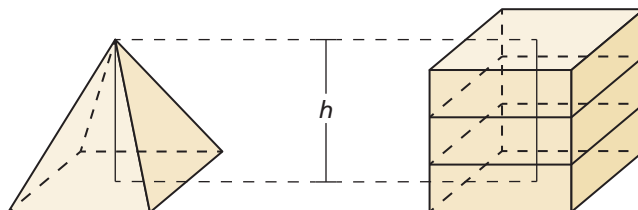


11.5 Volumes of Pyramids

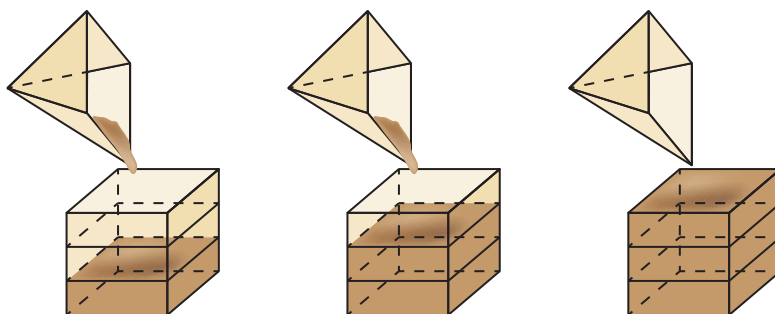
Essential Question How can you find the volume of a pyramid?

EXPLORATION 1 Finding the Volume of a Pyramid

Work with a partner. The pyramid and the prism have the same height and the same square base.



When the pyramid is filled with sand and poured into the prism, it takes three pyramids to fill the prism.



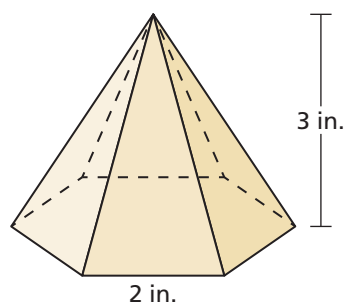
LOOKING FOR STRUCTURE

To be proficient in math, you need to look closely to discern a pattern or structure.

Use this information to write a formula for the volume V of a pyramid.

EXPLORATION 2 Finding the Volume of a Pyramid

Work with a partner. Use the formula you wrote in Exploration 1 to find the volume of the hexagonal pyramid.



Communicate Your Answer

- How can you find the volume of a pyramid?
- In Section 11.6, you will study volumes of cones. How do you think you could use a method similar to the one presented in Exploration 1 to write a formula for the volume of a cone? Explain your reasoning.

11.5 Lesson

Core Vocabulary

Previous

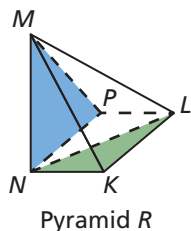
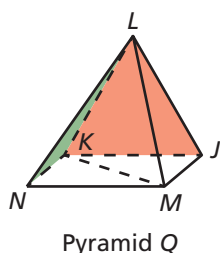
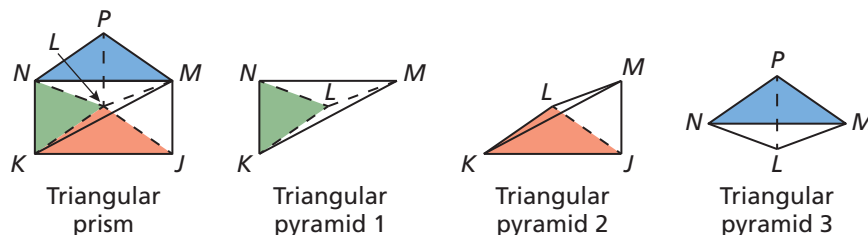
pyramid
composite solid

What You Will Learn

- Find volumes of pyramids.
- Use volumes of pyramids.

Finding Volumes of Pyramids

Consider a triangular prism with parallel, congruent bases $\triangle JKL$ and $\triangle MNP$. You can divide this triangular prism into three triangular pyramids.



You can combine triangular pyramids 1 and 2 to form a pyramid with a base that is a parallelogram, as shown at the left. Name this pyramid Q . Similarly, you can combine triangular pyramids 1 and 3 to form pyramid R with a base that is a parallelogram.

