following expressions are in simplest form? *(TEKS A.11.A)*

I. $x\sqrt{45x}$

II. $\sqrt[3]{\frac{4}{9}}$

III. $3x\sqrt{5x}$

IV. $2\sqrt[3]{x^2}$

Ⓕ I and II only

Ⓖ II and III only

Ⓗ I, III, and IV only

Ⓙ II, III, and IV only

8. The domain of the function $f(x) = 4x - 5$ is all integers in the interval $-3 < x \leq 3$. What is the range of the function? *(TEKS A.2.A)*

Ⓐ $-17, -13, -9, -5, -1, 3, 7$

Ⓑ $-13, -9, -5, -1, 3, 7$

Ⓒ $-17 < x \leq 7$

Ⓓ $-13 \leq x \leq 7$