

# 1 Geometric Modeling

- 1.1 Modeling with Area
- 1.2 Modeling with Volume
- 1.3 Cross Sections of Solids
- 1.4 Solids of Revolution



Silo (p. 27)



Sawmill (p. 20)



Road Salt (p. 12)



Corral (p. 5)

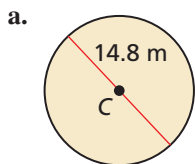


Population Density (p. 4)

# Maintaining Mathematical Proficiency

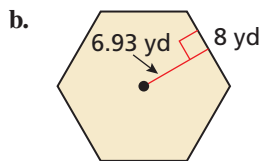
## Finding Areas of Two-Dimensional Figures

**Example 1** Find the area of the circle or regular polygon.



$$\begin{aligned}
 A &= \pi r^2 && \text{Write area formula.} \\
 &= \pi \cdot (7.4)^2 && \text{Substitute.} \\
 &= 54.76\pi && \text{Simplify.} \\
 &\approx 172.03 && \text{Use a calculator.}
 \end{aligned}$$

▶ The area is about 172.03 square meters.



$$\begin{aligned}
 A &= \frac{1}{2} a \cdot ns \\
 &= \frac{1}{2} (6.93) \cdot (6)(8) \\
 &= 3.465 \cdot 48 \\
 &= 166.32
 \end{aligned}$$

▶ The area is 166.32 square yards.

**Find the indicated measure.**

- area of a circle with (a) a radius of 9.2 centimeters and (b) a diameter of 50.5 inches
- area of a regular octagon with a perimeter of 80 feet and an apothem of 12.07 feet

## Finding Surface Areas and Volumes of Three-Dimensional Figures

**Example 2** Find the surface area and volume of the right cone.

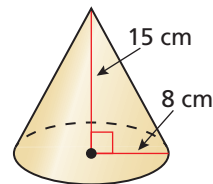
Using the Pythagorean Theorem, you can determine that the slant height  $\ell$  is 17 centimeters.

**Surface area**

$$\begin{aligned}
 S &= \pi r^2 + \pi r \ell && \text{Write formula.} \\
 &= \pi(8)^2 + \pi(8)(17) && \text{Substitute.} \\
 &= 200\pi \approx 628.32 && \text{Simplify.}
 \end{aligned}$$

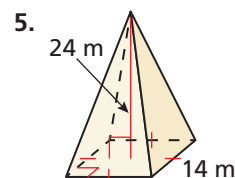
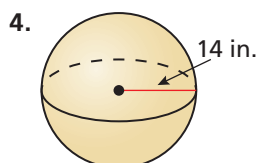
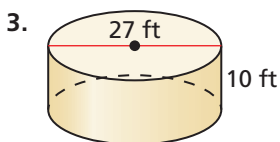
**Volume**

$$\begin{aligned}
 V &= \frac{1}{3} \pi r^2 h \\
 &= \frac{1}{3} \pi (8)^2 (15) \\
 &= 320\pi \approx 1005.31
 \end{aligned}$$



▶ The surface area is  $200\pi$ , or about 628.32 square centimeters and the volume is  $320\pi$ , or about 1005.31 cubic centimeters.

**Find the surface area and volume of the solid.**



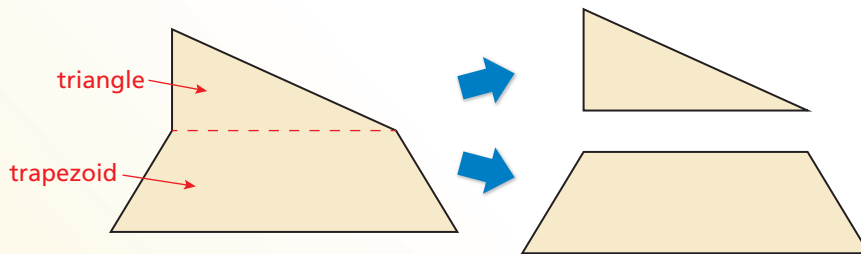
6. **ABSTRACT REASONING** A sphere with radius  $r$  fits perfectly inside a rectangular box. Can you determine the volume of the box? Explain your reasoning.

## Solving a Simpler Form of a Problem

### Core Concept

#### Composite Figures and Area

A **composite figure** is a figure that consists of triangles, squares, rectangles, and other two-dimensional figures. To find the area of a composite figure, separate it into figures with areas you know how to find. Then find the sum of the areas of those figures.



#### EXAMPLE 1 Finding the Area of a Composite Figure

Find the area of the composite figure.

#### SOLUTION

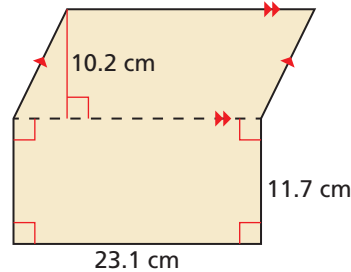
The figure consists of a parallelogram and a rectangle.

Area of parallelogram

$$\begin{aligned} A &= bh \\ &= (23.1)(10.2) \\ &= 235.62 \end{aligned}$$

Area of rectangle

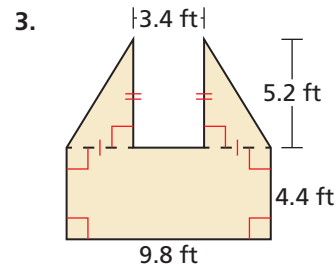
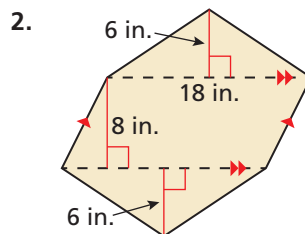
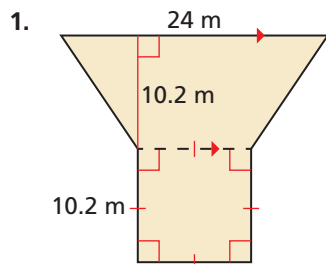
$$\begin{aligned} A &= \ell w \\ &= (23.1)(11.7) \\ &= 270.27 \end{aligned}$$



▶ The area of the figure is  $235.62 + 270.27 = 505.89$  square centimeters.

### Monitoring Progress

Find the area of the composite figure.



4. The concept above also applies when finding the surface area or volume of a composite solid. Why do you need to be careful when finding the surface area of a composite solid?

# 1.1 Modeling with Area

**Essential Question** How can you use the population and area of a region to describe how densely the region is populated?

## EXPLORATION 1 Exploring Population and Area

**Work with a partner.** Use the Internet to find the population and land area of each county in California. Then find the number of people per square mile for each county.

a. Mendocino County



b. Lake County



c. Yolo County



d. Napa County



e. Sonoma County



f. Marin County



## EXPLORATION 2 Analyzing Population and Area

**Work with a partner.** The six counties in Exploration 1 appear on a map as shown.

- Without calculating, how would you expect the number of people per square mile in the entire 6-county region to compare to the values for each individual county in Exploration 1?
- Use the populations and land areas in Exploration 1 to justify your answer in part (a).



## MODELING WITH MATHEMATICS

To be proficient in math, you need to interpret mathematical results in real-life contexts.

## Communicate Your Answer

- How can you use the population and area of a region to describe how densely the region is populated?
- Find the population and land area of the county in which you live. How densely populated is your county compared to the counties in Exploration 1?
- In Exploration 1, the two northern counties are less densely populated than the other four. What factors do you think might influence how densely a region is populated?

# 1.1 Lesson

## Core Vocabulary

population density,  $p$ . 4

**Previous**

perimeter

area

surface area



### REMEMBER

You can solve quadratic equations of the form  $x^2 = d$  by taking the square root of each side.

## What You Will Learn

- ▶ Use area formulas to solve problems.
- ▶ Use surface area formulas to solve problems.

## Using Area Formulas

The **population density** of a city, county, or state is a measure of how many people live within a given area.

$$\text{Population density} = \frac{\text{number of people}}{\text{area of land}}$$

Population density is usually given in terms of square miles but can be expressed using other units, such as city blocks.

### EXAMPLE 1 Finding a Population Density

The state of Nevada has a population of about 2.7 million people. Find the population density in people per square mile.

#### SOLUTION

**Step 1** Find the area of Nevada. It is approximately shaped like a trapezoid. Use the formula for the area of a trapezoid to estimate the area of Nevada.

$$A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(320)(200 + 490) = 110,400 \text{ mi}^2$$

**Step 2** Find the population density.

$$\text{Population density} = \frac{\text{number of people}}{\text{area of land}} = \frac{2,700,000}{110,400} \approx 24$$

- ▶ The population density is about 24 people per square mile.

### EXAMPLE 2 Using the Formula for Population Density

A circular region has a population of about 430,000 people and a population density of about 5475 people per square mile. Find the radius of the region.

#### SOLUTION

Use the formula for population density. Let  $r$  represent the radius of the region.

$$\text{Population density} = \frac{\text{number of people}}{\text{area of land}} \quad \text{Formula for population density}$$

$$5475 = \frac{430,000}{\pi r^2} \quad \text{Substitute.}$$

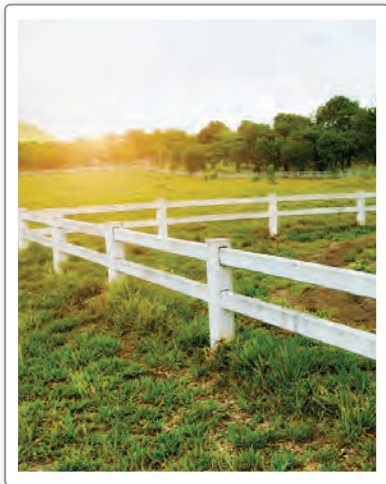
$$5475\pi r^2 = 430,000 \quad \text{Multiply each side by } \pi r^2.$$

$$r^2 = \frac{430,000}{5475\pi} \quad \text{Divide each side by } 5475\pi.$$

$$r = \sqrt{\frac{430,000}{5475\pi}} \quad \text{Take the positive square root of each side.}$$

$$r \approx 5 \quad \text{Use a calculator.}$$

- ▶ The radius of the region is about 5 miles.



### EXAMPLE 3 Using an Area Formula

You are designing a rectangular corral. A barn will form one side of the corral. The corral is to have an area of 450 square meters, but you want to minimize the amount of fencing that you need for the three sides of the corral not against the barn. This will include an opening that is 3 meters wide where a gate will be placed. How many meters of fencing do you need to build the corral?

#### SOLUTION

**Step 1** Use what you know about the area and perimeter of the corral to find an expression that represents the perimeter of the three sides that need fencing.

The area  $A$  of a corral of length  $\ell$  and width  $w$  is  $A = \ell w$ . So,  $450 = \ell w$ .

Solving for  $\ell$  gives  $\ell = \frac{450}{w}$ .

To minimize the amount of fencing you need, let a longer side of the corral be against the barn. So, the expression  $2w + \ell$  represents the perimeter of the three sides that need fencing. Using substitution, this expression can be rewritten as  $2w + \frac{450}{w}$ .

**Step 2** Use the *table* feature of a graphing calculator to create a table of values to find the width  $w$  that minimizes the value of  $2w + \frac{450}{w}$ . You may need to decrease the increment for the independent variable, as shown.

Increment of 4

X	Y1
4	120.5
8	72.25
12	61.5
16	60.125
20	62.5
24	66.75
28	72.071

X=16

Increment of 0.5

X	Y1
13.5	60.333
14	60.143
14.5	60.034
15	60
15.5	60.032
16	60.125
16.5	60.273

X=15

#### USING TECHNOLOGY

In the first table,  $y$ -values decrease and then increase. Scrolling through  $x$ -values greater than 28 shows that  $y$ -values continue to increase. So, the minimum occurs near  $x = 16$ .

