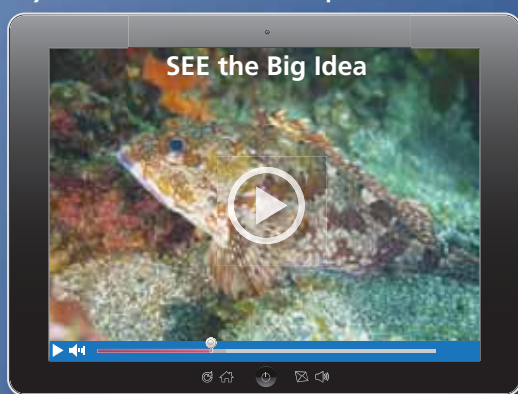


# 5 Solving Systems of Linear Equations

- 5.1 Solving Systems of Linear Equations by Graphing
- 5.2 Solving Systems of Linear Equations by Substitution
- 5.3 Solving Systems of Linear Equations by Elimination
- 5.4 Solving Special Systems of Linear Equations
- 5.5 Solving Equations by Graphing
- 5.6 Linear Inequalities in Two Variables
- 5.7 Systems of Linear Inequalities



Fishing (p. 265)



Fruit Salad (p. 255)



Pets (p. 250)



Roofing Contractor (p. 222)



Drama Club (p. 228)

**Mathematical Thinking:** *Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.*

# Maintaining Mathematical Proficiency

## Graphing Linear Functions (A.3.C)

**Example 1** Graph  $3 + y = \frac{1}{2}x$ .

**Step 1** Rewrite the equation in slope-intercept form.

$$y = \frac{1}{2}x - 3$$

**Step 2** Find the slope and the y-intercept.

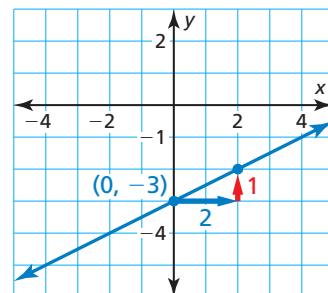
$$m = \frac{1}{2} \text{ and } b = -3$$

**Step 3** The y-intercept is  $-3$ . So, plot  $(0, -3)$ .

**Step 4** Use the slope to find another point on the line.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{1}{2}$$

Plot the point that is **2 units right** and **1 unit up** from  $(0, -3)$ . Draw a line through the two points.



**Graph the equation.**

1.  $y + 4 = x$
2.  $6x - y = -1$
3.  $4x + 5y = 20$
4.  $-2y + 12 = -3x$

## Solving and Graphing Linear Inequalities (7.10.B, A.5.B)

**Example 2** Solve  $2x - 17 \leq 8x - 5$ . Graph the solution.

$$2x - 17 \leq 8x - 5$$

Write the inequality.

$$\underline{+ 5} \qquad \underline{+ 5}$$

Add 5 to each side.

$$2x - 12 \leq 8x$$

Simplify.

$$\underline{- 2x} \qquad \underline{- 2x}$$

Subtract  $2x$  from each side.

$$-12 \leq 6x$$

Simplify.

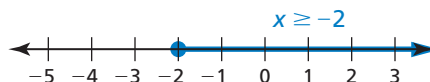
$$\underline{\frac{-12}{6}} \leq \underline{\frac{6x}{6}}$$

Divide each side by 6.

$$-2 \leq x$$

Simplify.

▶ The solution is  $x \geq -2$ .



**Solve the inequality. Graph the solution.**

5.  $m + 4 > 9$
6.  $24 \leq -6t$
7.  $2a - 5 \leq 13$
8.  $-5z + 1 < -14$
9.  $4k - 16 < k + 2$
10.  $7w + 12 \geq 2w - 3$

**11. ABSTRACT REASONING** The graphs of the linear functions  $g$  and  $h$  have different slopes. The value of both functions at  $x = a$  is  $b$ . When  $g$  and  $h$  are graphed in the same coordinate plane, what happens at the point  $(a, b)$ ?

# Mathematical Thinking

Mathematically proficient students select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems. (A.1.C)

## Using a Graphing Calculator

### Core Concept

#### Finding the Point of Intersection

You can use a graphing calculator to find the point of intersection, if it exists, of the graphs of two linear equations.

1. Enter the equations into a graphing calculator.
2. Graph the equations in an appropriate viewing window, so that the point of intersection is visible.
3. Use the *intersect* feature of the graphing calculator to find the point of intersection.

#### EXAMPLE 1 Using a Graphing Calculator

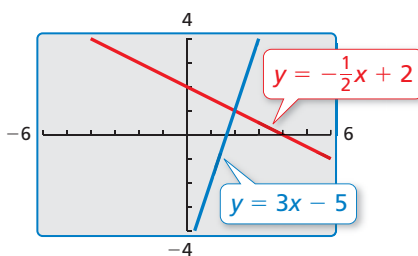
Use a graphing calculator to find the point of intersection, if it exists, of the graphs of the two linear equations.

$$y = -\frac{1}{2}x + 2 \quad \text{Equation 1}$$

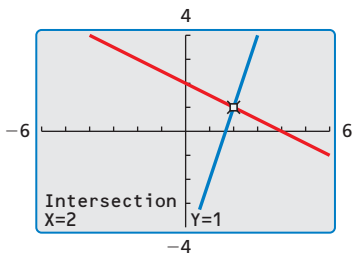
$$y = 3x - 5 \quad \text{Equation 2}$$

#### SOLUTION

The slopes of the lines are not the same, so you know that the lines intersect. Enter the equations into a graphing calculator. Then graph the equations in an appropriate viewing window.



Use the *intersect* feature to find the point of intersection of the lines.



► The point of intersection is (2, 1).

## Monitoring Progress

Use a graphing calculator to find the point of intersection of the graphs of the two linear equations.

1.  $y = -2x - 3$   
 $y = \frac{1}{2}x - 3$

2.  $y = -x + 1$   
 $y = x - 2$

3.  $3x - 2y = 2$   
 $2x - y = 2$

# 5.1 Solving Systems of Linear Equations by Graphing



## TEXAS ESSENTIAL KNOWLEDGE AND SKILLS

- A.2.I
- A.3.F
- A.3.G
- A.5.C

### APPLYING MATHEMATICS

To be proficient in math, you need to identify important quantities in real-life problems and map their relationships using tools such as diagrams, tables, and graphs.

**Essential Question** How can you solve a system of linear equations?

#### EXPLORATION 1 Writing a System of Linear Equations

**Work with a partner.** Your family opens a bed-and-breakfast. They spend \$600 preparing a bedroom to rent. The cost to your family for food and utilities is \$15 per night. They charge \$75 per night to rent the bedroom.

- a. Write an equation that represents the costs.

$$\text{Cost, } C \text{ (in dollars)} = \$15 \text{ per night} \cdot \text{Number of nights, } x + \$600$$

- b. Write an equation that represents the revenue (income).

$$\text{Revenue, } R \text{ (in dollars)} = \$75 \text{ per night} \cdot \text{Number of nights, } x$$

- c. A set of two (or more) linear equations is called a **system of linear equations**. Write the system of linear equations for this problem.

#### EXPLORATION 2 Using a Table or Graph to Solve a System

**Work with a partner.** Use the cost and revenue equations from Exploration 1 to determine how many nights your family needs to rent the bedroom before recovering the cost of preparing the bedroom. This is the *break-even point*.

- a. Copy and complete the table.

<b>x (nights)</b>	0	1	2	3	4	5	6	7	8	9	10	11
<b>C (dollars)</b>												
<b>R (dollars)</b>												

- b. How many nights does your family need to rent the bedroom before breaking even?
- c. In the same coordinate plane, graph the cost equation and the revenue equation from Exploration 1.
- d. Find the point of intersection of the two graphs. What does this point represent? How does this compare to the break-even point in part (b)? Explain.

### Communicate Your Answer

3. How can you solve a system of linear equations? How can you check your solution?
4. Solve each system by using a table or sketching a graph. Explain why you chose each method. Use a graphing calculator to check each solution.
- a.  $y = -4.3x - 1.3$   
 $y = 1.7x + 4.7$
- b.  $y = x$   
 $y = -3x + 8$
- c.  $y = -x - 1$   
 $y = 3x + 5$

# 5.1 Lesson

## Core Vocabulary

system of linear equations,  
p. 220  
solution of a system of linear  
equations, p. 220

**Previous**  
linear equation  
ordered pair

## What You Will Learn

- ▶ Check solutions of systems of linear equations.
- ▶ Solve systems of linear equations by graphing.
- ▶ Use systems of linear equations to solve real-life problems.

## Systems of Linear Equations

A **system of linear equations** is a set of two or more linear equations in the same variables. An example is shown below.

$$x + y = 7 \quad \text{Equation 1}$$

$$2x - 3y = -11 \quad \text{Equation 2}$$

A **solution of a system of linear equations** in two variables is an ordered pair that is a solution of each equation in the system.

### EXAMPLE 1 Checking Solutions

Tell whether the ordered pair is a solution of the system of linear equations.

a.  $(2, 5)$ ;  $x + y = 7$  Equation 1  
 $2x - 3y = -11$  Equation 2

b.  $(-2, 0)$ ;  $y = -2x - 4$  Equation 1  
 $y = x + 4$  Equation 2

### SOLUTION

a. Substitute 2 for  $x$  and 5 for  $y$  in each equation.

Equation 1

$$x + y = 7$$

$$2 + 5 \stackrel{?}{=} 7$$

$$7 = 7 \quad \checkmark$$

Equation 2

$$2x - 3y = -11$$

$$2(2) - 3(5) \stackrel{?}{=} -11$$

$$-11 = -11 \quad \checkmark$$

▶ Because the ordered pair  $(2, 5)$  is a solution of each equation, it is a solution of the linear system.

b. Substitute  $-2$  for  $x$  and  $0$  for  $y$  in each equation.

Equation 1

$$y = -2x - 4$$

$$0 \stackrel{?}{=} -2(-2) - 4$$

$$0 = 0 \quad \checkmark$$

Equation 2

$$y = x + 4$$

$$0 \stackrel{?}{=} -2 + 4$$

$$0 \neq 2 \quad \times$$

▶ The ordered pair  $(-2, 0)$  is a solution of the first equation, but it is not a solution of the second equation. So,  $(-2, 0)$  is *not* a solution of the linear system.

## READING

A system of linear equations is also called a *linear system*.

## Monitoring Progress



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Tell whether the ordered pair is a solution of the system of linear equations.

1.  $(1, -2)$ ;  $2x + y = 0$   
 $-x + 2y = 5$

2.  $(1, 4)$ ;  $y = 3x + 1$   
 $y = -x + 5$

## Solving Systems of Linear Equations by Graphing

The solution of a system of linear equations is the point of intersection of the graphs of the equations.

### Core Concept

#### Solving a System of Linear Equations by Graphing

- Step 1** Graph each equation in the same coordinate plane.
- Step 2** Estimate the point of intersection.
- Step 3** Check the point from Step 2 by substituting for  $x$  and  $y$  in each equation of the original system.

#### REMEMBER

Note that the linear equations are in slope-intercept form. You can use the method presented in Section 3.5 to graph the equations.

#### EXAMPLE 2

#### Solving a System of Linear Equations by Graphing

Solve the system of linear equations by graphing.

$$y = -2x + 5 \quad \text{Equation 1}$$

$$y = 4x - 1 \quad \text{Equation 2}$$

#### SOLUTION

**Step 1** Graph each equation.

**Step 2** Estimate the point of intersection.

The graphs appear to intersect at  $(1, 3)$ .

**Step 3** Check your point from Step 2.

Equation 1

$$y = -2x + 5$$

$$3 \stackrel{?}{=} -2(1) + 5$$

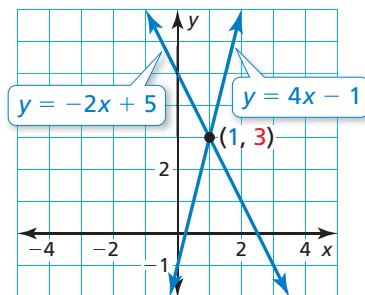
$$3 = 3 \quad \checkmark$$

Equation 2

$$y = 4x - 1$$

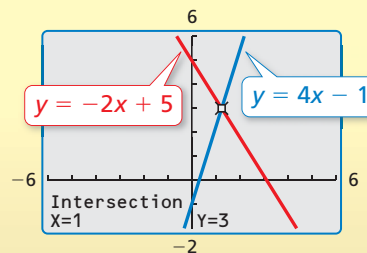
$$3 \stackrel{?}{=} 4(1) - 1$$

$$3 = 3 \quad \checkmark$$



▶ The solution is  $(1, 3)$ .

#### Check



#### Monitoring Progress



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Solve the system of linear equations by graphing.

3.  $y = x - 2$

$y = -x + 4$

4.  $y = \frac{1}{2}x + 3$

$y = -\frac{3}{2}x - 5$

5.  $2x + y = 5$

$3x - 2y = 4$



## Solving Real-Life Problems

### EXAMPLE 3 Modeling with Mathematics



A roofing contractor buys 30 bundles of shingles and 4 rolls of roofing paper for \$1040. In a second purchase (at the same prices), the contractor buys 8 bundles of shingles for \$256. Find the price per bundle of shingles and the price per roll of roofing paper.

#### SOLUTION

- 1. Understand the Problem** You know the total price of each purchase and how many of each item were purchased. You are asked to find the price of each item.
- 2. Make a Plan** Use a verbal model to write a system of linear equations that represents the problem. Then solve the system of linear equations.
- 3. Solve the Problem**

**Words**  $30 \cdot \text{Price per bundle} + 4 \cdot \text{Price per roll} = 1040$

$8 \cdot \text{Price per bundle} + 0 \cdot \text{Price per roll} = 256$

**Variables** Let  $x$  be the price (in dollars) per bundle and let  $y$  be the price (in dollars) per roll.

**System**  $30x + 4y = 1040$  Equation 1

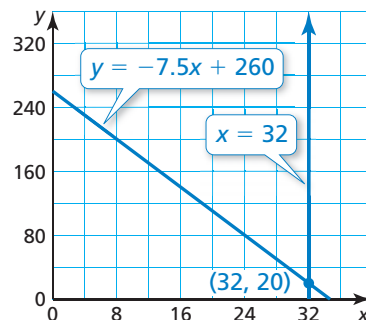
$8x = 256$  Equation 2

**Step 1** Graph each equation. Note that only the first quadrant is shown because  $x$  and  $y$  must be positive.

**Step 2** Estimate the point of intersection. The graphs appear to intersect at  $(32, 20)$ .

**Step 3** Check your point from Step 2.

<b>Equation 1</b>	<b>Equation 2</b>
$30x + 4y = 1040$	$8x = 256$
$30(32) + 4(20) \stackrel{?}{=} 1040$	$8(32) \stackrel{?}{=} 256$
$1040 = 1040$ ✓	$256 = 256$ ✓



► The solution is  $(32, 20)$ . So, the price per bundle of shingles is \$32, and the price per roll of roofing paper is \$20.

- 4. Look Back** You can use estimation to check that your solution is reasonable. A bundle of shingles costs about \$30. So, 30 bundles of shingles and 4 rolls of roofing paper (at \$20 per roll) cost about  $30(30) + 4(20) = \$980$ , and 8 bundles of shingles costs about  $8(30) = \$240$ . These prices are close to the given values, so the solution seems reasonable.

### Monitoring Progress Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

- You have a total of 18 math and science exercises for homework. You have six more math exercises than science exercises. How many exercises do you have in each subject?

# 5.1 Exercises

## Vocabulary and Core Concept Check

- VOCABULARY** Do the equations  $5y - 2x = 18$  and  $6x = -4y - 10$  form a system of linear equations? Explain.
- DIFFERENT WORDS, SAME QUESTION** Consider the system of linear equations  $-4x + 2y = 4$  and  $4x - y = -6$ . Which is different? Find “both” answers.

Solve the system of linear equations.

Solve each equation for  $y$ .

Find the point of intersection of the graphs of the equations.

Find an ordered pair that is a solution of each equation in the system.

## Monitoring Progress and Modeling with Mathematics

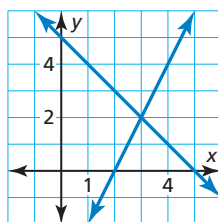
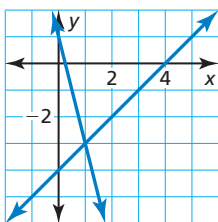
In Exercises 3–8, tell whether the ordered pair is a solution of the system of linear equations.

(See Example 1.)

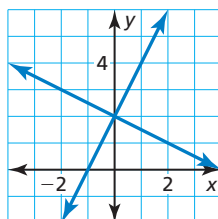
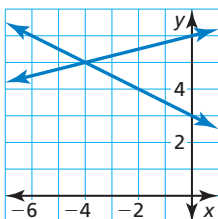
- $(2, 6)$ ;  $\begin{cases} x + y = 8 \\ 3x - y = 0 \end{cases}$
- $(8, 2)$ ;  $\begin{cases} x - y = 6 \\ 2x - 10y = 4 \end{cases}$
- $(-1, 3)$ ;  $\begin{cases} y = -7x - 4 \\ y = 8x + 5 \end{cases}$
- $(-4, -2)$ ;  $\begin{cases} y = 2x + 6 \\ y = -3x - 14 \end{cases}$
- $(-2, 1)$ ;  $\begin{cases} 6x + 5y = -7 \\ 2x - 4y = -8 \end{cases}$
- $(5, -6)$ ;  $\begin{cases} 6x + 3y = 12 \\ 4x + y = 14 \end{cases}$

In Exercises 9–12, use the graph to solve the system of linear equations. Check your solution.

- $\begin{cases} x - y = 4 \\ 4x + y = 1 \end{cases}$
- $\begin{cases} x + y = 5 \\ y - 2x = -4 \end{cases}$



- $\begin{cases} 6y + 3x = 18 \\ -x + 4y = 24 \end{cases}$
- $\begin{cases} 2x - y = -2 \\ 2x + 4y = 8 \end{cases}$



In Exercises 13–20, solve the system of linear equations by graphing. (See Example 2.)

- $\begin{cases} y = -x + 7 \\ y = x + 1 \end{cases}$
- $\begin{cases} y = -x + 4 \\ y = 2x - 8 \end{cases}$
- $\begin{cases} y = \frac{1}{3}x + 2 \\ y = \frac{2}{3}x + 5 \end{cases}$
- $\begin{cases} y = \frac{3}{4}x - 4 \\ y = -\frac{1}{2}x + 11 \end{cases}$
- $\begin{cases} 9x + 3y = -3 \\ 2x - y = -4 \end{cases}$
- $\begin{cases} 4x - 4y = 20 \\ y = -5 \end{cases}$
- $\begin{cases} x - 4y = -4 \\ -3x - 4y = 12 \end{cases}$
- $\begin{cases} 3y + 4x = 3 \\ x + 3y = -6 \end{cases}$

**ERROR ANALYSIS** In Exercises 21 and 22, describe and correct the error in solving the system of linear equations.

21. The solution of the linear system  $x - 3y = 6$  and  $2x - 3y = 3$  is  $(3, -1)$ .

22. The solution of the linear system  $y = 2x - 1$  and  $y = x + 1$  is  $x = 2$ .



**USING TOOLS** In Exercises 23–26, use a graphing calculator to solve the system of linear equations.

23.  $0.2x + 0.4y = 4$   
 $-0.6x + 0.6y = -3$
24.  $-1.6x - 3.2y = -24$   
 $2.6x + 2.6y = 26$
25.  $-7x + 6y = 0$   
 $0.5x + y = 2$
26.  $4x - y = 1.5$   
 $2x + y = 1.5$

27. **MODELING WITH MATHEMATICS** You have 40 minutes to exercise at the gym, and you want to burn 300 calories total using both machines. How much time should you spend on each machine? (See Example 3.)

Elliptical Trainer



8 calories per minute

Stationary Bike



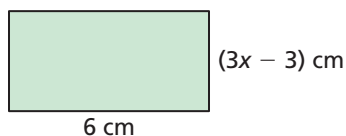
6 calories per minute

28. **MODELING WITH MATHEMATICS**

You sell small and large candles at a craft fair. You collect \$144 selling a total of 28 candles. How many of each type of candle did you sell?



29. **MATHEMATICAL CONNECTIONS** Write a linear equation that represents the area and a linear equation that represents the perimeter of the rectangle. Solve the system of linear equations by graphing. Interpret your solution.

