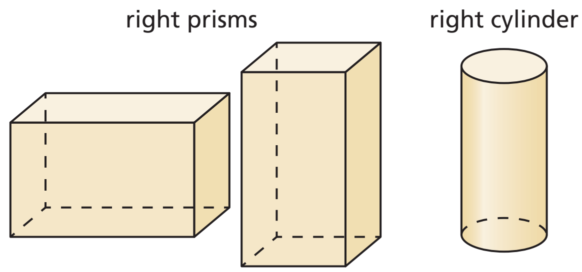
11.4 Volumes of Prisms and Cylinders

Essential Question How can you find the volume of a prism or cylinder that is not a right prism or right cylinder?

Recall that the volume V of a right prism or a right cylinder is equal to the product of the area of a base B and the height h.

$$V = Bh$$



EXPLORATION 1 **Finding Volume**

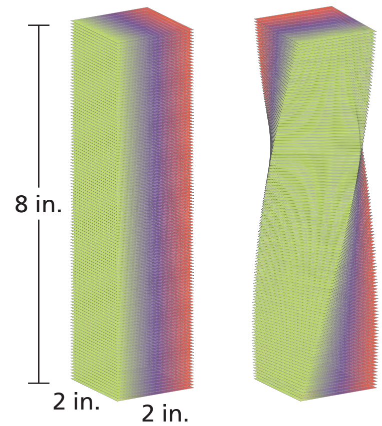
Work with a partner. Consider a stack of square papers that is in the form of a right prism.

a. What is the volume of the prism?

b. When you twist the stack of papers, as shown at the right, do you change the volume? Explain your reasoning.

c. Write a carefully worded conjecture that describes the conclusion you reached in part (b).

d. Use your conjecture to find the volume of the twisted stack of papers.



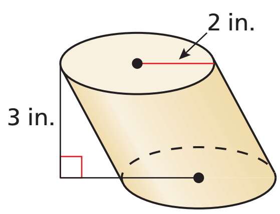
ATTENDING TO PRECISION

To be proficient in math, you need to communicate precisely to others.

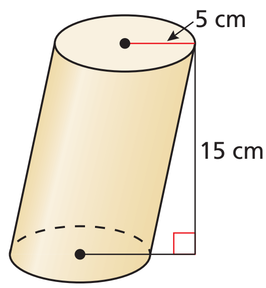
EXPLORATION 2 **Finding Volume**

Work with a partner. Use the conjecture you wrote in Exploration 1 to find the volume of the cylinder.

a.



b.



Communicate Your Answer

3. How can you find the volume of a prism or cylinder that is not a right prism or right cylinder?

4. In Exploration 1, would the conjecture you wrote change if the papers in each stack were not squares? Explain your reasoning.

11.4 Lesson

Core Vocabulary

polyhedron, *p. 666*
face, *p. 666*
edge, *p. 666*
vertex, *p. 666*
volume, *p. 667*
Cavalieri's Principle, *p. 667*
similar solids, *p. 669*

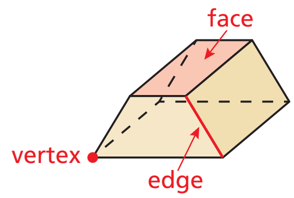
Previous
solid
prism
pyramid
cylinder
cone
sphere
base
composite solid

What You Will Learn

- Classify solids.
- Find volumes of prisms and cylinders.

Classifying Solids

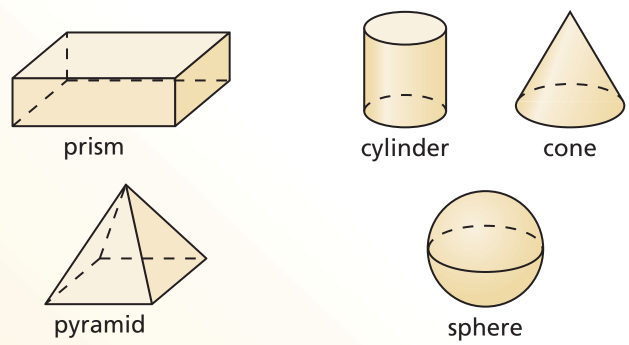
A three-dimensional figure, or solid, is bounded by flat or curved surfaces that enclose a single region of space. A **polyhedron** is a solid that is bounded by polygons, called **faces**. An **edge** of a polyhedron is a line segment formed by the intersection of two faces. A **vertex** of a polyhedron is a point where three or more edges meet. The plural of polyhedron is *polyhedra* or *polyhedrons*.