The width that minimizes the value of  $2w + \frac{450}{w}$  is 15 meters. So, the length of the corral is  $\ell = \frac{450}{w} = \frac{450}{15} = 30$  meters.

**Step 3** Sketch a diagram of the corral that includes the gate opening, as shown.



So, you need  $2w + \ell - 3 = 2(15) + 30 - 3 = 57$  meters of fencing.

#### Monitoring Progress Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

- About 58,000 people live in a circular region with a 2-mile radius. Find the population density in people per square mile.
- A circular region has a population of about 175,000 people and a population density of about 1318 people per square mile. Find the radius of the region.
- WHAT IF?** You want the corral to have an area of 800 square meters. How many meters of fencing do you need?

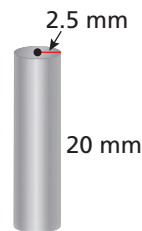
## Using Surface Area Formulas

### EXAMPLE 4 Using a Surface Area Formula



A manufacturer designs the bearing shown. To prevent corrosion, the manufacturer coats each bearing with an anticorrosive grease.

- A smaller bearing has linear dimensions that are one-half the dimensions of the bearing shown. Does the smaller bearing require one-half of the amount of grease that is used to coat the larger bearing? Explain.
- A smaller bearing has the same radius and a height that is one-half the height of the bearing shown. Is one-half of the amount of anticorrosive grease used to coat the larger bearing enough to coat this bearing? Explain.



### SOLUTION

The bearing is cylindrical. So, use the formula for the surface area of a cylinder to compare the bearings.

	Larger bearing	Smaller bearing
<b>Dimensions</b>	$r = 2.5 \text{ mm}, h = 20 \text{ mm}$	$r = 1.25 \text{ mm}, h = 10 \text{ mm}$
<b>Surface area</b>	$S = 2\pi r^2 + 2\pi r h$ $= 2\pi(2.5)^2 + 2\pi(2.5)(20)$ $= 112.5\pi \text{ mm}^2$	$S = 2\pi r^2 + 2\pi r h$ $= 2\pi(1.25)^2 + 2\pi(1.25)(10)$ $= 28.125\pi \text{ mm}^2$

The surface area of the smaller bearing is  $\frac{28.125\pi}{112.5\pi} = \frac{1}{4}$  times the surface area of the larger bearing.

- ▶ No, the smaller bearing requires only  $\frac{1}{4}$  of the amount of grease that is used to coat the larger bearing.

- b. The smaller bearing has a radius of 2.5 millimeters and a height of 10 millimeters. Its surface area is

$$S = 2\pi(2.5)^2 + 2\pi(2.5)(10)$$

$$= 62.5\pi \text{ mm}^2.$$

So, the surface area of the smaller bearing is  $\frac{62.5\pi}{112.5\pi} = \frac{5}{9}$  times the surface area of the larger bearing.

- ▶ No, because  $\frac{5}{9} > \frac{1}{2}$ , one-half of the amount of anticorrosive grease used to coat the larger bearing is not enough to coat the smaller bearing.

### ANALYZING MATHEMATICAL RELATIONSHIPS

Notice that while the surface area does not scale by a factor of  $\frac{1}{2}$ , the lateral surface area does scale by a factor of

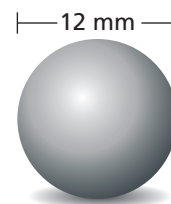
$$\frac{2\pi(2.5)(10)}{2\pi(2.5)(20)} = \frac{50\pi}{100\pi} = \frac{1}{2}.$$

### Monitoring Progress



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4. A manufacturer designs the bearing shown. To prevent corrosion, the manufacturer coats each bearing with an anticorrosive grease. Does a bearing with a diameter that is  $\frac{3}{2}$  times the diameter of the given bearing need  $\frac{3}{2}$  times the amount of grease to coat it? Explain.



## Vocabulary and Core Concept Check

- WRITING** Explain, in your own words, the difference between the population of a state and the population density of a state.
- WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

bears per square mile

video gamers per city

people per square kilometer

trees per acre

## Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, find the indicated measure.

(See Example 1.)

- The state of Kansas has a population of about 2.85 million people. Find the population density in people per square mile.



- About 210,000 people live in a circular region with a 12-mile radius. Find the population density in people per square mile.
- About 650,000 people live in a circular region with a 6-mile radius. Find the population density in people per square mile.
- Yellowstone National Park has an area of about 2.22 million acres. The table shows the estimated park populations for several animals. Find the population density in animals per acre for each animal.

Animal	Grizzly bear	Elk	Mule deer	Bighorn sheep
Population	445	20,000	2400	260

- A circular region with a 4-mile radius has a population density of 6366 people per square mile. Find the number of people who live in the region.
- Central Park in New York City is rectangular with a length of 2.5 miles and a width of 0.5 mile. During an afternoon, its population density is about 15 people per acre. Find the number of people in the park that afternoon. One acre is equal to  $\frac{1}{640}$  square mile.

- PROBLEM SOLVING** About 79,000 people live in a circular region with a population density of about 513 people per square mile. Find the radius of the region. (See Example 2.)
- PROBLEM SOLVING** About 1.15 million people live in a circular region with a population density of about 18,075 people per square kilometer. Find the radius of the region.
- ERROR ANALYSIS** Describe and correct the error in finding the number of people who live in a circular region with a 7.5-mile diameter and a population density of 1550 people per square mile.



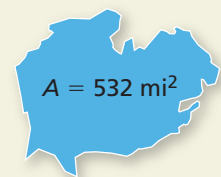
$$1550 = \frac{x}{\pi \cdot 7.5^2}$$

$$1550 = \frac{x}{56.25\pi}$$

$$273,908 \approx x$$

The number of people who live in the region is about 273,908.

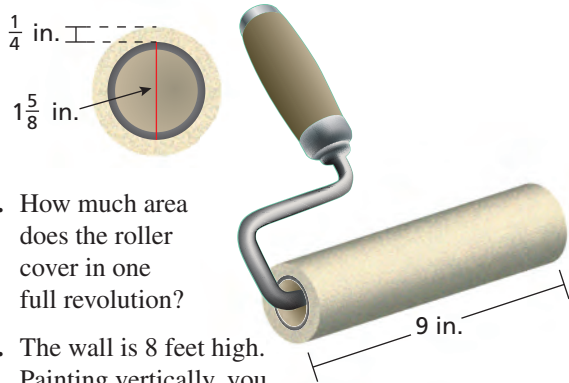
- HOW DO YOU SEE IT?** The two islands shown below with the given areas have the same population. Which has the greater population density? Explain.





- 13. MODELING WITH MATHEMATICS** A soccer field of length  $\ell$  and width  $w$  has a perimeter of 320 yards. (See Example 3.)
- Write an expression that represents the area of the soccer field in terms of  $\ell$ .
  - Use your expression from part (a) to determine the dimensions of the field that maximize the area. What do you notice?

- 14. MODELING WITH MATHEMATICS** You are using a 9-inch paint roller to paint a wall of your bedroom. The roller has a diameter of  $1\frac{5}{8}$  inches and a nap thickness of  $\frac{1}{4}$  inch, as shown. The *nap* is the fuzzy material on the surface of the roller.



- How much area does the roller cover in one full revolution?
- The wall is 8 feet high. Painting vertically, you start at the bottom of the wall and make 8 full revolutions with the roller. Are you more than halfway up the wall? Justify your answer.
- A 3-inch roller has the same diameter and nap thickness as the roller above. How much area does this roller cover in one full revolution? How does this compare to the area in part (a)?

- 17. MODELING WITH MATHEMATICS** A playground ball with a 16-inch diameter has a rubber coating on its surface. (See Example 4.)

- Does a ball with a diameter that is  $\frac{1}{4}$  times the diameter of the given ball need  $\frac{1}{4}$  times the amount of rubber coating? Explain.
- What is the radius of a ball that uses one-half of the amount of rubber coating used to cover the 16-inch ball?

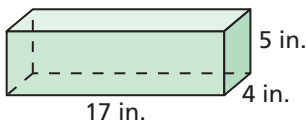
- 18. MODELING WITH MATHEMATICS** A cylindrical swimming pool has a diameter of 24 feet and a height of 4 feet. A smaller pool with the same height has a diameter of 12 feet. A vinyl liner covers the bottom and side of each pool.

- Does the smaller pool require one-half of the amount of vinyl liner that is used to cover the larger pool? Explain. If not, estimate the diameter of the cylindrical swimming pool that uses one-half of the amount of vinyl liner used to cover the larger pool. Assume the height of the pool is 4 feet.
- To install the vinyl liner, it costs \$1.95 per square foot. You have \$1000 in your budget to spend on the liner installation. What is the largest pool you can get without going over your liner budget? Assume the height of the pool is 4 feet.

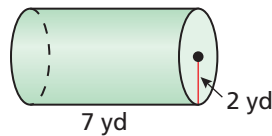
- 19. MAKING AN ARGUMENT** You ask your friend which U.S. states have the greatest population densities. Your friend says it must be California and Texas because they have the greatest populations. Is your friend correct? Explain.

In Exercises 15 and 16, describe how the change affects the surface area of the right prism or right cylinder.

- 15.** doubling all the linear dimensions



- 16.** tripling the radius



- 20. THOUGHT PROVOKING** Give an example from your everyday life of an object whose surface area changes. Sketch the object and determine what geometric shape can model it. Estimate the dimensions of the object before and after the change and explain how the surface area is affected.

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Evaluate the expression when  $x = -4$  and  $y = 3$ . (Skills Review Handbook)

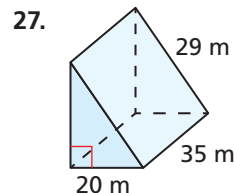
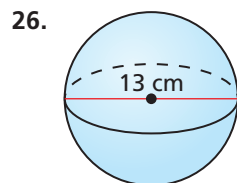
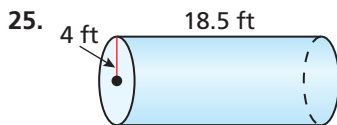
21.  $2x^2 + 5y^2$

22.  $\frac{1}{3}(8xy - y^2)$

23.  $-x^2 + 6x + 11y$

24.  $-5(y + x^3 + 10)$

Find the volume of the solid. (Skills Review Handbook)



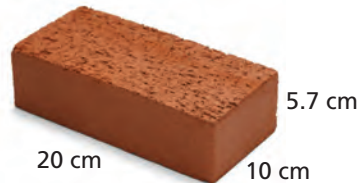
# 1.2 Modeling with Volume

**Essential Question** How can you use the mass and volume of an object to describe the density of the object?

## EXPLORATION 1 Finding Densities

**Work with a partner.** Approximate the volume of each object whose mass is given. Then find the mass per unit of volume, or *density*, of each object.

a. Brick: 2.3 kg



b. Log: 18.1 kg



c. Golf ball: 45.9 g



d. Cork: 2.6 g



## CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to justify your conclusions and communicate them to others.

## EXPLORATION 2 Analyzing Densities

**Work with a partner.** The objects in Exploration 1 with a density greater than 1 gram per cubic centimeter will sink in water. The objects with a density less than 1 gram per cubic centimeter will float in water. You place each object in Exploration 1 in a bucket of water.

- Which object(s) sink? float? Justify your answer.
- Would your answers in part (a) change when each object is cut in half and placed in water? Explain your reasoning.
- You dissolve enough salt in a bucket of water to cause one of the sunken objects to float. Which object is it and why do you think this happens?

