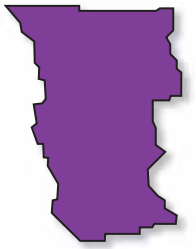
1.1 Modeling with Area

Essential Question How can you use the population and area of a region to describe how densely the region is populated?

EXPLORATION 1 Exploring Population and Area

Work with a partner. Use the Internet to find the population and land area of each county in California. Then find the number of people per square mile for each county.

a. Mendocino County



b. Lake County

c. Yolo County

d. Napa County

e. Sonoma County

f. Marin County

EXPLORATION 2 Analyzing Population and Area

Work with a partner. The six counties in Exploration 1 appear on a map as shown.

a. Without calculating, how would you expect the number of people per square mile in the entire 6-county region to compare to the values for each individual county in Exploration 1?

b. Use the populations and land areas in Exploration 1 to justify your answer in part (a).

Communicate Your Answer

3. How can you use the population and area of a region to describe how densely the region is populated?

4. Find the population and land area of the county in which you live. How densely populated is your county compared to the counties in Exploration 1?

5. In Exploration 1, the two northern counties are less densely populated than the other four. What factors do you think might influence how densely a region is populated?

MODELING WITH MATHEMATICS

To be proficient in math, you need to interpret mathematical results in real-life contexts.

1.1 Lesson

Core Vocabulary

population density, *p. 4*

Previous

perimeter
area
surface area

What You Will Learn

- Use area formulas to solve problems.
- Use surface area formulas to solve problems.

Using Area Formulas

The **population density** of a city, county, or state is a measure of how many people live within a given area.

$$\text{Population density} = \frac{\text{number of people}}{\text{area of land}}$$

Population density is usually given in terms of square miles but can be expressed using other units, such as city blocks.

EXAMPLE 1 Finding a Population Density

The state of Nevada has a population of about 2.7 million people. Find the population density in people per square mile.

SOLUTION

Step 1 Find the area of Nevada. It is approximately shaped like a trapezoid. Use the formula for the area of a trapezoid to estimate the area of Nevada.

$$A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(320)(200 + 490) = 110{,}400 \text{ mi}^2$$

Step 2 Find the population density.

$$\text{Population density} = \frac{\text{number of people}}{\text{area of land}} = \frac{2{,}700{,}000}{110{,}400} \approx 24$$

- The population density is about 24 people per square mile.

EXAMPLE 2 Using the Formula for Population Density

A circular region has a population of about 430,000 people and a population density of about 5475 people per square mile. Find the radius of the region.

SOLUTION

Use the formula for population density. Let r represent the radius of the region.

$$\text{Population density} = \frac{\text{number of people}}{\text{area of land}} \qquad \text{Formula for population density}$$

$$5475 = \frac{430{,}000}{\pi r^2} \qquad \text{Substitute.}$$

$$5475\pi r^2 = 430{,}000 \qquad \text{Multiply each side by } \pi r^2.$$

$$r^2 = \frac{430{,}000}{5475\pi} \qquad \text{Divide each side by } 5475\pi.$$

$$r = \sqrt{\frac{430{,}000}{5475\pi}} \qquad \text{Take the positive square root of each side.}$$

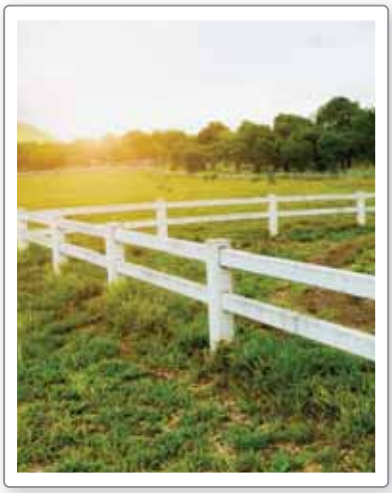
$$r \approx 5 \qquad \text{Use a calculator.}$$

- The radius of the region is about 5 miles.

REMEMBER

You can solve quadratic equations of the form $x^2 = d$ by taking the square root of each side.