In pyramid Q , diagonal \overline{KM} divides $\square JKNM$ into two congruent triangles, so the bases of triangular pyramids 1 and 2 are congruent. Similarly, you can divide any cross section parallel to $\square JKNM$ into two congruent triangles that are the cross sections of triangular pyramids 1 and 2.

By Cavalieri's Principle, triangular pyramids 1 and 2 have the same volume. Similarly, using pyramid R , you can show that triangular pyramids 1 and 3 have the same volume. By the Transitive Property of Equality, triangular pyramids 2 and 3 have the same volume.

The volume of each pyramid must be one-third the volume of the prism, or $V = \frac{1}{3}Bh$. You can generalize this formula to say that the volume of any pyramid with any base is equal to $\frac{1}{3}$ the volume of a prism with the same base and height because you can divide any polygon into triangles and any pyramid into triangular pyramids.

Core Concept

Volume of a Pyramid

The volume V of a pyramid is

$$V = \frac{1}{3}Bh$$

where B is the area of the base and h is the height.



EXAMPLE 1

Finding the Volume of a Pyramid

Find the volume of the pyramid.

SOLUTION

$$V = \frac{1}{3}Bh$$

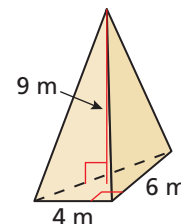
$$= \frac{1}{3}\left(\frac{1}{2} \cdot 4 \cdot 6\right)(9)$$

$$= 36$$

Formula for volume of a pyramid

Substitute.

Simplify.

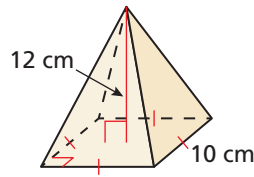


- The volume is 36 cubic meters.

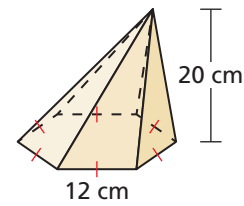


Find the volume of the pyramid.

1.



2.



Using Volumes of Pyramids

EXAMPLE 2

Using the Volume of a Pyramid



Khafre's Pyramid, Egypt

Originally, Khafre's Pyramid had a height of about 144 meters and a volume of about 2,218,800 cubic meters. Find the side length of the square base.

SOLUTION

$$V = \frac{1}{3}Bh$$

Formula for volume of a pyramid

$$2,218,800 \approx \frac{1}{3}x^2(144)$$

Substitute.

$$6,656,400 \approx 144x^2$$

Multiply each side by 3.

$$46,225 \approx x^2$$

Divide each side by 144.

$$215 \approx x$$

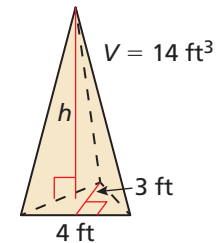
Find the positive square root.

Originally, the side length of the square base was about 215 meters.

EXAMPLE 3

Using the Volume of a Pyramid

Find the height of the triangular pyramid.



SOLUTION

The area of the base is $B = \frac{1}{2}(3)(4) = 6 \text{ ft}^2$ and the volume is $V = 14 \text{ ft}^3$.

$$V = \frac{1}{3}Bh$$

Formula for volume of a pyramid

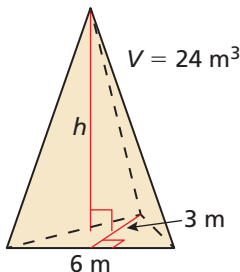
$$14 = \frac{1}{3}(6)h$$

Substitute.

$$7 = h$$

Solve for h .

The height is 7 feet.



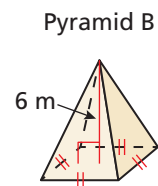
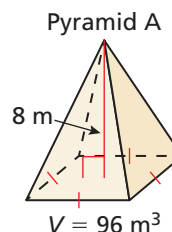
Monitoring Progress



- The volume of a square pyramid is 75 cubic meters and the height is 9 meters. Find the side length of the square base.
- Find the height of the triangular pyramid at the left.

EXAMPLE 4**Finding the Volume of a Similar Solid**

Pyramid A and pyramid B are similar.
Find the volume of pyramid B.

**SOLUTION**

The scale factor is $k = \frac{\text{Height of pyramid B}}{\text{Height of pyramid A}} = \frac{6}{8} = \frac{3}{4}$.