## Communicate Your Answer

- How can you use the mass and volume of an object to describe the density of the object?
- Use the Internet or some other reference to research the densities of water, mineral oil, and beeswax. You combine these substances in a bucket. How do you think the liquids interact? Where would the beeswax settle?

# 1.2 Lesson

## Core Vocabulary

density, p. 10

Previous

volume

square root

## What You Will Learn

- ▶ Use volume formulas to find densities.
- ▶ Use volume formulas to solve problems.

## Using Volume Formulas to Find Densities

**Density** is the amount of matter that an object has in a given unit of volume. The density of an object is calculated by dividing its mass by its volume.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

Different materials have different densities, so density can be used to distinguish between materials that look similar. For example, table salt and sugar look alike. However, table salt has a density of 2.16 grams per cubic centimeter, while sugar has a density of 1.58 grams per cubic centimeter.

### EXAMPLE 1 Using the Formula for Density

The diagram shows the dimensions of a standard gold bar at Fort Knox. Gold has a density of 19.3 grams per cubic centimeter. Find the mass of a standard gold bar to the nearest gram.

#### SOLUTION

**Step 1** Convert the dimensions to centimeters using 1 inch = 2.54 centimeters.

$$\text{Length } 7 \text{ in.} \cdot \frac{2.54 \text{ cm}}{1 \text{ in.}} = 17.78 \text{ cm}$$

$$\text{Width } 3.625 \text{ in.} \cdot \frac{2.54 \text{ cm}}{1 \text{ in.}} = 9.2075 \text{ cm}$$

$$\text{Height } 1.75 \text{ in.} \cdot \frac{2.54 \text{ cm}}{1 \text{ in.}} = 4.445 \text{ cm}$$

**Step 2** Find the volume.

The area of a base is  $B = 17.78(9.2075) = 163.70935 \text{ cm}^2$  and the height is  $h = 4.445 \text{ cm}$ .

$$V = Bh = 163.70935(4.445) \approx 727.69 \text{ cm}^3$$

**Step 3** Let  $x$  represent the mass in grams. Substitute the values for the volume and the density in the formula for density and solve for  $x$ .

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \quad \text{Formula for density}$$

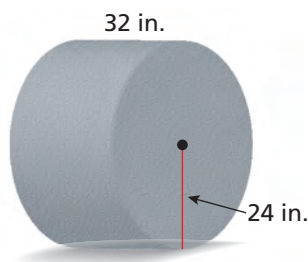
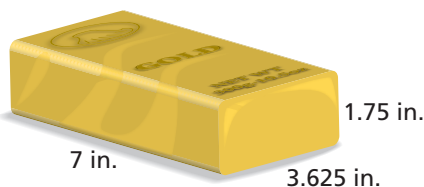
$$19.3 \approx \frac{x}{727.69} \quad \text{Substitute.}$$

$$14,044 \approx x \quad \text{Multiply each side by 727.69.}$$

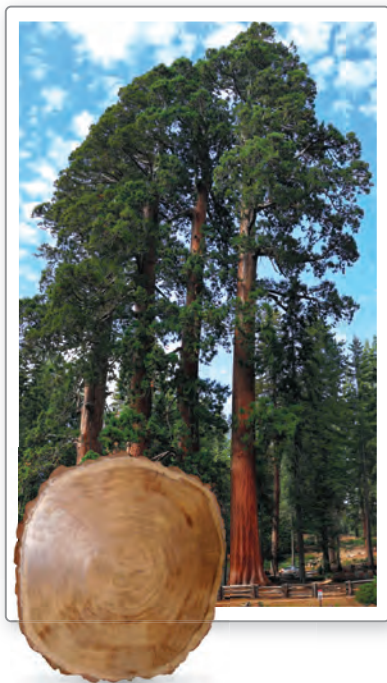
- ▶ The mass of a standard gold bar is about 14,044 grams.

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1. The diagram shows the dimensions of a concrete cylinder. Concrete has a density of 2.3 grams per cubic centimeter. Find the mass of the concrete cylinder to the nearest gram.



## Using Volume Formulas



### EXAMPLE 2 Using a Volume Formula

A tree harvester is often interested in the volume of a tree's trunk because most of the wood volume is located there. A tree harvester estimates the trunk of a sequoia tree to have a height of about 50 meters and a base diameter of about 0.8 meter.

- The wood of a sequoia tree has a density of about 450 kilograms per cubic meter. Find the mass of the trunk to the nearest kilogram.
- Each year, the tree trunk forms new cells that arrange themselves in concentric circles called *growth rings*. These rings indicate how much wood the tree produces annually. The harvester estimates that the trunk will put on a growth ring of about 1 centimeter thick and its height will increase by about 0.25 meter this year. How many cubic meters of wood does the tree trunk produce after one year? If the tree grows at a constant rate for the next five years, will it produce the same amount of wood each year? Explain.

### SOLUTION

- To estimate the volume of the tree trunk, assume that the trunk is cylindrical. So, the volume of the trunk is

$$V = \pi r^2 h = \pi(0.4)^2(50) = 8\pi \approx 25.13 \text{ m}^3.$$

Let  $x$  represent the mass in kilograms. Substitute the values for the volume and the density in the formula for density and solve for  $x$ .

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \quad \text{Formula for density}$$

$$450 \approx \frac{x}{25.13} \quad \text{Substitute.}$$

$$11,309 \approx x \quad \text{Multiply each side by 25.13.}$$

- The mass of the trunk is about 11,309 kilograms.

- Make a table that shows the trunk dimensions and volume for five years.

Year	1	2	3	4	5
Height (meters)	50.25	50.5	50.75	51	51.25
Base radius (meters)	0.41	0.42	0.43	0.44	0.45
Volume (cubic meters)	26.54	27.99	29.48	31.02	32.60

$$+ 1.45 \quad + 1.49 \quad + 1.54 \quad + 1.58$$

- The tree will produce about  $26.54 - 25.13 = 1.41$  cubic meters of wood after one year. The tree will not produce the same amount of wood each year for five years because the differences between the volumes from year to year are increasing.

### COMMON ERROR

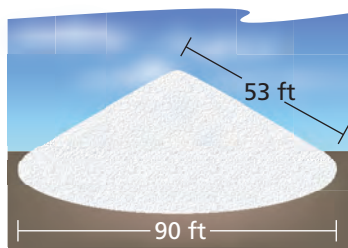
Because  $1 \text{ cm} = 0.01 \text{ m}$ , the trunk will have a diameter of  $0.8 + 0.01 + 0.01 = 0.82 \text{ m}$  for Year 1.



### Monitoring Progress Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

- WHAT IF?** The tree harvester makes the same growth estimates for the trunk of a sequoia tree that has a height of about 40 meters and a base diameter of about 0.75 meter. (a) Find the mass of the trunk to the nearest kilogram. (b) How many cubic meters of wood will the trunk gain after four years?

### EXAMPLE 3 Using a Volume Formula



Before a winter storm, a pile of road salt has the dimensions shown. After the storm, the linear dimensions of the pile are one-half of the original dimensions.

- How does this change affect the volume of the pile?
- A *lane mile* is an area of pavement that is one mile long and one lane wide. During the storm, about 400 pounds of road salt was used for every lane mile. Estimate the number of lane miles that were covered with road salt during the storm. A cubic foot of road salt weighs about 80 pounds.

### SOLUTION

- The pile of road salt is approximately shaped like a cone. Use the Pythagorean Theorem to find the height  $h$ . Then use the formula for the volume of a cone to find the volume of the pile before and after the storm.

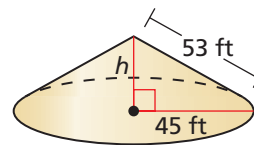
$$c^2 = a^2 + b^2 \quad \text{Pythagorean Theorem}$$

$$53^2 = h^2 + 45^2 \quad \text{Substitute.}$$

$$2809 = h^2 + 2025 \quad \text{Multiply.}$$

$$784 = h^2 \quad \text{Subtract 2025 from each side.}$$

$$28 = h \quad \text{Find the positive square root.}$$



### ANALYZING MATHEMATICAL RELATIONSHIPS

Notice that when all the linear dimensions are multiplied by  $k$ , the volume is multiplied by  $k^3$ .

	Before winter storm	After winter storm
<b>Dimensions</b>	$r = 45 \text{ ft}, h = 28 \text{ ft}$	$r = 22.5 \text{ ft}, h = 14 \text{ ft}$
<b>Volume</b>	$V = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi(45)^2(28)$ $= 18,900\pi \text{ ft}^3$	$V = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi(22.5)^2(14)$ $= 2362.5\pi \text{ ft}^3$

- ▶ The volume of the pile after the winter storm is  $\frac{2362.5\pi}{18,900\pi} = \frac{1}{8}$  times the original volume.

- During the storm,  $18,900\pi - 2362.5\pi = 16,537.5\pi$  cubic feet of road salt was used. Use conversions to find the number of lane miles covered with road salt during the storm.

$$\text{Pounds of road salt used: } 16,537.5\pi \text{ ft}^3 \cdot \frac{80 \text{ lb}}{1 \text{ ft}^3} = 1,323,000\pi \text{ lb}$$

$$\text{Lane miles covered: } 1,323,000\pi \text{ lb} \cdot \frac{1 \text{ lane mile}}{400 \text{ lb}} \approx 10,390.82 \text{ lane miles}$$

- ▶ So, about 10,400 lane miles were covered with road salt during the storm.

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- In Example 3, the department of transportation pays about \$31.50 for each ton of road salt. How much does the original pile of road salt cost?
- WHAT IF?** After a storm, the linear dimensions of the pile are  $\frac{1}{4}$  of the original dimensions. (a) How does this change affect the volume of the pile? (b) Estimate the number of lane miles that were covered with road salt during the storm.

# 1.2 Exercises

## Vocabulary and Core Concept Check

1. **VOCABULARY** What do you obtain when you multiply an object's density by its volume?

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

What is the mass per unit of volume?

What is the mass in kilograms?

What is the mass divided by the volume?

What is the density?

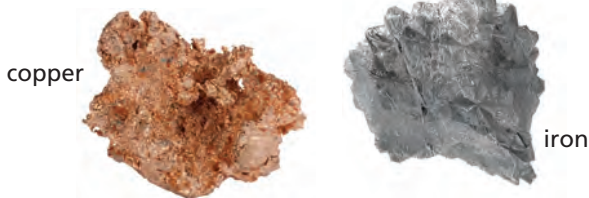
Tourmaline



Mass = 6.2 g  
Volume = 2 cm<sup>3</sup>

## Monitoring Progress and Modeling with Mathematics

3. **PROBLEM SOLVING** A piece of copper with a volume of 8.25 cubic centimeters has a mass of 73.92 grams. A piece of iron with a volume of 5 cubic centimeters has a mass of 39.35 grams. Which metal has the greater density?



4. **PROBLEM SOLVING** The United States has minted one-dollar silver coins called the American Eagle Silver Bullion Coin since 1986. Each coin has a diameter of 40.6 millimeters and is 2.98 millimeters thick. The density of silver is 10.5 grams per cubic centimeter. What is the mass of an American Eagle Silver Bullion Coin to the nearest gram? (See Example 1.)



5. **ERROR ANALYSIS** Describe and correct the error in finding the density of an object that has a mass of 24 grams and a volume of 28.3 cubic centimeters.



$$\text{density} = \frac{28.3}{24} \approx 1.18$$

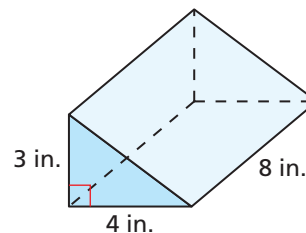
So, the density is about 1.18 cubic centimeters per gram.