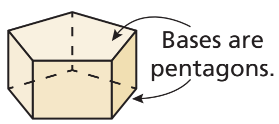
Core Concept

Types of Solids

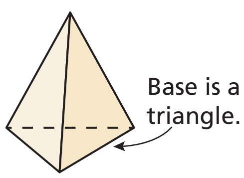
Polyhedra | **Not Polyhedra**



Pentagonal prism



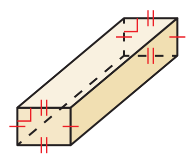
Triangular pyramid

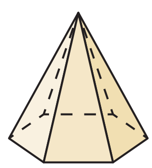


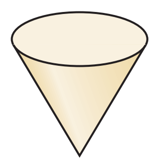
To name a prism or a pyramid, use the shape of the *base*. The two bases of a prism are congruent polygons in parallel planes. For example, the bases of a pentagonal prism are pentagons. The base of a pyramid is a polygon. For example, the base of a triangular pyramid is a triangle.

EXAMPLE 1 Classifying Solids

Tell whether each solid is a polyhedron. If it is, name the polyhedron.

a. 

b. 

c. 

SOLUTION

a. The solid is formed by polygons, so it is a polyhedron. The two bases are congruent rectangles, so it is a rectangular prism.

b. The solid is formed by polygons, so it is a polyhedron. The base is a hexagon, so it is a hexagonal pyramid.

c. The cone has a curved surface, so it is not a polyhedron.

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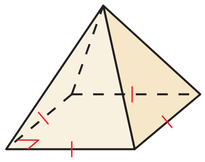
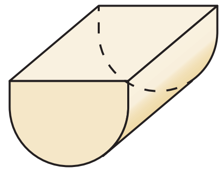
Tell whether the solid is a polyhedron. If it is, name the polyhedron.

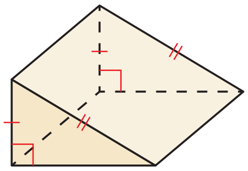
1.



2.

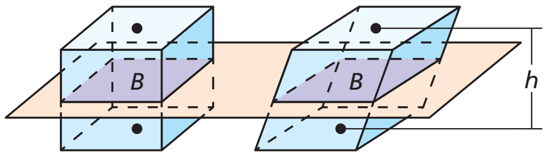


3.



Finding Volumes of Prisms and Cylinders

The **volume** of a solid is the number of cubic units contained in its interior. Volume is measured in cubic units, such as cubic centimeters (cm^3). **Cavalieri's Principle**, named after Bonaventura Cavalieri (1598–1647), states that if two solids have the same height and the same cross-sectional area at every level, then they have the same volume. The prisms below have equal heights h and equal cross-sectional areas B at every level. By Cavalieri's Principle, the prisms have the same volume.



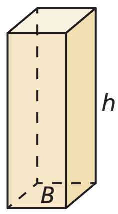
Core Concept

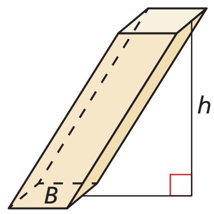
Volume of a Prism

The volume V of a prism is

$$V = Bh$$

where B is the area of a base and h is the height.

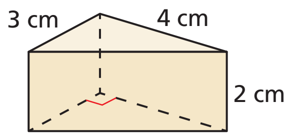




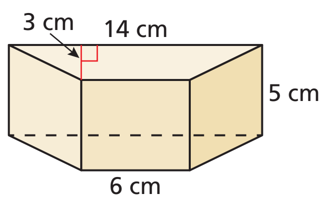
EXAMPLE 2 Finding Volumes of Prisms

Find the volume of each prism.

a.



b.



