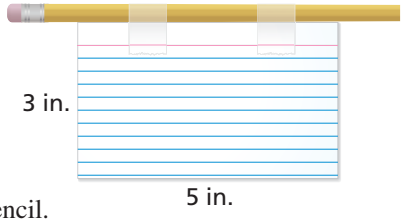
1.4 Solids of Revolution

Essential Question How can you create a solid of revolution?

A **solid of revolution** is a three-dimensional figure that is formed by rotating a two-dimensional shape around an axis.

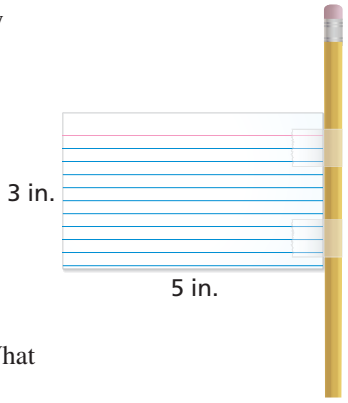
EXPLORATION 1 Creating Solids of Revolution

Work with a partner. Tape the 5-inch side of a 3-inch by 5-inch index card to a pencil, as shown.



a. Rotate the pencil. What type of solid is produced by the rotating index card? What are its dimensions?

b. Tape the 3-inch side of the index card to the pencil. Rotate the pencil. What type of solid is produced by the rotating index card? What are its dimensions?



c. Do the solids in parts (a) and (b) have the same surface area? the same volume? Justify your answers.

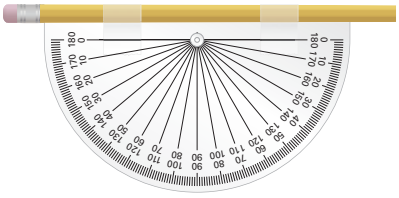
d. Cut the index card in half along its diagonal. Tape the 5-inch leg of the triangle formed to a pencil. Rotate the pencil. What type of solid is produced? What are its dimensions?

e. Tape the 3-inch leg to a pencil. Rotate the pencil. What type of solid is produced? What are its dimensions?

f. Do the solids in parts (d) and (e) have the same surface area? the same volume? Justify your answers.

> **USING TOOLS STRATEGICALLY**
> To be proficient in math, you need to use appropriate tools strategically, including real objects.

EXPLORATION 2 Creating Solids of Revolution

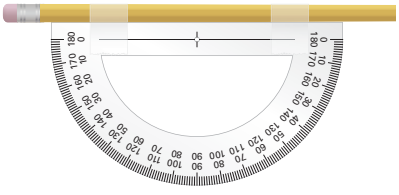


Work with a partner. Tape the straight side of a protractor, similar to the one at the left, to a pencil, as shown.

a. Rotate the pencil. What type of solid is produced by the rotating protractor? What are its dimensions?

b. Find the surface area and volume of the solid produced in part (a).

c. Tape the straight side of a protractor, similar to the one at the right, to a pencil, as shown. Rotate the pencil. Is the solid produced by this rotating protractor different from the solid in part (a)? Explain. Draw a diagram to support your answer.



d. Describe a method you might use to approximate the volume of the solid in part (c).

Communicate Your Answer

3. How can you create a solid of revolution?

4. Give some examples of real-life objects that are solids of revolution.

1.4 Lesson

Core Vocabulary

solid of revolution, *p. 24*
axis of revolution, *p. 24*

Previous
surface area
volume

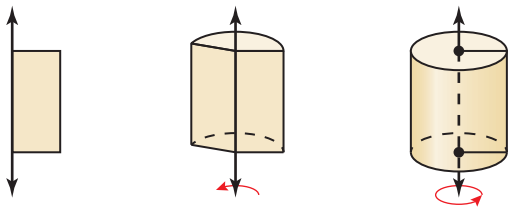
What You Will Learn

- Sketch and describe solids of revolution.
- Find surface areas and volumes of solids of revolution.
- Form solids of revolution in the coordinate plane.