6. **PROBLEM SOLVING** The height of a tree trunk is 20 meters and the base diameter is 0.5 meter. (See Example 2.)

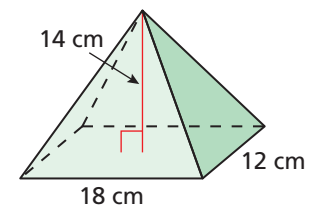
- The wood has a density of 380 kilograms per cubic meter. Find the mass of the trunk to the nearest kilogram.
- The trunk puts on a growth ring of 4 millimeters and its height increases by 0.2 meter this year. How many cubic meters of wood does the tree trunk produce? The tree grows at a constant rate for the next five years. Does the tree produce the same amount of wood each year? Explain.

In Exercises 7 and 8, describe how the change affects the volume of the prism or pyramid.

7. tripling all the linear dimensions



8. multiplying the height by  $\frac{3}{2}$



9. **PROBLEM SOLVING** A conical pile of road salt has a diameter of 112 feet and a slant height of 65 feet. After a storm, the linear dimensions of the pile are  $\frac{1}{3}$  of the original dimensions. (See Example 3.)

- How does this change affect the volume of the pile?
- During the storm, 350 pounds of road salt was used for every lane mile. Estimate the number of lane miles that were covered with salt. How many lane miles can be covered with the remaining salt? A cubic foot of road salt weighs about 80 pounds.

10. **HOW DO YOU SEE IT?** The two stone blocks shown below with the given densities have the same volume. Which block has a greater mass? Explain.

Granite:  $2.7 \text{ g/cm}^3$

Sandstone:  $2.3 \text{ g/cm}^3$



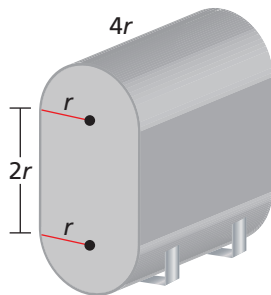
11. **MODELING WITH MATHEMATICS** A pool in the shape of a rectangular prism is 6 meters long and 3 meters wide. The water in the pool is 1 meter deep.

- The density of water is about 1 gram per cubic centimeter. Find the number of kilograms of water in the pool.
- You add 6000 kilograms of water to the pool. What is the depth of the water in the pool?

12. **MODELING WITH MATHEMATICS** A British thermal unit (Btu) is the amount of heat needed to raise the temperature of 1 pound of liquid water by  $1^\circ\text{F}$ . There are about 1000 Btu per cubic foot of natural gas and about 140,000 Btu per gallon of heating oil.

- In 2010, electricity-generating power plants paid \$5.27 per 1000 cubic feet of natural gas and \$56.35 per 42-gallon barrel of heating oil. Express the cost of each fuel in dollars per million Btu.

- The tank shown can be used to store heating oil. Write a formula for the volume of the tank.



- You pay \$3.75 per gallon of heating oil to fill a new tank in which  $r$  is 1 foot. Compare your cost for heating oil in dollars per million Btu to a power plant's cost in part (a). How many Btu can be produced from a full tank of heating oil?

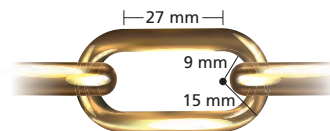
13. **MAKING AN ARGUMENT** As ocean depth increases, water molecules get pushed closer together due to the weight of the water above. Your friend says that the density of water increases as depth increases. Is your friend correct? Explain.

14. **THOUGHT PROVOKING** You place two cans of regular soda and two cans of diet soda in a container full of water. The two regular cans sink, but the two diet cans float.



Use the Internet to research the contents of regular soda and diet soda. Then make a conjecture about why the diet cans float, but the regular cans sink. Include a discussion of *density* and *buoyancy* in your explanation.

15. **MODELING WITH MATHEMATICS** Links of a chain are made from cylindrical metal rods with a diameter of 6 millimeters. The density of the metal is about 8 grams per cubic centimeter.



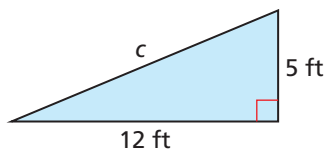
- To approximate the length of a rod used to make a link, should you use the perimeter around the inside of the link? the outside? the average of these perimeters? Explain your reasoning. Then approximate the mass of a chain with 100 links.
- Approximate the length of a taut chain with 100 links. Explain your procedure.

## Maintaining Mathematical Proficiency

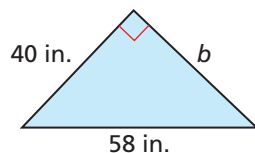
Reviewing what you learned in previous grades and lessons

Find the missing length of the triangle. (*Skills Review Handbook*)

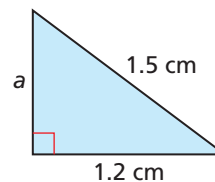
16.



17.



18.



# 1.1–1.2 What Did You Learn?

## Core Vocabulary

population density, *p. 4*

density, *p. 10*

## Core Concepts

### Section 1.1

Finding a Population Density, *p. 4*

### Section 1.2

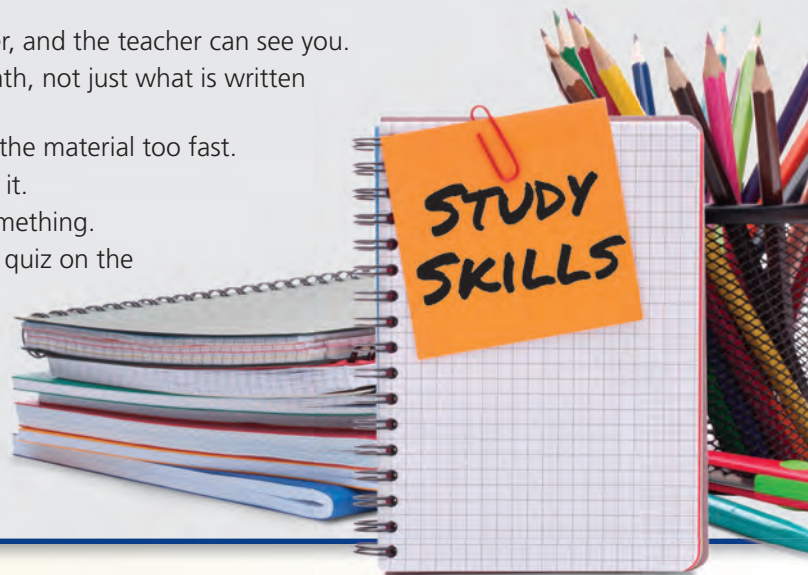
Finding a Density, *p. 10*

## Mathematical Practices

1. In Exercise 17(b) on page 8, explain the steps you used to find the radius of the ball.
2. You have a classmate who is confused about the difference between the mass of an object and the weight of an object in Exercise 4 on page 13. What resources can you use to help your friend figure it out?
3. In Exercise 7 on page 13, explain why the given change results in a volume that is 27 times the original volume.

## Taking Control of Your Class Time

1. Sit where you can easily see and hear the teacher, and the teacher can see you.
2. Pay attention to what the teacher says about math, not just what is written on the board.
3. Ask a question if the teacher is moving through the material too fast.
4. Try to memorize new information while learning it.
5. Ask for clarification if you do not understand something.
6. Think as intensely as if you were going to take a quiz on the material at the end of class.
7. Volunteer when the teacher asks for someone to go up to the board.
8. At the end of class, identify concepts or problems for which you still need clarification.
9. Use the tutorials at [BigIdeasMath.com](http://BigIdeasMath.com) for additional help.





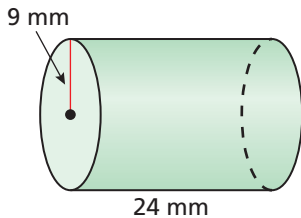
# 1.1–1.2 Quiz

- The state of Wyoming has a population of about 564,000 people. The population density of the state is about 2.16 people per square kilometer. Find the width of Wyoming. (*Section 1.1*)
- A circular region with a 3-mile radius has a population density of about 6195 people per square mile. Find the number of people who live in the region. (*Section 1.1*)

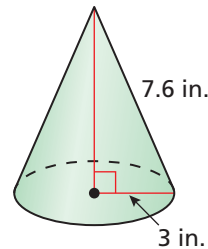


Describe how the change affects the surface area of the right cylinder or right cone. (*Section 1.1*)

3. multiplying all the linear dimensions by  $\frac{1}{3}$

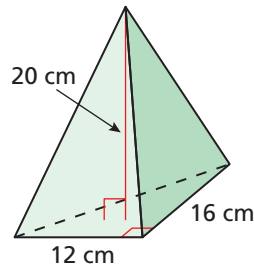
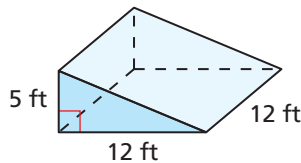


4. doubling the radius



Describe how the change affects the volume of the prism or pyramid. (*Section 1.2*)

5. tripling the base and the height of the triangular bases
6. multiplying all the linear dimensions by  $\frac{1}{4}$



7. You make the ball of aluminum foil shown. (*Section 1.1 and Section 1.2*)

- You add foil to the ball so its diameter is  $\frac{3}{2}$  times the original diameter. How does this change affect the surface area and volume of the ball?
- Aluminum foil has a density of about 2.7 grams per cubic centimeter. Find the mass of the ball before and after you add the foil.



8. An ice sculptor is carving the block of ice shown. (*Section 1.2*)

- The block has a mass of about 745 kilograms. Find the density of the block of ice. Is it greater than the density of water? Explain. The density of water is about 1 gram per cubic centimeter.
- Another rectangular block of ice has a mass of about 298 kilograms. Give two possible sets of dimensions for the block of ice.



## 1.3 Cross Sections of Solids

**Essential Question** How can you use a piece of food to create a real-life cross section?

Imagine cutting through a piece of food. The intersection formed by the cut you make and the piece of food is called a **cross section**. The shape of the cross section depends on the angle of the cut you make.

### EXPLORATION 1 Describing Cross Sections

**Work with a partner.** Describe the shapes of the cross sections that are formed by making cuts into each given food so that it is separated into two congruent parts.

a. wheel of cheese



b. watermelon



c. stick of butter



d. cucumber



### EXPLORATION 2 Forming Cross Sections

**Work with a partner.** Describe how you can slice the portion of cheese so that the cross section formed is the given shape.

- a. triangle
- b. rectangle
- c. trapezoid



### ATTENDING TO PRECISION

To be proficient in math, you need to communicate precisely with others.

### Communicate Your Answer

3. How can you use a piece of food to create a real-life cross section?
4. Is there more than one way to slice the portion of cheese in Exploration 2 to form a triangular cross section? Explain. Use drawings to support your answer.

# 1.3 Lesson

## Core Vocabulary

cross section, p. 18

### Previous

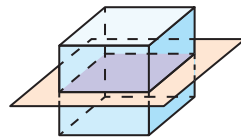
plane  
Pythagorean Theorem  
square root

## What You Will Learn

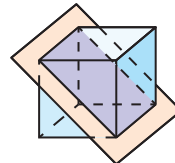
- Describe cross sections.
- Draw cross sections.
- Solve real-life problems involving cross sections.

