# 6.3 Multiplying and Dividing Rational Expressions

**Essential Question** How can you determine the excluded values in a product or quotient of two rational expressions?

You can multiply and divide rational expressions in much the same way that you multiply and divide fractions. Values that make the denominator of an expression zero are *excluded values*.

$$\frac{1}{\cancel{x}} \cdot \frac{\cancel{x}}{x+1} = \frac{1}{x+1},\ x \neq 0$$ Product of rational expressions

$$\frac{1}{x} \div \frac{x}{x+1} = \frac{1}{x} \cdot \frac{x+1}{x} = \frac{x+1}{x^2},\ x \neq -1$$ Quotient of rational expressions

## EXPLORATION 1 Multiplying and Dividing Rational Expressions

**Work with a partner.** Find the product or quotient of the two rational expressions. Then match the product or quotient with its excluded values. Explain your reasoning.

| Product or Quotient | Excluded Values |
|---|---|
| **a.** $\frac{1}{x-1} \cdot \frac{x-2}{x+1} =$ | **A.** $-1$, 0, and 2 |
| **b.** $\frac{1}{x-1} \cdot \frac{-1}{x-1} =$ | **B.** $-2$ and 1 |
| **c.** $\frac{1}{x-2} \cdot \frac{x-2}{x+1} =$ | **C.** $-2$, 0, and 1 |
| **d.** $\frac{x+2}{x-1} \cdot \frac{-x}{x+2} =$ | **D.** $-1$ and 2 |
| **e.** $\frac{x}{x+2} \div \frac{x+1}{x+2} =$ | **E.** $-1$, 0, and 1 |
| **f.** $\frac{x}{x-2} \div \frac{x+1}{x} =$ | **F.** $-1$ and 1 |
| **g.** $\frac{x}{x+2} \div \frac{x}{x-1} =$ | **G.** $-2$ and $-1$ |
| **h.** $\frac{x+2}{x} \div \frac{x+1}{x-1} =$ | **H.** 1 |

**REASONING ABSTRACTLY**

To be proficient in math, you need to know and flexibly use different properties of operations and objects.

## EXPLORATION 2 Writing a Product or Quotient

**Work with a partner.** Write a product or quotient of rational expressions that has the given excluded values. Justify your answer.

**a.** $-1$ **b.** $-1$ and 3 **c.** $-1$, 0, and 3

## Communicate Your Answer

**3.** How can you determine the excluded values in a product or quotient of two rational expressions?

**4.** Is it possible for the product or quotient of two rational expressions to have *no* excluded values? Explain your reasoning. If it is possible, give an example.

# 6.3 Lesson

## Core Vocabulary

rational expression, *p. 324*
simplified form of a rational expression, *p. 324*

***Previous***
fractions
polynomials
domain
equivalent expressions
reciprocal

## What You Will Learn

- Simplify rational expressions.
- Multiply rational expressions.
- Divide rational expressions.

## Simplifying Rational Expressions

A **rational expression** is a fraction whose numerator and denominator are nonzero polynomials. The *domain* of a rational expression excludes values that make the denominator zero. A rational expression is in **simplified form** when its numerator and denominator have no common factors (other than $\pm 1$).

## Core Concept

### Simplifying Rational Expressions

Let $a$, $b$, and $c$ be expressions with $b \neq 0$ and $c \neq 0$.

**Property** $\frac{a\cancel{c}}{b\cancel{c}} = \frac{a}{b}$ Divide out common factor $c$.

**Examples** $\frac{15}{65} = \frac{3 \cdot \cancel{5}}{13 \cdot \cancel{5}} = \frac{3}{13}$ Divide out common factor 5.

$\frac{4\cancel{(x+3)}}{(x+3)\cancel{(x+3)}} = \frac{4}{x+3}$ Divide out common factor $x + 3$.

### STUDY TIP

Notice that you can divide out common factors in the second expression at the right. You cannot, however, divide out like terms in the third expression.