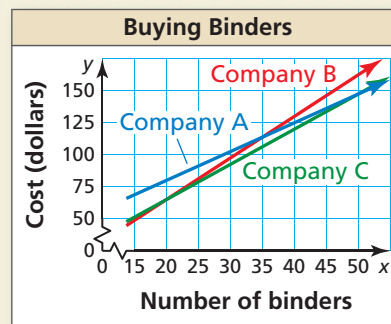


30. **THOUGHT PROVOKING** Your friend's bank account balance (in dollars) is represented by the equation  $y = 25x + 250$ , where  $x$  is the number of months. Graph this equation. After 6 months, you want to have the same account balance as your friend. Write a linear equation that represents your account balance. Interpret the slope and  $y$ -intercept of the line that represents your account balance.

31. **COMPARING METHODS** Consider the equation  $x + 2 = 3x - 4$ .

- Solve the equation using algebra.
- Solve the system of linear equations  $y = x + 2$  and  $y = 3x - 4$  by graphing.
- How is the linear system and the solution in part (b) related to the original equation and the solution in part (a)?

32. **HOW DO YOU SEE IT?** A teacher is purchasing binders for students. The graph shows the total costs of ordering  $x$  binders from three different companies.



- For what numbers of binders are the costs the same at two different companies? Explain.
- How do your answers in part (a) relate to systems of linear equations?

33. **MAKING AN ARGUMENT** You and a friend are going hiking but start at different locations. You start at the trailhead and walk 5 miles per hour. Your friend starts 3 miles from the trailhead and walks 3 miles per hour.



- Write and graph a system of linear equations that represents this situation.
- Your friend says that after an hour of hiking you will both be at the same location on the trail. Is your friend correct? Use the graph from part (a) to explain your answer.

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the literal equation for  $y$ . (Section 1.4)

34.  $10x + 5y = 5x + 20$

35.  $9x + 18 = 6y - 3x$

36.  $\frac{3}{4}x + \frac{1}{4}y = 5$

# 5.2 Solving Systems of Linear Equations by Substitution



TEXAS ESSENTIAL  
KNOWLEDGE AND SKILLS

A.2.I  
A.5.C

**Essential Question** How can you use substitution to solve a system of linear equations?

## EXPLORATION 1 Using Substitution to Solve Systems

**Work with a partner.** Solve each system of linear equations using two methods.

### Method 1 Solve for $x$ first.

Solve for  $x$  in one of the equations. Substitute the expression for  $x$  into the other equation to find  $y$ . Then substitute the value of  $y$  into one of the original equations to find  $x$ .

### Method 2 Solve for $y$ first.

Solve for  $y$  in one of the equations. Substitute the expression for  $y$  into the other equation to find  $x$ . Then substitute the value of  $x$  into one of the original equations to find  $y$ .

Is the solution the same using both methods? Explain which method you would prefer to use for each system.

a.  $x + y = -7$   
 $-5x + y = 5$

b.  $x - 6y = -11$   
 $3x + 2y = 7$

c.  $4x + y = -1$   
 $3x - 5y = -18$

## EXPLORATION 2 Writing and Solving a System of Equations

**Work with a partner.**

- Write a random ordered pair with integer coordinates. One way to do this is to use a graphing calculator. The ordered pair generated at the right is  $(-2, -3)$ .
- Write a system of linear equations that has your ordered pair as its solution.
- Exchange systems with your partner and use one of the methods from Exploration 1 to solve the system. Explain your choice of method.

Choose two random integers between  $-5$  and  $5$ .

```
randInt(-5,5,2)  
{-2 -3}
```

## USING PRECISE MATHEMATICAL LANGUAGE

To be proficient in math, you need to communicate precisely with others.

## Communicate Your Answer

3. How can you use substitution to solve a system of linear equations?

4. Use one of the methods from Exploration 1 to solve each system of linear equations. Explain your choice of method. Check your solutions.

a.  $x + 2y = -7$   
 $2x - y = -9$

b.  $x - 2y = -6$   
 $2x + y = -2$

c.  $-3x + 2y = -10$   
 $-2x + y = -6$

d.  $3x + 2y = 13$   
 $x - 3y = -3$

e.  $3x - 2y = 9$   
 $-x - 3y = 8$

f.  $3x - y = -6$   
 $4x + 5y = 11$

## 5.2 Lesson

### Core Vocabulary

#### Previous

system of linear equations  
solution of a system of  
linear equations

## What You Will Learn

- ▶ Solve systems of linear equations by substitution.
- ▶ Use systems of linear equations to solve real-life problems.

## Solving Linear Systems by Substitution

Another way to solve a system of linear equations is to use substitution.

### Core Concept

#### Solving a System of Linear Equations by Substitution

**Step 1** Solve one of the equations for one of the variables.

**Step 2** Substitute the expression from Step 1 into the other equation and solve for the other variable.

**Step 3** Substitute the value from Step 2 into one of the original equations and solve.

#### EXAMPLE 1

#### Solving a System of Linear Equations by Substitution

Solve the system of linear equations by substitution.

$$y = -2x - 9 \quad \text{Equation 1}$$

$$6x - 5y = -19 \quad \text{Equation 2}$$

#### SOLUTION

**Step 1** Equation 1 is already solved for  $y$ .

**Step 2** Substitute  $-2x - 9$  for  $y$  in Equation 2 and solve for  $x$ .

$$6x - 5y = -19 \quad \text{Equation 2}$$

$$6x - 5(-2x - 9) = -19 \quad \text{Substitute } -2x - 9 \text{ for } y.$$

$$6x + 10x + 45 = -19 \quad \text{Distributive Property}$$

$$16x + 45 = -19 \quad \text{Combine like terms.}$$

$$16x = -64 \quad \text{Subtract 45 from each side.}$$

$$x = -4 \quad \text{Divide each side by 16.}$$

**Step 3** Substitute  $-4$  for  $x$  in Equation 1 and solve for  $y$ .

$$y = -2x - 9 \quad \text{Equation 1}$$

$$= -2(-4) - 9 \quad \text{Substitute } -4 \text{ for } x.$$

$$= 8 - 9 \quad \text{Multiply.}$$

$$= -1 \quad \text{Subtract.}$$

▶ The solution is  $(-4, -1)$ .

#### Check

##### Equation 1

$$y = -2x - 9$$

$$-1 \stackrel{?}{=} -2(-4) - 9$$

$$-1 = -1 \quad \checkmark$$

##### Equation 2

$$6x - 5y = -19$$

$$6(-4) - 5(-1) \stackrel{?}{=} -19$$

$$-19 = -19 \quad \checkmark$$

## Monitoring Progress



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Solve the system of linear equations by substitution. Check your solution.

1.  $y = 3x + 14$

$$y = -4x$$

2.  $3x + 2y = 0$

$$y = \frac{1}{2}x - 1$$

3.  $x = 6y - 7$

$$4x + y = -3$$

**EXAMPLE 2****Solving a System of Linear Equations by Substitution****ANOTHER WAY**

You could also begin by solving for  $x$  in Equation 1, solving for  $y$  in Equation 2, or solving for  $x$  in Equation 2.

Solve the system of linear equations by substitution.

$$-x + y = 3 \quad \text{Equation 1}$$

$$3x + y = -1 \quad \text{Equation 2}$$

**SOLUTION**

**Step 1** Solve for  $y$  in Equation 1.

$$y = x + 3 \quad \text{Revised Equation 1}$$

**Step 2** Substitute  $x + 3$  for  $y$  in Equation 2 and solve for  $x$ .

$$3x + y = -1 \quad \text{Equation 2}$$

$$3x + (x + 3) = -1 \quad \text{Substitute } x + 3 \text{ for } y.$$

$$4x + 3 = -1 \quad \text{Combine like terms.}$$

$$4x = -4 \quad \text{Subtract 3 from each side.}$$

$$x = -1 \quad \text{Divide each side by 4.}$$

**Step 3** Substitute  $-1$  for  $x$  in Equation 1 and solve for  $y$ .

$$-x + y = 3 \quad \text{Equation 1}$$

$$-(-1) + y = 3 \quad \text{Substitute } -1 \text{ for } x.$$

$$y = 2 \quad \text{Subtract 1 from each side.}$$

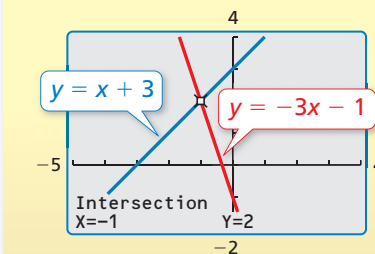
► The solution is  $(-1, 2)$ .

**Algebraic Check****Equation 1**

$$\begin{aligned} -x + y &= 3 \\ -(-1) + 2 &\stackrel{?}{=} 3 \\ 3 &= 3 \quad \checkmark \end{aligned}$$

**Equation 2**

$$\begin{aligned} 3x + y &= -1 \\ 3(-1) + 2 &\stackrel{?}{=} -1 \\ -1 &= -1 \quad \checkmark \end{aligned}$$

**Graphical Check****Monitoring Progress**

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Solve the system of linear equations by substitution. Check your solution.

4.  $x + y = -2$

$$-3x + y = 6$$

6.  $2x - y = -5$

$$3x - y = 1$$

5.  $-x + y = -4$

$$4x - y = 10$$

7.  $x - 2y = 7$

$$3x - 2y = 3$$

## Solving Real-Life Problems

### EXAMPLE 3 Modeling with Mathematics



A drama club earns \$1040 from a production. An adult ticket costs twice as much as a student ticket. Write a system of linear equations that represents this situation. What is the price of each type of ticket?

Tickets sold	
Type	Number
adult	64
student	132

#### SOLUTION

- 1. Understand the Problem** You know the amount earned, the total numbers of adult and student tickets sold, and the relationship between the price of an adult ticket and the price of a student ticket. You are asked to write a system of linear equations that represents the situation and find the price of each type of ticket.
- 2. Make a Plan** Use a verbal model to write a system of linear equations that represents the problem. Then solve the system of linear equations.
- 3. Solve the Problem**

**Words**  $64 \cdot \text{Adult ticket price} + 132 \cdot \text{Student ticket price} = 1040$

$\text{Adult ticket price} = 2 \cdot \text{Student ticket price}$

**Variables** Let  $x$  be the price (in dollars) of an adult ticket and let  $y$  be the price (in dollars) of a student ticket.

**System**  $64x + 132y = 1040$  Equation 1

$x = 2y$  Equation 2

**Step 1** Equation 2 is already solved for  $x$ .

**Step 2** Substitute  $2y$  for  $x$  in Equation 1 and solve for  $y$ .

$$64x + 132y = 1040 \quad \text{Equation 1}$$

$$64(2y) + 132y = 1040 \quad \text{Substitute } 2y \text{ for } x.$$

$$260y = 1040 \quad \text{Simplify.}$$

$$y = 4 \quad \text{Simplify.}$$

**Step 3** Substitute 4 for  $y$  in Equation 2 and solve for  $x$ .

$$x = 2y \quad \text{Equation 2}$$

$$x = 2(4) \quad \text{Substitute 4 for } y.$$

$$x = 8 \quad \text{Simplify.}$$

▶ The solution is  $(8, 4)$ . So, an adult ticket costs \$8 and a student ticket costs \$4.

- 4. Look Back** To check that your solution is correct, substitute the values of  $x$  and  $y$  into both of the original equations and simplify.

$$64(8) + 132(4) = 1040 \quad 8 = 2(4)$$

$$1040 = 1040 \quad \checkmark \quad 8 = 8 \quad \checkmark$$

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8. There are a total of 64 students in a drama club and a yearbook club. The drama club has 10 more students than the yearbook club. Write a system of linear equations that represents this situation. How many students are in each club?

#### STUDY TIP

You can use either of the original equations to solve for  $x$ . However, using Equation 2 requires fewer calculations.



## 5.2 Exercises

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### Vocabulary and Core Concept Check

- WRITING** Describe how to solve a system of linear equations by substitution.
- NUMBER SENSE** When solving a system of linear equations by substitution, how do you decide which variable to solve for in Step 1?

### Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, tell which equation you would choose to solve for one of the variables. Explain.

- |                                   |                                    |
|-----------------------------------|------------------------------------|
| 3. $x + 4y = 30$<br>$x - 2y = 0$  | 4. $3x - y = 0$<br>$2x + y = -10$  |
| 5. $5x + 3y = 11$<br>$5x - y = 5$ | 6. $3x - 2y = 19$<br>$x + y = 8$   |
| 7. $x - y = -3$<br>$4x + 3y = -5$ | 8. $3x + 5y = 25$<br>$x - 2y = -6$ |

In Exercises 9–16, solve the system of linear equations by substitution. Check your solution. (See Examples 1 and 2.)

- |                                    |                                       |
|------------------------------------|---------------------------------------|
| 9. $x = 17 - 4y$<br>$y = x - 2$    | 10. $6x - 9 = y$<br>$y = -3x$         |
| 11. $x = 16 - 4y$<br>$3x + 4y = 8$ | 12. $-5x + 3y = 51$<br>$y = 10x - 8$  |
| 13. $2x = 12$<br>$x - 5y = -29$    | 14. $2x - y = 23$<br>$x - 9 = -1$     |
| 15. $5x + 2y = 9$<br>$x + y = -3$  | 16. $11x - 7y = -14$<br>$x - 2y = -4$ |

17. **ERROR ANALYSIS** Describe and correct the error in solving for one of the variables in the linear system  $8x + 2y = -12$  and  $5x - y = 4$ .

**X**

Step 1  $5x - y = 4$   
 $-y = -5x + 4$   
 $y = 5x - 4$

Step 2  $5x - (5x - 4) = 4$   
 $5x - 5x + 4 = 4$   
 $4 = 4$

18. **ERROR ANALYSIS** Describe and correct the error in solving for one of the variables in the linear system  $4x + 2y = 6$  and  $3x + y = 9$ .

**X**

Step 1  $3x + y = 9$   
 $y = 9 - 3x$

Step 2  $4x + 2(9 - 3x) = 6$   
 $4x + 18 - 6x = 6$   
 $-2x = -12$   
 $x = 6$

Step 3  $3x + y = 9$   
 $3x + 6 = 9$   
 $3x = 3$   
 $x = 1$

19. **MODELING WITH MATHEMATICS** A farmer plants corn and wheat on a 180-acre farm. The farmer wants to plant three times as many acres of corn as wheat. Write a system of linear equations that represents this situation. How many acres of each crop should the farmer plant? (See Example 3.)
20. **MODELING WITH MATHEMATICS** A company that offers tubing trips down a river rents tubes for a person to use and “cooler” tubes to carry food and water. A group spends \$270 to rent a total of 15 tubes. Write a system of linear equations that represents this situation. How many of each type of tube does the group rent?





In Exercises 21–24, write a system of linear equations that has the ordered pair as its solution.

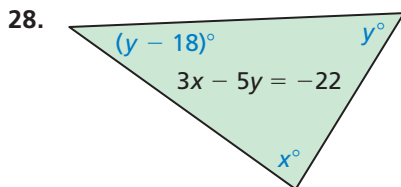
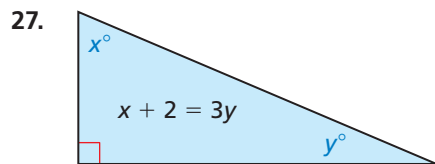
21. (3, 5)                      22. (-2, 8)  
 23. (-4, -12)                24. (15, -25)

25. **PROBLEM SOLVING** A math test is worth 100 points and has 38 problems. Each problem is worth either 5 points or 2 points. How many problems of each point value are on the test?

26. **PROBLEM SOLVING** An investor owns shares of Stock A and Stock B. The investor owns a total of 200 shares with a total value of \$4000. How many shares of each stock does the investor own?

Stock	Price
A	\$9.50
B	\$27.00

**MATHEMATICAL CONNECTIONS** In Exercises 27 and 28, (a) write an equation that represents the sum of the angle measures of the triangle and (b) use your equation and the equation shown to find the values of  $x$  and  $y$ .



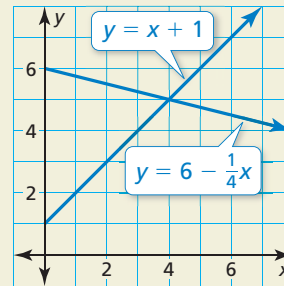
29. **REASONING** Find the values of  $a$  and  $b$  so that the solution of the linear system is  $(-9, 1)$ .

$$\begin{aligned} ax + by &= -31 && \text{Equation 1} \\ ax - by &= -41 && \text{Equation 2} \end{aligned}$$

30. **MAKING AN ARGUMENT** Your friend says that given a linear system with an equation of a horizontal line and an equation of a vertical line, you cannot solve the system by substitution. Is your friend correct? Explain.

31. **OPEN-ENDED** Write a system of linear equations in which  $(3, -5)$  is a solution of Equation 1 but not a solution of Equation 2, and  $(-1, 7)$  is a solution of the system.

32. **HOW DO YOU SEE IT?** The graphs of two linear equations are shown.



- At what point do the lines appear to intersect?
- Could you solve a system of linear equations by substitution to check your answer in part (a)? Explain.

33. **REPEATED REASONING** A radio station plays a total of 272 pop, rock, and hip-hop songs during a day. The number of pop songs is 3 times the number of rock songs. The number of hip-hop songs is 32 more than the number of rock songs. How many of each type of song does the radio station play?

34. **THOUGHT PROVOKING** You have \$2.65 in coins. Write a system of equations that represents this situation. Use variables to represent the number of each type of coin.

35. **NUMBER SENSE** The sum of the digits of a two-digit number is 11. When the digits are reversed, the number increases by 27. Find the original number.

## Maintaining Mathematical Proficiency Reviewing what you learned in previous grades and lessons

Find the sum or difference. (*Skills Review Handbook*)

- |                            |                              |
|----------------------------|------------------------------|
| 36. $(x - 4) + (2x - 7)$   | 37. $(5y - 12) + (-5y - 1)$  |
| 38. $(t - 8) - (t + 15)$   | 39. $(6d + 2) - (3d - 3)$    |
| 40. $4(m + 2) + 3(6m - 4)$ | 41. $2(5v + 6) - 6(-9v + 2)$ |

# 5.3 Solving Systems of Linear Equations by Elimination



TEXAS ESSENTIAL  
KNOWLEDGE AND SKILLS

A.2.I  
A.5.C

## USING PROBLEM-SOLVING STRATEGIES

To be proficient in math, you need to monitor and evaluate your progress and change course using a different solution method, if necessary.

**Essential Question** How can you use elimination to solve a system of linear equations?

### EXPLORATION 1 Writing and Solving a System of Equations

**Work with a partner.** You purchase a drink and a sandwich for \$4.50. Your friend purchases a drink and five sandwiches for \$16.50. You want to determine the price of a drink and the price of a sandwich.

- a. Let  $x$  represent the price (in dollars) of one drink. Let  $y$  represent the price (in dollars) of one sandwich. Write a system of equations for the situation. Use the following verbal model.

$$\begin{array}{c} \text{Number} \\ \text{of drinks} \end{array} \cdot \begin{array}{c} \text{Price} \\ \text{per drink} \end{array} + \begin{array}{c} \text{Number of} \\ \text{sandwiches} \end{array} \cdot \begin{array}{c} \text{Price per} \\ \text{sandwich} \end{array} = \begin{array}{c} \text{Total} \\ \text{price} \end{array}$$

Label one of the equations Equation 1 and the other equation Equation 2.

- b. Subtract Equation 1 from Equation 2. Explain how you can use the result to solve the system of equations. Then find and interpret the solution.

### EXPLORATION 2 Using Elimination to Solve Systems

**Work with a partner.** Solve each system of linear equations using two methods.

**Method 1 Subtract.** Subtract Equation 2 from Equation 1. Then use the result to solve the system.

**Method 2 Add.** Add the two equations. Then use the result to solve the system.

Is the solution the same using both methods? Which method do you prefer?

a. $3x - y = 6$	b. $2x + y = 6$	c. $x - 2y = -7$
$3x + y = 0$	$2x - y = 2$	$x + 2y = 5$

### EXPLORATION 3 Using Elimination to Solve a System

**Work with a partner.**

$$2x + y = 7 \quad \text{Equation 1}$$

$$x + 5y = 17 \quad \text{Equation 2}$$

- a. Can you eliminate a variable by adding or subtracting the equations as they are? If not, what do you need to do to one or both equations so that you can?
- b. Solve the system individually. Then exchange solutions with your partner and compare and check the solutions.

## Communicate Your Answer

- How can you use elimination to solve a system of linear equations?
- When can you add or subtract the equations in a system to solve the system? When do you have to multiply first? Justify your answers with examples.
- In Exploration 3, why can you multiply an equation in the system by a constant and not change the solution of the system? Explain your reasoning.