## Describing Cross Sections

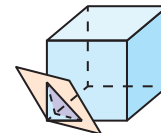
Imagine a plane slicing through a solid. The intersection of the plane and the solid is called a **cross section**. For example, three different cross sections of a cube are shown below.



square



rectangle



triangle

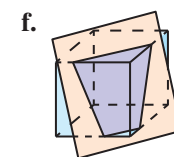
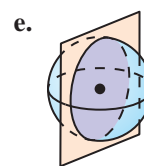
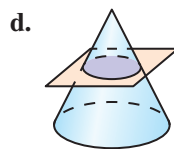
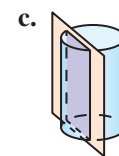
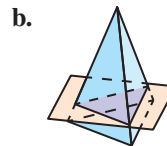
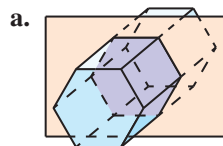
## STUDY TIP

To help you better visualize a cross section, rotate the solid mentally so you are looking directly at the cross section.

### EXAMPLE 1

## Describing Cross Sections

Describe the shape formed by the intersection of the plane and the solid.



## SOLUTION

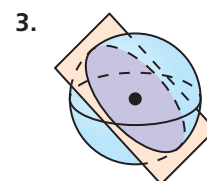
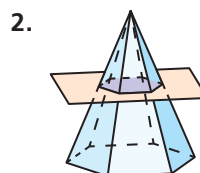
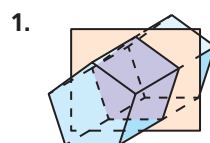
- a. The cross section is a hexagon.
- b. The cross section is a triangle.
- c. The cross section is a rectangle.
- d. The cross section is a circle.
- e. The cross section is a circle.
- f. The cross section is a trapezoid.

## Monitoring Progress



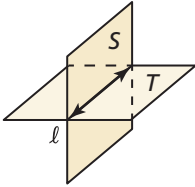
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Describe the shape formed by the intersection of the plane and the solid.



## REMEMBER

An example of the Plane Intersection Postulate is shown below, where the intersection of plane  $S$  and plane  $T$  is line  $\ell$ .

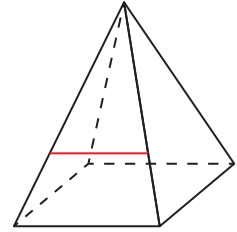


## Drawing Cross Sections

The Plane Intersection Postulate states that if two planes intersect, then their intersection is a line. This postulate can help you when drawing a cross section.

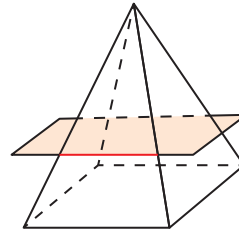
### EXAMPLE 2 Drawing a Cross Section

Draw the cross section formed by a plane parallel to the base that intersects the red line segment drawn on the square pyramid. What is the shape of the cross section?

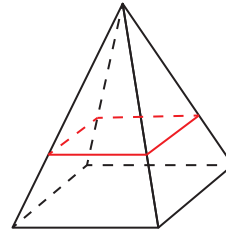


### SOLUTION

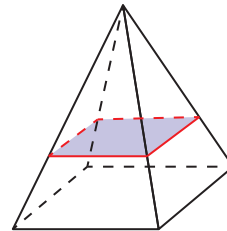
**Step 1** Visualize a horizontal plane parallel to the base that intersects the lateral face and passes through the red line segment.



**Step 2** The horizontal plane is parallel to the base of the pyramid. So, draw each pair of parallel line segments where the plane intersects the lateral faces of the pyramid.



**Step 3** Shade the cross section.



► The cross section is a square.

## Monitoring Progress



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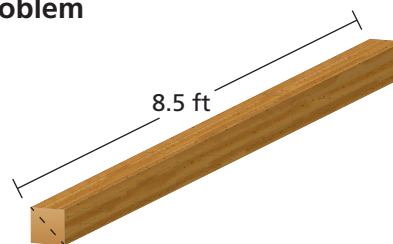
- WHAT IF?** Draw the cross section formed by a plane perpendicular to the base that intersects the vertex of the square pyramid in Example 2. What is the shape of the cross section?
- REASONING** Describe how a plane can intersect the pyramid in Example 2 so that it forms a cross section that is (a) a trapezoid and (b) a line segment.

## Solving Real-Life Problems

### EXAMPLE 3 Solving a Real-Life Problem



A machine at a sawmill cuts a 4-inch by 4-inch piece of wood lengthwise along its diagonal, as shown. Find the perimeter and area of the cross section formed by the cut.



### SOLUTION

- 1. Understand the Problem** You know that the piece of wood is shaped like a rectangular prism with a length of 8.5 feet and a width and height of 4 inches. You are asked to calculate the perimeter and area of the cross section formed when an 8.5-foot cut is made along its diagonal.
- 2. Make a Plan** Determine the shape and the dimensions of the cross section. Then use the dimensions to calculate the perimeter and area of the cross section.
- 3. Solve the Problem** Draw a diagram of the cross section. It is a rectangle with a length of 8.5 feet, or 102 inches.



Use the Pythagorean Theorem to find its width. The length and width of the end of the piece of wood is 4 inches.

$$c^2 = a^2 + b^2 \quad \text{Pythagorean Theorem}$$

$$c^2 = 4^2 + 4^2 \quad \text{Substitute.}$$

$$c^2 = 16 + 16 \quad \text{Multiply.}$$

$$c^2 = 32 \quad \text{Add.}$$

$$c = \sqrt{32} \quad \text{Find the positive square root.}$$

$$c = 4\sqrt{2} \quad \text{Simplify.}$$

The width of the rectangular cross section is  $4\sqrt{2}$  inches.

#### Perimeter of cross section

$$\begin{aligned} P &= 2\ell + 2w \\ &= 2(102) + 2(4\sqrt{2}) \\ &= 204 + 8\sqrt{2} \\ &\approx 215.31 \end{aligned}$$

#### Area of cross section

$$\begin{aligned} A &= \ell w \\ &= 102 \cdot 4\sqrt{2} \\ &= 408\sqrt{2} \\ &\approx 577 \end{aligned}$$

▶ The perimeter of the cross section is about 215.31 inches and the area of the cross section is about 577 square inches.

- 4. Look Back** You can use estimation to check that your answer is reasonable. The length of the rectangular cross section is about 9 feet and its width is about 0.5 foot.

**Perimeter of cross section:**  $P = 2\ell + 2w = 2(9) + 2(0.5) = 19 \text{ ft} = 228 \text{ in.}$  ✓

**Area of cross section:**  $A = \ell w = 9 \cdot 0.5 = 4.5 \text{ ft}^2 = 648 \text{ in.}^2$  ✓

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6. A 6-inch by 6-inch piece of wood that is 10.25 feet long is cut lengthwise along its diagonal. Find the perimeter and area of the cross section formed by the cut.

# 1.3 Exercises

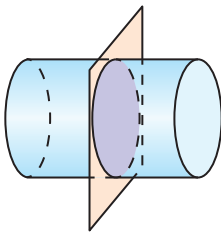
## Vocabulary and Core Concept Check

- COMPLETE THE SENTENCE** The intersection of a plane and a solid is called a \_\_\_\_\_.
- WRITING** Can a plane intersect a rectangular prism and form a cross section that is a circle? Explain.

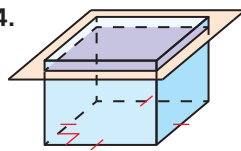
## Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, describe the shape formed by the intersection of the plane and the solid. (See Example 1.)

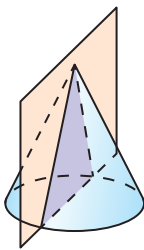
3.



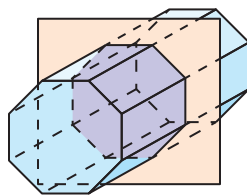
4.



5.

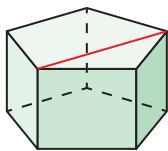


6.

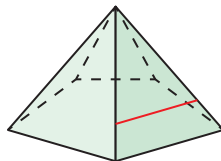


In Exercises 7–10, draw the cross section formed by the described plane that intersects the red line segment drawn on the solid. What is the shape of the cross section? (See Example 2.)

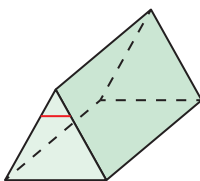
7. plane is perpendicular to base



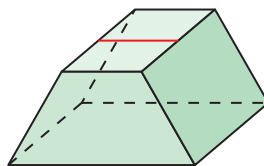
8. plane is parallel to base



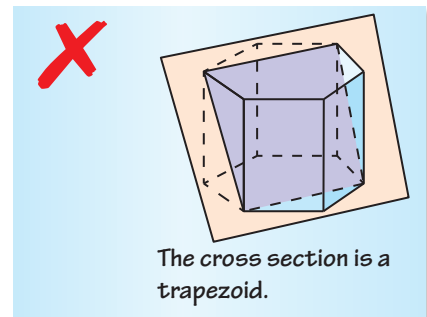
9. plane is parallel to bottom face



10. plane is perpendicular to bottom face



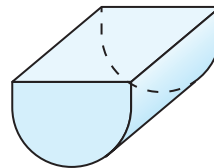
11. **ERROR ANALYSIS** Describe and correct the error in describing the shape formed by the intersection of the plane and the regular hexagonal prism.



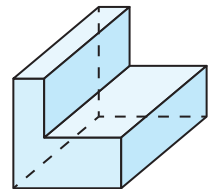
12. **OPEN-ENDED** Give an example of a solid from which a triangular, hexagonal, and trapezoidal cross section can be formed.

In Exercises 13–18, draw the cross section formed by a vertical plane that divides the solid into two congruent parts. Is there more than one way to use a vertical plane to divide the figure into two congruent parts? If so, does the cross section change? Explain.

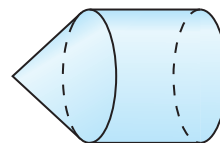
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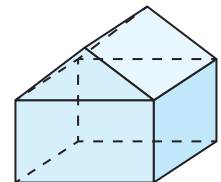
14.



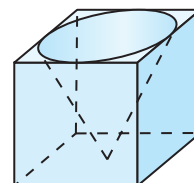
15.



16.



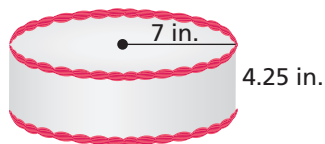
17.



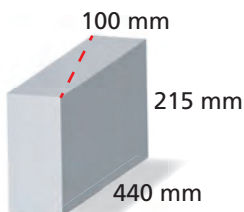
18.



19. **PROBLEM SOLVING** You cut the cake vertically to make two congruent parts. (See Example 3.)



- Find the perimeter and area of the cross section formed by the cut.
  - Find the surface area of the cake that is not frosted before the cut. How does the unfrosted surface area change after the cut?
  - Can the cake be cut another way to make two congruent parts? If so, find the perimeter and area of the cross section formed by the cut.
20. **PROBLEM SOLVING** A mason uses a concrete saw to cut the block along the indicated diagonal.



- Identify the solids formed by the cut.
  - Find the perimeter and area of the cross section formed by the cut.
  - The block has a density of about 0.002 gram per cubic millimeter. Find the mass of the block to the nearest gram.
21. **PROBLEM SOLVING** Use the figure shown.

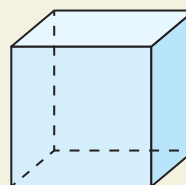


- One of the hexagonal pipes is cut vertically so that it is divided into two congruent parts. Draw two possible cross sections.
- How many different ways can a pipe be cut lengthwise to form two congruent parts? Explain.

22. **PROBLEM SOLVING** A regular octagonal pyramid is intersected by a plane perpendicular to its base. The plane passes through its vertex so it is divided into two congruent parts. Draw the cross section. Is there more than one way to divide the pyramid into two congruent parts? If so, does the shape of the cross section change? Explain.