Simplifying a rational expression usually requires two steps. First, factor the numerator and denominator. Then, divide out any factors that are common to both the numerator and denominator. Here is an example:

$$\frac{x^2 + 7x}{x^2} = \frac{x(x+7)}{x \cdot x} = \frac{x+7}{x}$$

### EXAMPLE 1 Simplifying a Rational Expression

Simplify $\frac{x^2 - 4x - 12}{x^2 - 4}$.

#### SOLUTION

$$\frac{x^2 - 4x - 12}{x^2 - 4} = \frac{(x+2)(x-6)}{(x+2)(x-2)}$$ Factor numerator and denominator.

$$= \frac{\cancel{(x+2)}(x-6)}{\cancel{(x+2)}(x-2)}$$ Divide out common factor.

$$= \frac{x-6}{x-2}, \quad x \neq -2$$ Simplified form

The original expression is undefined when $x = -2$. To make the original and simplified expressions equivalent, restrict the domain of the simplified expression by excluding $x = -2$. Both expressions are undefined when $x = 2$, so it is not necessary to list it.

### COMMON ERROR

Do not divide out variable terms that are not factors.

$\frac{x-6}{x-2} \neq \frac{-6}{-2}$

## Monitoring Progress

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**Simplify the rational expression, if possible.**

**1.** $\frac{2(x+1)}{(x+1)(x+3)}$ **2.** $\frac{x+4}{x^2-16}$ **3.** $\frac{4}{x(x+2)}$ **4.** $\frac{x^2-2x-3}{x^2-x-6}$

## Multiplying Rational Expressions

The rule for multiplying rational expressions is the same as the rule for multiplying numerical fractions: multiply numerators, multiply denominators, and write the new fraction in simplified form. Similar to rational numbers, rational expressions are closed under multiplication.

### Core Concept

**Multiplying Rational Expressions**

Let $a$, $b$, $c$, and $d$ be expressions with $b \neq 0$ and $d \neq 0$.

**Property** $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ Simplify $\frac{ac}{bd}$ if possible.

**Example** $\frac{5x^2}{2xy^2} \cdot \frac{6xy^3}{10y} = \frac{30x^3y^3}{20xy^3} = \frac{\cancel{10} \cdot 3 \cdot \cancel{x} \cdot x^2 \cdot \cancel{y^3}}{\cancel{10} \cdot 2 \cdot \cancel{x} \cdot \cancel{y^3}} = \frac{3x^2}{2}, \quad x \neq 0, y \neq 0$

### ANOTHER WAY

In Example 2, you can first simplify each rational expression, then multiply, and finally simplify the result.

$$\frac{8x^3y}{2xy^2} \cdot \frac{7x^4y^3}{4y}$$
$$= \frac{4x^2}{y} \cdot \frac{7x^4y^2}{4}$$
$$= \frac{\cancel{4} \cdot 7 \cdot x^6 \cdot \cancel{y} \cdot y}{\cancel{4} \cdot \cancel{y}}$$
$$= 7x^6y, \quad x \neq 0, y \neq 0$$

### EXAMPLE 2 Multiplying Rational Expressions

Find the product $\frac{8x^3y}{2xy^2} \cdot \frac{7x^4y^3}{4y}$.

**SOLUTION**

$$\frac{8x^3y}{2xy^2} \cdot \frac{7x^4y^3}{4y} = \frac{56x^7y^4}{8xy^3}$$ Multiply numerators and denominators.

$$= \frac{\cancel{8} \cdot 7 \cdot \cancel{x} \cdot x^6 \cdot \cancel{y^3} \cdot y}{\cancel{8} \cdot \cancel{x} \cdot \cancel{y^3}}$$ Factor and divide out common factors.

$$= 7x^6y, \quad x \neq 0, y \neq 0$$ Simplified form

### EXAMPLE 3 Multiplying Rational Expressions

Find the product $\frac{3x - 3x^2}{x^2 + 4x - 5} \cdot \frac{x^2 + x - 20}{3x}$.