## 5.3 Lesson

### What You Will Learn

- ▶ Solve systems of linear equations by elimination.
- ▶ Use systems of linear equations to solve real-life problems.

### Core Vocabulary

Previous  
coefficient

### Solving Linear Systems by Elimination

#### Core Concept

##### Solving a System of Linear Equations by Elimination

- Step 1** Multiply, if necessary, one or both equations by a constant so at least one pair of like terms has the same or opposite coefficients.
- Step 2** Add or subtract the equations to eliminate one of the variables.
- Step 3** Solve the resulting equation.
- Step 4** Substitute the value from Step 3 into one of the original equations and solve for the other variable.

You can use elimination to solve a system of equations because replacing one equation in the system with the sum of that equation and a multiple of the other produces a system that has the same solution. Here is why.

Consider System 1. In this system,  $a$  and  $c$  are algebraic expressions, and  $b$  and  $d$  are constants. Begin by multiplying each side of Equation 2 by a constant  $k$ . By the Multiplication Property of Equality,  $kc = kd$ . You can rewrite Equation 1 as Equation 3 by adding  $kc$  on the left and  $kd$  on the right. You can rewrite Equation 3 as Equation 1 by subtracting  $kc$  on the left and  $kd$  on the right. Because you can rewrite either system as the other, System 1 and System 2 have the same solution.

##### System 1

$$\begin{aligned} a &= b && \text{Equation 1} \\ c &= d && \text{Equation 2} \end{aligned}$$

##### System 2

$$\begin{aligned} a + kc &= b + kd && \text{Equation 3} \\ c &= d && \text{Equation 2} \end{aligned}$$

#### EXAMPLE 1

##### Solving a System of Linear Equations by Elimination

Solve the system of linear equations by elimination.

$$\begin{aligned} 3x + 2y &= 4 && \text{Equation 1} \\ 3x - 2y &= -4 && \text{Equation 2} \end{aligned}$$

#### SOLUTION

**Step 1** Because the coefficients of the  $y$ -terms are opposites, you do not need to multiply either equation by a constant.

**Step 2** Add the equations.

$$\begin{aligned} 3x + 2y &= 4 && \text{Equation 1} \\ \underline{3x - 2y} &= \underline{-4} && \text{Equation 2} \\ 6x &= 0 && \text{Add the equations.} \end{aligned}$$

**Step 3** Solve for  $x$ .

$$\begin{aligned} 6x &= 0 && \text{Resulting equation from Step 2} \\ x &= 0 && \text{Divide each side by 6.} \end{aligned}$$

**Step 4** Substitute 0 for  $x$  in one of the original equations and solve for  $y$ .

$$\begin{aligned} 3x + 2y &= 4 && \text{Equation 1} \\ 3(0) + 2y &= 4 && \text{Substitute 0 for } x. \\ y &= 2 && \text{Solve for } y. \end{aligned}$$

▶ The solution is  $(0, 2)$ .

#### Check

##### Equation 1

$$\begin{aligned} 3x + 2y &= 4 \\ 3(0) + 2(2) &\stackrel{?}{=} 4 \\ 4 &= 4 \quad \checkmark \end{aligned}$$

##### Equation 2

$$\begin{aligned} 3x - 2y &= -4 \\ 3(0) - 2(2) &\stackrel{?}{=} -4 \\ -4 &= -4 \quad \checkmark \end{aligned}$$

**EXAMPLE 2****Solving a System of Linear Equations by Elimination**

Solve the system of linear equations by elimination.

$$-10x + 3y = 1 \quad \text{Equation 1}$$

$$-5x - 6y = 23 \quad \text{Equation 2}$$

**ANOTHER WAY**

To use subtraction to eliminate one of the variables, multiply Equation 2 by 2 and then subtract the equations.

$$\begin{array}{r} -10x + 3y = 1 \\ -(-10x - 12y = 46) \\ \hline 15y = -45 \end{array}$$

**SOLUTION****Step 1** Multiply Equation 2 by  $-2$  so that the coefficients of the  $x$ -terms are opposites.

$$-10x + 3y = 1$$

$$-10x + 3y = 1 \quad \text{Equation 1}$$

$$-5x - 6y = 23$$

**Multiply by  $-2$ .**

$$10x + 12y = -46 \quad \text{Revised Equation 2}$$

**Step 2** Add the equations.

$$-10x + 3y = 1$$

Equation 1

$$\underline{10x + 12y = -46}$$

Revised Equation 2

$$15y = -45$$

Add the equations.

**Step 3** Solve for  $y$ .

$$15y = -45$$

Resulting equation from Step 2

$$y = -3$$

Divide each side by 15.

**Step 4** Substitute  $-3$  for  $y$  in one of the original equations and solve for  $x$ .

$$-5x - 6y = 23$$

Equation 2

$$-5x - 6(-3) = 23$$

Substitute  $-3$  for  $y$ .

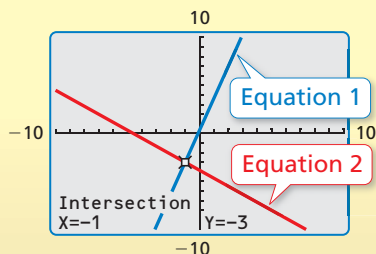
$$-5x + 18 = 23$$

Multiply.

$$-5x = 5$$

Subtract 18 from each side.

$$x = -1$$

Divide each side by  $-5$ .▶ The solution is  $(-1, -3)$ .**Check****Monitoring Progress**Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

Solve the system of linear equations by elimination. Check your solution.

1.  $3x + 2y = 7$

2.  $x - 3y = 24$

3.  $x + 4y = 22$

$-3x + 4y = 5$

$3x + y = 12$

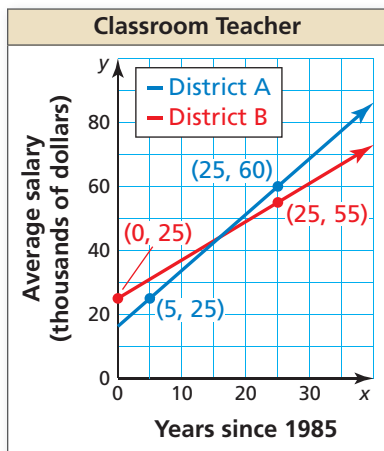
$4x + y = 13$

**Concept Summary****Methods for Solving Systems of Linear Equations**

Method	When to Use
Graphing ( <i>Lesson 5.1</i> )	To estimate solutions
Substitution ( <i>Lesson 5.2</i> )	When one of the variables in one of the equations has a coefficient of 1 or $-1$
Elimination ( <i>Lesson 5.3</i> )	When at least one pair of like terms has the same or opposite coefficients
Elimination (Multiply First) ( <i>Lesson 5.3</i> )	When one of the variables cannot be eliminated by adding or subtracting the equations

## Solving Real-Life Problems

### EXAMPLE 3 Modeling with Mathematics



The graph represents the average salaries of classroom teachers in two school districts. During what year were the average salaries in the two districts equal? What was the average salary in both districts in that year?

#### SOLUTION

- Understand the Problem** You know two points on each line in the graph. You are asked to determine the year in which the average salaries were equal and then determine the average salary in that year.
- Make a Plan** Use the points in the graph to write a system of linear equations. Then solve the system of linear equations.
- Solve the Problem** Find the slope of each line.

$$\text{District A: } m = \frac{60 - 25}{25 - 5} = \frac{35}{20} = \frac{7}{4} \quad \text{District B: } m = \frac{55 - 25}{25 - 0} = \frac{30}{25} = \frac{6}{5}$$

Use each slope and a point on each line to write equations of the lines.

#### District A

$$y - y_1 = m(x - x_1) \quad \text{Write the point-slope form.}$$

$$y - 25 = \frac{7}{4}(x - 5) \quad \text{Substitute for } m, x_1, \text{ and } y_1.$$

$$-7x + 4y = 65 \quad \text{Write in standard form.}$$

$$\text{System} \quad -7x + 4y = 65 \quad \text{Equation 1}$$

$$-6x + 5y = 125 \quad \text{Equation 2}$$

#### District B

$$y - y_1 = m(x - x_1)$$

$$y - 25 = \frac{6}{5}(x - 0)$$

$$-6x + 5y = 125$$

**Step 1** Multiply Equation 1 by  $-5$ . Multiply Equation 2 by 4.

$$-7x + 4y = 65 \quad \text{Multiply by } -5. \quad 35x - 20y = -325 \quad \text{Revised Equation 1}$$

$$-6x + 5y = 125 \quad \text{Multiply by } 4. \quad -24x + 20y = 500 \quad \text{Revised Equation 2}$$

**Step 2** Add the equations.

$$35x - 20y = -325 \quad \text{Revised Equation 1}$$

$$-24x + 20y = 500 \quad \text{Revised Equation 2}$$

$$11x = 175 \quad \text{Add the equations.}$$

**Step 3** Solving the equation  $11x = 175$  gives  $x = \frac{175}{11} \approx 15.9$ .

**Step 4** Substitute  $\frac{175}{11}$  for  $x$  in one of the original equations and solve for  $y$ .

$$-7x + 4y = 65 \quad \text{Equation 1}$$

$$-7\left(\frac{175}{11}\right) + 4y = 65 \quad \text{Substitute } \frac{175}{11} \text{ for } x.$$

$$y \approx 44 \quad \text{Solve for } y.$$

► The solution is about  $(15.9, 44)$ . Because  $x \approx 15.9$  corresponds to the year 2000, the average salary in both districts was about \$44,000 in 2000.

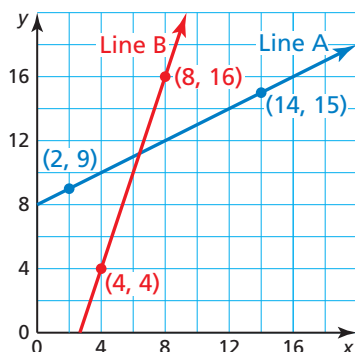
- Look Back** Using the graph, the point of intersection appears to be about  $(15, 45)$ . So, the solution of  $(15.9, 44)$  is reasonable.

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- Write and solve a system of linear equations represented by the graph at the left.

#### STUDY TIP

In Example 3, both equations are multiplied by a constant so that the coefficients of the  $y$ -terms are opposites.



# 5.3 Exercises

## Vocabulary and Core Concept Check

- OPEN-ENDED** Give an example of a system of linear equations that can be solved by first adding the equations to eliminate one variable.
- WRITING** Explain how to solve the system of linear equations by elimination.
 
$$\begin{array}{r} 2x - 3y = -4 \quad \text{Equation 1} \\ -5x + 9y = 7 \quad \text{Equation 2} \end{array}$$

## Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, solve the system of linear equations by elimination. Check your solution. (See Example 1.)

- |                                       |                                      |
|---------------------------------------|--------------------------------------|
| 3. $x + 2y = 13$<br>$-x + y = 5$      | 4. $9x + y = 2$<br>$-4x - y = -17$   |
| 5. $5x + 6y = 50$<br>$x - 6y = -26$   | 6. $-x + y = 4$<br>$x + 3y = 4$      |
| 7. $-3x - 5y = -7$<br>$-4x + 5y = 14$ | 8. $4x - 9y = -21$<br>$-4x - 3y = 9$ |
| 9. $-y - 10 = 6x$<br>$5x + y = -10$   | 10. $3x - 30 = y$<br>$7y - 6 = 3x$   |

In Exercises 11–18, solve the system of linear equations by elimination. Check your solution. (See Examples 2 and 3.)

- |  |  |
|--|--|
| 11. $x + y = 2$<br>$2x + 7y = 9$       | 12. $8x - 5y = 11$<br>$4x - 3y = 5$    |
| 13. $11x - 20y = 28$<br>$3x + 4y = 36$ | 14. $10x - 9y = 46$<br>$-2x + 3y = 10$ |
| 15. $4x - 3y = 8$<br>$5x - 2y = -11$   | 16. $-2x - 5y = 9$<br>$3x + 11y = 4$   |
| 17. $9x + 2y = 39$<br>$6x + 13y = -9$  | 18. $12x - 7y = -2$<br>$8x + 11y = 30$ |
19. **ERROR ANALYSIS** Describe and correct the error in solving for one of the variables in the linear system  $5x - 7y = 16$  and  $x + 7y = 8$ .

**X**

$$\begin{array}{r} 5x - 7y = 16 \\ x + 7y = 8 \\ \hline 4x = 24 \\ x = 6 \end{array}$$

20. **ERROR ANALYSIS** Describe and correct the error in solving for one of the variables in the linear system  $4x + 3y = 8$  and  $x - 2y = -13$ .

**X**

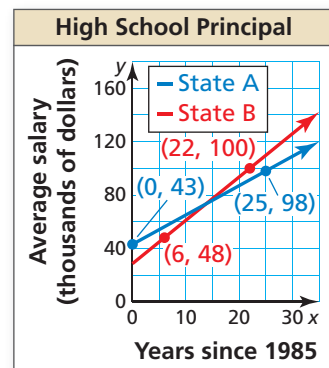
$$\begin{array}{r} 4x + 3y = 8 \\ x - 2y = -13 \quad \text{Multiply by } -4 \rightarrow \\ \hline 4x + 3y = 8 \\ -4x + 8y = -52 \\ \hline 11y = -44 \\ y = -4 \end{array}$$

21. **MODELING WITH MATHEMATICS** A service center charges a fee of  $x$  dollars for an oil change plus  $y$  dollars per quart of oil used. A sample of its sales record is shown. Write a system of linear equations that represents this situation. Find the fee and cost per quart of oil.

	A	B	C
1	Customer	Oil Tank Size (quarts)	Total Cost
2	A	5	\$22.45
3	B	7	\$25.45

22. **MODELING WITH MATHEMATICS**

The graph represents the average salaries of high school principals in two states. During what year were the average salaries in the two states equal? What was the average salary in both states in that year?





In Exercises 23–26, solve the system of linear equations using any method. Explain why you chose the method.

23.  $3x + 2y = 4$   
 $2y = 8 - 5x$

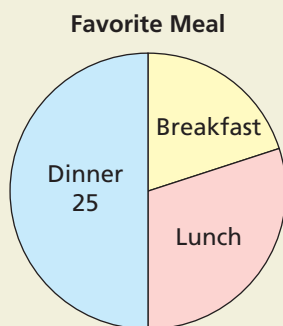
24.  $-6y + 2 = -4x$   
 $y - 2 = x$

25.  $y - x = 2$   
 $y = -\frac{1}{4}x + 7$

26.  $3x + y = \frac{1}{3}$   
 $2x - 3y = \frac{8}{3}$

27. **WRITING** For what values of  $a$  can you solve the linear system  $ax + 3y = 2$  and  $4x + 5y = 6$  by elimination without multiplying first? Explain.

28. **HOW DO YOU SEE IT?** The circle graph shows the results of a survey in which 50 students were asked about their favorite meal.

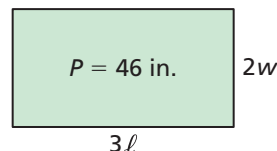


- Estimate the numbers of students who chose breakfast and lunch.
- The number of students who chose lunch was 5 more than the number of students who chose breakfast. Write a system of linear equations that represents the numbers of students who chose breakfast and lunch.
- Explain how you can solve the linear system in part (b) to check your answers in part (a).

29. **MAKING AN ARGUMENT** Your friend says that any system of equations that can be solved by elimination can be solved by substitution in an equal or fewer number of steps. Is your friend correct? Explain.

30. **THOUGHT PROVOKING** Write a system of linear equations that can be added to eliminate a variable or subtracted to eliminate a variable.

31. **MATHEMATICAL CONNECTIONS** A rectangle has a perimeter of 18 inches. A new rectangle is formed by doubling the width  $w$  and tripling the length  $\ell$ , as shown. The new rectangle has a perimeter  $P$  of 46 inches.



- Write and solve a system of linear equations to find the length and width of the original rectangle.
  - Find the length and width of the new rectangle.
32. **CRITICAL THINKING** Refer to the discussion of System 1 and System 2 on page 232. Without solving, explain why the two systems shown have the same solution.

System 1	System 2
$3x - 2y = 8$ Equation 1	$5x = 20$ Equation 3
$x + y = 6$ Equation 2	$x + y = 6$ Equation 2

33. **PROBLEM SOLVING** You are making 6 quarts of fruit punch for a party. You have bottles of 100% fruit juice and 20% fruit juice. How many quarts of each type of juice should you mix to make 6 quarts of 80% fruit juice?
34. **PROBLEM SOLVING** A motorboat takes 40 minutes to travel 20 miles downstream. The return trip takes 60 minutes. What is the speed of the current?
35. **CRITICAL THINKING** Solve for  $x$ ,  $y$ , and  $z$  in the system of equations. Explain your steps.

$x + 7y + 3z = 29$	Equation 1
$3z + x - 2y = -7$	Equation 2
$5y = 10 - 2x$	Equation 3

## Maintaining Mathematical Proficiency Reviewing what you learned in previous grades and lessons

Solve the equation. Determine whether the equation has *one solution*, *no solution*, or *infinitely many solutions*. (Section 1.3)

36.  $5d - 8 = 1 + 5d$

37.  $9 + 4t = 12 - 4t$

38.  $3n + 2 = 2(n - 3)$

39.  $-3(4 - 2v) = 6v - 12$

Write an equation of the line that passes through the given point and is parallel to the given line. (Section 4.4)

40.  $(4, -1)$ ;  $y = -2x + 7$

41.  $(0, 6)$ ;  $y = 5x - 3$

42.  $(-5, -2)$ ;  $y = \frac{2}{3}x + 1$

# 5.4 Solving Special Systems of Linear Equations



TEXAS ESSENTIAL KNOWLEDGE AND SKILLS

A.2.I  
A.3.F  
A.5.C

**Essential Question** Can a system of linear equations have no solution or infinitely many solutions?

## EXPLORATION 1 Using a Table to Solve a System

**Work with a partner.** You invest \$450 for equipment to make skateboards. The materials for each skateboard cost \$20. You sell each skateboard for \$20.

- a. Write the cost and revenue equations. Then copy and complete the table for your cost  $C$  and your revenue  $R$ .

$x$ (skateboards)	0	1	2	3	4	5	6	7	8	9	10
$C$ (dollars)											
$R$ (dollars)											

### APPLYING MATHEMATICS

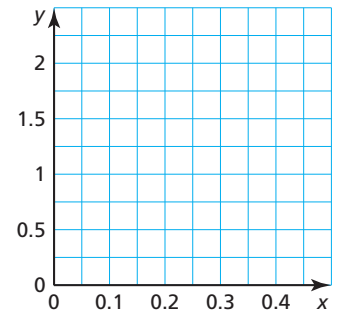
To be proficient in math, you need to interpret mathematical results in real-life contexts.

- b. When will your company break even? What is wrong?

## EXPLORATION 2 Writing and Analyzing a System

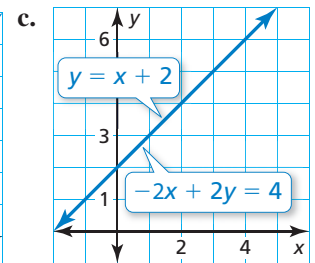
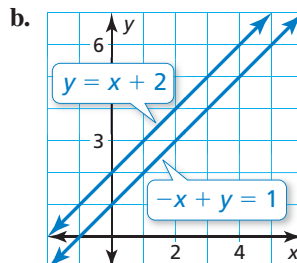
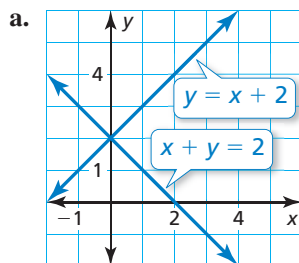
**Work with a partner.** A necklace and matching bracelet have two types of beads. The necklace has 40 small beads and 6 large beads and weighs 10 grams. The bracelet has 20 small beads and 3 large beads and weighs 5 grams. The threads holding the beads have no significant weight.

- a. Write a system of linear equations that represents the situation. Let  $x$  be the weight (in grams) of a small bead and let  $y$  be the weight (in grams) of a large bead.
- b. Graph the system in the coordinate plane shown. What do you notice about the two lines?
- c. Can you find the weight of each type of bead? Explain your reasoning.



### Communicate Your Answer

3. Can a system of linear equations have no solution or infinitely many solutions? Give examples to support your answers.
4. Does the system of linear equations represented by each graph have *no solution*, *one solution*, or *infinitely many solutions*? Explain.



# 5.4 Lesson

## What You Will Learn

- ▶ Determine the numbers of solutions of linear systems.
- ▶ Use linear systems to solve real-life problems.