23. **MAKING AN ARGUMENT** Your friend says that any plane that intersects a sphere forms a circular cross section. Is your friend correct? Explain.

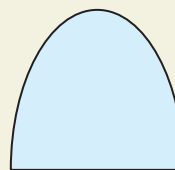
24. **HOW DO YOU SEE IT?** Draw a plane that intersects the cube to form the given cross section.



- |                       |                         |
|-----------------------|-------------------------|
| a. isosceles triangle | b. equilateral triangle |
| c. scalene triangle   | d. parallelogram        |
| e. pentagon           | f. hexagon              |

25. **REASONING** A plane intersects a sphere 7 meters from the center of the sphere. The radius of the sphere is 25 meters. Draw a diagram to represent this situation. Then find the area of the cross section to the nearest tenth.

26. **THOUGHT PROVOKING** Describe a solid that can be intersected by a plane to form the cross section shown. Explain how you form the cross section.

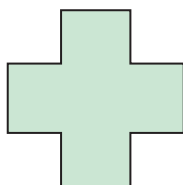


## Maintaining Mathematical Proficiency

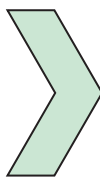
Reviewing what you learned in previous grades and lessons

Tell how many lines of symmetry the figure has. (Skills Review Handbook)

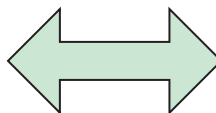
27.



28.



29.



30.



Graph the inequality in a coordinate plane. (Skills Review Handbook)

31.  $x > 0$

32.  $x \leq \frac{1}{2}$

33.  $y \geq 0$

34.  $y < -2.5$

# 1.4 Solids of Revolution

## Essential Question

How can you create a solid of revolution?

A **solid of revolution** is a three-dimensional figure that is formed by rotating a two-dimensional shape around an axis.

### EXPLORATION 1 Creating Solids of Revolution

**Work with a partner.** Tape the 5-inch side of a 3-inch by 5-inch index card to a pencil, as shown.

a. Rotate the pencil. What type of solid is produced by the rotating index card? What are its dimensions?

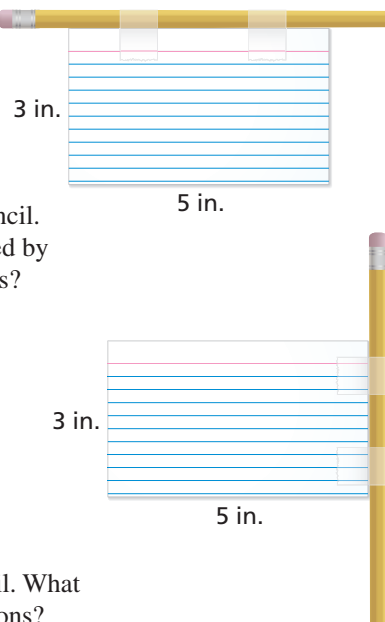
b. Tape the 3-inch side of the index card to the pencil. Rotate the pencil. What type of solid is produced by the rotating index card? What are its dimensions?

c. Do the solids in parts (a) and (b) have the same surface area? the same volume? Justify your answers.

d. Cut the index card in half along its diagonal. Tape the 5-inch leg of the triangle formed to a pencil. Rotate the pencil. What type of solid is produced? What are its dimensions?

e. Tape the 3-inch leg to a pencil. Rotate the pencil. What type of solid is produced? What are its dimensions?

f. Do the solids in parts (d) and (e) have the same surface area? the same volume? Justify your answers.



### USING TOOLS STRATEGICALLY

To be proficient in math, you need to use appropriate tools strategically, including real objects.

### EXPLORATION 2 Creating Solids of Revolution

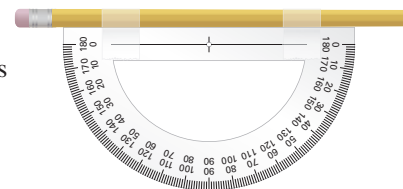
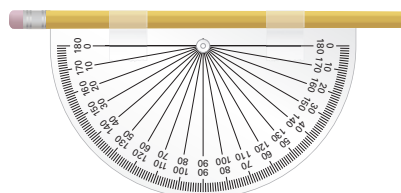
**Work with a partner.** Tape the straight side of a protractor, similar to the one at the left, to a pencil, as shown.

a. Rotate the pencil. What type of solid is produced by the rotating protractor? What are its dimensions?

b. Find the surface area and volume of the solid produced in part (a).

c. Tape the straight side of a protractor, similar to the one at the right, to a pencil, as shown. Rotate the pencil. Is the solid produced by this rotating protractor different from the solid in part (a)? Explain. Draw a diagram to support your answer.

d. Describe a method you might use to approximate the volume of the solid in part (c).



## Communicate Your Answer

- How can you create a solid of revolution?
- Give some examples of real-life objects that are solids of revolution.



# 1.4 Lesson

## Core Vocabulary

solid of revolution, p. 24  
axis of revolution, p. 24

**Previous**  
surface area  
volume

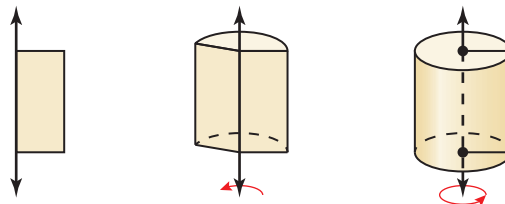
## What You Will Learn

- ▶ Sketch and describe solids of revolution.
- ▶ Find surface areas and volumes of solids of revolution.
- ▶ Form solids of revolution in the coordinate plane.

## Sketching and Describing Solids of Revolution

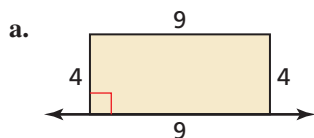
A **solid of revolution** is a three-dimensional figure that is formed by rotating a two-dimensional shape around an axis. The line around which the shape is rotated is called the **axis of revolution**.

For example, when you rotate a rectangle around a line that contains one of its sides, the solid of revolution that is produced is a cylinder.

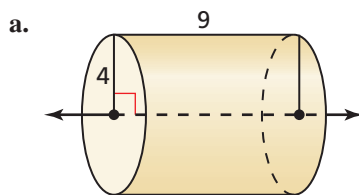


### EXAMPLE 1 Sketching and Describing Solids of Revolution

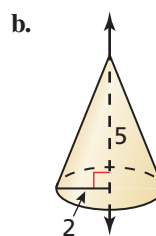
Sketch the solid produced by rotating the figure around the given axis. Then identify and describe the solid.



### SOLUTION



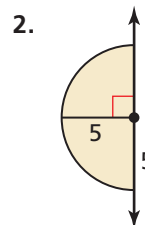
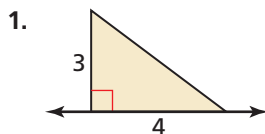
- ▶ The solid is a cylinder with a height of 9 units and a base radius of 4 units.



- ▶ The solid is a cone with a height of 5 units and a base radius of 2 units.

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Sketch the solid produced by rotating the figure around the given axis. Then identify and describe the solid.





### EXAMPLE 2 Sketching a Two-Dimensional Shape and Axis

Most vases are solids of revolution. Sketch a two-dimensional shape and an axis of revolution that forms the vase shown.

#### SOLUTION

The two-dimensional shape should match the outline of one side of the vase.



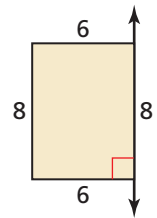
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- Sketch a two-dimensional shape and an axis of revolution that forms the bird bath shown.

## Finding Surface Areas and Volumes of Solids of Revolution

### EXAMPLE 3 Finding the Surface Area and Volume of a Solid of Revolution

Sketch and describe the solid produced by rotating the figure around the given axis. Then find its surface area and volume.



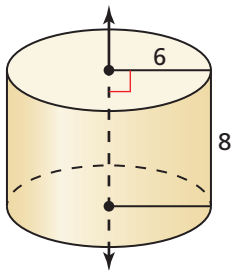
#### SOLUTION

The solid is a cylinder with a height of 8 units and a base radius of 6 units.

**Surface area:**  $S = 2\pi r^2 + 2\pi rh = 2\pi(6)^2 + 2\pi(6)(8) = 168\pi \approx 527.79$

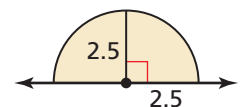
**Volume:**  $V = \pi r^2 h = \pi(6)^2(8) = 288\pi \approx 904.78$

- The cylinder has a surface area of about 527.79 square units and a volume of about 904.78 cubic units.



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- Sketch and describe the solid produced by rotating the figure around the given axis. Then find its surface area and volume.



# Forming Solids of Revolution in the Coordinate Plane

## EXAMPLE 4 Forming a Solid of Revolution

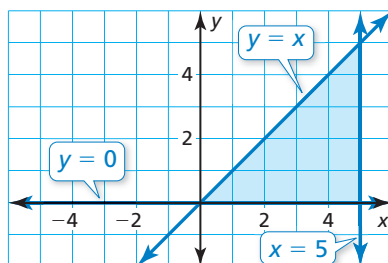
### REMEMBER

When  $(a, b)$  is reflected in the  $y$ -axis, then its image is the point  $(-a, b)$ .

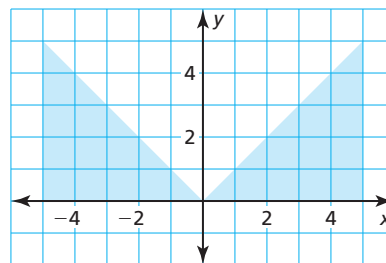
Sketch and describe the solid that is produced when the region enclosed by  $y = 0$ ,  $y = x$ , and  $x = 5$  is rotated around the  $y$ -axis. Then find the volume of the solid.

### SOLUTION

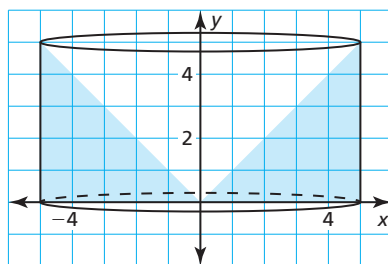
**Step 1** Graph each equation and determine the region that will be rotated around the  $y$ -axis.



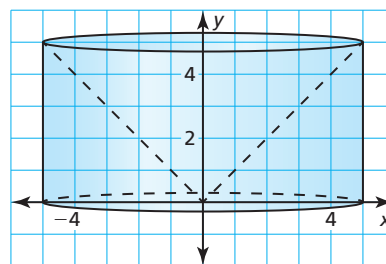
**Step 2** Reflect the region in the  $y$ -axis.



**Step 3** Connect the vertices of the triangles using curved lines.



**Step 4** The composite solid consists of a cylinder with a cone removed.



**Step 5** Find the volume of the composite solid. The cylinder and the cone both have a height of 5 units and a base radius of 5 units.

$$\begin{aligned}
 \text{Volume of solid} &= \text{Volume of cylinder} - \text{Volume of cone} \\
 &= \pi r^2 h - \frac{1}{3} \pi r^2 h \\
 &= \pi \cdot 5^2 \cdot 5 - \frac{1}{3} \pi \cdot 5^2 \cdot 5 \\
 &= 125\pi - \frac{125}{3}\pi \\
 &= \frac{250}{3}\pi \\
 &\approx 261.80
 \end{aligned}$$

Write formulas.

Substitute.

Simplify.

Subtract.

Use a calculator.

▶ The volume of the solid is  $\frac{250}{3}\pi$ , or about 261.80 cubic units.