SOLUTION

a. The area of a base is $B = \frac{1}{2}(3)(4) = 6\ \text{cm}^2$ and the height is $h = 2$ cm.

$$V = Bh = 6(2) = 12$$

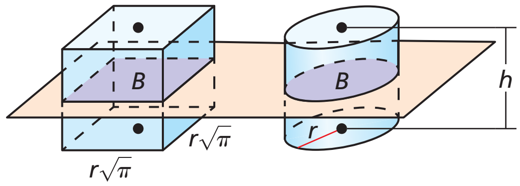
▶ The volume is 12 cubic centimeters.

b. The area of a base is $B = \frac{1}{2}(3)(6 + 14) = 30\ \text{cm}^2$ and the height is $h = 5$ cm.

$$V = Bh = 30(5) = 150$$

▶ The volume is 150 cubic centimeters.

Consider a cylinder with height h and base radius r and a rectangular prism with the same height that has a square base with sides of length $r\sqrt{\pi}$.



The cylinder and the prism have the same cross-sectional area, πr^2, at every level and the same height. By Cavalieri's Principle, the prism and the cylinder have the same volume. The volume of the prism is $V = Bh = \pi r^2 h$, so the volume of the cylinder is also $V = Bh = \pi r^2 h$.

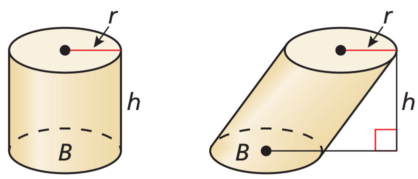
Core Concept

Volume of a Cylinder

The volume V of a cylinder is

$$V = Bh = \pi r^2 h$$

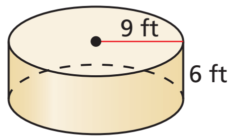
where B is the area of a base, h is the height, and r is the radius of a base.



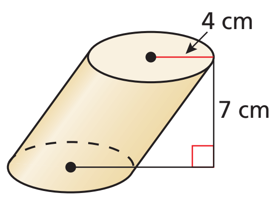
EXAMPLE 3 Finding Volumes of Cylinders

Find the volume of each cylinder.

a.



b.



SOLUTION

a. The dimensions of the cylinder are $r = 9$ ft and $h = 6$ ft.

$$V = \pi r^2 h = \pi(9)^2(6) = 486\pi \approx 1526.81$$

▶ The volume is 486π, or about 1526.81 cubic feet.

b. The dimensions of the cylinder are $r = 4$ cm and $h = 7$ cm.

$$V = \pi r^2 h = \pi(4)^2(7) = 112\pi \approx 351.86$$

▶ The volume is 112π, or about 351.86 cubic centimeters.

Monitoring Progress

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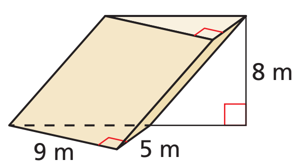
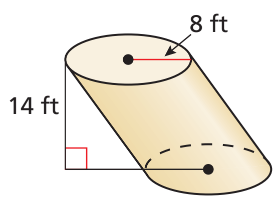
Find the volume of the solid.

4.



5.



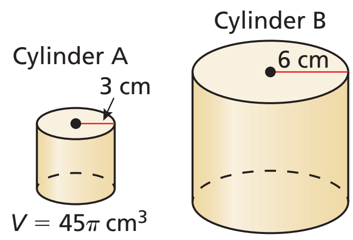
Core Concept

Similar Solids

Two solids of the same type with equal ratios of corresponding linear measures, such as heights or radii, are called **similar solids**. The ratio of the corresponding linear measures of two similar solids is called the *scale factor*. If two similar solids have a scale factor of k, then the ratio of their volumes is equal to k^3.

EXAMPLE 4 Finding the Volume of a Similar Solid

Cylinder A and cylinder B are similar. Find the volume of cylinder B.



SOLUTION

The scale factor is $k = \dfrac{\text{Radius of cylinder B}}{\text{Radius of cylinder A}}$

$= \dfrac{6}{3} = 2.$

Use the scale factor to find the volume of cylinder B.

$\dfrac{\text{Volume of cylinder B}}{\text{Volume of cylinder A}} = k^3$ — The ratio of the volumes is k^3.

$\dfrac{\text{Volume of cylinder B}}{45\pi} = 2^3$ — Substitute.

Volume of cylinder B $= 360\pi$ — Solve for volume of cylinder B.

▶ The volume of cylinder B is 360π cubic centimeters.

COMMON ERROR

Be sure to write the ratio of the volumes in the same order you wrote the ratio of the radii.