### Core Vocabulary

Previous  
parallel

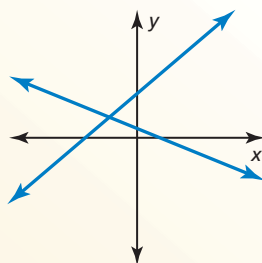
## The Numbers of Solutions of Linear Systems

### Core Concept

#### Solutions of Systems of Linear Equations

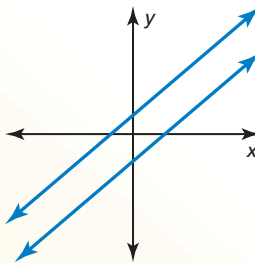
A system of linear equations can have *one solution*, *no solution*, or *infinitely many solutions*.

##### One solution



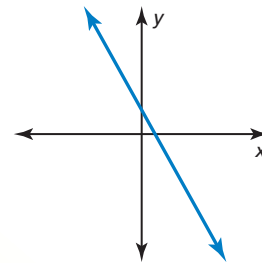
The lines intersect.

##### No solution



The lines are parallel.

##### Infinitely many solutions



The lines are the same.

### ANOTHER WAY

You can solve some linear systems by inspection. In Example 1, notice you can rewrite the system as

$$\begin{aligned} -2x + y &= 1 \\ -2x + y &= -5. \end{aligned}$$

This system has no solution because  $-2x + y$  cannot be equal to both 1 and  $-5$ .

### EXAMPLE 1 Solving a System: No Solution

Solve the system of linear equations.

$$\begin{aligned} y &= 2x + 1 && \text{Equation 1} \\ y &= 2x - 5 && \text{Equation 2} \end{aligned}$$

#### SOLUTION

**Method 1** Solve by graphing.

Graph each equation.

The lines have the same slope and different  $y$ -intercepts. So, the lines are parallel.

Because parallel lines do not intersect, there is no point that is a solution of both equations.

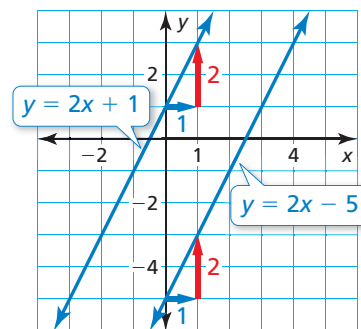
- ▶ So, the system of linear equations has no solution.

**Method 2** Solve by substitution.

Substitute  $2x - 5$  for  $y$  in Equation 1.

$$\begin{aligned} y &= 2x + 1 && \text{Equation 1} \\ 2x - 5 &= 2x + 1 && \text{Substitute } 2x - 5 \text{ for } y. \\ -5 &= 1 && \text{Subtract } 2x \text{ from each side.} \end{aligned}$$

- ▶ The equation  $-5 = 1$  is never true. So, the system of linear equations has no solution.



### STUDY TIP

A linear system with no solution is called an *inconsistent system*.

## ANOTHER WAY

You can also solve the linear system by graphing. The lines have the same slope and the same  $y$ -intercept. So, the lines are the same, which means all points on the line are solutions of both equations.

## STUDY TIP

A linear system with infinitely many solutions is called a *consistent dependent system*.

### EXAMPLE 2 Solving a System: Infinitely Many Solutions

Solve the system of linear equations.

$$-2x + y = 3 \quad \text{Equation 1}$$

$$-4x + 2y = 6 \quad \text{Equation 2}$$

#### SOLUTION

Solve by elimination.

**Step 1** Multiply Equation 1 by  $-2$ .

$$-2x + y = 3 \quad \text{Multiply by } -2. \rightarrow 4x - 2y = -6 \quad \text{Revised Equation 1}$$

$$-4x + 2y = 6 \quad \text{Equation 2}$$

**Step 2** Add the equations.

$$4x - 2y = -6 \quad \text{Revised Equation 1}$$

$$\underline{-4x + 2y = 6} \quad \text{Equation 2}$$

$$0 = 0 \quad \text{Add the equations.}$$

▶ The equation  $0 = 0$  is always true. So, the solutions are all the points on the line  $-2x + y = 3$ . The system of linear equations has infinitely many solutions.

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Solve the system of linear equations.

1.  $x + y = 3$

$$2x + 2y = 6$$

2.  $y = -x + 3$

$$2x + 2y = 4$$

## Solving Real-Life Problems

### EXAMPLE 3 Modeling with Mathematics

An athletic director is comparing the costs of renting two banquet halls for an awards banquet. Write a system of linear equations that represents this situation. If the cost patterns continue, will the cost of Hall A ever equal the cost of Hall B?

#### SOLUTION

**Words** Total cost = Cost per hour • Number of hours + Initial cost

**Variables** Let  $y$  be the cost (in dollars) and let  $x$  be the number of hours.

**System**  $y = 100x + 75$  Equation 1 - Cost of Hall A

$$y = 100x + 100 \quad \text{Equation 2 - Cost of Hall B}$$

The equations are in slope-intercept form. The graphs of the equations have the same slope but different  $y$ -intercepts. There is no solution because the lines are parallel.

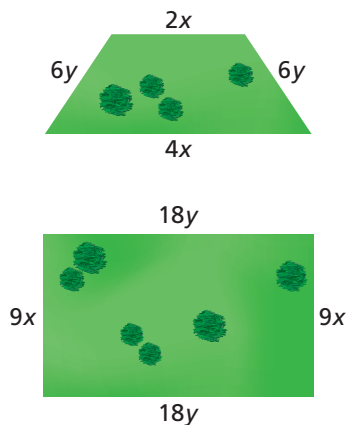
▶ So, the cost of Hall A will never equal the cost of Hall B.

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3. **WHAT IF?** What happens to the solution in Example 3 when the cost per hour for Hall A is \$125?

Cost (dollars)		
Hours	Hall A	Hall B
0	75	100
1	175	200
2	275	300
3	375	400

### EXAMPLE 4 Modeling with Mathematics



The perimeter of the trapezoidal piece of land is 48 kilometers. The perimeter of the rectangular piece of land is 144 kilometers. Write and solve a system of linear equations to find the values of  $x$  and  $y$ .

#### SOLUTION

- Understand the Problem** You know the perimeter of each piece of land and the side lengths in terms of  $x$  or  $y$ . You are asked to write and solve a system of linear equations to find the values of  $x$  and  $y$ .
- Make a Plan** Use the figures and the definition of perimeter to write a system of linear equations that represents the problem. Then solve the system of linear equations.
- Solve the Problem**

**Perimeter of trapezoid**

$$2x + 4x + 6y + 6y = 48$$

$$6x + 12y = 48 \quad \text{Equation 1}$$

**System**  $6x + 12y = 48 \quad \text{Equation 1}$

$$18x + 36y = 144 \quad \text{Equation 2}$$

**Perimeter of rectangle**

$$9x + 9x + 18y + 18y = 144$$

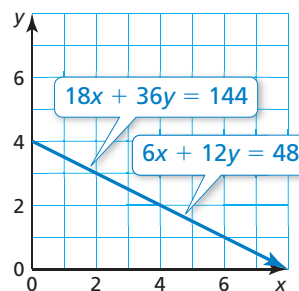
$$18x + 36y = 144 \quad \text{Equation 2}$$

**Method 1** Solve by graphing.

Graph each equation.

The lines have the same slope and the same  $y$ -intercept. So, the lines are the same.

In this context,  $x$  and  $y$  must be positive. Because the lines are the same, all the points on the line in Quadrant I are solutions of both equations.



► So, the system of linear equations has infinitely many solutions.

**Method 2** Solve by elimination.

Multiply Equation 1 by  $-3$  and add the equations.

$$\begin{array}{r} 6x + 12y = 48 \\ 18x + 36y = 144 \\ \hline -18x - 36y = -144 \quad \text{Revised Equation 1} \\ 18x + 36y = 144 \quad \text{Equation 2} \\ \hline 0 = 0 \quad \text{Add the equations.} \end{array}$$

► The equation  $0 = 0$  is always true. In this context,  $x$  and  $y$  must be positive. So, the solutions are all the points on the line  $6x + 12y = 48$  in Quadrant I. The system of linear equations has infinitely many solutions.

- Look Back** Choose a few of the ordered pairs  $(x, y)$  that are solutions of Equation 1. You should find that no matter which ordered pairs you choose, they will also be solutions of Equation 2. So, *infinitely many solutions* seems reasonable.

**Monitoring Progress**  Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

- WHAT IF?** What happens to the solution in Example 4 when the perimeter of the trapezoidal piece of land is 96 kilometers? Explain.

# 5.4 Exercises

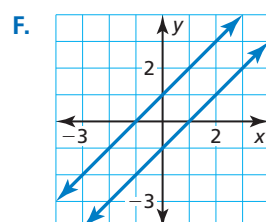
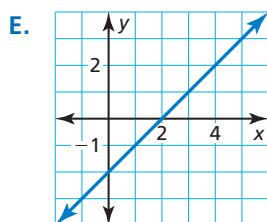
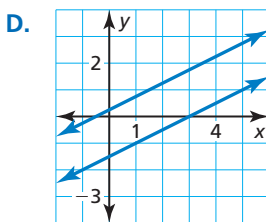
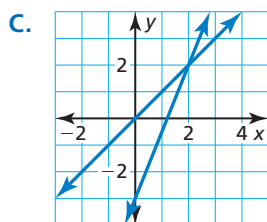
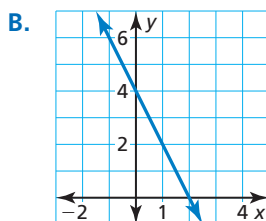
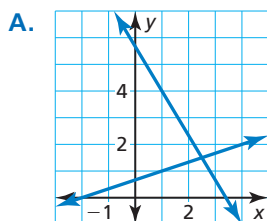
## Vocabulary and Core Concept Check

- REASONING** Is it possible for a system of linear equations to have exactly two solutions? Explain.
- WRITING** Compare the graph of a system of linear equations that has infinitely many solutions and the graph of a system of linear equations that has no solution.

## Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, match the system of linear equations with its graph. Then determine whether the system has *one solution*, *no solution*, or *infinitely many solutions*.

- |                                    |                                    |
|------------------------------------|------------------------------------|
| 3. $-x + y = 1$<br>$x - y = 1$     | 4. $2x - 2y = 4$<br>$-x + y = -2$  |
| 5. $2x + y = 4$<br>$-4x - 2y = -8$ | 6. $x - y = 0$<br>$5x - 2y = 6$    |
| 7. $-2x + 4y = 1$<br>$3x - 6y = 9$ | 8. $5x + 3y = 17$<br>$x - 3y = -2$ |



In Exercises 9–16, solve the system of linear equations. (See Examples 1 and 2.)

- |                                    |                                    |
|------------------------------------|------------------------------------|
| 9. $y = -2x - 4$<br>$y = 2x - 4$   | 10. $y = -6x - 8$<br>$y = -6x + 8$ |
| 11. $3x - y = 6$<br>$-3x + y = -6$ | 12. $-x + 2y = 7$<br>$x - 2y = 7$  |

- |   |                                       |
|---|---------------------------------------|
| 13. $4x + 4y = -8$<br>$-2x - 2y = 4$    | 14. $15x - 5y = -20$<br>$-3x + y = 4$ |
| 15. $9x - 15y = 24$<br>$6x - 10y = -16$ | 16. $3x - 2y = -5$<br>$4x + 5y = 47$  |

In Exercises 17–22, use only the slopes and y-intercepts of the graphs of the equations to determine whether the system of linear equations has *one solution*, *no solution*, or *infinitely many solutions*. Explain.

- |                                       |                                       |
|---------------------------------------|---------------------------------------|
| 17. $y = 7x + 13$<br>$-21x + 3y = 39$ | 18. $y = -6x - 2$<br>$12x + 2y = -6$  |
| 19. $4x + 3y = 27$<br>$4x - 3y = -27$ | 20. $-7x + 7y = 1$<br>$2x - 2y = -18$ |
| 21. $-18x + 6y = 24$<br>$3x - y = -2$ | 22. $2x - 2y = 16$<br>$3x - 6y = 30$  |

**ERROR ANALYSIS** In Exercises 23 and 24, describe and correct the error in solving the system of linear equations.

23.

$-4x + y = 4$   
 $4x + y = 12$

The lines do not intersect. So, the system has no solution.

24.

$y = 3x - 8$   
 $y = 3x - 12$

The lines have the same slope. So, the system has infinitely many solutions.



25. **MODELING WITH MATHEMATICS** The table shows the distances two groups have traveled at different times during a canoeing excursion. The groups continue traveling at their current rates until they reach the same destination. Let  $d$  be the distance traveled and  $t$  be the time since 1 P.M. Write a system of linear equations that represents this situation. Will Group B catch up to Group A before reaching the destination? Explain. (See Example 3.)

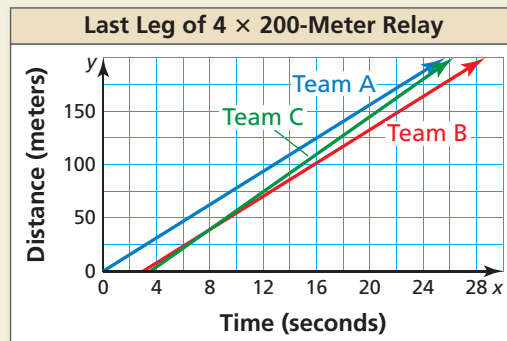
	Distance Traveled (miles)			
	1 P.M.	2 P.M.	3 P.M.	4 P.M.
Group A	3	9	15	21
Group B	1	7	13	19

26. **MODELING WITH MATHEMATICS** A \$6-bag of trail mix contains 3 cups of dried fruit and 4 cups of almonds. A \$9-bag contains  $4\frac{1}{2}$  cups of dried fruit and 6 cups of almonds. Write and solve a system of linear equations to find the price of 1 cup of dried fruit and 1 cup of almonds. (See Example 4.)
27. **PROBLEM SOLVING** A train travels from New York City to Washington, D.C., and then back to New York City. The table shows the number of tickets purchased for each leg of the trip. The cost per ticket is the same for each leg of the trip. Is there enough information to determine the cost of one coach ticket? Explain.

Destination	Coach tickets	Business class tickets	Money collected (dollars)
Washington, D.C.	150	80	22,860
New York City	170	100	27,280

28. **THOUGHT PROVOKING** Write a system of three linear equations in two variables so that any two of the equations have exactly one solution, but the entire system of equations has no solution.
29. **REASONING** In a system of linear equations, one equation has a slope of 2 and the other equation has a slope of  $-\frac{1}{3}$ . How many solutions does the system have? Explain.

30. **HOW DO YOU SEE IT?** The graph shows information about the last leg of a  $4 \times 200$ -meter relay for three relay teams. Team A's runner ran about 7.8 meters per second, Team B's runner ran about 7.8 meters per second, and Team C's runner ran about 8.8 meters per second.



- Estimate the distance at which Team C's runner passed Team B's runner.
  - If the race was longer, could Team C's runner have passed Team A's runner? Explain.
  - If the race was longer, could Team B's runner have passed Team A's runner? Explain.
31. **ABSTRACT REASONING** Consider the system of linear equations  $y = ax + 4$  and  $y = bx - 2$ , where  $a$  and  $b$  are real numbers. Determine whether each statement is *always*, *sometimes*, or *never* true. Explain your reasoning.
- The system has infinitely many solutions.
  - The system has no solution.
  - When  $a < b$ , the system has one solution.
32. **MAKING AN ARGUMENT** One admission to an ice skating rink costs  $x$  dollars, and renting a pair of ice skates costs  $y$  dollars. Your friend says she can determine the exact cost of one admission and one skate rental. Is your friend correct? Explain.

D&G ICE RINK			
Client No.	Table No.	Server Name	
	240796		
3	Admissions		
2	Skate Rentals		
Total			\$ 38.00

D&G ICE RINK			
Client No.	Table No.	Server Name	
	240797		
15	Admissions		
10	Skate Rentals		
Total			\$ 190.00

## Maintaining Mathematical Proficiency Reviewing what you learned in previous grades and lessons

Solve the system of linear equations by graphing. (Section 5.1)

33.  $y = x - 6$   
 $y = -x + 10$

34.  $y = 2x + 3$   
 $y = -3x - 7$

35.  $2x + y = 6$   
 $3x - 2y = 16$

## 5.1–5.4 What Did You Learn?

### Core Vocabulary

system of linear equations, *p.* 220

solution of a system of linear equations, *p.* 220

### Core Concepts

#### Section 5.1

Solving a System of Linear Equations by Graphing, *p.* 221

#### Section 5.2

Solving a System of Linear Equations by Substitution, *p.* 226

#### Section 5.3

Solving a System of Linear Equations by Elimination, *p.* 232

#### Section 5.4

Solutions of Systems of Linear Equations, *p.* 238

### Mathematical Thinking

1. Describe the given information in Exercise 33 on page 230 and your plan for finding the solution.
2. Describe another real-life situation similar to Exercise 21 on page 235 and the mathematics that you can apply to solve the problem.
3. What question(s) can you ask your friend to help her understand the error in the statement she made in Exercise 32 on page 242?

### Study Skills

## Analyzing Your Errors

#### Study Errors

**What Happens:** You do not study the right material or you do not learn it well enough to remember it on a test without resources such as notes.

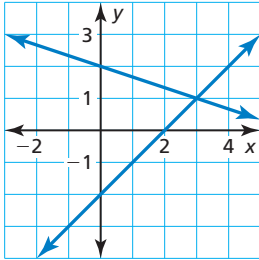
**How to Avoid This Error:** Take a practice test. Work with a study group. Discuss the topics on the test with your teacher. Do not try to learn a whole chapter's worth of material in one night.



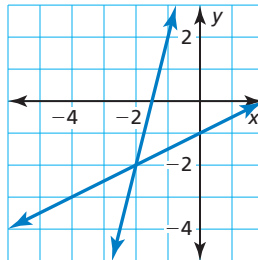
# 5.1–5.4 Quiz

Use the graph to solve the system of linear equations. Check your solution. (Section 5.1)

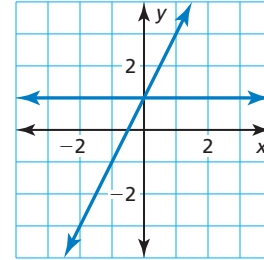
1.  $y = -\frac{1}{3}x + 2$   
 $y = x - 2$



2.  $y = \frac{1}{2}x - 1$   
 $y = 4x + 6$



3.  $y = 1$   
 $y = 2x + 1$



Solve the system of linear equations by substitution. Check your solution. (Section 5.2)

4.  $y = x - 4$   
 $-2x + y = 18$

5.  $2y + x = -4$   
 $y - x = -5$

6.  $3x - 5y = 13$   
 $x + 4y = 10$

Solve the system of linear equations by elimination. Check your solution. (Section 5.3)

7.  $x + y = 4$   
 $-3x - y = -8$

8.  $x + 3y = 1$   
 $5x + 6y = 14$

9.  $2x - 3y = -5$   
 $5x + 2y = 16$

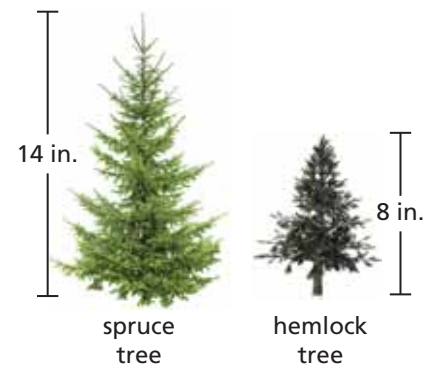
Solve the system of linear equations. (Section 5.4)

10.  $x - y = 1$   
 $x - y = 6$

11.  $6x + 2y = 16$   
 $2x - y = 2$

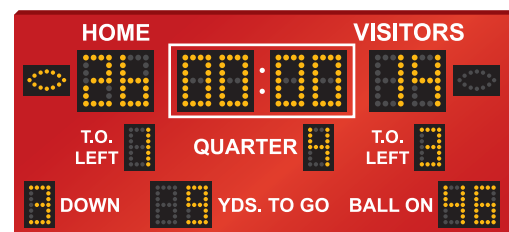
12.  $3x - 3y = -2$   
 $-6x + 6y = 4$

13. You plant a spruce tree that grows 4 inches per year and a hemlock tree that grows 6 inches per year. The initial heights are shown. (Section 5.1)
- Write a system of linear equations that represents this situation.
  - Solve the system by graphing. Interpret your solution.