Sketching and Describing Solids of Revolution

A **solid of revolution** is a three-dimensional figure that is formed by rotating a two-dimensional shape around an axis. The line around which the shape is rotated is called the **axis of revolution**.

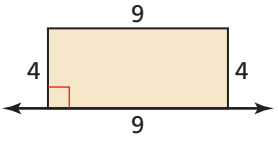
For example, when you rotate a rectangle around a line that contains one of its sides, the solid of revolution that is produced is a cylinder.



EXAMPLE 1 Sketching and Describing Solids of Revolution

Sketch the solid produced by rotating the figure around the given axis. Then identify and describe the solid.

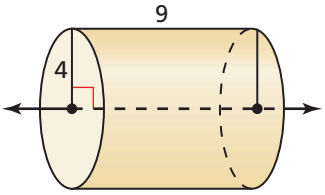
a.



b.

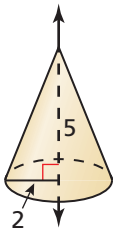
SOLUTION

a.



▶ The solid is a cylinder with a height of 9 units and a base radius of 4 units.

b.



▶ The solid is a cone with a height of 5 units and a base radius of 2 units.

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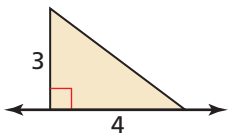
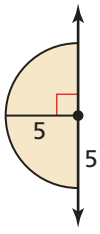
Sketch the solid produced by rotating the figure around the given axis. Then identify and describe the solid.

1.



2.



EXAMPLE 2 Sketching a Two-Dimensional Shape and Axis

Most vases are solids of revolution. Sketch a two-dimensional shape and an axis of revolution that forms the vase shown.

SOLUTION

The two-dimesional shape should match the outline of one side of the vase.

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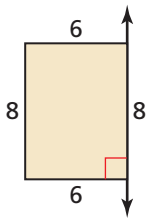
3. Sketch a two-dimensional shape and an axis of revolution that forms the bird bath shown.

Finding Surface Areas and Volumes of Solids of Revolution

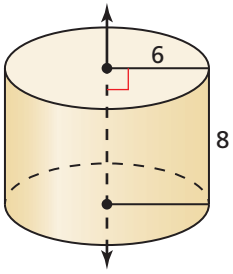
EXAMPLE 3 Finding the Surface Area and Volume of a Solid of Revolution

Sketch and describe the solid produced by rotating the figure around the given axis. Then find its surface area and volume.



SOLUTION

The solid is a cylinder with a height of 8 units and a base radius of 6 units.



Surface area: $S = 2\pi r^2 + 2\pi rh = 2\pi(6)^2 + 2\pi(6)(8) = 168\pi \approx 527.79$

Volume: $V = \pi r^2 h = \pi(6)^2(8) = 288\pi \approx 904.78$

▶ The cylinder has a surface area of about 527.79 square units and a volume of about 904.78 cubic units.

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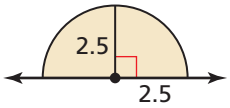
4. Sketch and describe the solid produced by rotating the figure around the given axis. Then find its surface area and volume.



Forming Solids of Revolution in the Coordinate Plane

EXAMPLE 4 **Forming a Solid of Revolution**

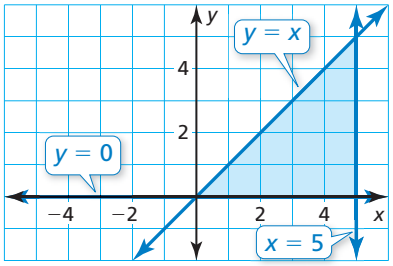
Sketch and describe the solid that is produced when the region enclosed by $y = 0$, $y = x$, and $x = 5$ is rotated around the y-axis. Then find the volume of the solid.