**SOLUTION**

$$\frac{3x - 3x^2}{x^2 + 4x - 5} \cdot \frac{x^2 + x - 20}{3x} = \frac{3x(1 - x)}{(x - 1)(x + 5)} \cdot \frac{(x + 5)(x - 4)}{3x}$$ Factor numerators and denominators.

$$= \frac{3x(1 - x)(x + 5)(x - 4)}{(x - 1)(x + 5)(3x)}$$ Multiply numerators and denominators.

$$= \frac{3x(-1)(x - 1)(x + 5)(x - 4)}{(x - 1)(x + 5)(3x)}$$ Rewrite $1 - x$ as $(-1)(x - 1)$.

$$= \frac{\cancel{3x}(-1)\cancel{(x - 1)}\cancel{(x + 5)}(x - 4)}{\cancel{(x - 1)}\cancel{(x + 5)}\cancel{(3x)}}$$ Divide out common factors.

$$= -x + 4, \quad x \neq -5, x \neq 0, x \neq 1$$ Simplified form

**Check**

| X | Y1 | Y2 |
|---|---|---|
| −5 | ERROR | 9 |
| −4 | 8 | 8 |
| −3 | 7 | 7 |
| −2 | 6 | 6 |
| −1 | 5 | 5 |
| 0 | ERROR | 4 |
| 1 | ERROR | 3 |

X=−4

Check the simplified expression. Enter the original expression as $y_1$ and the simplified expression as $y_2$ in a graphing calculator. Then use the *table* feature to compare the values of the two expressions. The values of $y_1$ and $y_2$ are the same, except when $x = -5$, $x = 0$, and $x = 1$. So, when these values are excluded from the domain of the simplified expression, it is equivalent to the original expression.

## EXAMPLE 4 Multiplying a Rational Expression by a Polynomial

**STUDY TIP**

Notice that $x^2 + 3x + 9$ does not equal zero for any real value of $x$. So, no values must be excluded from the domain to make the simplified form equivalent to the original.

Find the product $\frac{x+2}{x^3-27} \cdot (x^2 + 3x + 9)$.

### SOLUTION

$$\frac{x+2}{x^3-27} \cdot (x^2+3x+9) = \frac{x+2}{x^3-27} \cdot \frac{x^2+3x+9}{1}$$ Write polynomial as a rational expression.

$$= \frac{(x+2)(x^2+3x+9)}{(x-3)(x^2+3x+9)}$$ Multiply. Factor denominator.

$$= \frac{(x+2)\cancel{(x^2+3x+9)}}{(x-3)\cancel{(x^2+3x+9)}}$$ Divide out common factor.

$$= \frac{x+2}{x-3}$$ Simplified form

### Monitoring Progress

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**Find the product.**

**5.** $\frac{3x^5y^2}{8xy} \cdot \frac{6xy^2}{9x^3y}$

**6.** $\frac{2x^2-10x}{x^2-25} \cdot \frac{x+3}{2x^2}$

**7.** $\frac{x+5}{x^3-1} \cdot (x^2+x+1)$

## Dividing Rational Expressions

To divide one rational expression by another, multiply the first rational expression by the reciprocal of the second rational expression. Rational expressions are closed under nonzero division.

### Core Concept

**Dividing Rational Expressions**

Let $a$, $b$, $c$, and $d$ be expressions with $b \neq 0$, $c \neq 0$, and $d \neq 0$.

**Property** $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$ Simplify $\frac{ad}{bc}$ if possible.

**Example** $\frac{7}{x+1} \div \frac{x+2}{2x-3} = \frac{7}{x+1} \cdot \frac{2x-3}{x+2} = \frac{7(2x-3)}{(x+1)(x+2)}, x \neq \frac{3}{2}$

## EXAMPLE 5 Dividing Rational Expressions

Find the quotient $\frac{7x}{2x-10} \div \frac{x^2-6x}{x^2-11x+30}$.