EXAMPLE 3 Using an Area Formula



You are designing a rectangular corral. A barn will form one side of the corral. The corral is to have an area of 450 square meters, but you want to minimize the amount of fencing that you need for the three sides of the corral not against the barn. This will include an opening that is 3 meters wide where a gate will be placed. How many meters of fencing do you need to build the corral?

SOLUTION

Step 1 Use what you know about the area and perimeter of the corral to find an expression that represents the perimeter of the three sides that need fencing.

The area A of a corral of length ℓ and width w is $A = \ell w$. So, $450 = \ell w$. Solving for ℓ gives $\ell = \frac{450}{w}$.

To minimize the amount of fencing you need, let a longer side of the corral be against the barn. So, the expression $2w + \ell$ represents the perimeter of the three sides that need fencing. Using substitution, this expression can be rewritten as $2w + \frac{450}{w}$.

Step 2 Use the *table* feature of a graphing calculator to create a table of values to find the width w that minimizes the value of $2w + \frac{450}{w}$. You may need to decrease the increment for the independent variable, as shown.

Increment of 4

X	Y1
4	120.5
8	72.25
12	61.5
16	60.125
20	62.5
24	66.75
28	72.071

X=16

Increment of 0.5

X	Y1
13.5	60.333
14	60.143
14.5	60.034
15	60
15.5	60.032
16	60.125
16.5	60.273

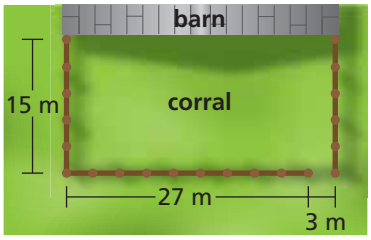
X=15

USING TECHNOLOGY

In the first table, y-values decrease and then increase. Scrolling through x-values greater than 28 shows that y-values continue to increase. So, the minimum occurs near $x = 16$.

The width that minimizes the value of $2w + \frac{450}{w}$ is 15 meters. So, the length of the corral is $\ell = \frac{450}{w} = \frac{450}{15} = 30$ meters.

Step 3 Sketch a diagram of the corral that includes the gate opening, as shown.



▶ So, you need $2w + \ell - 3 = 2(15) + 30 - 3 = 57$ meters of fencing.

Monitoring Progress

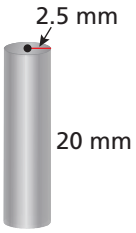
Help in English and Spanish at *BigIdeasMath.com*

1. About 58,000 people live in a circular region with a 2-mile radius. Find the population density in people per square mile.
2. A circular region has a population of about 175,000 people and a population density of about 1318 people per square mile. Find the radius of the region.
3. **WHAT IF?** You want the corral to have an area of 800 square meters. How many meters of fencing do you need?

Using Surface Area Formulas

EXAMPLE 4 Using a Surface Area Formula

A manufacturer designs the bearing shown. To prevent corrosion, the manufacturer coats each bearing with an anticorrosive grease.



a. A smaller bearing has linear dimensions that are one-half the dimensions of the bearing shown. Does the smaller bearing require one-half of the amount of grease that is used to coat the larger bearing? Explain.

b. A smaller bearing has the same radius and a height that is one-half the height of the bearing shown. Is one-half of the amount of anticorrosive grease used to coat the larger bearing enough to coat this bearing? Explain.

SOLUTION

The bearing is cylindrical. So, use the formula for the surface area of a cylinder to compare the bearings.

a.

	Larger bearing	Smaller bearing
Dimensions	$r = 2.5$ mm, $h = 20$ mm	$r = 1.25$ mm, $h = 10$ mm
Surface area	$S = 2\pi r^2 + 2\pi rh$ $= 2\pi(2.5)^2 + 2\pi(2.5)(20)$ $= 112.5\pi$ mm^2	$S = 2\pi r^2 + 2\pi rh$ $= 2\pi(1.25)^2 + 2\pi(1.25)(10)$ $= 28.125\pi$ mm^2

The surface area of the smaller bearing is $\frac{28.125\pi}{112.5\pi} = \frac{1}{4}$ times the surface area of the larger bearing.