14. It takes you 3 hours to drive to a concert 135 miles away. You drive 55 miles per hour on highways and 40 miles per hour on the rest of the roads. (Section 5.1, Section 5.2, and Section 5.3)
- How much time do you spend driving at each speed?
  - How many miles do you drive on highways? the rest of the roads?

15. In a football game, all of the home team's points are from 7-point touchdowns and 3-point field goals. The team scores six times. Write and solve a system of linear equations to find the numbers of touchdowns and field goals that the home team scores. (Section 5.1, Section 5.2, and Section 5.3)



# 5.5 Solving Equations by Graphing



TEXAS ESSENTIAL  
KNOWLEDGE AND SKILLS  
A.5.A

**Essential Question** How can you use a system of linear equations to solve an equation with variables on both sides?

Previously, you learned how to use algebra to solve equations with variables on both sides. Another way is to use a system of linear equations.

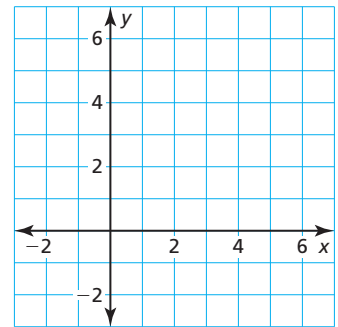
## EXPLORATION 1 Solving an Equation by Graphing

**Work with a partner.** Solve  $2x - 1 = -\frac{1}{2}x + 4$  by graphing.

- Use the left side to write a linear equation. Then use the right side to write another linear equation.
- Graph the two linear equations from part (a). Find the  $x$ -value of the point of intersection. Check that the  $x$ -value is the solution of

$$2x - 1 = -\frac{1}{2}x + 4.$$

- Explain why this “graphical method” works.



### SELECTING TOOLS

To be proficient in math, you need to consider the available tools, which may include pencil and paper or a graphing calculator, when solving a mathematical problem.

## EXPLORATION 2 Solving Equations Algebraically and Graphically

**Work with a partner.** Solve each equation using two methods.

**Method 1** Use an algebraic method.

**Method 2** Use a graphical method.

Is the solution the same using both methods?

- |   |  |
|---|--|
| a. $\frac{1}{2}x + 4 = -\frac{1}{4}x + 1$ | b. $\frac{2}{3}x + 4 = \frac{1}{3}x + 3$ |
| c. $-\frac{2}{3}x - 1 = \frac{1}{3}x - 4$ | d. $\frac{4}{5}x + \frac{7}{5} = 3x - 3$ |
| e. $-x + 2.5 = 2x - 0.5$                  | f. $-3x + 1.5 = x + 1.5$                 |

## Communicate Your Answer

- How can you use a system of linear equations to solve an equation with variables on both sides?
- Compare the algebraic method and the graphical method for solving a linear equation with variables on both sides. Describe the advantages and disadvantages of each method.

# 5.5 Lesson

## What You Will Learn

- ▶ Solve linear equations by graphing.
- ▶ Use linear equations to solve real-life problems.

### Core Vocabulary

Previous  
absolute value equation

## Solving Linear Equations by Graphing

You can use a system of linear equations to solve an equation with variables on both sides.

### Core Concept

#### Solving Linear Equations by Graphing

**Step 1** To solve the equation  $ax + b = cx + d$ , write two linear equations.

$$ax + b = cx + d$$

$y = ax + b$  and  $y = cx + d$

**Step 2** Graph the system of linear equations. The  $x$ -value of the solution of the system of linear equations is the solution of the equation  $ax + b = cx + d$ .

#### EXAMPLE 1 Solving an Equation by Graphing

Solve  $-x + 1 = 2x - 5$  by graphing. Check your solution.

#### SOLUTION

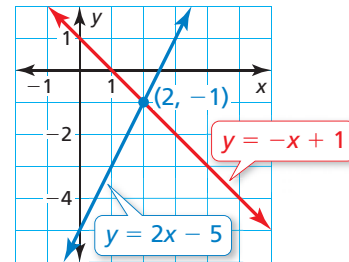
**Step 1** Write a system of linear equations using each side of the original equation.

$$-x + 1 = 2x - 5$$

$y = -x + 1$  and  $y = 2x - 5$

**Step 2** Graph the system.

$y = -x + 1$  Equation 1  
 $y = 2x - 5$  Equation 2



The graphs intersect at  $(2, -1)$ .

▶ So, the solution of the equation is  $x = 2$ .

#### Check

$$\begin{aligned} -x + 1 &= 2x - 5 \\ -(2) + 1 &\stackrel{?}{=} 2(2) - 5 \\ -1 &= -1 \quad \checkmark \end{aligned}$$

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Solve the equation by graphing. Check your solution.

1.  $\frac{1}{2}x - 3 = 2x$

2.  $-4 + 9x = -3x + 2$

## Solving Real-Life Problems

### EXAMPLE 2 Modeling with Mathematics



You are an ecologist studying the populations of two types of fish in a lake. Use the information in the table to predict when the populations of the two types of fish will be equal.

Type	Current population	Change per year
A	5750	-250
B	2825	400

#### SOLUTION

- Understand the Problem** You know the current population of each type of fish and the annual change in each population. You are asked to determine when the populations of the two types of fish will be equal.
- Make a Plan** Use a verbal model to write an equation that represents the problem. Then solve the equation by graphing.
- Solve the Problem**

Words

$$\overbrace{\text{Change per year} \cdot \text{Years} + \text{Current population}}^{\text{Type A}} = \overbrace{\text{Change per year} \cdot \text{Years} + \text{Current population}}^{\text{Type B}}$$

**Variable** Let  $x$  be the number of years.

**Equation**  $-250x + 5750 = 400x + 2825$

Solve the equation by graphing.

**Step 1** Write a system of linear equations using each side of the original equation.

$$-250x + 5750 = 400x + 2825$$

$$y = -250x + 5750 \quad y = 400x + 2825$$

**Step 2** Graph the system.

$$y = -250x + 5750 \quad \text{Equation 1}$$

$$y = 400x + 2825 \quad \text{Equation 2}$$

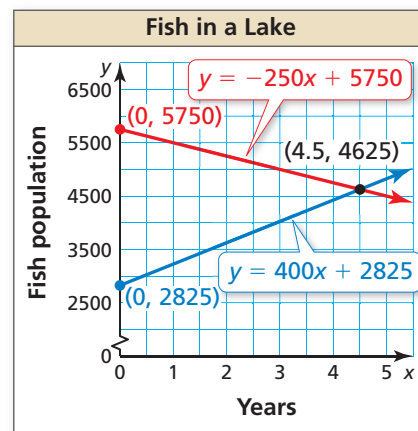
The graphs appear to intersect at  $(4.5, 4625)$ . Check this solution in each equation of the linear system, as shown.

► So, the populations of the two types of fish will be equal in 4.5 years.

**4. Look Back** To check that your solution is correct, verify that  $x = 4.5$  is the solution of the original equation.

$$-250(4.5) + 5750 = 400(4.5) + 2825$$

$$4625 = 4625 \quad \checkmark$$



#### Check

$$y = -250x + 5750$$

$$4625 \stackrel{?}{=} -250(4.5) + 5750$$

$$4625 = 4625 \quad \checkmark$$

$$y = 400x + 2825$$

$$4625 \stackrel{?}{=} 400(4.5) + 2825$$

$$4625 = 4625 \quad \checkmark$$

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- WHAT IF?** Type C has a current population of 3500 and grows by 500 each year. Predict when the population of Type C will be equal to the population of Type A.



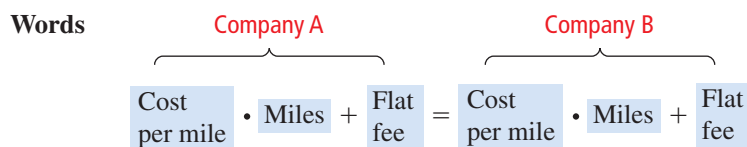


### EXAMPLE 3 Modeling with Mathematics

Your family needs to rent a car for a week while on vacation. Company A charges \$3.25 per mile plus a flat fee of \$125 per week. Company B charges \$3 per mile plus a flat fee of \$150 per week. After how many miles of travel are the total costs the same at both companies?

#### SOLUTION

- 1. Understand the Problem** You know the costs of renting a car from two companies. You are asked to determine how many miles of travel will result in the same total costs at both companies.
- 2. Make a Plan** Use a verbal model to write an equation that represents the problem. Then solve the equation by graphing.
- 3. Solve the Problem**

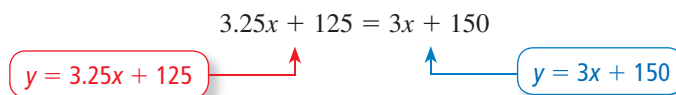


**Variable** Let  $x$  be the number of miles traveled.

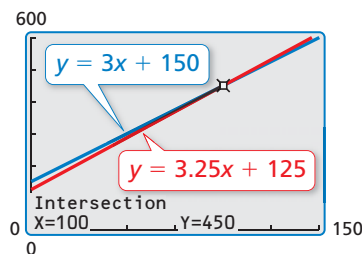
**Equation**  $3.25x + 125 = 3x + 150$

Solve the equation by graphing.

**Step 1** Write a system of linear equations using each side of the original equation.



**Step 2** Use a graphing calculator to graph the system.



Because the graphs intersect at  $(100, 450)$ , the solution of the equation is  $x = 100$ .

► So, the total costs are the same after 100 miles.

- 4. Look Back** One way to check your solution is to solve the equation algebraically, as shown.

#### Check

$$\begin{aligned}
 3.25x + 125 &= 3x + 150 \\
 0.25x + 125 &= 150 \\
 0.25x &= 25 \\
 x &= 100
 \end{aligned}$$

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- 4. WHAT IF?** Company C charges \$3.30 per mile plus a flat fee of \$115 per week. After how many miles are the total costs the same at Company A and Company C?

# 5.5 Exercises

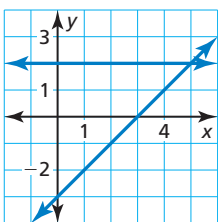
## Vocabulary and Core Concept Check

- REASONING** The graphs of the equations  $y = 3x - 20$  and  $y = -2x + 10$  intersect at the point  $(6, -2)$ . Without solving, find the solution of the equation  $3x - 20 = -2x + 10$ .
- WRITING** Consider the equation  $ax + b = cx + d$ , where  $c = 0$ . Can you solve the equation by graphing a system of linear equations? Explain.

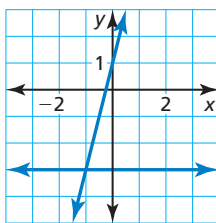
## Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, use the graph to solve the equation. Check your solution.

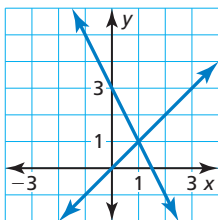
3.  $x - 3 = 2$



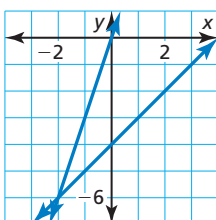
4.  $-3 = 4x + 1$



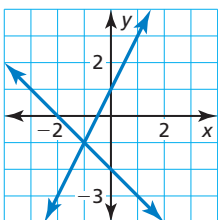
5.  $-2x + 3 = x$



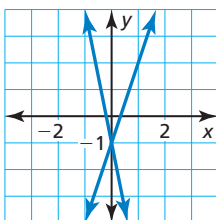
6.  $x - 4 = 3x$



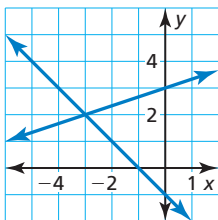
7.  $2x + 1 = -x - 2$



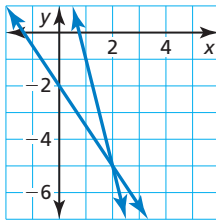
8.  $-5x - 1 = 3x - 1$



9.  $-x - 1 = \frac{1}{3}x + 3$



10.  $-\frac{3}{2}x - 2 = -4x + 3$



In Exercises 11–18, solve the equation by graphing. Check your solution. (See Example 1.)

11.  $x + 4 = -x$

12.  $4x = x + 3$

13.  $x + 5 = -2x - 4$

14.  $-2x + 6 = 5x - 1$

15.  $\frac{1}{2}x - 2 = 9 - 5x$

16.  $-5 + \frac{1}{4}x = 3x + 6$

17.  $5x - 7 = 2(x + 1)$

18.  $-6(x + 4) = -3x - 6$

In Exercises 19–24, solve the equation by graphing. Determine whether the equation has *one solution*, *no solution*, or *infinitely many solutions*.

19.  $3x - 1 = -x + 7$

20.  $5x - 4 = 5x + 1$

21.  $-4(2 - x) = 4x - 8$

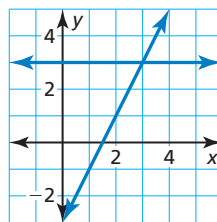
22.  $-2x - 3 = 2(x - 2)$

23.  $-x - 5 = -\frac{1}{3}(3x + 5)$

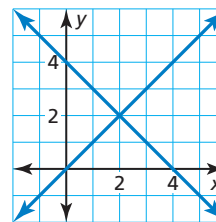
24.  $\frac{1}{2}(8x + 3) = 4x + \frac{3}{2}$

In Exercises 25–28, write an equation that has the same solution as the linear system represented by the graph.

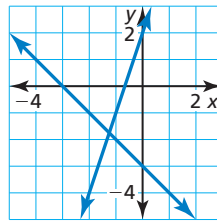
25.



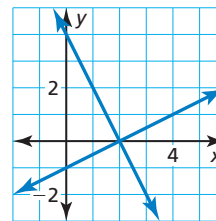
26.



27.



28.



**USING TOOLS** In Exercises 29 and 30, use a graphing calculator to solve the equation.

29.  $0.7x + 0.5 = -0.2x - 1.3$

30.  $2.1x + 0.6 = -1.4x + 6.9$

31. **MODELING WITH MATHEMATICS** You need to hire a catering company to serve meals to guests at a wedding reception. Company A charges \$500 plus \$20 per guest. Company B charges \$800 plus \$16 per guest. For how many guests are the total costs the same at both companies? (See Examples 2 and 3.)

32. **MODELING WITH MATHEMATICS** Your dog is 16 years old in dog years. Your cat is 28 years old in cat years. For every human year, your dog ages by 7 dog years and your cat ages by 4 cat years. In how many human years will both pets be the same age in their respective types of years?



33. **MODELING WITH MATHEMATICS** You and a friend race across a field to a fence. Your friend has a 50-meter head start. The equations shown represent you and your friend's distances  $d$  (in meters) from the fence  $t$  seconds after the race begins. Find the time at which you catch up to your friend.

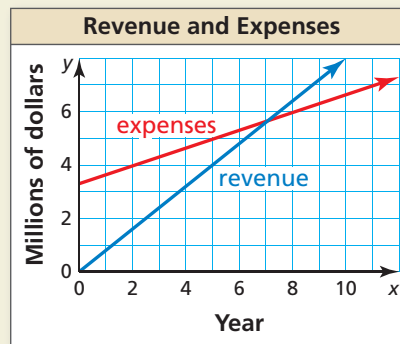
You:  $d = -5t + 200$

Your friend:  $d = -3\frac{1}{3}t + 150$

34. **MAKING AN ARGUMENT** The graphs of  $y = -x + 4$  and  $y = 2x - 8$  intersect at the point  $(4, 0)$ . So, your friend says the solution of the equation  $-x + 4 = 2x - 8$  is  $(4, 0)$ . Is your friend correct? Explain.

35. **OPEN-ENDED** Find values for  $m$  and  $b$  so that the solution of the equation  $mx + b = -2x - 1$  is  $x = -3$ .

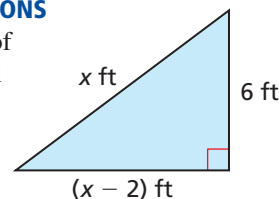
36. **HOW DO YOU SEE IT?** The graph shows the total revenue and expenses of a company  $x$  years after it opens for business.



- Estimate the point of intersection of the graphs.
- Interpret your answer in part (a).

37. **MATHEMATICAL CONNECTIONS**

The value of the perimeter of the triangle (in feet) is equal to the value of the area of the triangle (in square feet). Use a graph to find  $x$ .



38. **THOUGHT PROVOKING** A car has an initial value of \$20,000 and decreases in value at a rate of \$1500 per year. Describe a different car that will be worth the same amount as this car in exactly 5 years. Specify the initial value and the rate at which the value decreases.

39. **ABSTRACT REASONING** Use a graph to determine the sign of the solution of the equation  $ax + b = cx + d$  in each situation.

- $0 < b < d$  and  $a < c$
- $d < b < 0$  and  $a < c$

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Graph the inequality. (Section 2.1)

40.  $y > 5$

41.  $x \leq -2$

42.  $n \geq 9$

43.  $c < -6$

Use the graphs of  $f$  and  $g$  to describe the transformation from the graph of  $f$  to the graph of  $g$ . (Section 3.7)

44.  $f(x) = x - 5$ ;  $g(x) = f(x + 2)$

45.  $f(x) = 6x$ ;  $g(x) = -f(x)$

46.  $f(x) = -2x + 1$ ;  $g(x) = f(4x)$

47.  $f(x) = \frac{1}{2}x - 2$ ;  $g(x) = f(x - 1)$

# 5.6 Linear Inequalities in Two Variables



TEXAS ESSENTIAL  
KNOWLEDGE AND SKILLS

A.2.H  
A.3.D

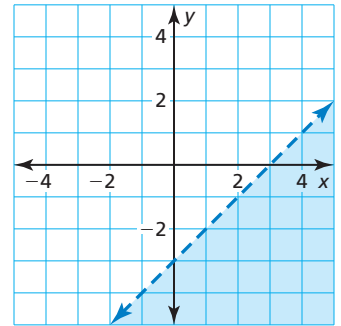
**Essential Question** How can you write and graph a linear inequality in two variables?

A **solution of a linear inequality in two variables** is an ordered pair  $(x, y)$  that makes the inequality true. The **graph of a linear inequality** in two variables shows all the solutions of the inequality in a coordinate plane.

## EXPLORATION 1 Writing a Linear Inequality in Two Variables

**Work with a partner.**

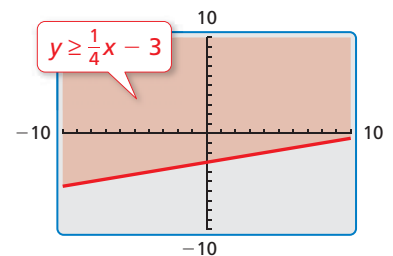
- Write an equation represented by the dashed line.
- The solutions of an inequality are represented by the shaded region. In words, describe the solutions of the inequality.
- Write an inequality represented by the graph. Which inequality symbol did you use? Explain your reasoning.



## EXPLORATION 2 Using a Graphing Calculator

**Work with a partner.** Use a graphing calculator to graph  $y \geq \frac{1}{4}x - 3$ .

- Enter the equation  $y = \frac{1}{4}x - 3$  into your calculator.
- The inequality has the symbol  $\geq$ . So, the region to be shaded is above the graph of  $y = \frac{1}{4}x - 3$ , as shown. Verify this by testing a point in this region, such as  $(0, 0)$ , to make sure it is a solution of the inequality.



### SELECTING TOOLS

To be proficient in math, you need to use technological tools to explore and deepen your understanding of concepts.

Because the inequality symbol is *greater than or equal to*, the line is solid and not dashed. Some graphing calculators always use a solid line when graphing inequalities. In this case, you have to determine whether the line should be solid or dashed, based on the inequality symbol used in the original inequality.

## EXPLORATION 3 Graphing Linear Inequalities in Two Variables

**Work with a partner.** Graph each linear inequality in two variables. Explain your steps. Use a graphing calculator to check your graphs.

- $y > x + 5$
- $y \leq -\frac{1}{2}x + 1$
- $y \geq -x - 5$

## Communicate Your Answer

- How can you write and graph a linear inequality in two variables?
- Give an example of a real-life situation that can be modeled using a linear inequality in two variables.

# 5.6 Lesson

## Core Vocabulary

linear inequality in two variables, p. 252  
 solution of a linear inequality in two variables, p. 252  
 graph of a linear inequality, p. 252  
 half-planes, p. 252

**Previous**  
 ordered pair

## What You Will Learn

- ▶ Check solutions of linear inequalities.
- ▶ Graph linear inequalities in two variables.
- ▶ Write linear inequalities in two variables.
- ▶ Use linear inequalities to solve real-life problems.