### SOLUTION

$$\frac{7x}{2x-10} \div \frac{x^2-6x}{x^2-11x+30} = \frac{7x}{2x-10} \cdot \frac{x^2-11x+30}{x^2-6x}$$ Multiply by reciprocal.

$$= \frac{7x}{2(x-5)} \cdot \frac{(x-5)(x-6)}{x(x-6)}$$ Factor.

$$= \frac{7\cancel{x}\cancel{(x-5)}\cancel{(x-6)}}{2\cancel{(x-5)}\cancel{(x)}\cancel{(x-6)}}$$ Multiply. Divide out common factors.

$$= \frac{7}{2}, \quad x \neq 0, x \neq 5, x \neq 6$$ Simplified form

## EXAMPLE 6 Dividing a Rational Expression by a Polynomial

Find the quotient $\frac{6x^2 + x - 15}{4x^2} \div (3x^2 + 5x)$.

### SOLUTION

$$\frac{6x^2 + x - 15}{4x^2} \div (3x^2 + 5x) = \frac{6x^2 + x - 15}{4x^2} \cdot \frac{1}{3x^2 + 5x}$$ Multiply by reciprocal.

$$= \frac{(3x + 5)(2x - 3)}{4x^2} \cdot \frac{1}{x(3x + 5)}$$ Factor.

$$= \frac{\cancel{(3x + 5)}(2x - 3)}{4x^2(x)\cancel{(3x + 5)}}$$ Multiply. Divide out common factor.

$$= \frac{2x - 3}{4x^3}, \quad x \neq -\frac{5}{3}$$ Simplified form

## EXAMPLE 7 Solving a Real-Life Problem

The total annual amount $I$ (in millions of dollars) of personal income earned in Alabama and its annual population $P$ (in millions) can be modeled by

$$I = \frac{6922t + 106{,}947}{0.0063t + 1}$$

and

$$P = 0.0343t + 4.432$$

where $t$ represents the year, with $t = 1$ corresponding to 2001. Find a model $M$ for the annual per capita income. (Per capita means per person.) Estimate the per capita income in 2010. (Assume $t > 0$.)

### SOLUTION

To find a model $M$ for the annual per capita income, divide the total amount $I$ by the population $P$.

$$M = \frac{6922t + 106{,}947}{0.0063t + 1} \div (0.0343t + 4.432)$$ Divide $I$ by $P$.

$$= \frac{6922t + 106{,}947}{0.0063t + 1} \cdot \frac{1}{0.0343t + 4.432}$$ Multiply by reciprocal.

$$= \frac{6922t + 106{,}947}{(0.0063t + 1)(0.0343t + 4.432)}$$ Multiply.

To estimate Alabama's per capita income in 2010, let $t = 10$ in the model.

$$M = \frac{6922 \cdot 10 + 106{,}947}{(0.0063 \cdot 10 + 1)(0.0343 \cdot 10 + 4.432)}$$ Substitute 10 for $t$.

$$\approx 34{,}707$$ Use a calculator.

▶ In 2010, the per capita income in Alabama was about \$34,707.

## Monitoring Progress

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**Find the quotient.**

**8.** $\frac{4x}{5x - 20} \div \frac{x^2 - 2x}{x^2 - 6x + 8}$

**9.** $\frac{2x^2 + 3x - 5}{6x} \div (2x^2 + 5x)$

# 6.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

## Vocabulary and Core Concept Check

**1. WRITING** Describe how to multiply and divide two rational expressions.

**2. WHICH ONE DOESN'T BELONG?** Which rational expression does *not* belong with the other three? Explain your reasoning.

| | | | |
|---|---|---|---|
| $\frac{x-4}{x^2}$ | $\frac{x^2+4x-12}{x^2+6x}$ | $\frac{9+x}{3x^2}$ | $\frac{x^2-x-12}{x^2-6x}$ |