Use the scale factor to find the volume of pyramid B.

$$\frac{\text{Volume of pyramid B}}{\text{Volume of pyramid A}} = k^3 \quad \text{The ratio of the volumes is } k^3.$$

$$\frac{\text{Volume of pyramid B}}{96} = \left(\frac{3}{4}\right)^3 \quad \text{Substitute.}$$

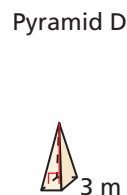
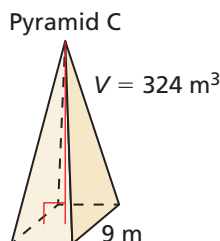
$$\text{Volume of pyramid B} = 40.5 \quad \text{Solve for volume of pyramid B.}$$

▶ The volume of pyramid B is 40.5 cubic meters.

Monitoring Progress

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5. Pyramid C and pyramid D are similar. Find the volume of pyramid D.

**EXAMPLE 5****Finding the Volume of a Composite Solid**

Find the volume of the composite solid.

SOLUTION

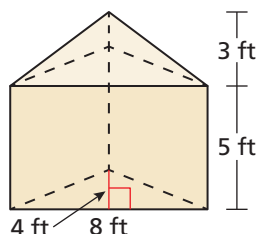
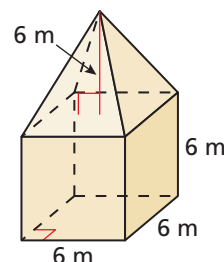
$$\text{Volume of solid} = \text{Volume of cube} + \text{Volume of pyramid}$$

$$= s^3 + \frac{1}{3}Bh \quad \text{Write formulas.}$$

$$= 6^3 + \frac{1}{3}(6)^2 \cdot 6 \quad \text{Substitute.}$$

$$= 216 + 72 \quad \text{Simplify.}$$

$$= 288 \quad \text{Add.}$$



▶ The volume is 288 cubic meters.

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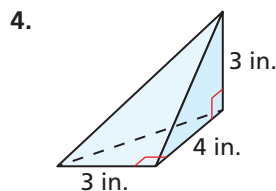
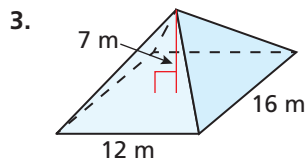
6. Find the volume of the composite solid.

Vocabulary and Core Concept Check

- VOCABULARY** Explain the difference between a triangular prism and a triangular pyramid.
- REASONING** A square pyramid and a cube have the same base and height. Compare the volume of the square pyramid to the volume of the cube.

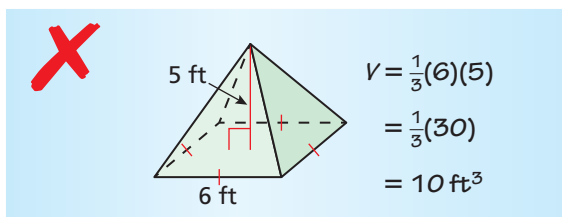
Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, find the volume of the pyramid.
(See Example 1.)



In Exercises 5–8, find the indicated measure.
(See Example 2.)

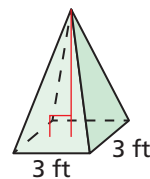
- A pyramid with a square base has a volume of 120 cubic meters and a height of 10 meters. Find the side length of the square base.
- A pyramid with a square base has a volume of 912 cubic feet and a height of 19 feet. Find the side length of the square base.
- A pyramid with a rectangular base has a volume of 480 cubic inches and a height of 10 inches. The width of the rectangular base is 9 inches. Find the length of the rectangular base.
- A pyramid with a rectangular base has a volume of 105 cubic centimeters and a height of 15 centimeters. The length of the rectangular base is 7 centimeters. Find the width of the rectangular base.
- ERROR ANALYSIS** Describe and correct the error in finding the volume of the pyramid.



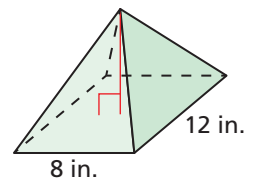
- OPEN-ENDED** Give an example of a pyramid and a prism that have the same base and the same volume. Explain your reasoning.

In Exercises 11–14, find the height of the pyramid.
(See Example 3.)