## Linear Inequalities

A **linear inequality in two variables**,  $x$  and  $y$ , can be written as

$$ax + by < c \quad ax + by \leq c \quad ax + by > c \quad ax + by \geq c$$

where  $a$ ,  $b$ , and  $c$  are real numbers. A **solution of a linear inequality in two variables** is an ordered pair  $(x, y)$  that makes the inequality true.

### EXAMPLE 1 Checking Solutions

Tell whether the ordered pair is a solution of the inequality.

- a.  $2x + y < -3$ ;  $(-1, 9)$                       b.  $x - 3y \geq 8$ ;  $(2, -2)$

### SOLUTION

- a.  $2x + y < -3$                       Write the inequality.  
 $2(-1) + 9 \stackrel{?}{<} -3$                       Substitute  $-1$  for  $x$  and  $9$  for  $y$ .  
 $7 \not< -3$  ✗                      Simplify.  $7$  is *not* less than  $-3$ .

▶ So,  $(-1, 9)$  is *not* a solution of the inequality.

- b.  $x - 3y \geq 8$                       Write the inequality.  
 $2 - 3(-2) \stackrel{?}{\geq} 8$                       Substitute  $2$  for  $x$  and  $-2$  for  $y$ .  
 $8 \geq 8$  ✓                      Simplify.  $8$  is equal to  $8$ .

▶ So,  $(2, -2)$  is a solution of the inequality.

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Tell whether the ordered pair is a solution of the inequality.

1.  $x + y > 0$ ;  $(-2, 2)$                       2.  $4x - y \geq 5$ ;  $(0, 0)$   
 3.  $5x - 2y \leq -1$ ;  $(-4, -1)$                       4.  $-2x - 3y < 15$ ;  $(5, -7)$

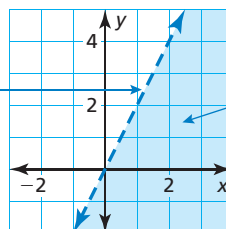
## Graphing Linear Inequalities in Two Variables

The **graph of a linear inequality** in two variables shows all the solutions of the inequality in a coordinate plane.

### READING

A dashed boundary line means that points on the line are *not* solutions. A solid boundary line means that points on the line are solutions.

All solutions of  $y < 2x$  lie on one side of the boundary line  $y = 2x$ .



The boundary line divides the coordinate plane into two **half-planes**. The shaded half-plane is the graph of  $y < 2x$ .

## Core Concept

### Graphing a Linear Inequality in Two Variables

- Step 1** Graph the boundary line for the inequality. Use a dashed line for  $<$  or  $>$ . Use a solid line for  $\leq$  or  $\geq$ .
- Step 2** Test a point that is not on the boundary line to determine whether it is a solution of the inequality.
- Step 3** When the test point is a solution, shade the half-plane that contains the point. When the test point is *not* a solution, shade the half-plane that does *not* contain the point.

### STUDY TIP

It is often convenient to use the origin as a test point. However, you must choose a different test point when the origin is on the boundary line.

### EXAMPLE 2 Graphing a Linear Inequality in One Variable

Graph  $y \leq 2$  in a coordinate plane.

#### SOLUTION

**Step 1** Graph  $y = 2$ . Use a solid line because the inequality symbol is  $\leq$ .

**Step 2** Test  $(0, 0)$ .

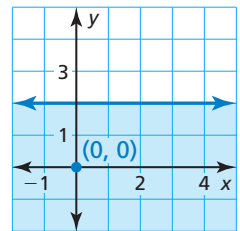
$$y \leq 2$$

Write the inequality.

$$0 \leq 2 \quad \checkmark$$

Substitute.

**Step 3** Because  $(0, 0)$  is a solution, shade the half-plane that contains  $(0, 0)$ .



### EXAMPLE 3 Graphing a Linear Inequality in Two Variables

Graph  $-x + 2y > 2$  in a coordinate plane.

#### SOLUTION

**Step 1** Graph  $-x + 2y = 2$ , or  $y = \frac{1}{2}x + 1$ . Use a dashed line because the inequality symbol is  $>$ .

**Step 2** Test  $(0, 0)$ .

$$-x + 2y > 2$$

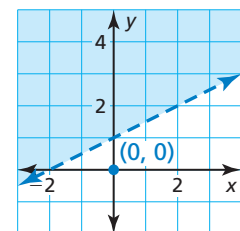
Write the inequality.

$$-(0) + 2(0) \stackrel{?}{>} 2$$

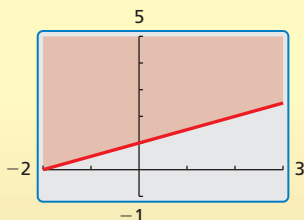
Substitute.

$$0 \not> 2 \quad \times \quad \text{Simplify.}$$

**Step 3** Because  $(0, 0)$  is *not* a solution, shade the half-plane that does *not* contain  $(0, 0)$ .



#### Check



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Graph the inequality in a coordinate plane.

5.  $y > -1$

6.  $x \leq -4$

7.  $x + y \leq -4$

8.  $x - 2y < 0$



# Writing Linear Inequalities in Two Variables

## Core Concept

### Writing a Linear Inequality in Two Variables Using a Graph

Write an equation in slope-intercept form of the boundary line.

If the shaded half-plane is *above* the boundary line, then

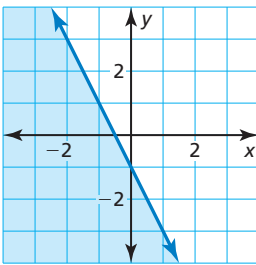
- replace = with  $>$  when the boundary line is *dashed*.
- replace = with  $\geq$  when the boundary line is *solid*.

If the shaded half-plane is *below* the boundary line, then

- replace = with  $<$  when the boundary line is *dashed*.
- replace = with  $\leq$  when the boundary line is *solid*.

		Boundary Line	
		Dashed	Solid
Half-plane	Above	$>$	$\geq$
	Below	$<$	$\leq$

### EXAMPLE 4 Writing a Linear Inequality Using a Graph



Write an inequality that represents the graph.

#### SOLUTION

The boundary line has a slope of  $-2$  and a  $y$ -intercept of  $-1$ . So, an equation of the boundary line is  $y = -2x - 1$ . The shaded half-plane is *below* the boundary line and the boundary line is *solid*. So, replace = with  $\leq$ .

► The inequality  $y \leq -2x - 1$  represents the graph.

### EXAMPLE 5 Writing a Linear Inequality Using a Table

Flavor	Quantity (gallons)
vanilla	20
chocolate	15
strawberry	$x$
banana	$y$
sherbet	18

An ice cream truck can carry at most 75 gallons of ice cream. The table shows the inventory on the truck. Write an inequality that represents the numbers of gallons of strawberry and banana ice cream on the truck.

#### SOLUTION

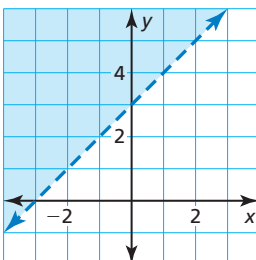
Use the table to write an inequality that represents the problem.

$$20 + 15 + x + y + 18 \leq 75 \quad \text{The total number of gallons is less than or equal to 75.}$$

$$x + y \leq 22 \quad \text{Isolate the variable terms on one side.}$$

► The inequality  $x + y \leq 22$  represents the numbers of gallons of strawberry and banana ice cream on the truck.

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- Write an inequality that represents the graph.
- For a weight loss competition, the blue team needs to lose more than 55 pounds to win. Write an inequality that represents the amounts (in pounds) of weight Team Members B and E must lose so that the blue team wins.

Blue team member	A	B	C	D	E	F
Weight lost (pounds)	10	$y$	8	13	$x$	5

## Solving Real-Life Problems

### EXAMPLE 6 Modeling with Mathematics



You can spend at most \$10 on grapes and apples for a fruit salad. Grapes cost \$2.50 per pound, and apples cost \$1 per pound. Write and graph an inequality that represents the amounts of grapes and apples you can buy. Identify and interpret two solutions of the inequality.

#### SOLUTION

- 1. Understand the Problem** You know the most that you can spend and the prices per pound for grapes and apples. You are asked to write and graph an inequality and then identify and interpret two solutions.
- 2. Make a Plan** Use a verbal model to write an inequality that represents the problem. Then graph the inequality. Use the graph to identify two solutions. Then interpret the solutions.
- 3. Solve the Problem**

**Words**      Cost per pound of grapes   •   Pounds of grapes   +   Cost per pound of apples   •   Pounds of apples   ≤   Amount you can spend

**Variables** Let  $x$  be pounds of grapes and  $y$  be pounds of apples.

**Inequality**  $2.50 \cdot x + 1 \cdot y \leq 10$

**Step 1** Graph  $2.5x + y = 10$ , or  $y = -2.5x + 10$ . Use a solid line because the inequality symbol is  $\leq$ . Restrict the graph to positive values of  $x$  and  $y$  because negative values do not make sense in this real-life context.

**Step 2** Test  $(0, 0)$ .

$$\begin{array}{ll} 2.5x + y \leq 10 & \text{Write the inequality.} \\ 2.5(0) + 0 \stackrel{?}{\leq} 10 & \text{Substitute.} \\ 0 \leq 10 \quad \checkmark & \text{Simplify.} \end{array}$$

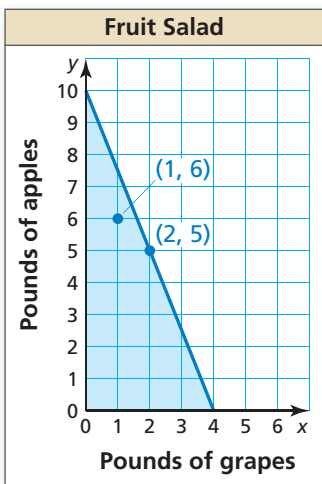
**Step 3** Because  $(0, 0)$  is a solution, shade the half-plane that contains  $(0, 0)$ .

- ▶ One possible solution is  $(1, 6)$  because it lies in the shaded half-plane. Another possible solution is  $(2, 5)$  because it lies on the solid line. So, you can buy 1 pound of grapes and 6 pounds of apples, or 2 pounds of grapes and 5 pounds of apples.

- 4. Look Back** Check your solutions by substituting them into the original inequality, as shown.

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- You can spend at most \$12 on red peppers and tomatoes for salsa. Red peppers cost \$4 per pound, and tomatoes cost \$3 per pound. Write and graph an inequality that represents the amounts of red peppers and tomatoes you can buy. Identify and interpret two solutions of the inequality.



#### Check

$$\begin{array}{l} 2.5x + y \leq 10 \\ 2.5(1) + 6 \stackrel{?}{\leq} 10 \\ 8.5 \leq 10 \quad \checkmark \\ \\ 2.5x + y \leq 10 \\ 2.5(2) + 5 \stackrel{?}{\leq} 10 \\ 10 \leq 10 \quad \checkmark \end{array}$$

# 5.6 Exercises

## Vocabulary and Core Concept Check

- VOCABULARY** How can you tell whether an ordered pair is a solution of a linear inequality?
- WRITING** Compare the graph of a linear inequality in two variables with the graph of a linear equation in two variables.

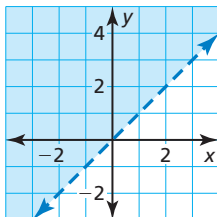
## Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, tell whether the ordered pair is a solution of the inequality. (See Example 1.)

- $x + y < 7$ ; (2, 3)
- $x - y \leq 0$ ; (5, 2)
- $x + 3y \geq -2$ ; (-9, 2)
- $8x + y > -6$ ; (-1, 2)
- $-6x + 4y \leq 6$ ; (-3, -3)
- $3x - 5y \geq 2$ ; (-1, -1)
- $-x - 6y > 12$ ; (-8, 2)
- $-4x - 8y < 15$ ; (-6, 3)

In Exercises 11–16, tell whether the ordered pair is a solution of the inequality whose graph is shown.

- (0, -1)
- (-1, 3)
- (1, 4)
- (0, 0)
- (3, 3)
- (2, 1)



17. **MODELING WITH MATHEMATICS** A carpenter has at most \$250 to spend on lumber. The inequality  $8x + 12y \leq 250$  represents the numbers  $x$  of 2-by-8 boards and the numbers  $y$  of 4-by-4 boards the carpenter can buy. Can the carpenter buy twelve 2-by-8 boards and fourteen 4-by-4 boards? Explain.



4 in. x 4 in. x 8 ft  
\$12 each

2 in. x 8 in. x 8 ft  
\$8 each

18. **MODELING WITH MATHEMATICS** The inequality  $3x + 2y \geq 93$  represents the numbers  $x$  of multiple-choice questions and the numbers  $y$  of matching questions you can answer correctly to receive an A on a test. You answer 20 multiple-choice questions and 18 matching questions correctly. Do you receive an A on the test? Explain.

In Exercises 19–24, graph the inequality in a coordinate plane. (See Example 2.)

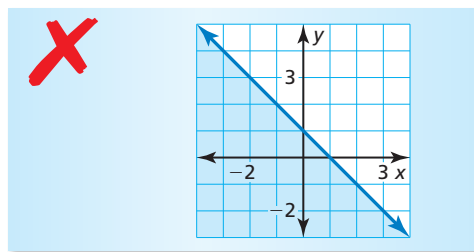
- $y \leq 5$
- $y > 6$
- $x < 2$
- $x \geq -3$
- $y > -7$
- $x < 9$

In Exercises 25–30, graph the inequality in a coordinate plane. (See Example 3.)

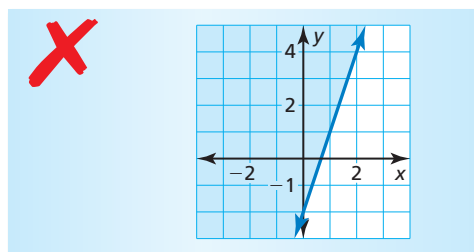
- $y > -2x - 4$
- $y \leq 3x - 1$
- $-4x + y < -7$
- $3x - y \geq 5$
- $5x - 2y \leq 6$
- $-x + 4y > -12$

**ERROR ANALYSIS** In Exercises 31 and 32, describe and correct the error in graphing the inequality.

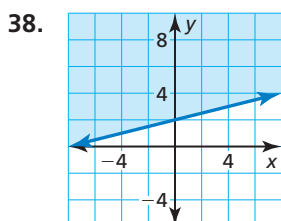
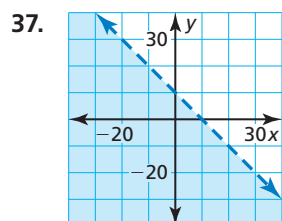
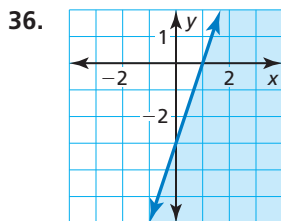
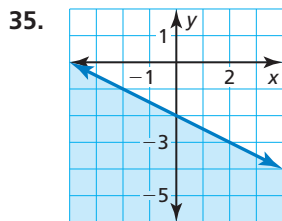
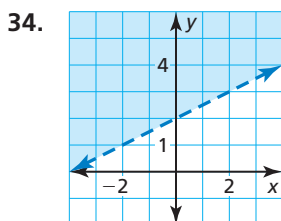
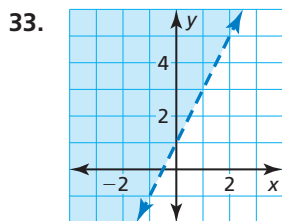
31.  $y < -x + 1$



32.  $y \leq 3x - 2$



In Exercises 33–38, write an inequality that represents the graph. (See Example 4.)



39. **MODELING WITH MATHEMATICS** A restaurant employee works at least 40 hours per week. The table shows the employee's weekly work schedule. Write an inequality that represents the numbers of hours the employee spends cooking and washing dishes. (See Example 5.)

Task	Hours
cleaning	6
cooking	$x$
ordering inventory	2
stocking food	3
washing dishes	$y$

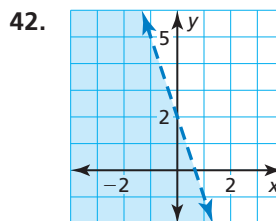
40. **MODELING WITH MATHEMATICS** You are allowed to watch TV for less than 10 hours total on school nights. The table shows the numbers of hours you watch TV. Write an inequality that represents the numbers of hours you can watch TV on Sunday and Wednesday.

School night	Hours
Sunday	$x$
Monday	1.5
Tuesday	0.5
Wednesday	$y$
Thursday	2.5

41. **MODELING WITH MATHEMATICS** A department store offers a 10% discount on any purchase over \$200. The table shows the costs of items in a purchase at the store. Write an inequality that represents the costs of the shirt and shoes that will qualify the purchase for the discount.

Item	Cost
backpack	\$54.99
jeans	\$39.99
shirt	$x$
shoes	$y$
video game	\$49.95

**ERROR ANALYSIS** In Exercises 42 and 43, describe and correct the error in writing the inequality.



$y > -3x + 2$

43. You eat no more than 2500 calories a day. The table shows the numbers of calories you eat at each meal.

Meal	Calories
breakfast	770
lunch	$x$
snack	150
dinner	$y$
snack	250

$x + y \leq 3670$

44. **MODELING WITH MATHEMATICS** You have at most \$20 to spend at an arcade. Arcade games cost \$0.75 each, and snacks cost \$2.25 each. Write and graph an inequality that represents the numbers of games you can play and snacks you can buy. Identify and interpret two solutions of the inequality. (See Example 6.)

45. **MODELING WITH MATHEMATICS** A drama club must sell at least \$1500 worth of tickets to cover the expenses of producing a play. Write and graph an inequality that represents how many adult and student tickets the club must sell. Identify and interpret two solutions of the inequality.



46. **MODELING WITH MATHEMATICS** A shipping company is delivering ceramic pots. The company earns \$1 for each unbroken ceramic pot and is fined \$4 for each broken ceramic pot. The company wants to earn at least \$2000 for this delivery. Write and graph an inequality that represents the numbers of unbroken and broken ceramic pots the company must deliver. Identify and interpret two solutions of the inequality.

47. **MODELING WITH MATHEMATICS** A clothing store sells T-shirts for \$20 and skirts for \$25. The store wants to sell a minimum of \$800 worth of T-shirts and skirts. Write and graph an inequality that represents the numbers of T-shirts and skirts the store must sell. Identify and interpret two solutions of the inequality.

48. **PROBLEM SOLVING** Large boxes weigh 75 pounds, and small boxes weigh 40 pounds.

- a. Write and graph an inequality that represents the numbers of large and small boxes a 200-pound delivery person can take on the elevator.
- b. Explain why some solutions of the inequality might not be practical in real life.

Weight limit:  
200 lb



49. **WRITING** Can you always use  $(0, 0)$  as a test point when graphing an inequality? Explain.

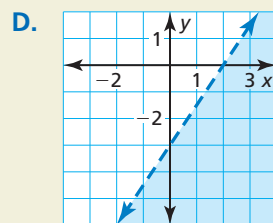
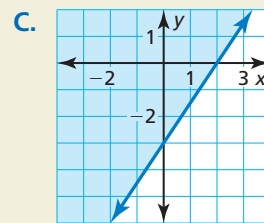
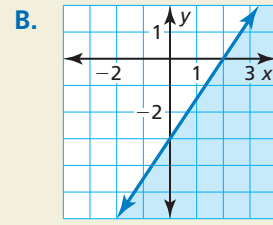
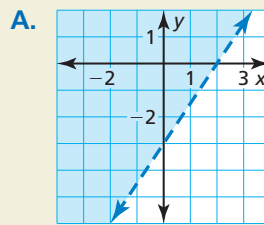
50. **HOW DO YOU SEE IT?** Match each inequality with its graph.

a.  $3x - 2y \leq 6$

b.  $3x - 2y < 6$

c.  $3x - 2y > 6$

d.  $3x - 2y \geq 6$



51. **REASONING** When graphing a linear inequality in two variables, why must you choose a test point that is *not* on the boundary line?

52. **THOUGHT PROVOKING** Write a linear inequality in two variables that has the following two properties.

- $(0, 0)$ ,  $(0, -1)$ , and  $(0, 1)$  are not solutions.
- $(1, 1)$ ,  $(3, -1)$ , and  $(-1, 3)$  are solutions.

**CRITICAL THINKING** In Exercises 53 and 54, write and graph an inequality whose graph is described by the given information.

53. The points  $(2, 5)$  and  $(-3, -5)$  lie on the boundary line. The points  $(6, 5)$  and  $(-2, -3)$  are solutions of the inequality.
54. The points  $(-7, -16)$  and  $(1, 8)$  lie on the boundary line. The points  $(-7, 0)$  and  $(3, 14)$  are *not* solutions of the inequality.