## Monitoring Progress and Modeling with Mathematics

**In Exercises 3–10, simplify the expression, if possible.** *(See Example 1.)*

**3.** $\frac{2x^2}{3x^2-4x}$

**4.** $\frac{7x^3-x^2}{2x^3}$

**5.** $\frac{x^2-3x-18}{x^2-7x+6}$

**6.** $\frac{x^2+13x+36}{x^2-7x+10}$

**7.** $\frac{x^2+11x+18}{x^3+8}$

**8.** $\frac{x^2-7x+12}{x^3-27}$

**9.** $\frac{32x^4-50}{4x^3-12x^2-5x+15}$

**10.** $\frac{3x^3-3x^2+7x-7}{27x^4-147}$

**In Exercises 11–20, find the product.** *(See Examples 2, 3, and 4.)*

**11.** $\frac{4xy^3}{x^2y} \cdot \frac{y}{8x}$

**12.** $\frac{48x^5y^3}{y^4} \cdot \frac{x^2y}{6x^3y^2}$

**13.** $\frac{x^2(x-4)}{x-3} \cdot \frac{(x-3)(x+6)}{x^3}$

**14.** $\frac{x^3(x+5)}{x-9} \cdot \frac{(x-9)(x+8)}{3x^3}$

**15.** $\frac{x^2-3x}{x-2} \cdot \frac{x^2+x-6}{x}$

**16.** $\frac{x^2-4x}{x-1} \cdot \frac{x^2+3x-4}{2x}$

**17.** $\frac{x^2+3x-4}{x^2+4x+4} \cdot \frac{2x^2+4x}{x^2-4x+3}$

**18.** $\frac{x^2-x-6}{4x^3} \cdot \frac{2x^2+2x}{x^2+5x+6}$

**19.** $\frac{x^2+5x-36}{x^2-49} \cdot (x^2-11x+28)$

**20.** $\frac{x^2-x-12}{x^2-16} \cdot (x^2+2x-8)$

**21. ERROR ANALYSIS** Describe and correct the error in simplifying the rational expression.

✗

$$\frac{x^2 + \overset{2}{\cancel{16}}x + \overset{3}{\cancel{48}}}{x^2 + \underset{1}{\cancel{8}}x + \underset{1}{\cancel{16}}} = \frac{x^2+2x+3}{x^2+x+1}$$

**22. ERROR ANALYSIS** Describe and correct the error in finding the product.

✗

$$\frac{x^2-25}{3-x} \cdot \frac{x-3}{x+5} = \frac{(x+5)(x-5)}{3-x} \cdot \frac{x-3}{x+5}$$

$$= \frac{\cancel{(x+5)}(x-5)\cancel{(x-3)}}{\cancel{(3-x)}\cancel{(x+5)}}$$

$$= x-5,\ x \neq 3,\ x \neq -5$$

**23. USING STRUCTURE** Which rational expression is in simplified form?

Ⓐ $\frac{x^2-x-6}{x^2+3x+2}$

Ⓑ $\frac{x^2+6x+8}{x^2+2x-3}$

Ⓒ $\frac{x^2-6x+9}{x^2-2x-3}$

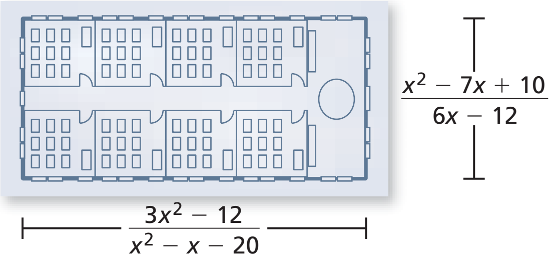
Ⓓ $\frac{x^2+3x-4}{x^2+x-2}$

**24. COMPARING METHODS** Find the product below by multiplying the numerators and denominators, then simplifying. Then find the product by simplifying each expression, then multiplying. Which method do you prefer? Explain.