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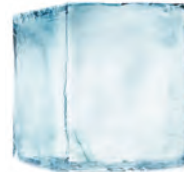
- WHAT IF?** Does the solid change when the region is rotated around the  $x$ -axis? Explain.
- Sketch and describe the solid that is produced when the region enclosed by  $x = 0$ ,  $y = -x$ , and  $y = -3$  is rotated around the  $x$ -axis. Then find the volume of the solid.

### ANOTHER WAY

You can also simplify  $\pi r^2 h - \frac{1}{3} \pi r^2 h$  to obtain  $\frac{2}{3} \pi r^2 h$ . Then substitute 5 for  $r$  and  $h$ , and evaluate.

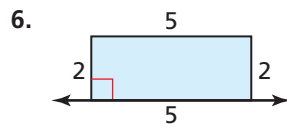
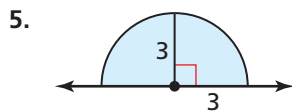
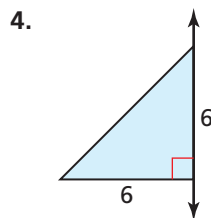
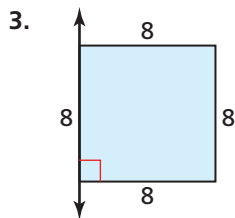
## Vocabulary and Core Concept Check

- COMPLETE THE SENTENCE** When you rotate a square around a line that contains one of its sides, the solid of revolution that is produced is a \_\_\_\_\_.
- WHICH ONE DOESN'T BELONG?** Which object does *not* belong with the other three? Explain your reasoning.

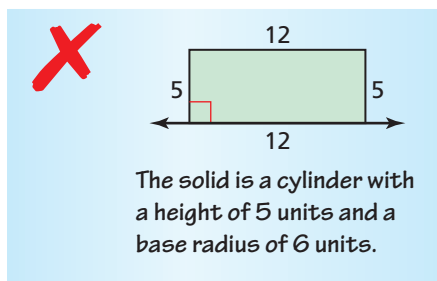


## Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, sketch the solid produced by rotating the figure around the given axis. Then identify and describe the solid. (See Example 1.)



7. **ERROR ANALYSIS** Describe and correct the error in identifying and describing the solid produced by rotating the figure around the given axis.



8. **REASONING** Can you form any solid by rotating a two-dimensional figure around an axis? Explain.

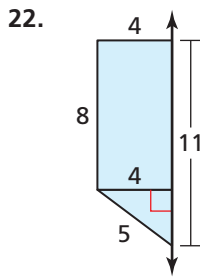
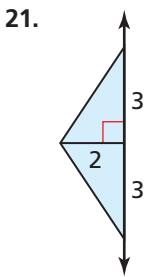
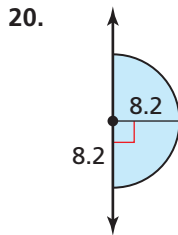
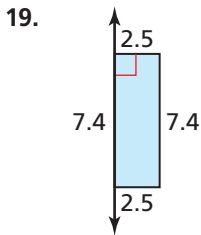
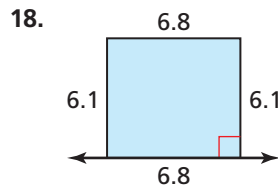
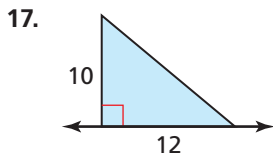
In Exercises 9–12, sketch the solid of revolution. Then identify and describe the solid.

- a square with side length 4 rotated around one side
- a rectangle with length 6 and width 3 rotated around one of its shorter sides
- a right triangle with legs of lengths 6 and 9 rotated around its longer leg
- a semicircle with radius 10 rotated around its diameter

In Exercises 13–16, sketch a two-dimensional shape and an axis of revolution that forms the object shown. (See Example 2.)



In Exercises 17–22, sketch and describe the solid produced by rotating the figure around the given axis. Then find its surface area and volume. (See Example 3.)



In Exercises 23–26, sketch and describe the solid that is produced when the region enclosed by the given equations is rotated around the given axis. Then find the volume of the solid. (See Example 4.)

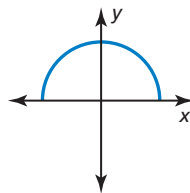
23.  $x = 0, y = 0, y = x + 3; x$ -axis

24.  $x = 0, y = 0, y = -2x + 5; y$ -axis

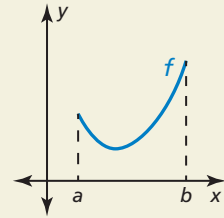
25.  $x = 3, y = 0, y = \frac{1}{2}x; y$ -axis

26.  $x = -4, y = 0, y = x; x$ -axis

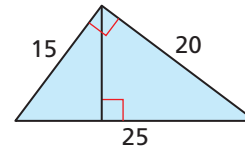
27. **MAKING AN ARGUMENT** Your friend says when you rotate the figure shown around either the  $x$ -axis or the  $y$ -axis, the resulting solid is a sphere. Is your friend correct? Explain.



28. **HOW DO YOU SEE IT?** The figure shows the graph of a function  $f$  on an interval  $[a, b]$ . Sketch the solid produced when the region enclosed by the graph of  $f$  and the equations  $x = a$ ,  $x = b$ , and  $y = 0$  is rotated around the  $x$ -axis.



29. **CRITICAL THINKING** A right triangle has sides with lengths 15, 20, and 25, as shown. Describe the three solids formed when the triangle is rotated around each of its sides. Then find the volumes of the solids. Give your answers in terms of  $\pi$ .



30. **THOUGHT PROVOKING** Write a system of equations whose enclosed region, when rotated around the  $x$ -axis or  $y$ -axis, produces the same solid with the same dimensions.

31. **REASONING** The solid shown is a type of *torus*.



a. Sketch a two-dimensional shape and an axis of revolution that forms the torus.

b. Which solid can you use to “construct” a torus similar to the one above? Explain, in your words, how to manipulate the solid to form the torus. You can think of the surface of the solid you choose as being stretchable.

32. **CRITICAL THINKING** A  $30^\circ$ - $30^\circ$ - $120^\circ$  isosceles triangle has two legs of length 4 units. When it is rotated around an axis that contains one leg, what is the volume of the solid of revolution?

## Maintaining Mathematical Proficiency Reviewing what you learned in previous grades and lessons

Determine whether the ordered pair is a solution of the equation. (Skills Review Handbook)

33.  $f(x) = 8x - 3; (-5, -37)$

34.  $h(x) = 2x^2 - 7x - 1; (3, 2)$

35.  $n(x) = -5x^2 - 4x; (1.5, -17.25)$

36.  $p(x) = |6x + 5|; (-1, 11)$

37. A circular region has a population of about 2.5 million people and a population density of about 9824 people per square mile. Find the radius of the region. (Section 1.1)

# 1.3–1.4 What Did You Learn?

## Core Vocabulary

cross section, *p. 18*

solid of revolution, *p. 24*

axis of revolution, *p. 24*

## Core Concepts

### Section 1.3

Cross Section of a Solid, *p. 18*

Drawing a Cross Section, *p. 19*

### Section 1.4

Solids of Revolution, *p. 24*

Surface Area and Volume of a  
Solid of Revolution, *p. 25*

Solids of Revolution  
in the Coordinate Plane, *p. 26*

## Mathematical Practices

1. What question(s) can you ask your friend to help her understand the error in the statement she made in Exercise 23 on page 22?
2. Describe the given information in Exercise 25 on page 22 and your plan for finding the solution.
3. Describe the overall step-by-step process you used to solve Exercise 32 on page 28.

## Performance Task:

# Population Density

Population density is a measure of how many people live within a given area. Locations of new schools are decided using geometric models based on population density. How can these models help officials draw new attendance boundaries when the schools are built?

To explore the answer to this question and more, check out the Performance Task and Real-Life STEM video at [BigIdeasMath.com](http://BigIdeasMath.com).



**1.1** Modeling with Area (pp. 3–8)

About 210,000 people live in a circular region with a 4.5-mile radius. Find the population density in people per square mile.

**Step 1** Find the area of the region.

$$A = \pi r^2 = \pi \cdot (4.5)^2 = 20.25\pi \text{ mi}^2$$

**Step 2** Find the population density.

$$\text{Population density} = \frac{\text{number of people}}{\text{area of land}} = \frac{210,000}{20.25\pi} \approx 3301$$

▶ The population density is about 3301 people per square mile.

- About 1.75 million people live in a circular region with a 15-mile diameter. Find the population density in people per square mile.
- A circular region has a population of about 15,500 people and a population density of about 775 people per square kilometer. Find the radius of the region.
- A furniture designer applies a thin glaze to the entire marble stool. How much of that amount is needed for a stool whose linear dimensions are
  - $\frac{7}{8}$  times the dimensions of the given stool?
  - $\frac{5}{4}$  times the dimensions of the given stool?

**1.2** Modeling with Volume (pp. 9–14)

The diagram shows the dimensions of a bar of platinum. Platinum has a density of 21.4 grams per cubic centimeter. Find the mass of the bar to the nearest gram.

**Step 1** Convert the dimensions to centimeters using 1 millimeter = 0.1 centimeter.

**Length** 4.97 cm    **Width** 2.85 cm    **Height** 0.34 cm

**Step 2** Find the volume.

$$V = Bh = 4.97(2.85)(0.34) = 4.81593 \text{ cm}^3$$

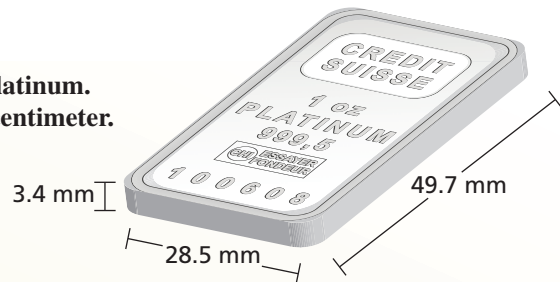
**Step 3** Let  $x$  represent the mass in grams. Substitute the values for the volume and the density in the formula for density and solve for  $x$ .

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \quad \text{Formula for density}$$

$$21.4 = \frac{x}{4.81593} \quad \text{Substitute.}$$

$$103 \approx x \quad \text{Multiply each side by 4.81593.}$$

▶ The mass of the bar of platinum is about 103 grams.



4. A bar of platinum has a mass of about 1000 grams. It has a width of 51 millimeters and a height of 9.7 millimeters. Find the length of the bar to the nearest millimeter.

**Describe how the change affects the volume of the platinum bar in the example.**

5. double the height  
 6. double the length and width  
 7. multiply the length by  $\frac{1}{3}$   
 8. A part for a toy train is made by drilling a hole that has a diameter of 0.6 centimeter through a wooden ball that has a diameter of 4 centimeters.

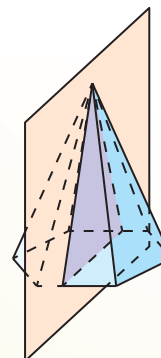
- a. Estimate the volume of the wooden ball after the hole is made. Explain your reasoning.  
 b. Do the surface area and volume of the wooden ball decrease after the hole is made? Explain.



### 1.3 Cross Sections of Solids (pp. 17–22)

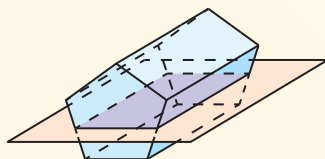
**Describe the shape formed by the intersection of the plane and the solid.**

- The cross section is a triangle.