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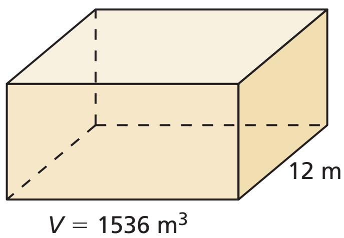
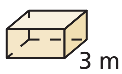
Prism C



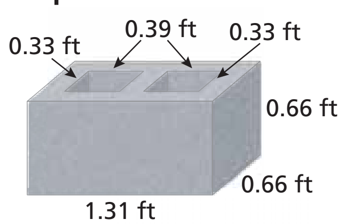
Prism D



6. Prism C and prism D are similar. Find the volume of prism D.

EXAMPLE 5 Finding the Volume of a Composite Solid

Find the volume of the concrete block.



SOLUTION

To find the area of the base, subtract two times the area of the small rectangle from the large rectangle.

$B = $ Area of large rectangle $- 2 \cdot$ Area of small rectangle

$= 1.31(0.66) - 2(0.33)(0.39)$

$= 0.6072$

Using the formula for the volume of a prism, the volume is

$V = Bh = 0.6072(0.66) \approx 0.40.$

▶ The volume is about 0.40 cubic foot.

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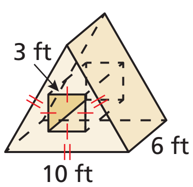


7. Find the volume of the composite solid.

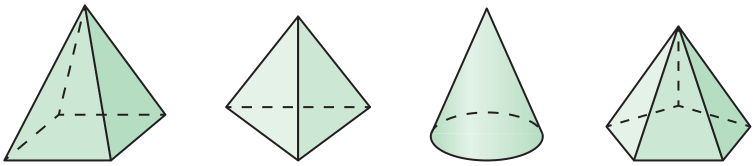
11.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

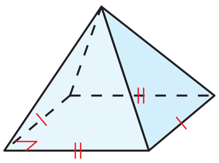
1. **VOCABULARY** In what type of units is the volume of a solid measured?

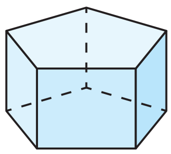
2. **WHICH ONE DOESN'T BELONG?** Which solid does *not* belong with the other three? Explain your reasoning.

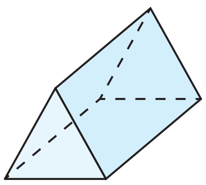


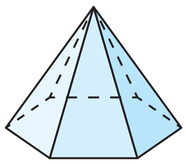
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, match the polyhedron with its name.

3. 

4. 

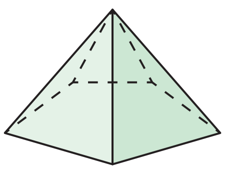
5. 

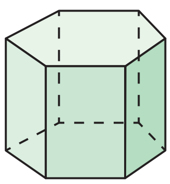
6. 

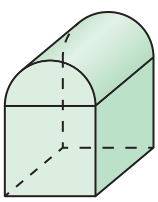
A. triangular prism **B.** rectangular pyramid

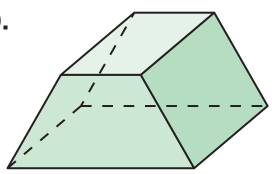
C. hexagonal pyramid **D.** pentagonal prism

In Exercises 7–10, tell whether the solid is a polyhedron. If it is, name the polyhedron. *(See Example 1.)*

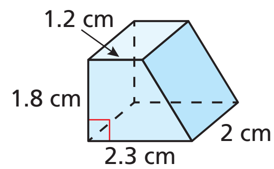
7. 

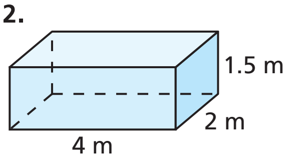
8. 

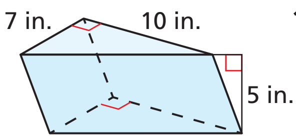
9. 

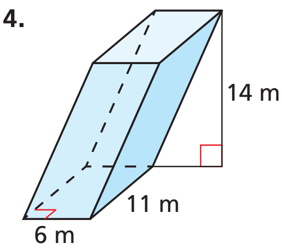
10. 