$$\frac{4x^2y}{2x^3} \cdot \frac{12y^4}{24x^2}$$

**25. WRITING** Compare the function
$$f(x) = \frac{(3x - 7)(x + 6)}{(3x - 7)}$$ to the function $g(x) = x + 6$.

**26. MODELING WITH MATHEMATICS** Write a model in terms of $x$ for the total area of the base of the building.




**In Exercises 27–34, find the quotient.** *(See Examples 5 and 6.)*

**27.** $\frac{32x^3y}{y^8} \div \frac{y^7}{8x^4}$

**28.** $\frac{2xyz}{x^3z^3} \div \frac{6y^4}{2x^2z^2}$

**29.** $\frac{x^2 - x - 6}{2x^4 - 6x^3} \div \frac{x + 2}{4x^3}$

**30.** $\frac{2x^2 - 12x}{x^2 - 7x + 6} \div \frac{2x}{3x - 3}$

**31.** $\frac{x^2 - x - 6}{x + 4} \div (x^2 - 6x + 9)$

**32.** $\frac{x^2 - 5x - 36}{x + 2} \div (x^2 - 18x + 81)$

**33.** $\frac{x^2 + 9x + 18}{x^2 + 6x + 8} \div \frac{x^2 - 3x - 18}{x^2 + 2x - 8}$

**34.** $\frac{x^2 - 3x - 40}{x^2 + 8x - 20} \div \frac{x^2 + 13x + 40}{x^2 + 12x + 20}$

**In Exercises 35 and 36, use the following information.**

*Manufacturers often package products in a way that uses the least amount of material. One measure of the efficiency of a package is the ratio of its surface area S to its volume V. The smaller the ratio, the more efficient the packaging.*

**35.** You are examining three cylindrical containers.

**a.** Write an expression for the efficiency ratio $\frac{S}{V}$ of a cylinder.

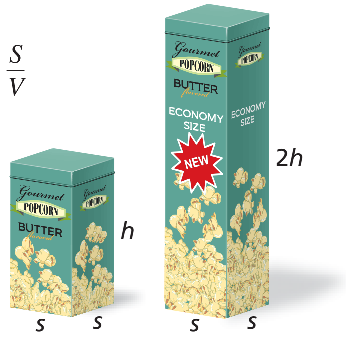
**b.** Find the efficiency ratio for each cylindrical can listed in the table. Rank the three cans according to efficiency.

| | Soup | Coffee | Paint |
|---|---|---|---|
| **Height, *h*** | 10.2 cm | 15.9 cm | 19.4 cm |
| **Radius, *r*** | 3.4 cm | 7.8 cm | 8.4 cm |

**36.** A popcorn company is designing a new tin with the same square base and twice the height of the old tin.

**a.** Write an expression for the efficiency ratio $\frac{S}{V}$ of each tin.

**b.** Did the company make a good decision by creating the new tin? Explain.




**37. MODELING WITH MATHEMATICS** The total amount $I$ (in millions of dollars) of healthcare expenditures and the residential population $P$ (in millions) in the United States can be modeled by

$$I = \frac{171{,}000t + 1{,}361{,}000}{1 + 0.018t} \quad \text{and}$$

$$P = 2.96t + 278.649$$

where $t$ is the number of years since 2000. Find a model $M$ for the annual healthcare expenditures per resident. Estimate the annual healthcare expenditures per resident in 2010. *(See Example 7.)*

**38. MODELING WITH MATHEMATICS** The total amount $I$ (in millions of dollars) of school expenditures from prekindergarten to a college level and the enrollment $P$ (in millions) in prekindergarten through college in the United States can be modeled by

$$I = \frac{17{,}913t + 709{,}569}{1 - 0.028t} \text{ and } P = 0.5906t + 70.219$$

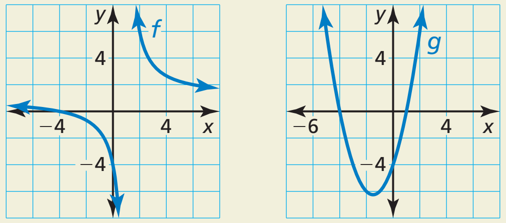
where $t$ is the number of years since 2001. Find a model $M$ for the annual education expenditures per student. Estimate the annual education expenditures per student in 2009.

