1.2 Modeling with Volume

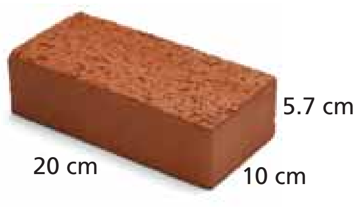
Essential Question

How can you use the mass and volume of an object to describe the density of the object?

EXPLORATION 1 Finding Densities

Work with a partner. Approximate the volume of each object whose mass is given. Then find the mass per unit of volume, or *density*, of each object.

a. Brick: 2.3 kg



b. Log: 18.1 kg

c. Golf ball: 45.9 g

d. Cork: 2.6 g

CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to justify your conclusions and communicate them to others.

EXPLORATION 2 Analyzing Densities

Work with a partner. The objects in Exploration 1 with a density greater than 1 gram per cubic centimeter will sink in water. The objects with a density less than 1 gram per cubic centimeter will float in water. You place each object in Exploration 1 in a bucket of water.

a. Which object(s) sink? float? Justify your answer.

b. Would your answers in part (a) change when each object is cut in half and placed in water? Explain your reasoning.

c. You dissolve enough salt in a bucket of water to cause one of the sunken objects to float. Which object is it and why do you think this happens?

Communicate Your Answer

3. How can you use the mass and volume of an object to describe the density of the object?

4. Use the Internet or some other reference to research the densities of water, mineral oil, and beeswax. You combine these substances in a bucket. How do you think the liquids interact? Where would the beeswax settle?

1.2 Lesson

Core Vocabulary

density, *p. 10*

Previous
volume
square root

What You Will Learn

- Use volume formulas to find densities.
- Use volume formulas to solve problems.

Using Volume Formulas to Find Densities

Density is the amount of matter that an object has in a given unit of volume. The density of an object is calculated by dividing its mass by its volume.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

Different materials have different densities, so density can be used to distinguish between materials that look similar. For example, table salt and sugar look alike. However, table salt has a density of 2.16 grams per cubic centimeter, while sugar has a density of 1.58 grams per cubic centimeter.

EXAMPLE 1 Using the Formula for Density

The diagram shows the dimensions of a standard gold bar at Fort Knox. Gold has a density of 19.3 grams per cubic centimeter. Find the mass of a standard gold bar to the nearest gram.

SOLUTION

Step 1 Convert the dimensions to centimeters using 1 inch = 2.54 centimeters.

Length $7\ \cancel{\text{in.}} \cdot \dfrac{2.54\text{ cm}}{1\ \cancel{\text{in.}}} = 17.78\text{ cm}$

Width $3.625\ \cancel{\text{in.}} \cdot \dfrac{2.54\text{ cm}}{1\ \cancel{\text{in.}}} = 9.2075\text{ cm}$

Height $1.75\ \cancel{\text{in.}} \cdot \dfrac{2.54\text{ cm}}{1\ \cancel{\text{in.}}} = 4.445\text{ cm}$

Step 2 Find the volume.

The area of a base is $B = 17.78(9.2075) = 163.70935\text{ cm}^2$ and the height is $h = 4.445$ cm.

$$V = Bh = 163.70935(4.445) \approx 727.69\text{ cm}^3$$

Step 3 Let x represent the mass in grams. Substitute the values for the volume and the density in the formula for density and solve for x.

$\text{Density} = \dfrac{\text{Mass}}{\text{Volume}}$ Formula for density

$19.3 \approx \dfrac{x}{727.69}$ Substitute.

$14{,}044 \approx x$ Multiply each side by 727.69.

▶ The mass of a standard gold bar is about 14,044 grams.