REMEMBER

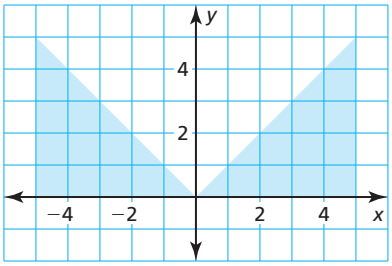
When (a, b) is reflected in the y-axis, then its image is the point $(-a, b)$.

SOLUTION

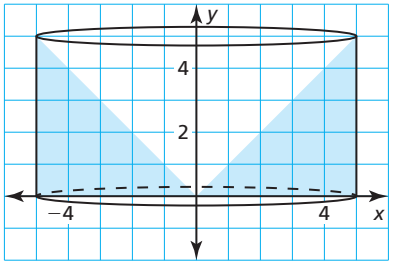
Step 1 Graph each equation and determine the region that will be rotated around the y-axis.



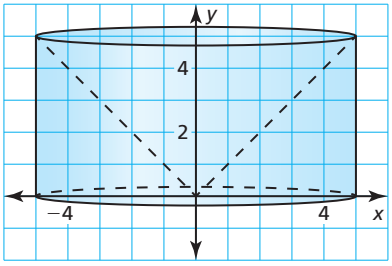
Step 2 Reflect the region in the y-axis.



Step 3 Connect the vertices of the triangles using curved lines.



Step 4 The composite solid consists of a cylinder with a cone removed.



ANOTHER WAY

You can also simplify $\pi r^2 h - \frac{1}{3}\pi r^2 h$ to obtain $\frac{2}{3}\pi r^2 h$. Then substitute 5 for r and h, and evaluate.

Step 5 Find the volume of the composite solid. The cylinder and the cone both have a height of 5 units and a base radius of 5 units.

$$\text{Volume of solid} = \text{Volume of cylinder} - \text{Volume of cone}$$

$= \pi r^2 h - \frac{1}{3}\pi r^2 h$ Write formulas.

$= \pi \cdot 5^2 \cdot 5 - \frac{1}{3}\pi \cdot 5^2 \cdot 5$ Substitute.

$= 125\pi - \frac{125}{3}\pi$ Simplify.

$= \frac{250}{3}\pi$ Subtract.

≈ 261.80 Use a calculator.

▶ The volume of the solid is $\frac{250}{3}\pi$, or about 261.80 cubic units.

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5. **WHAT IF?** Does the solid change when the region is rotated around the x-axis? Explain.

6. Sketch and describe the solid that is produced when the region enclosed by $x = 0$, $y = -x$, and $y = -3$ is rotated around the x-axis. Then find the volume of the solid.

1.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

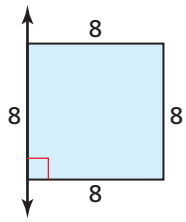
1. **COMPLETE THE SENTENCE** When you rotate a square around a line that contains one of its sides, the solid of revolution that is produced is a _____.

2. **WHICH ONE DOESN'T BELONG?** Which object does *not* belong with the other three? Explain your reasoning.




Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, sketch the solid produced by rotating the figure around the given axis. Then identify and describe the solid. *(See Example 1.)*

3.


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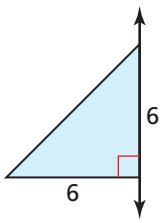

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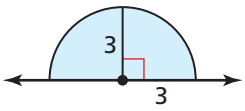
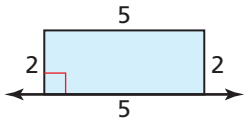
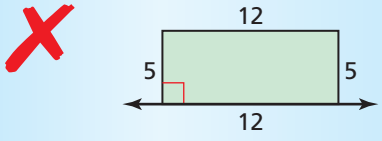


6.


7. **ERROR ANALYSIS** Describe and correct the error in identifying and describing the solid produced by rotating the figure around the given axis.



The solid is a cylinder with a height of 5 units and a base radius of 6 units.

8. **REASONING** Can you form any solid by rotating a two-dimensional figure around an axis? Explain.

In Exercises 9–12, sketch the solid of revolution. Then identify and describe the solid.