In Exercises 11–14, find the volume of the prism. *(See Example 2.)*

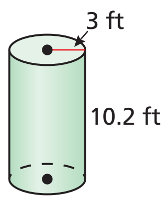
11. 

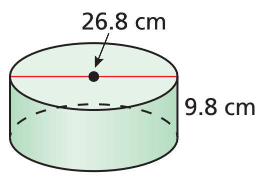
12. 

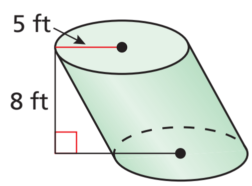
13. 

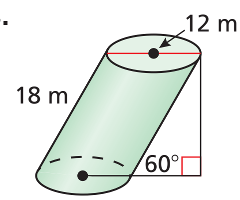
14. 

In Exercises 15–18, find the volume of the cylinder. *(See Example 3.)*

15. 

16. 

17. 

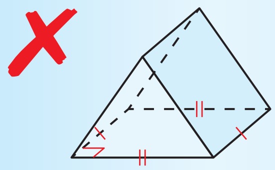
18. 

In Exercises 19 and 20, make a sketch of the solid and find its volume. Round your answer to the nearest hundredth.

19. A prism has a height of 11.2 centimeters and an equilateral triangle for a base, where each base edge is 8 centimeters.

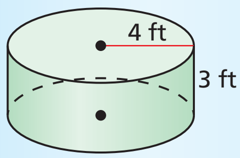
20. A pentagonal prism has a height of 9 feet and each base edge is 3 feet.

21. **ERROR ANALYSIS** Describe and correct the error in identifying the solid.



The solid is a rectangular pyramid.

22. **ERROR ANALYSIS** Describe and correct the error in finding the volume of the cylinder.



$V = 2\pi rh$

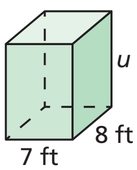
$= 2\pi(4)(3)$

$= 24\pi$

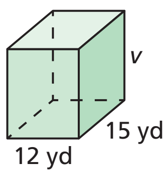
So, the volume of the cylinder is 24π cubic feet.

In Exercises 23–28, find the missing dimension of the prism or cylinder.

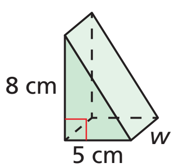
23. Volume $= 560 \text{ ft}^3$



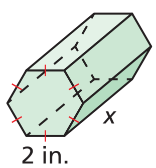
24. Volume $= 2700 \text{ yd}^3$



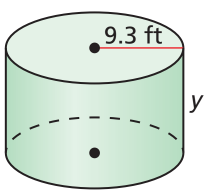
25. Volume $= 80 \text{ cm}^3$



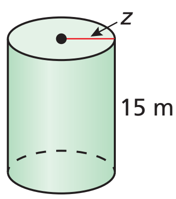
26. Volume $= 72.66 \text{ in.}^3$



27. Volume $= 3000 \text{ ft}^3$

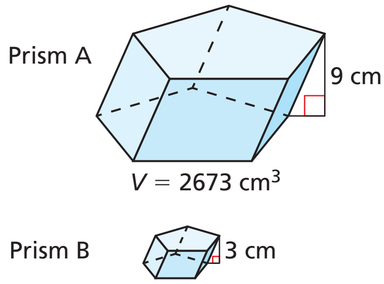


28. Volume $= 1696.5 \text{ m}^3$



In Exercises 29 and 30, the solids are similar. Find the volume of solid B. *(See Example 4.)*

29.



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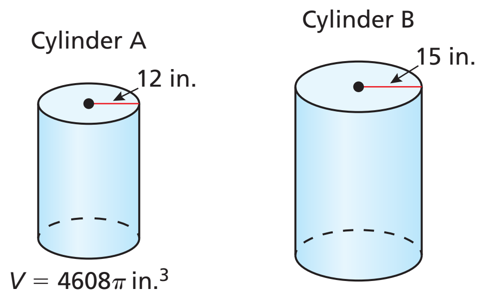
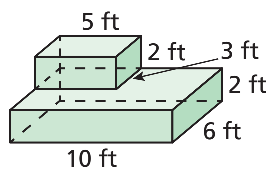


In Exercises 31–34, find the volume of the composite solid. *(See Example 5.)*

31.