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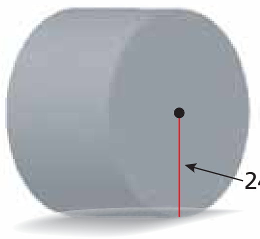
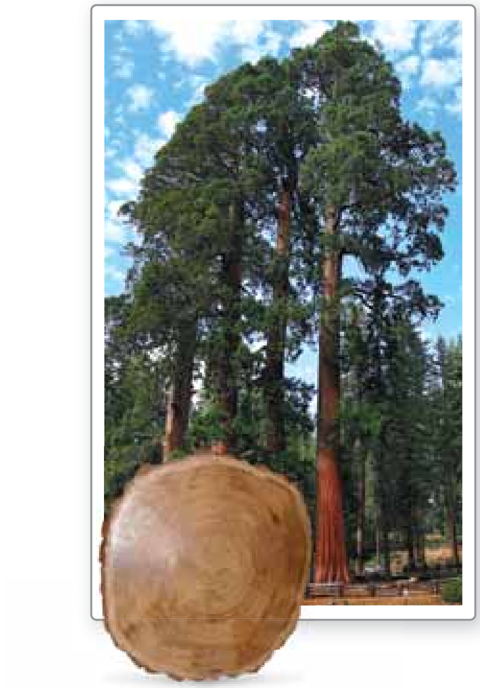


1. The diagram shows the dimensions of a concrete cylinder. Concrete has a density of 2.3 grams per cubic centimeter. Find the mass of the concrete cylinder to the nearest gram.

Using Volume Formulas



EXAMPLE 2 Using a Volume Formula

A tree harvester is often interested in the volume of a tree's trunk because most of the wood volume is located there. A tree harvester estimates the trunk of a sequoia tree to have a height of about 50 meters and a base diameter of about 0.8 meter.

a. The wood of a sequoia tree has a density of about 450 kilograms per cubic meter. Find the mass of the trunk to the nearest kilogram.

b. Each year, the tree trunk forms new cells that arrange themselves in concentric circles called *growth rings*. These rings indicate how much wood the tree produces annually. The harvester estimates that the trunk will put on a growth ring of about 1 centimeter thick and its height will increase by about 0.25 meter this year. How many cubic meters of wood does the tree trunk produce after one year? If the tree grows at a constant rate for the next five years, will it produce the same amount of wood each year? Explain.

SOLUTION

a. To estimate the volume of the tree trunk, assume that the trunk is cylindrical. So, the volume of the trunk is

$$V = \pi r^2 h = \pi(0.4)^2(50) = 8\pi \approx 25.13 \text{ m}^3.$$

Let x represent the mass in kilograms. Substitute the values for the volume and the density in the formula for density and solve for x.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \qquad \text{Formula for density}$$

$$450 \approx \frac{x}{25.13} \qquad \text{Substitute.}$$

$$11{,}309 \approx x \qquad \text{Multiply each side by 25.13.}$$

▶ The mass of the trunk is about 11,309 kilograms.

COMMON ERROR

Because 1 cm = 0.01 m, the trunk will have a diameter of 0.8 + 0.01 + 0.01 = 0.82 m for Year 1.

b. Make a table that shows the trunk dimensions and volume for five years.

Year	1	2	3	4	5
Height (meters)	50.25	50.5	50.75	51	51.25
Base radius (meters)	0.41	0.42	0.43	0.44	0.45
Volume (cubic meters)	26.54	27.99	29.48	31.02	32.60

+ 1.45 + 1.49 + 1.54 + 1.58

▶ The tree will produce about 26.54 − 25.13 = 1.41 cubic meters of wood after one year. The tree will not produce the same amount of wood each year for five years because the differences between the volumes from year to year are increasing.

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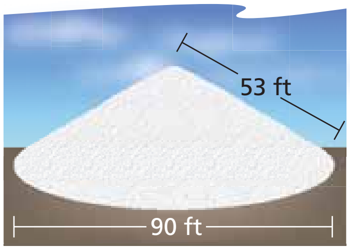
2. **WHAT IF?** The tree harvester makes the same growth estimates for the trunk of a sequoia tree that has a height of about 40 meters and a base diameter of about 0.75 meter. (a) Find the mass of the trunk to the nearest kilogram. (b) How many cubic meters of wood will the trunk gain after four years?



EXAMPLE 3 Using a Volume Formula

Before a winter storm, a pile of road salt has the dimensions shown. After the storm, the linear dimensions of the pile are one-half of the original dimensions.

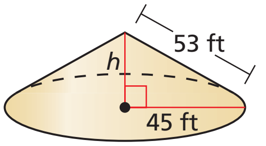
a. How does this change affect the volume of the pile?

b. A *lane mile* is an area of pavement that is one mile long and one lane wide. During the storm, about 400 pounds of road salt was used for every lane mile. Estimate the number of lane miles that were covered with road salt during the storm. A cubic foot of road salt weighs about 80 pounds.