**39. USING EQUATIONS** Refer to the population model $P$ in Exercise 37.

**a.** Interpret the meaning of the coefficient of $t$.

**b.** Interpret the meaning of the constant term.

**40.** **HOW DO YOU SEE IT?** Use the graphs of $f$ and $g$ to determine the excluded values of the functions $h(x) = (fg)(x)$ and $k(x) = \left(\frac{f}{g}\right)(x)$. Explain your reasoning.




**41.** **DRAWING CONCLUSIONS** Complete the table for the function $y = \frac{x+4}{x^2-16}$. Then use the *trace* feature of a graphing calculator to explain the behavior of the function at $x = -4$.

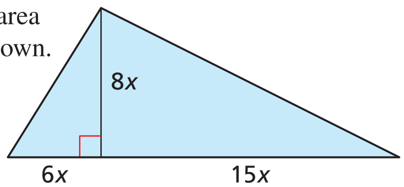
| $x$ | $y$ |
|---|---|
| −3.5 | |
| −3.8 | |
| −3.9 | |
| −4.1 | |
| −4.2 | |

**42.** **MAKING AN ARGUMENT** You and your friend are asked to state the domain of the expression below.

$$\frac{x^2 + 6x - 27}{x^2 + 4x - 45}$$

Your friend claims the domain is all real numbers except 5. You claim the domain is all real numbers except −9 and 5. Who is correct? Explain.

**43.** **MATHEMATICAL CONNECTIONS** Find the ratio of the perimeter to the area of the triangle shown.




**44.** **CRITICAL THINKING** Find the expression that makes the following statement true. Assume $x \neq -2$ and $x \neq 5$.

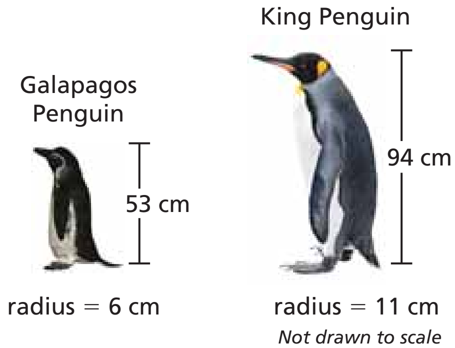
$$\frac{x-5}{x^2+2x-35} \div \frac{\square}{x^2-3x-10} = \frac{x+2}{x+7}$$

**USING STRUCTURE** **In Exercises 45 and 46, perform the indicated operations.**

**45.** $\frac{2x^2+x-15}{2x^2-11x-21} \cdot (6x+9) \div \frac{2x-5}{3x-21}$

**46.** $(x^3+8) \cdot \frac{x-2}{x^2-2x+4} \div \frac{x^2-4}{x-6}$

**47.** **REASONING** Animals that live in temperatures several degrees colder than their bodies must avoid losing heat to survive. Animals can better conserve body heat as their surface area to volume ratios decrease. Find the surface area to volume ratio of each penguin shown by using cylinders to approximate their shapes. Which penguin is better equipped to live in a colder environment? Explain your reasoning.




*Not drawn to scale*

**48.** **THOUGHT PROVOKING** Is it possible to write two radical functions whose product when graphed is a parabola and whose quotient when graphed is a hyperbola? Justify your answer.

**49.** **REASONING** Find two rational functions $f$ and $g$ that have the stated product and quotient.

$$(fg)(x) = x^2, \left(\frac{f}{g}\right)(x) = \frac{(x-1)^2}{(x+2)^2}$$

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

**Solve the equation. Check your solution.** *(Skills Review Handbook)*

**50.** $\frac{1}{2}x + 4 = \frac{3}{2}x + 5$ **51.** $\frac{1}{3}x - 2 = \frac{3}{4}x$ **52.** $\frac{1}{4}x - \frac{3}{5} = \frac{9}{2}x - \frac{4}{5}$ **53.** $\frac{1}{2}x + \frac{1}{3} = \frac{3}{4}x - \frac{1}{5}$

**Write the prime factorization of the number. If the number is prime, then write *prime*.** *(Skills Review Handbook)*

**54.** 42 **55.** 91 **56.** 72 **57.** 79