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write the next three terms of the arithmetic sequence. (Section 4.7)

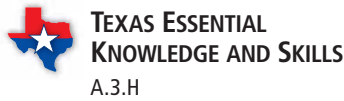
55.  $0, 8, 16, 24, 32, \dots$

56.  $-5, -8, -11, -14, -17, \dots$

57.  $3.6, 2.8, 2, 1.2, 0.4, \dots$

58.  $-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots$

# 5.7 Systems of Linear Inequalities



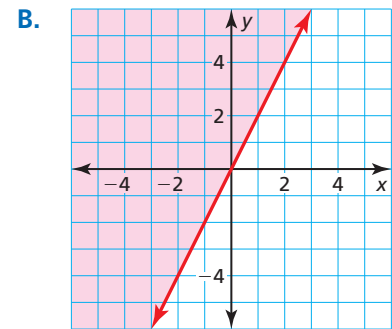
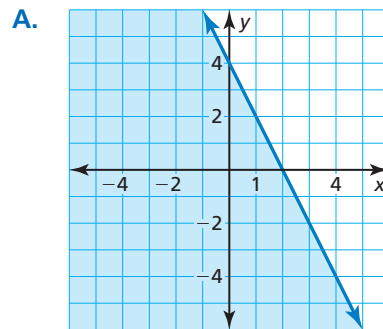
**Essential Question** How can you graph a system of linear inequalities?

## EXPLORATION 1 Graphing Linear Inequalities

**Work with a partner.** Match each linear inequality with its graph. Explain your reasoning.

$2x + y \leq 4$       Inequality 1

$2x - y \leq 0$       Inequality 2



## EXPLORATION 2 Graphing a System of Linear Inequalities

**Work with a partner.** Consider the linear inequalities given in Exploration 1.

$2x + y \leq 4$       Inequality 1

$2x - y \leq 0$       Inequality 2

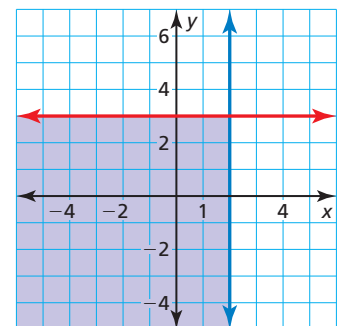
- Use two different colors to graph the inequalities in the same coordinate plane. What is the result?
- Describe each of the shaded regions of the graph. What does the unshaded region represent?

### USING PROBLEM-SOLVING STRATEGIES

To be proficient in math, you need to explain to yourself the meaning of a problem.

## Communicate Your Answer

- How can you graph a system of linear inequalities?
- When graphing a system of linear inequalities, which region represents the solution of the system?
- Do you think all systems of linear inequalities have a solution? Explain your reasoning.
- Write a system of linear inequalities represented by the graph.





## 5.7 Lesson

### Core Vocabulary

system of linear inequalities,  
p. 260  
solution of a system of linear  
inequalities, p. 260  
graph of a system of linear  
inequalities, p. 261

### Previous

linear inequality in two  
variables

## What You Will Learn

- ▶ Check solutions of systems of linear inequalities.
- ▶ Graph systems of linear inequalities.
- ▶ Write systems of linear inequalities.
- ▶ Use systems of linear inequalities to solve real-life problems.

## Systems of Linear Inequalities

A **system of linear inequalities** is a set of two or more linear inequalities in the same variables. An example is shown below.

$$y < x + 2 \quad \text{Inequality 1}$$

$$y \geq 2x - 1 \quad \text{Inequality 2}$$

A **solution of a system of linear inequalities** in two variables is an ordered pair that is a solution of each inequality in the system.

### EXAMPLE 1 Checking Solutions

Tell whether each ordered pair is a solution of the system of linear inequalities.

$$y < 2x \quad \text{Inequality 1}$$

$$y \geq x + 1 \quad \text{Inequality 2}$$

a. (3, 5)

b. (-2, 0)

### SOLUTION

a. Substitute 3 for  $x$  and 5 for  $y$  in each inequality.

$$\text{Inequality 1}$$

$$y < 2x$$

$$5 \stackrel{?}{<} 2(3)$$

$$5 < 6 \quad \checkmark$$

$$\text{Inequality 2}$$

$$y \geq x + 1$$

$$5 \stackrel{?}{\geq} 3 + 1$$

$$5 \geq 4 \quad \checkmark$$

- ▶ Because the ordered pair (3, 5) is a solution of each inequality, it is a solution of the system.

b. Substitute  $-2$  for  $x$  and  $0$  for  $y$  in each inequality.

$$\text{Inequality 1}$$

$$y < 2x$$

$$0 \stackrel{?}{<} 2(-2)$$

$$0 \not< -4 \quad \times$$

$$\text{Inequality 2}$$

$$y \geq x + 1$$

$$0 \stackrel{?}{\geq} -2 + 1$$

$$0 \geq -1 \quad \checkmark$$

- ▶ Because  $(-2, 0)$  is not a solution of each inequality, it is *not* a solution of the system.

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Tell whether the ordered pair is a solution of the system of linear inequalities.

1.  $(-1, 5)$ ;  $y < 5$   
 $y > x - 4$

2.  $(1, 4)$ ;  $y \geq 3x + 1$   
 $y > x - 1$

# Graphing Systems of Linear Inequalities

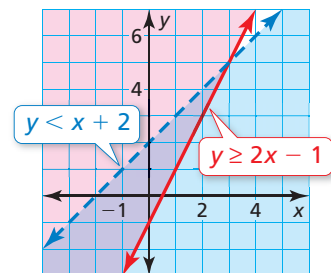
The **graph of a system of linear inequalities** is the graph of all the solutions of the system.

## Core Concept

### Graphing a System of Linear Inequalities

**Step 1** Graph each inequality in the same coordinate plane.

**Step 2** Find the intersection of the half-planes that are solutions of the inequalities. This intersection is the graph of the system.



### Check

Verify that  $(-3, 1)$  is a solution of each inequality.

**Inequality 1**

$$y \leq 3$$

$$1 \leq 3 \quad \checkmark$$

**Inequality 2**

$$y > x + 2$$

$$1 \stackrel{?}{>} -3 + 2$$

$$1 > -1 \quad \checkmark$$

### EXAMPLE 2 Graphing a System of Linear Inequalities

Graph the system of linear inequalities.

$$y \leq 3$$

**Inequality 1**

$$y > x + 2$$

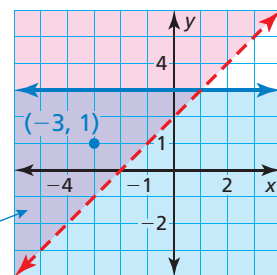
**Inequality 2**

### SOLUTION

**Step 1** Graph each inequality.

**Step 2** Find the intersection of the half-planes. One solution is  $(-3, 1)$ .

The solution is the purple-shaded region.



### EXAMPLE 3 Graphing a System of Linear Inequalities: No Solution

Graph the system of linear inequalities.

$$2x + y < -1$$

**Inequality 1**

$$2x + y > 3$$

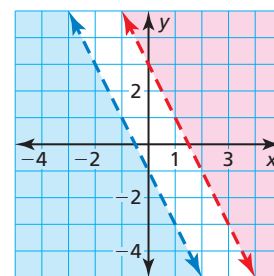
**Inequality 2**

### SOLUTION

**Step 1** Graph each inequality.

**Step 2** Find the intersection of the half-planes. Notice that the lines are parallel, and the half-planes do not intersect.

► So, the system has no solution.



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Graph the system of linear inequalities.

3.  $y \geq -x + 4$   
 $x + y \leq 0$

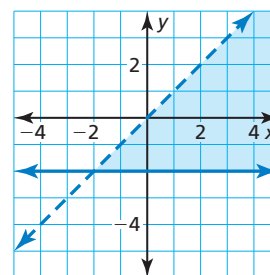
4.  $y > 2x - 3$   
 $y \geq \frac{1}{2}x + 1$

5.  $-2x + y < 4$   
 $2x + y > 4$

## Writing Systems of Linear Inequalities

### EXAMPLE 4 Writing a System of Linear Inequalities

Write a system of linear inequalities represented by the graph.



#### SOLUTION

**Inequality 1** The horizontal boundary line passes through  $(0, -2)$ . So, an equation of the line is  $y = -2$ . The shaded region is *above* the *solid* boundary line, so the inequality is  $y \geq -2$ .

**Inequality 2** The slope of the other boundary line is 1, and the  $y$ -intercept is 0. So, an equation of the line is  $y = x$ . The shaded region is *below* the *dashed* boundary line, so the inequality is  $y < x$ .

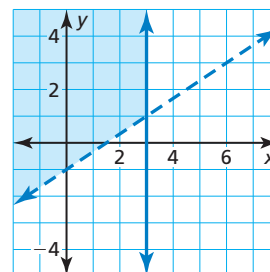
► The system of linear inequalities represented by the graph is

$$y \geq -2 \quad \text{Inequality 1}$$

$$y < x. \quad \text{Inequality 2}$$

### EXAMPLE 5 Writing a System of Linear Inequalities

Write a system of linear inequalities represented by the graph.



#### SOLUTION

**Inequality 1** The vertical boundary line passes through  $(3, 0)$ . So, an equation of the line is  $x = 3$ . The shaded region is to the *left* of the *solid* boundary line, so the inequality is  $x \leq 3$ .

**Inequality 2** The slope of the other boundary line is  $\frac{2}{3}$ , and the  $y$ -intercept is  $-1$ . So, an equation of the line is  $y = \frac{2}{3}x - 1$ . The shaded region is *above* the *dashed* boundary line, so the inequality is  $y > \frac{2}{3}x - 1$ .

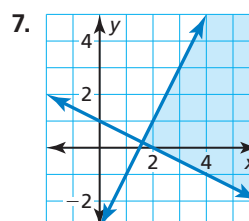
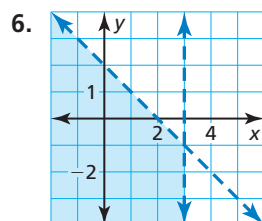
► The system of linear inequalities represented by the graph is

$$x \leq 3 \quad \text{Inequality 1}$$

$$y > \frac{2}{3}x - 1. \quad \text{Inequality 2}$$

### Monitoring Progress Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

Write a system of linear inequalities represented by the graph.



## Solving Real-Life Problems

### EXAMPLE 6 Modeling with Mathematics

You have at most 8 hours to spend at the mall and at the beach. You want to spend at least 2 hours at the mall and more than 4 hours at the beach. Write and graph a system that represents the situation. How much time can you spend at each location?



### SOLUTION

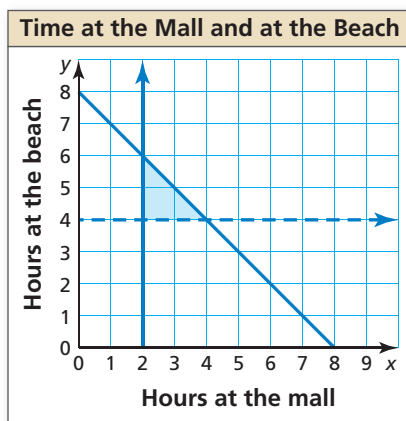
- Understand the Problem** You know the total amount of time you can spend at the mall and at the beach. You also know how much time you want to spend at each location. You are asked to write and graph a system that represents the situation and determine how much time you can spend at each location.
- Make a Plan** Use the given information to write a system of linear inequalities. Then graph the system and identify an ordered pair in the solution region.
- Solve the Problem** Let  $x$  be the number of hours at the mall and let  $y$  be the number of hours at the beach.

$$x + y \leq 8 \quad \text{at most 8 hours at the mall and at the beach}$$

$$x \geq 2 \quad \text{at least 2 hours at the mall}$$

$$y > 4 \quad \text{more than 4 hours at the beach}$$

Graph the system.



### Check

$$x + y \leq 8$$

$$2.5 + 5 \stackrel{?}{\leq} 8$$

$$7.5 \leq 8 \quad \checkmark$$

$$x \geq 2$$

$$2.5 \geq 2 \quad \checkmark$$

$$y > 4$$

$$5 > 4 \quad \checkmark$$

One ordered pair in the solution region is  $(2.5, 5)$ .

► So, you can spend 2.5 hours at the mall and 5 hours at the beach.

- Look Back** Check your solution by substituting it into the inequalities in the system, as shown.

### Monitoring Progress



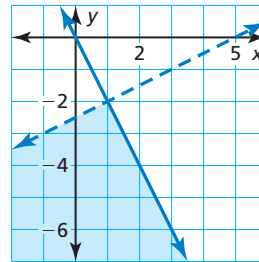
Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

- Name another solution of Example 6.
- WHAT IF?** You want to spend at least 3 hours at the mall. How does this change the system? Is  $(2.5, 5)$  still a solution? Explain.

# 5.7 Exercises

## Vocabulary and Core Concept Check

- VOCABULARY** How can you verify that an ordered pair is a solution of a system of linear inequalities?
- WHICH ONE DOESN'T BELONG?** Use the graph shown. Which of the ordered pairs does *not* belong with the other three? Explain your reasoning.



(1, -2)

(0, -4)

(-1, -6)

(2, -4)

## Monitoring Progress and Modeling with Mathematics

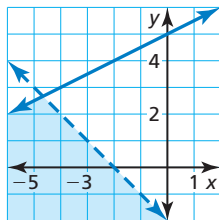
In Exercises 3–6, tell whether the ordered pair is a solution of the system of linear inequalities.

3.  $(-4, 3)$

4.  $(-3, -1)$

5.  $(-2, 5)$

6.  $(1, 1)$



In Exercises 7–10, tell whether the ordered pair is a solution of the system of linear inequalities.

(See Example 1.)

7.  $(-5, 2); \begin{cases} y < 4 \\ y > x + 3 \end{cases}$

8.  $(1, -1); \begin{cases} y > -2 \\ y > x - 5 \end{cases}$

9.  $(0, 0); \begin{cases} y \leq x + 7 \\ y \geq 2x + 3 \end{cases}$

10.  $(4, -3); \begin{cases} y \leq -x + 1 \\ y \leq 5x - 2 \end{cases}$

In Exercises 11–20, graph the system of linear inequalities. (See Examples 2 and 3.)

11.  $\begin{cases} y > -3 \\ y \geq 5x \end{cases}$

12.  $\begin{cases} y < -1 \\ x > 4 \end{cases}$

13.  $\begin{cases} y < -2 \\ y > 2 \end{cases}$

14.  $\begin{cases} y < x - 1 \\ y \geq x + 1 \end{cases}$

15.  $\begin{cases} y \geq -5 \\ y - 1 < 3x \end{cases}$

16.  $\begin{cases} x + y > 4 \\ y \geq \frac{3}{2}x - 9 \end{cases}$

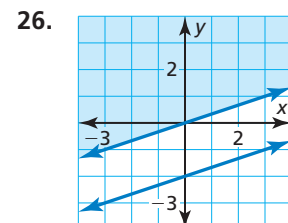
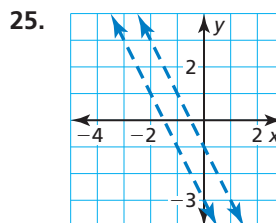
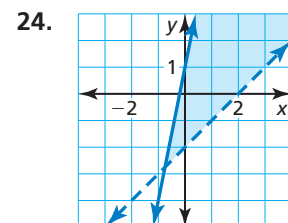
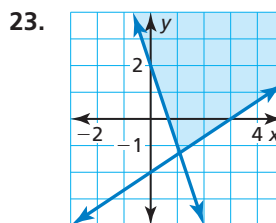
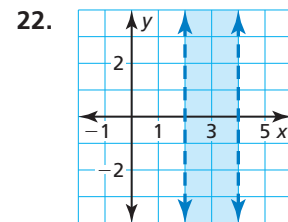
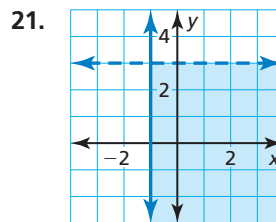
17.  $\begin{cases} x + y > 1 \\ -x - y < -3 \end{cases}$

18.  $\begin{cases} 2x + y \leq 5 \\ y + 2 \geq -2x \end{cases}$

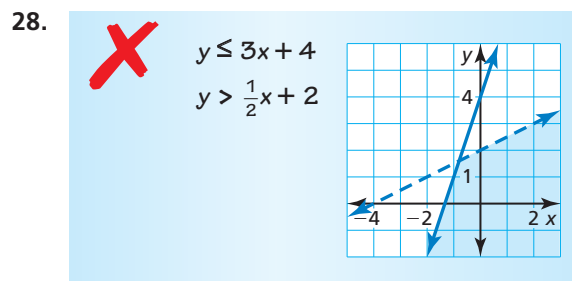
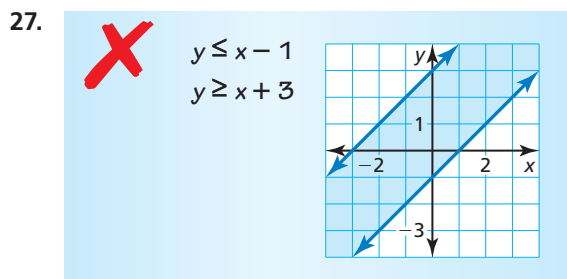
19.  $\begin{cases} x < 4 \\ y > 1 \\ y \geq -x + 1 \end{cases}$

20.  $\begin{cases} x + y \leq 10 \\ x - y \geq 2 \\ y > 2 \end{cases}$

In Exercises 21–26, write a system of linear inequalities represented by the graph. (See Examples 4 and 5.)



**ERROR ANALYSIS** In Exercises 27 and 28, describe and correct the error in graphing the system of linear inequalities.



29. **MODELING WITH MATHEMATICS** You can spend at most \$21 on fruit. Blueberries cost \$4 per pound, and strawberries cost \$3 per pound. You need at least 3 pounds of fruit to make muffins. (See Example 6.)

- Write and graph a system of linear inequalities that represents the situation.
- Identify and interpret a solution of the system.
- Use the graph to determine whether you can buy 4 pounds of blueberries and 1 pound of strawberries.



30. **MODELING WITH MATHEMATICS** You earn \$10 per hour working as a manager at a grocery store. You are required to work at the grocery store at least 8 hours per week. You also teach music lessons for \$15 per hour. You need to earn at least \$120 per week, but you do not want to work more than 20 hours per week.

- Write and graph a system of linear inequalities that represents the situation.
- Identify and interpret a solution of the system.
- Use the graph to determine whether you can work 8 hours at the grocery store and teach 1 hour of music lessons.

31. **MODELING WITH MATHEMATICS** You are fishing for surfperch and rockfish, which are species of bottomfish. Gaming laws allow you to catch no more than 15 surfperch per day, no more than 10 rockfish per day, and no more than 20 total bottomfish per day.

- Write and graph a system of linear inequalities that represents the situation.
- Use the graph to determine whether you can catch 11 surfperch and 9 rockfish in 1 day.



surfperch



rockfish

32. **REASONING** Describe the intersection of the half-planes of the system shown.

$$x - y \leq 4$$

$$x - y \geq 4$$

33. **MATHEMATICAL CONNECTIONS** The following points are the vertices of a shaded rectangle.

$$(-1, 1), (6, 1), (6, -3), (-1, -3)$$

- Write a system of linear inequalities represented by the shaded rectangle.
- Find the area of the rectangle.

34. **MATHEMATICAL CONNECTIONS** The following points are the vertices of a shaded triangle.

$$(2, 5), (6, -3), (-2, -3)$$

- Write a system of linear inequalities represented by the shaded triangle.
- Find the area of the triangle.

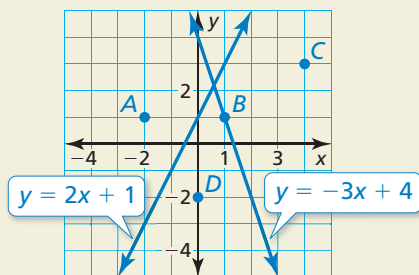
35. **PROBLEM SOLVING** You plan to spend less than half of your monthly \$2000 paycheck on housing and savings. You want to spend at least 10% of your paycheck on savings and at most 30% of it on housing. How much money can you spend on savings and housing?

36. **PROBLEM SOLVING** On a road trip with a friend, you drive about 70 miles per hour, and your friend drives about 60 miles per hour. The plan is to drive less than 15 hours and at least 600 miles each day. Your friend will drive more hours than you. How many hours can you and your friend each drive in 1 day?



37. **WRITING** How are solving systems of linear inequalities and solving systems of linear equations similar? How are they different?