SOLUTION

a. The pile of road salt is approximately shaped like a cone. Use the Pythagorean Theorem to find the height h. Then use the formula for the volume of a cone to find the volume of the pile before and after the storm.

$c^2 = a^2 + b^2$	Pythagorean Theorem
$53^2 = h^2 + 45^2$	Substitute.
$2809 = h^2 + 2025$	Multiply.
$784 = h^2$	Subtract 2025 from each side.
$28 = h$	Find the positive square root.



	Before winter storm	After winter storm
Dimensions	$r = 45$ ft, $h = 28$ ft	$r = 22.5$ ft, $h = 14$ ft
Volume	$V = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi(45)^2(28)$ $= 18{,}900\pi$ ft^3	$V = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi(22.5)^2(14)$ $= 2362.5\pi$ ft^3

ANALYZING MATHEMATICAL RELATIONSHIPS

Notice that when all the linear dimensions are multiplied by k, the volume is multiplied by k^3.

▶ The volume of the pile after the winter storm is $\frac{2362.5\pi}{18{,}900\pi} = \frac{1}{8}$ times the original volume.

b. During the storm, $18{,}900\pi - 2362.5\pi = 16{,}537.5\pi$ cubic feet of road salt was used. Use conversions to find the number of lane miles covered with road salt during the storm.

Pounds of road salt used: $16{,}537.5\pi \cancel{\text{ft}^3} \cdot \frac{80 \text{ lb}}{1 \cancel{\text{ft}^3}} = 1{,}323{,}000\pi$ lb

Lane miles covered: $1{,}323{,}000\pi \cancel{\text{lb}} \cdot \frac{1 \text{ lane mile}}{400 \cancel{\text{lb}}} \approx 10{,}390.82$ lane miles

▶ So, about 10,400 lane miles were covered with road salt during the storm.

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3. In Example 3, the department of transportation pays about \$31.50 for each ton of road salt. How much does the original pile of road salt cost?

4. **WHAT IF?** After a storm, the linear dimensions of the pile are $\frac{1}{4}$ of the original dimensions. (a) How does this change affect the volume of the pile? (b) Estimate the number of lane miles that were covered with road salt during the storm.

1.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** What do you obtain when you multiply an object's density by its volume?

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

What is the mass per unit of volume?	What is the mass in kilograms?
What is the mass divided by the volume?	What is the density?

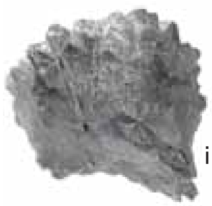
Tourmaline

Mass = 6.2 g
Volume = 2 cm^3

Monitoring Progress and Modeling with Mathematics

3. **PROBLEM SOLVING** A piece of copper with a volume of 8.25 cubic centimeters has a mass of 73.92 grams. A piece of iron with a volume of 5 cubic centimeters has a mass of 39.35 grams. Which metal has the greater density?

copper



iron

4. **PROBLEM SOLVING** The United States has minted one-dollar silver coins called the American Eagle Silver Bullion Coin since 1986. Each coin has a diameter of 40.6 millimeters and is 2.98 millimeters thick. The density of silver is 10.5 grams per cubic centimeter. What is the mass of an American Eagle Silver Bullion Coin to the nearest gram? *(See Example 1.)*

5. **ERROR ANALYSIS** Describe and correct the error in finding the density of an object that has a mass of 24 grams and a volume of 28.3 cubic centimeters.

$$\text{density} = \frac{28.3}{24} \approx 1.18$$

So, the density is about 1.18 cubic centimeters per gram.

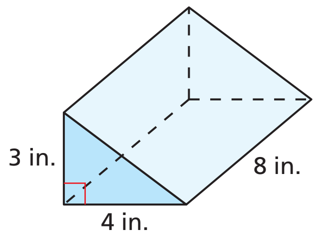
6. **PROBLEM SOLVING** The height of a tree trunk is 20 meters and the base diameter is 0.5 meter. *(See Example 2.)*

   a. The wood has a density of 380 kilograms per cubic meter. Find the mass of the trunk to the nearest kilogram.