9. a square with side length 4 rotated around one side

10. a rectangle with length 6 and width 3 rotated around one of its shorter sides

11. a right triangle with legs of lengths 6 and 9 rotated around its longer leg

12. a semicircle with radius 10 rotated around its diameter

In Exercises 13–16, sketch a two-dimensional shape and an axis of revolution that forms the object shown. *(See Example 2.)*

13.

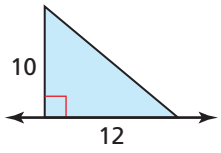

14.


15.


16.


In Exercises 17–22, sketch and describe the solid produced by rotating the figure around the given axis. Then find its surface area and volume. *(See Example 3.)*

17.



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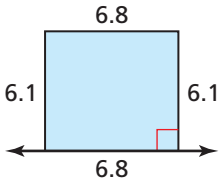

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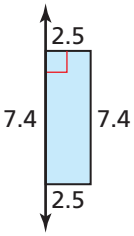

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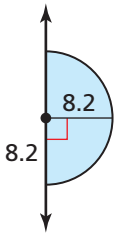

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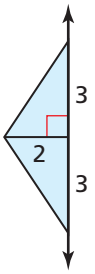

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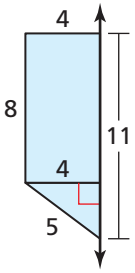


In Exercises 23–26, sketch and describe the solid that is produced when the region enclosed by the given equations is rotated around the given axis. Then find the volume of the solid. *(See Example 4.)*

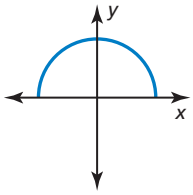
23. $x = 0, y = 0, y = x + 3$; x-axis

24. $x = 0, y = 0, y = -2x + 5$; y-axis

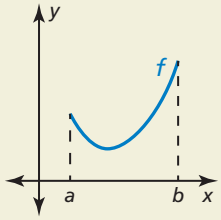
25. $x = 3, y = 0, y = \frac{1}{2}x$; y-axis

26. $x = -4, y = 0, y = x$; x-axis

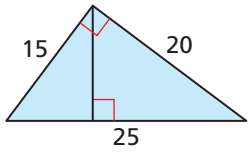
27. MAKING AN ARGUMENT Your friend says when you rotate the figure shown around either the x-axis or the y-axis, the resulting solid is a sphere. Is your friend correct? Explain.



28. HOW DO YOU SEE IT? The figure shows the graph of a function f on an interval $[a, b]$. Sketch the solid produced when the region enclosed by the graph of f and the equations $x = a$, $x = b$, and $y = 0$ is rotated around the x-axis.



29. CRITICAL THINKING A right triangle has sides with lengths 15, 20, and 25, as shown. Describe the three solids formed when the triangle is rotated around each of its sides. Then find the volumes of the solids. Give your answers in terms of π.



30. THOUGHT PROVOKING Write a system of equations whose enclosed region, when rotated around the x-axis or y-axis, produces the same solid with the same dimensions.

31. REASONING The solid shown is a type of *torus*.

a. Sketch a two-dimensional shape and an axis of revolution that forms the torus.

b. Which solid can you use to "construct" a torus similar to the one above? Explain, in your words, how to manipulate the solid to form the torus. You can think of the surface of the solid you choose as being stretchable.

32. CRITICAL THINKING A 30°-30°-120° isosceles triangle has two legs of length 4 units. When it is rotated around an axis that contains one leg, what is the volume of the solid of revolution?

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Determine whether the ordered pair is a solution of the equation. *(Skills Review Handbook)*

33. $f(x) = 8x - 3$; $(-5, -37)$

34. $h(x) = 2x^2 - 7x - 1$; $(3, 2)$

35. $n(x) = -5x^2 - 4x$; $(1.5, -17.25)$

36. $p(x) = |6x + 5|$; $(-1, 11)$

37. A circular region has a population of about 2.5 million people and a population density of about 9824 people per square mile. Find the radius of the region. *(Section 1.1)*