38. **HOW DO YOU SEE IT?** The graphs of two linear equations are shown.



Replace the equal signs with inequality symbols to create a system of linear inequalities that has point C as a solution, but not points A, B, and D. Explain your reasoning.

$$y \quad \square \quad -3x + 4$$

$$y \quad \square \quad 2x + 1$$

39. **OPEN-ENDED** Write a real-life problem that can be represented by a system of linear inequalities. Write the system of linear inequalities and graph the system.

40. **MAKING AN ARGUMENT** Your friend says that a system of linear inequalities in which the boundary lines are parallel must have no solution. Is your friend correct? Explain.

41. **CRITICAL THINKING** Is it possible for the solution set of a system of linear inequalities to be all real numbers? Explain your reasoning.

**OPEN-ENDED** In Exercises 42–44, write a system of linear inequalities with the given characteristic.

42. All solutions are in Quadrant I.

43. All solutions have one positive coordinate and one negative coordinate.

44. There are no solutions.

45. **OPEN-ENDED** One inequality in a system is  $-4x + 2y > 6$ . Write another inequality so the system has (a) no solution and (b) infinitely many solutions.

46. **THOUGHT PROVOKING** You receive a gift certificate for a clothing store and plan to use it to buy T-shirts and sweatshirts. Describe a situation in which you can buy 9 T-shirts and 1 sweatshirt, but you cannot buy 3 T-shirts and 8 sweatshirts. Write and graph a system of linear inequalities that represents the situation.

47. **CRITICAL THINKING** Write a system of linear inequalities that has exactly one solution.

48. **MODELING WITH MATHEMATICS** You make necklaces and key chains to sell at a craft fair. The table shows the amounts of time and money it takes to make a necklace and a key chain, and the amounts of time and money you have available for making them.

	Necklace	Key chain	Available
Time to make (hours)	0.5	0.25	20
Cost to make (dollars)	2	3	120

a. Write and graph a system of four linear inequalities that represents the number  $x$  of necklaces and the number  $y$  of key chains that you can make.

b. Find the vertices (corner points) of the graph of the system.

c. You sell each necklace for \$10 and each key chain for \$8. The revenue  $R$  is given by the equation  $R = 10x + 8y$ . Find the revenue corresponding to each ordered pair in part (b). Which vertex results in the maximum revenue?

## Maintaining Mathematical Proficiency Reviewing what you learned in previous grades and lessons

Write the product using exponents. (*Skills Review Handbook*)

49.  $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

50.  $(-13) \cdot (-13) \cdot (-13)$

51.  $x \cdot x \cdot x \cdot x \cdot x \cdot x$

Write an equation of the line with the given slope and y-intercept. (*Section 4.1*)

52. slope: 1

53. slope:  $-3$

54. slope:  $-\frac{1}{4}$

55. slope:  $\frac{4}{3}$

y-intercept:  $-6$

y-intercept: 5

y-intercept:  $-1$

y-intercept: 0

## 5.5–5.7 What Did You Learn?

### Core Vocabulary

linear inequality in two variables, *p.* 252  
solution of a linear inequality in two variables,  
*p.* 252  
graph of a linear inequality, *p.* 252

half-planes, *p.* 252  
system of linear inequalities, *p.* 260  
solution of a system of linear inequalities, *p.* 260  
graph of a system of linear inequalities, *p.* 261

### Core Concepts

#### Section 5.5

Solving Linear Equations by Graphing, *p.* 246

#### Section 5.6

Graphing a Linear Inequality in Two Variables, *p.* 253  
Writing a Linear Inequality in Two Variables, *p.* 254

#### Section 5.7

Graphing a System of Linear Inequalities, *p.* 261  
Writing a System of Linear Inequalities, *p.* 262

### Mathematical Thinking

1. Describe how to solve Exercise 39 on page 250 algebraically.
2. Why is it important to be precise when answering part (a) of Exercise 48 on page 258?
3. Describe the overall step-by-step process you used to solve Exercise 35 on page 265.

### Performance Task

## Prize Patrol

You have been selected to drive a prize patrol cart and place prizes on the competing teams' predetermined paths. You know the teams' routes and you can only make one pass. Where will you place the prizes so that each team will have a chance to find a prize on their route?

To explore the answer to this question and more, go to [BigIdeasMath.com](http://BigIdeasMath.com).



# 5 Chapter Review

## 5.1 Solving Systems of Linear Equations by Graphing (pp. 219–224)

Solve the system by graphing.

$$y = x - 2 \quad \text{Equation 1}$$

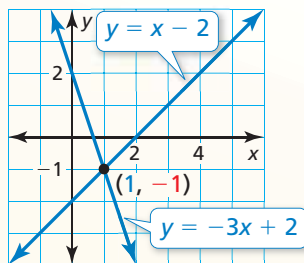
$$y = -3x + 2 \quad \text{Equation 2}$$

**Step 1** Graph each equation.

**Step 2** Estimate the point of intersection.  
The graphs appear to intersect at  $(1, -1)$ .

**Step 3** Check your point from Step 2.

Equation 1	Equation 2
$y = x - 2$	$y = -3x + 2$
$-1 \stackrel{?}{=} 1 - 2$	$-1 \stackrel{?}{=} -3(1) + 2$
$-1 = -1$ ✓	$-1 = -1$ ✓



▶ The solution is  $(1, -1)$ .

Solve the system of linear equations by graphing.

- |                  |                  |                   |
|------------------|------------------|-------------------|
| 1. $y = -3x + 1$ | 2. $y = -4x + 3$ | 3. $5x + 5y = 15$ |
| $y = x - 7$      | $4x - 2y = 6$    | $2x - 2y = 10$    |

## 5.2 Solving Systems of Linear Equations by Substitution (pp. 225–230)

Solve the system by substitution.

$$-2x + y = -8 \quad \text{Equation 1}$$

$$7x + y = 10 \quad \text{Equation 2}$$

**Step 1** Solve for  $y$  in Equation 1.

$$y = 2x - 8 \quad \text{Revised Equation 1}$$

**Step 2** Substitute  $2x - 8$  for  $y$  in Equation 2 and solve for  $x$ .

$7x + y = 10$	Equation 2
$7x + (2x - 8) = 10$	Substitute $2x - 8$ for $y$ .
$9x - 8 = 10$	Combine like terms.
$9x = 18$	Add 8 to each side.
$x = 2$	Divide each side by 9.

**Step 3** Substituting 2 for  $x$  in Equation 1 and solving for  $y$  gives  $y = -4$ .

▶ The solution is  $(2, -4)$ .

Solve the system of linear equations by substitution. Check your solution.

- |                  |                 |                  |
|------------------|-----------------|------------------|
| 4. $3x + y = -9$ | 5. $x + 4y = 6$ | 6. $2x + 3y = 4$ |
| $y = 5x + 7$     | $x - y = 1$     | $y + 3x = 6$     |

7. You spend \$20 total on tubes of paint and disposable brushes for an art project. Tubes of paint cost \$4.00 each and paintbrushes cost \$0.50 each. You purchase twice as many brushes as tubes of paint. How many brushes and tubes of paint do you purchase?

### 5.3 Solving Systems of Linear Equations by Elimination (pp. 231–236)

Solve the system by elimination.

$$\begin{array}{rcl} 4x + 6y = -8 & \text{Equation 1} \\ x - 2y = -2 & \text{Equation 2} \end{array}$$

**Step 1** Multiply Equation 2 by 3 so that the coefficients of the  $y$ -terms are opposites.

$$\begin{array}{rcl} 4x + 6y = -8 & \text{Equation 1} \\ x - 2y = -2 & \xrightarrow{\text{Multiply by 3.}} & 3x - 6y = -6 & \text{Revised Equation 2} \end{array}$$

**Step 2** Add the equations.

$$\begin{array}{rcl} 4x + 6y = -8 & \text{Equation 1} \\ \underline{3x - 6y = -6} & \text{Revised Equation 2} \\ 7x & = & -14 & \text{Add the equations.} \end{array}$$

**Step 3** Solve for  $x$ .

$$\begin{array}{rcl} 7x = -14 & \text{Resulting equation from Step 2} \\ x = -2 & \text{Divide each side by 7.} \end{array}$$

**Step 4** Substitute  $-2$  for  $x$  in one of the original equations and solve for  $y$ .

$$\begin{array}{rcl} 4x + 6y = -8 & \text{Equation 1} \\ 4(-2) + 6y = -8 & \text{Substitute } -2 \text{ for } x. \\ -8 + 6y = -8 & \text{Multiply.} \\ y = 0 & \text{Solve for } y. \end{array}$$

▶ The solution is  $(-2, 0)$ .

#### Check

Equation 1

$$4x + 6y = -8$$

$$4(-2) + 6(0) \stackrel{?}{=} -8$$

$$-8 = -8 \quad \checkmark$$

Equation 2

$$x - 2y = -2$$

$$(-2) - 2(0) \stackrel{?}{=} -2$$

$$-2 = -2 \quad \checkmark$$

Solve the system of linear equations by elimination. Check your solution.

8.  $9x - 2y = 34$

$$5x + 2y = -6$$

9.  $x + 6y = 28$

$$2x - 3y = -19$$

10.  $8x - 7y = -3$

$$6x - 5y = -1$$

### 5.4 Solving Special Systems of Linear Equations (pp. 237–242)

Solve the system.

$$\begin{array}{rcl} 4x + 2y = -14 & \text{Equation 1} \\ y = -2x - 6 & \text{Equation 2} \end{array}$$

Solve by substitution. Substitute  $-2x - 6$  for  $y$  in Equation 1.

$$\begin{array}{rcl} 4x + 2y = -14 & \text{Equation 1} \\ 4x + 2(-2x - 6) = -14 & \text{Substitute } -2x - 6 \text{ for } y. \\ 4x - 4x - 12 = -14 & \text{Distributive Property} \\ -12 = -14 \quad \times & \text{Combine like terms.} \end{array}$$

▶ The equation  $-12 = -14$  is never true. So, the system has no solution.

Solve the system of linear equations.

11.  $x = y + 2$

$$-3x + 3y = 6$$

12.  $3x - 6y = -9$

$$-5x + 10y = 10$$

13.  $-4x + 4y = 32$

$$3x + 24 = 3y$$

## 5.5 Solving Equations by Graphing (pp. 245–250)

Solve  $3x - 1 = -2x + 4$  by graphing. Check your solution.

**Step 1** Write a system of linear equations using each side of the original equation.

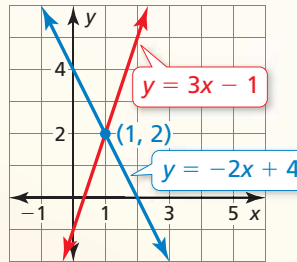
$$y = 3x - 1 \rightarrow 3x - 1 = -2x + 4 \leftarrow y = -2x + 4$$

**Step 2** Graph the system.

$$\begin{aligned} y &= 3x - 1 && \text{Equation 1} \\ y &= -2x + 4 && \text{Equation 2} \end{aligned}$$

The graphs intersect at  $(1, 2)$ .

► So, the solution of the equation is  $x = 1$ .



**Check**

$$\begin{aligned} 3x - 1 &= -2x + 4 \\ 3(1) - 1 &\stackrel{?}{=} -2(1) + 4 \\ 2 &= 2 \quad \checkmark \end{aligned}$$

Solve the equation by graphing. Check your solution(s).

14.  $x + 1 = -x - 9$

15.  $2x - 8 = x + 5$

16.  $\frac{1}{3}x + 5 = -2x - 2$

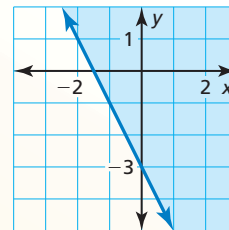
## 5.6 Linear Inequalities in Two Variables (pp. 251–258)

Graph  $4x + 2y \geq -6$  in a coordinate plane.

**Step 1** Graph  $4x + 2y = -6$ , or  $y = -2x - 3$ . Use a solid line because the inequality symbol is  $\geq$ .

**Step 2** Test  $(0, 0)$ .

$$\begin{aligned} 4x + 2y &\geq -6 && \text{Write the inequality.} \\ 4(0) + 2(0) &\stackrel{?}{\geq} -6 && \text{Substitute.} \\ 0 &\geq -6 \quad \checkmark && \text{Simplify.} \end{aligned}$$



**Step 3** Because  $(0, 0)$  is a solution, shade the half-plane that contains  $(0, 0)$ .

Graph the inequality in a coordinate plane.

17.  $y > -4$

18.  $-9x + 3y \geq 3$

19.  $5x + 10y < 40$

## 5.7 Systems of Linear Inequalities (pp. 259–266)

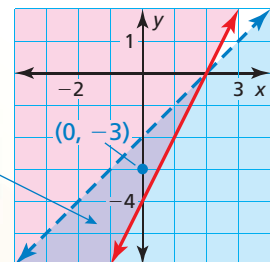
Graph the system.  $y < x - 2$  Inequality 1  
 $y \geq 2x - 4$  Inequality 2

**Step 1** Graph each inequality.

**Step 2** Find the intersection of the half-planes. One solution is  $(0, -3)$ .

Graph the system of linear inequalities.

The solution is the purple-shaded region.



20.  $y \leq x - 3$   
 $y \geq x + 1$

21.  $y > -2x + 3$   
 $y \geq \frac{1}{4}x - 1$

22.  $x + 3y > 6$   
 $2x + y < 7$

# 5 Chapter Test

Solve the system of linear equations using any method. Explain why you chose the method.

1.  $8x + 3y = -9$   
 $-8x + y = 29$

2.  $\frac{1}{2}x + y = -6$   
 $y = \frac{3}{5}x + 5$

3.  $y = 4x + 4$   
 $-8x + 2y = 8$

4.  $x = y - 11$   
 $x - 3y = 1$

5.  $6x - 4y = 9$   
 $9x - 6y = 15$

6.  $y = 5x - 7$   
 $-4x + y = -1$

7. Write a system of linear inequalities so the points  $(1, 2)$  and  $(4, -3)$  are solutions of the system, but the point  $(-2, 8)$  is not a solution of the system.

Graph the system of linear inequalities.

8.  $y > -1$   
 $x < 3$

9.  $y > \frac{1}{2}x + 4$   
 $2y \leq x + 4$

10.  $x + y < 1$   
 $5x + y > 4$

11.  $y \geq -\frac{2}{3}x + 1$   
 $-3x + y > -2$

12. You pay \$45.50 for 10 gallons of gasoline and 2 quarts of oil at a gas station. Your friend pays \$22.75 for 5 gallons of the same gasoline and 1 quart of the same oil.

- Is there enough information to determine the cost of 1 gallon of gasoline and 1 quart of oil? Explain.
- The receipt shown is for buying the same gasoline and same oil. Is there now enough information to determine the cost of 1 gallon of gasoline and 1 quart of oil? Explain.
- Determine the cost of 1 gallon of gasoline and 1 quart of oil.

WELCOME	
DATE 11/12/13	16:25
PUMP # 03	
PRODUCT:	REGUNL
GALLONS:	8,00
2 QUARTS OIL	
TOTAL:	\$38,40

THANK YOU  
HAVE A NICE DAY

13. Describe the advantages and disadvantages of solving a system of linear equations by graphing.
14. You have at most \$60 to spend on trophies and medals to give as prizes for a contest.
- Write and graph an inequality that represents the numbers of trophies and medals you can buy. Identify and interpret a solution of the inequality.
  - You want to purchase at least 6 items. Write and graph a system that represents the situation. How many of each item can you buy?
15. Compare the slopes and y-intercepts of the graphs of the equations in the linear system  $8x + 4y = 12$  and  $3y = -6x - 15$  to determine whether the system has one solution, no solution, or infinitely many solutions. Explain.

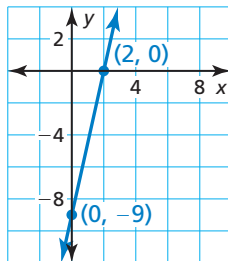




# 5 Standards Assessment

1. The graph of which equation is shown? (TEKS A.2.B)

- (A)  $9x - 2y = -18$
- (B)  $-9x - 2y = 18$
- (C)  $9x + 2y = 18$
- (D)  $-9x + 2y = -18$



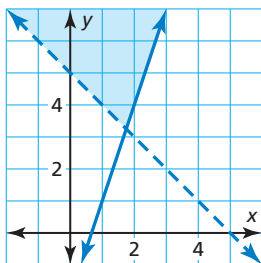
2. A van rental company rents out 6-, 8-, 12-, and 16-passenger vans. The function  $C(x) = 100 + 5x$  represents the cost  $C$  (in dollars) of renting an  $x$ -passenger van for a day. Which of the following numbers is in the range of the function? (TEKS A.2.A)

- (F) 6
- (G) 100
- (H) 130
- (J) 150

3. **GRIDDED ANSWER** The students in the graduating classes at three high schools in a school district have to pay for their cap-and-gown sets and extra tassels. At one high school, students pay \$3262 for 215 cap-and-gown sets and 72 extra tassels. At another high school, students pay \$3346 for 221 cap-and-gown sets and 72 extra tassels. How much (in dollars) do students at the third high school pay for 218 cap-and-gown sets and 56 extra tassels? (TEKS A.2.I, TEKS A.5.C)

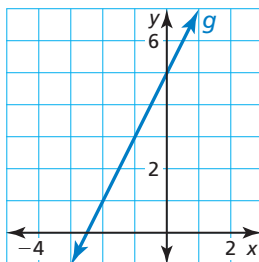
4. Which of the following points is *not* a solution of the system of linear inequalities represented by the graph? (TEKS A.3.H)

- (A) (4, 10)
- (B) (5, 8)
- (C) (-2, 8)
- (D) (0, 10)



5. Consider the function  $f(x) = 2x - 1$ . Which of the following functions are represented by the graph? (TEKS A.3.E)

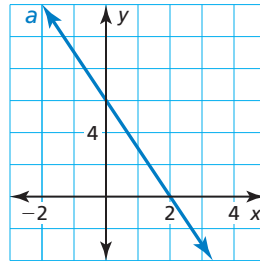
- I.  $g(x) = f(x) + 6$
- II.  $g(x) = f(x - 3)$
- III.  $g(x) = f(x + 3)$
- IV.  $g(x) = f(2x)$



- (F) I and II only
- (G) I and III only
- (H) I, II, and III only
- (J) I, II, III, and IV

6. Line  $b$  is perpendicular to line  $a$  and passes through the point  $(-9, 2)$ . Which of the following is an equation for line  $b$ ? (TEKS A.2.F)

- (A)  $y = \frac{1}{3}x + 5$   
 (B)  $y = -\frac{1}{3}x - 1$   
 (C)  $y = \frac{1}{3}x - \frac{29}{3}$   
 (D)  $y = -\frac{1}{3}x + 1$



7. Which system of linear equations has no solution? (TEKS A.5.C)

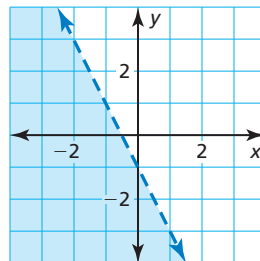
- (F)  $y - 2x = 5$   
 $4y - 10x = 20$   
 (H)  $2y = 6x + 4$   
 $y - \frac{1}{2} = 3x$   
 (G)  $y = -4x + 6$   
 $y = 8x - 12$   
 (J)  $y + 1 = 4x$   
 $y + 1 = \frac{1}{4}x$

8. A car dealership offers interest-free car loans for one day only. During this day, a salesperson at the dealership sells two cars. One of the clients decides to pay off a \$17,424 car in 36 monthly payments of \$484. The other client decides to pay off a \$15,840 car in 48 monthly payments of \$330. Which system of equations can you use to determine the number  $x$  of months after which both clients will have the same loan balance  $y$  (in dollars)? (TEKS A.2.I)

- (A)  $y = -484x$   
 $y = -330x$   
 (C)  $y = -484x + 15,840$   
 $y = -330x + 17,424$   
 (B)  $y = -484x + 17,424$   
 $y = -330x + 15,840$   
 (D)  $y = 484x + 17,424$   
 $y = 330x + 15,840$

9. The graph of which linear inequality is shown? (TEKS A.2.H)

- (F)  $y < -2x - 1$   
 (G)  $y \leq -2x - 1$   
 (H)  $y > -2x - 1$   
 (J)  $y \geq -2x - 1$



10. Simplify the expression  $4(2x + 4) - 6(x + 1)$ . (TEKS A.10.D)

- (A)  $2x + 10$   
 (C)  $2x + 17$   
 (B)  $2x + 15$   
 (D)  $2x + 22$