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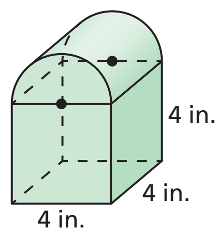

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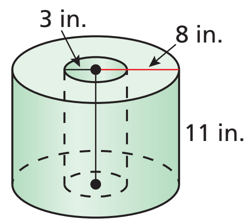

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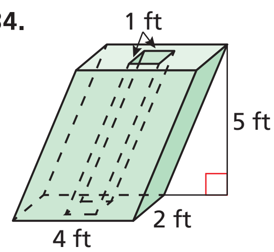
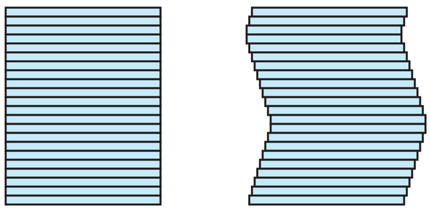


35. **MODELING WITH MATHEMATICS** The Great Blue Hole is a cylindrical trench located off the coast of Belize. It is approximately 1000 feet wide and 400 feet deep. About how many gallons of water does the Great Blue Hole contain? (1 $\text{ft}^3 \approx 7.48$ gallons)

36. **COMPARING METHODS** The *Volume Addition Postulate* states that the volume of a solid is the sum of the volumes of all its nonoverlapping parts. Use this postulate to find the volume of the block of concrete in Example 5 by subtracting the volume of each hole from the volume of the large rectangular prism. Which method do you prefer? Explain your reasoning.

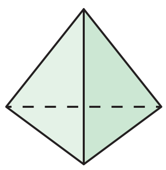
37. **WRITING** Both of the figures shown are made up of the same number of congruent rectangles. Explain how Cavalieri's Principle can be adapted to compare the areas of these figures.



38. **HOW DO YOU SEE IT?** Each stack of memo papers contains 500 equally-sized sheets of paper. Compare their volumes. Explain your reasoning.

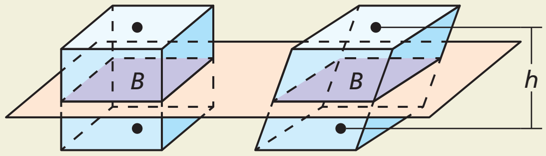
39. **OPEN-ENDED** Sketch two rectangular prisms that have volumes of 100 cubic inches but different surface areas. Include dimensions in your sketches.

40. **MAKING AN ARGUMENT** Your friend says that the polyhedron shown is a triangular prism. Your cousin says that it is a triangular pyramid. Who is correct? Explain your reasoning.



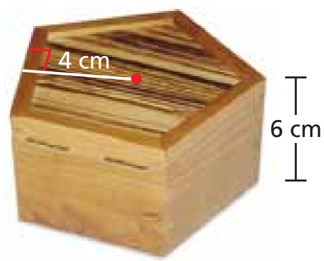
41. **MAKING AN ARGUMENT** A prism and a cylinder have the same height and different cross-sectional areas. Your friend claims that the two solids have the same volume by Cavalieri's Principle. Is your friend correct? Explain your reasoning.

42. **THOUGHT PROVOKING** Cavalieri's Principle states that the two solids shown below have the same volume. Do they also have the same surface area? Explain your reasoning.



43. **PROBLEM SOLVING** A barn is in the shape of a pentagonal prism with the dimensions shown. The volume of the barn is 9072 cubic feet. Find the dimensions of each half of the roof.

44. **PROBLEM SOLVING** A wooden box is in the shape of a regular pentagonal prism. The sides, top, and bottom of the box are 1 centimeter thick. Approximate the volume of wood used to construct the box. Round your answer to the nearest tenth.

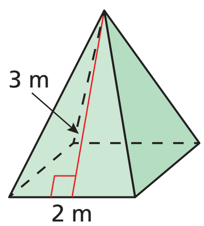


Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the surface area of the regular pyramid. *(Skills Review Handbook)*

45.



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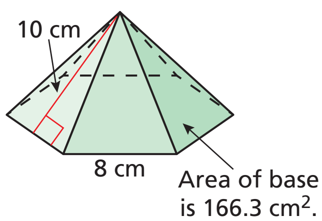

47.