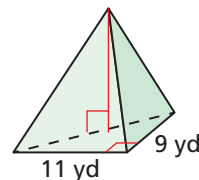
11. Volume = 15 ft^3



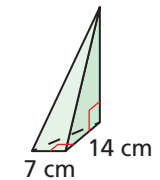
12. Volume = 224 in.^3



13. Volume = 198 yd^3

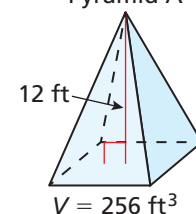


14. Volume = 392 cm^3



In Exercises 15 and 16, the pyramids are similar. Find the volume of pyramid B. (See Example 4.)

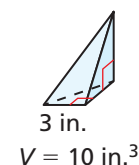
15. Pyramid A



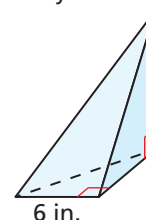
Pyramid B



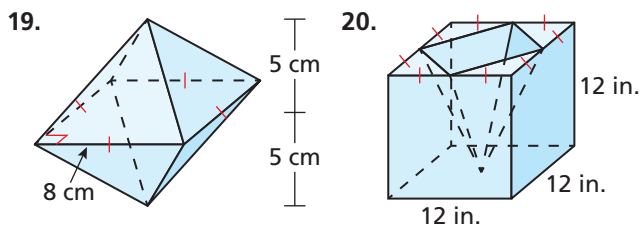
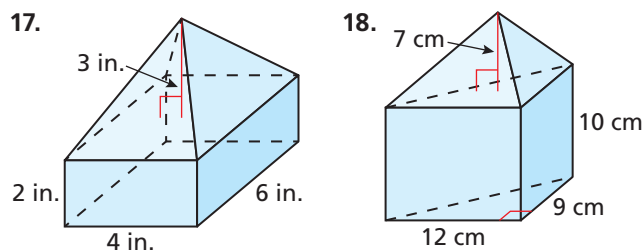
16. Pyramid A



Pyramid B



In Exercises 17–20, find the volume of the composite solid. (See Example 5.)

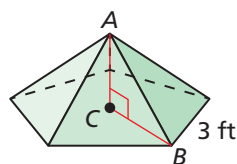


21. **ABSTRACT REASONING** A pyramid has a height of 8 feet and a square base with a side length of 6 feet.
- How does the volume of the pyramid change when the base stays the same and the height is doubled?
 - How does the volume of the pyramid change when the height stays the same and the side length of the base is doubled?
 - Are your answers to parts (a) and (b) true for any square pyramid? Explain your reasoning.

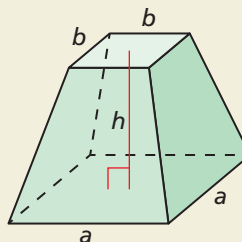
22. **HOW DO YOU SEE IT?** The cube shown is formed by three pyramids, each with the same square base and the same height. How could you use this to verify the formula for the volume of a pyramid?



23. **CRITICAL THINKING** Find the volume of the regular pentagonal pyramid. Round your answer to the nearest hundredth. In the diagram, $m\angle ABC = 35^\circ$.



24. **THOUGHT PROVOKING** A frustum of a pyramid is the part of the pyramid that lies between the base and a plane parallel to the base, as shown. Write a formula for the volume of the frustum of a square pyramid in terms of a , b , and h . (Hint: Consider the “missing” top of the pyramid and use similar triangles.)



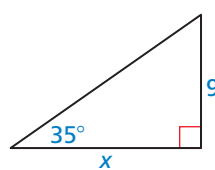
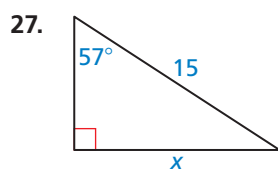
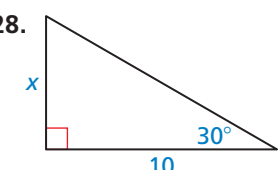
25. **MODELING WITH MATHEMATICS** Nautical deck prisms were used as a safe way to illuminate decks on ships. The deck prism shown here is composed of the following three solids: a regular hexagonal prism with an edge length of 3.5 inches and a height of 1.5 inches, a regular hexagonal prism with an edge length of 3.25 inches and a height of 0.25 inch, and a regular hexagonal pyramid with an edge length of 3 inches and a height of 3 inches. Find the volume of the deck prism.



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the value of x . Round your answer to the nearest tenth. (Section 9.4 and Section 9.5)

26. 
27. 
28. 
29. 