▶ No, the smaller bearing requires only $\frac{1}{4}$ of the amount of grease that is used to coat the larger bearing.

ANALYZING MATHEMATICAL RELATIONSHIPS

Notice that while the surface area does not scale by a factor of $\frac{1}{2}$, the lateral surface area does scale by a factor of $\frac{2\pi(2.5)(10)}{2\pi(2.5)(20)} = \frac{50\pi}{100\pi} = \frac{1}{2}$.

b. The smaller bearing has a radius of 2.5 millimeters and a height of 10 millimeters. Its surface area is

$$S = 2\pi(2.5)^2 + 2\pi(2.5)(10)$$
$$= 62.5\pi \text{ mm}^2.$$

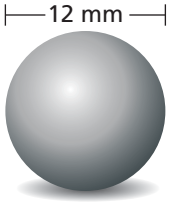
So, the surface area of the smaller bearing is $\frac{62.5\pi}{112.5\pi} = \frac{5}{9}$ times the surface area of the larger bearing.

▶ No, because $\frac{5}{9} > \frac{1}{2}$, one-half of the amount of anticorrosive grease used to coat the larger bearing is not enough to coat the smaller bearing.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

4. A manufacturer designs the bearing shown. To prevent corrosion, the manufacturer coats each bearing with an anticorrosive grease. Does a bearing with a diameter that is $\frac{3}{2}$ times the diameter of the given bearing need $\frac{3}{2}$ times the amount of grease to coat it? Explain.



1.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Explain, in your own words, the difference between the population of a state and the population density of a state.

2. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

bears per square mile | video gamers per city | people per square kilometer | trees per acre

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, find the indicated measure. *(See Example 1.)*

3. The state of Kansas has a population of about 2.85 million people. Find the population density in people per square mile.

4. About 210,000 people live in a circular region with a 12-mile radius. Find the population density in people per square mile.

5. About 650,000 people live in a circular region with a 6-mile radius. Find the population density in people per square mile.

6. Yellowstone National Park has an area of about 2.22 million acres. The table shows the estimated park populations for several animals. Find the population density in animals per acre for each animal.

Animal	Grizzly bear	Elk	Mule deer	Bighorn sheep
Population	445	20,000	2400	260

7. A circular region with a 4-mile radius has a population density of 6366 people per square mile. Find the number of people who live in the region.

8. Central Park in New York City is rectangular with a length of 2.5 miles and a width of 0.5 mile. During an afternoon, its population density is about 15 people per acre. Find the number of people in the park that afternoon. One acre is equal to $\frac{1}{640}$ square mile.

9. **PROBLEM SOLVING** About 79,000 people live in a circular region with a population density of about 513 people per square mile. Find the radius of the region. *(See Example 2.)*

10. **PROBLEM SOLVING** About 1.15 million people live in a circular region with a population density of about 18,075 people per square kilometer. Find the radius of the region.

11. **ERROR ANALYSIS** Describe and correct the error in finding the number of people who live in a circular region with a 7.5-mile diameter and a population density of 1550 people per square mile.

✗

$$1550 = \frac{x}{\pi \cdot 7.5^2}$$

$$1550 = \frac{x}{56.25\pi}$$

$$273{,}908 \approx x$$

The number of people who live in the region is about 273,908.

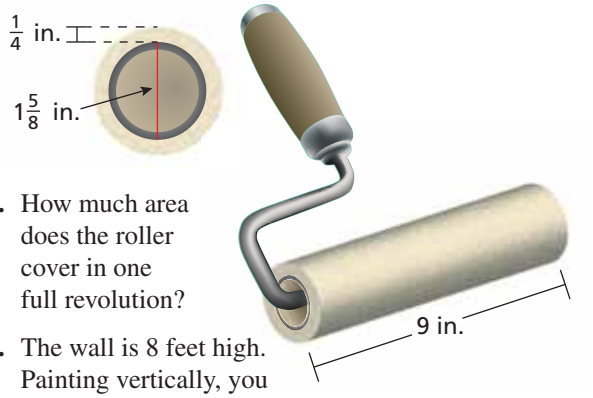
12. **HOW DO YOU SEE IT?** The two islands shown below with the given areas have the same population. Which has the greater population density? Explain.