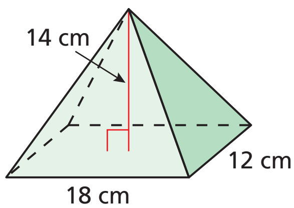
   b. The trunk puts on a growth ring of 4 millimeters and its height increases by 0.2 meter this year. How many cubic meters of wood does the tree trunk produce? The tree grows at a constant rate for the next five years. Does the tree produce the same amount of wood each year? Explain.

In Exercises 7 and 8, describe how the change affects the volume of the prism or pyramid.

7. tripling all the linear dimensions



8. multiplying the height by $\frac{3}{2}$



9. **PROBLEM SOLVING** A conical pile of road salt has a diameter of 112 feet and a slant height of 65 feet. After a storm, the linear dimensions of the pile are $\frac{1}{3}$ of the original dimensions. *(See Example 3.)*

   a. How does this change affect the volume of the pile?

   b. During the storm, 350 pounds of road salt was used for every lane mile. Estimate the number of lane miles that were covered with salt. How many lane miles can be covered with the remaining salt? A cubic foot of road salt weighs about 80 pounds.

10. HOW DO YOU SEE IT? The two stone blocks shown below with the given densities have the same volume. Which block has a greater mass? Explain.

Granite: 2.7 g/cm^3

Sandstone: 2.3 g/cm^3

11. MODELING WITH MATHEMATICS A pool in the shape of a rectangular prism is 6 meters long and 3 meters wide. The water in the pool is 1 meter deep.

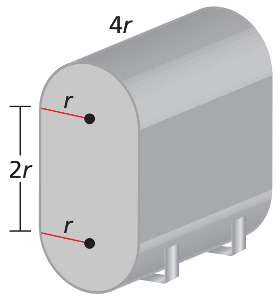
a. The density of water is about 1 gram per cubic centimeter. Find the number of kilograms of water in the pool.

b. You add 6000 kilograms of water to the pool. What is the depth of the water in the pool?

12. MODELING WITH MATHEMATICS A British thermal unit (Btu) is the amount of heat needed to raise the temperature of 1 pound of liquid water by 1°F. There are about 1000 Btu per cubic foot of natural gas and about 140,000 Btu per gallon of heating oil.

a. In 2010, electricity-generating power plants paid \$5.27 per 1000 cubic feet of natural gas and \$56.35 per 42-gallon barrel of heating oil. Express the cost of each fuel in dollars per million Btu.

b. The tank shown can be used to store heating oil. Write a formula for the volume of the tank.



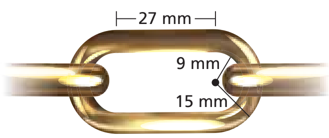
c. You pay \$3.75 per gallon of heating oil to fill a new tank in which r is 1 foot. Compare your cost for heating oil in dollars per million Btu to a power plant's cost in part (a). How many Btu can be produced from a full tank of heating oil?

13. MAKING AN ARGUMENT As ocean depth increases, water molecules get pushed closer together due to the weight of the water above. Your friend says that the density of water increases as depth increases. Is your friend correct? Explain.

14. THOUGHT PROVOKING You place two cans of regular soda and two cans of diet soda in a container full of water. The two regular cans sink, but the two diet cans float.

Use the Internet to research the contents of regular soda and diet soda. Then make a conjecture about why the diet cans float, but the regular cans sink. Include a discussion of *density* and *buoyancy* in your explanation.

15. MODELING WITH MATHEMATICS Links of a chain are made from cylindrical metal rods with a diameter of 6 millimeters. The density of the metal is about 8 grams per cubic centimeter.



a. To approximate the length of a rod used to make a link, should you use the perimeter around the inside of the link? the outside? the average of these perimeters? Explain your reasoning. Then approximate the mass of a chain with 100 links.

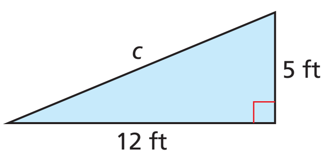
b. Approximate the length of a taut chain with 100 links. Explain your procedure.

Maintaining Mathematical Proficiency

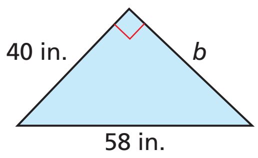
Reviewing what you learned in previous grades and lessons

Find the missing length of the triangle. *(Skills Review Handbook)*

16.



17.



18.