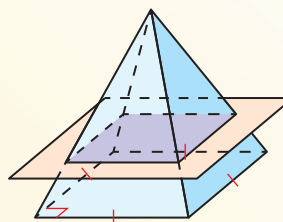


**Describe the shape formed by the intersection of the plane and the solid.**

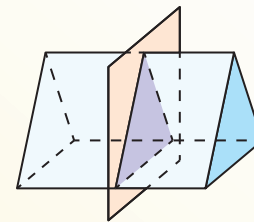
9.



10.

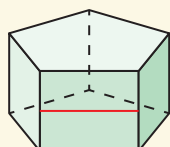


11.

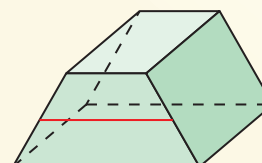


**Draw the cross section formed by the described plane that intersects the red line segment drawn on the solid. What is the shape of the cross section?**

12. plane is parallel to base



13. plane is parallel to bottom face

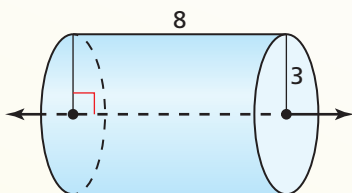


14. Describe and draw two cross sections that can be formed by a plane intersecting the solid in Exercise 12. The shapes of the cross sections should be different than the shape of the cross section in Exercise 12.



## 1.4 Solids of Revolution (pp. 23–28)

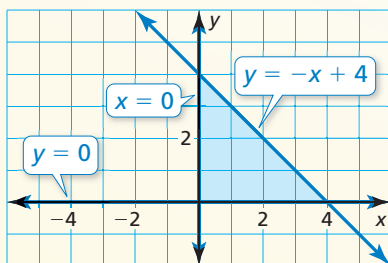
- a. Sketch the solid produced by rotating the figure around the given axis. Then identify and describe the solid.



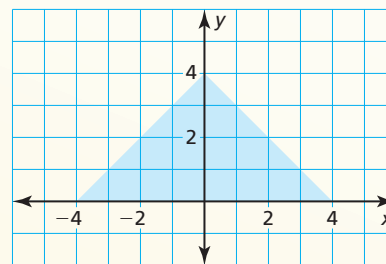
► The solid is a cylinder with a height of 8 units and a base radius of 3 units.

- b. Sketch and describe the solid that is produced when the region enclosed by  $y = 0$ ,  $x = 0$ , and  $y = -x + 4$  is rotated around the  $y$ -axis. Then find the volume of the solid.

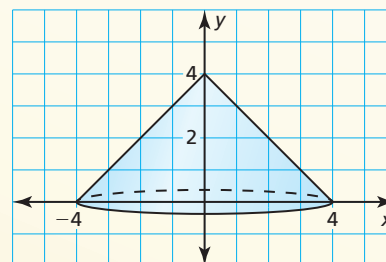
**Step 1** Graph each equation and determine the region that will be rotated around the  $y$ -axis.



**Step 2** Reflect the region in the  $y$ -axis.



**Step 3** Connect the vertices of the triangles using curved lines. The solid is a cone with a height of 4 units and a base radius of 4 units.



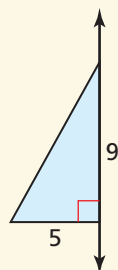
**Step 4** Find the volume of the cone.

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \cdot 4^2 \cdot 4 = \frac{64}{3}\pi \approx 67.02$$

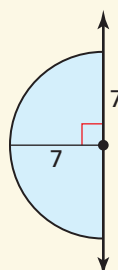
► The volume of the cone is  $\frac{64}{3}\pi$ , or about 67.02 cubic units.

Sketch and describe the solid produced by rotating the figure around the given axis. Then find its surface area and volume.

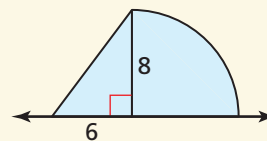
15.



16.



17.

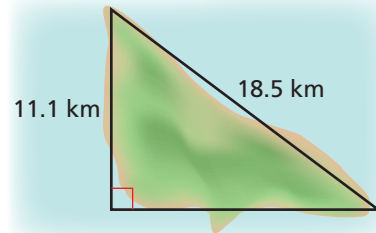


18. Sketch and describe the solid that is produced when the region enclosed by  $y = 0$ ,  $y = x$ , and  $x = 2$  is rotated around the  $y$ -axis. Then find the volume of the solid.

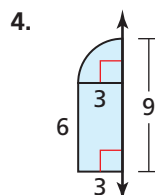
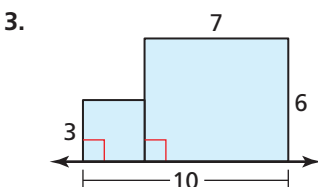
# 1

## Chapter Test

- The island shown has a population of 12,175 people. Find the population density in people per square kilometer.
- You slice the bagel shown in half horizontally to split with your friend. Describe the shape of the cross section you make. What does the cross section look like if you cut the bagel in half vertically?

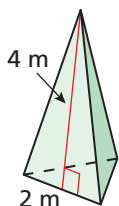


Sketch and describe the solid produced by rotating the figure around the given axis. Then find its surface area and volume.

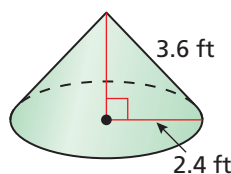


Describe how the change affects the surface area of the regular pyramid or right cone.

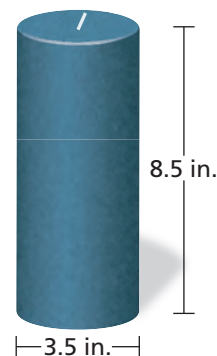
5. tripling all the linear dimensions



6. multiplying all the linear dimensions by  $\frac{2}{3}$



7. The candle has a mass of 1200 grams.
- What is the density of the candle wax in grams per cubic centimeter?
  - After three hours of burning, the height of the candle is about 8.3 inches. How does this change affect the surface area and volume of the candle?
  - How much mass, to the nearest gram, did the candle lose after 3 hours of burning? If this rate remains constant, estimate the number of hours of burning time the candle will have.



8. Sketch a two-dimensional shape and an axis of revolution that forms the water container shown.

9. A circular region with a 3-mile radius has a population density of about 1000 people per square mile. Find the number of people who live in the region.

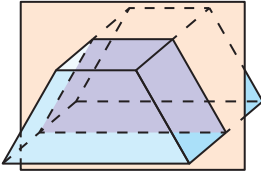
10. Sketch and describe the solid that is produced when the region enclosed by  $y = 0$ ,  $y = -x + 8$ ,  $x = 0$ , and  $x = 5$  is rotated around the  $y$ -axis. Then find the volume of the solid.

# 1

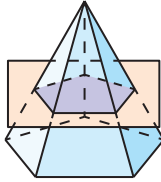
# Cumulative Assessment

1. Identify the shape formed by the intersection of the plane and the solid.

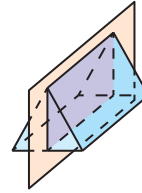
a.



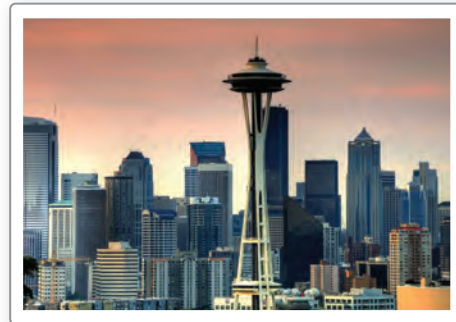
b.



c.



2. About 19,400 people live in a circular region with a 5-mile radius. Find the population density in people per square mile.



3. What is the volume of the solid that is produced when the region enclosed by  $y = 0$ ,  $y = 3.5$ ,  $x = 0$ , and  $x = 8$  is rotated around the  $x$ -axis?

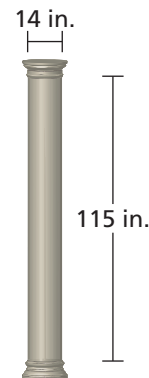
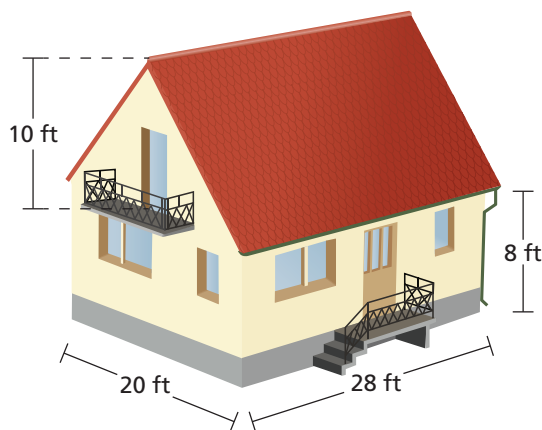
(A) about 87.96 cubic units

(B) about 307.88 cubic units

(C) about 703.72 cubic units

(D) about 1231.5 cubic units

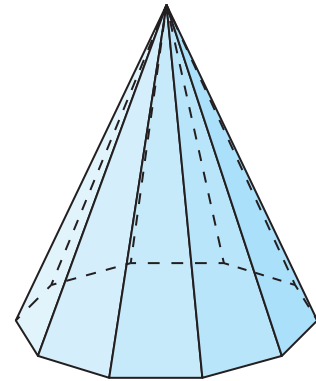
4. You are vacationing at the cottage shown and after arriving, realize that you need to increase the inside air temperature. To raise the temperature to your desired level, you need 7 Btu per cubic foot. How many Btu do you need to raise the temperature of the cottage?



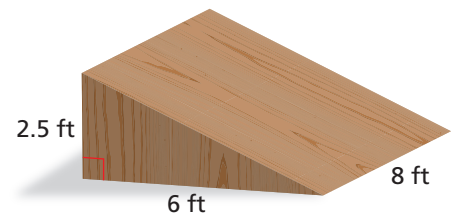
5. Marble has a density of 2.56 grams per cubic centimeter. What is the mass of the cylindrical portion of the marble pillar shown to the nearest kilogram?

6. The regular decagonal pyramid shown is intersected by a plane perpendicular to its base. The plane passes through its vertex so it is divided into two congruent parts. What is the shape of the cross section?

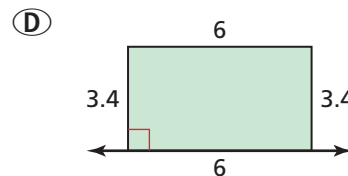
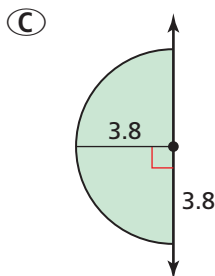
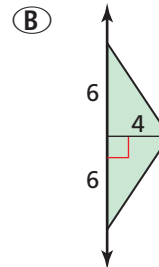
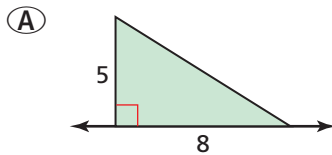
- (A) triangle    (B) rectangle  
 (C) trapezoid    (D) decagon



7. You design the ramp shown. The outer surface of the ramp, including the bottom, is covered with plywood. Your friend wants to double the base and height of the triangular sides of your ramp. Your friend claims that the new ramp will use no more than twice the amount of plywood as your ramp. Do you support your friend's claim? Explain your reasoning.



8. Which figure has the greatest volume when it is rotated around the given axis?



9. A silo similar to the one shown has a height of 60 feet and a diameter of 18 feet. It is filled with grain to a height of about 30 feet. How many bushels of grain are currently in the silo? One bushel of grain is about 1.25 cubic feet.