13. MODELING WITH MATHEMATICS A soccer field of length ℓ and width w has a perimeter of 320 yards. *(See Example 3.)*

a. Write an expression that represents the area of the soccer field in terms of ℓ.

b. Use your expression from part (a) to determine the dimensions of the field that maximize the area. What do you notice?

14. MODELING WITH MATHEMATICS You are using a 9-inch paint roller to paint a wall of your bedroom. The roller has a diameter of $1\frac{5}{8}$ inches and a nap thickness of $\frac{1}{4}$ inch, as shown. The *nap* is the fuzzy material on the surface of the roller.



a. How much area does the roller cover in one full revolution?

b. The wall is 8 feet high. Painting vertically, you start at the bottom of the wall and make 8 full revolutions with the roller. Are you more than halfway up the wall? Justify your answer.

c. A 3-inch roller has the same diameter and nap thickness as the roller above. How much area does this roller cover in one full revolution? How does this compare to the area in part (a)?

17. MODELING WITH MATHEMATICS A playground ball with a 16-inch diameter has a rubber coating on its surface. *(See Example 4.)*

a. Does a ball with a diameter that is $\frac{1}{4}$ times the diameter of the given ball need $\frac{1}{4}$ times the amount of rubber coating? Explain.

b. What is the radius of a ball that uses one-half of the amount of rubber coating used to cover the 16-inch ball?

18. MODELING WITH MATHEMATICS A cylindrical swimming pool has a diameter of 24 feet and a height of 4 feet. A smaller pool with the same height has a diameter of 12 feet. A vinyl liner covers the bottom and side of each pool.

a. Does the smaller pool require one-half of the amount of vinyl liner that is used to cover the larger pool? Explain. If not, estimate the diameter of the cylindrical swimming pool that uses one-half of the amount of vinyl liner used to cover the larger pool. Assume the height of the pool is 4 feet.

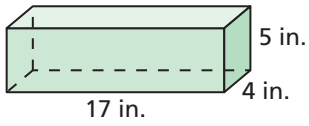
b. To install the vinyl liner, it costs \$1.95 per square foot. You have \$1000 in your budget to spend on the liner installation. What is the largest pool you can get without going over your liner budget? Assume the height of the pool is 4 feet.

19. MAKING AN ARGUMENT You ask your friend which U.S. states have the greatest population densities. Your friend says it must be California and Texas because they have the greatest populations. Is your friend correct? Explain.

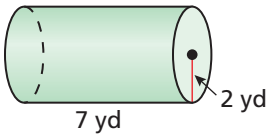
20. THOUGHT PROVOKING Give an example from your everyday life of an object whose surface area changes. Sketch the object and determine what geometric shape can model it. Estimate the dimensions of the object before and after the change and explain how the surface area is affected.

In Exercises 15 and 16, describe how the change affects the surface area of the right prism or right cylinder.

15. doubling all the linear dimensions



16. tripling the radius



Maintaining Mathematical Proficiency

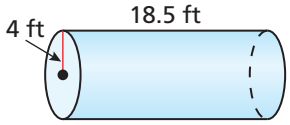
Reviewing what you learned in previous grades and lessons

Evaluate the expression when $x = -4$ and $y = 3$. *(Skills Review Handbook)*

21. $2x^2 + 5y^2$ **22.** $\frac{1}{3}(8xy - y^2)$ **23.** $-x^2 + 6x + 11y$ **24.** $-5(y + x^3 + 10)$

Find the volume of the solid. *(Skills Review Handbook)*

25.



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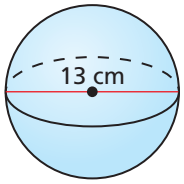

27.