10.6 Segment Relationships in Circles

Essential Question

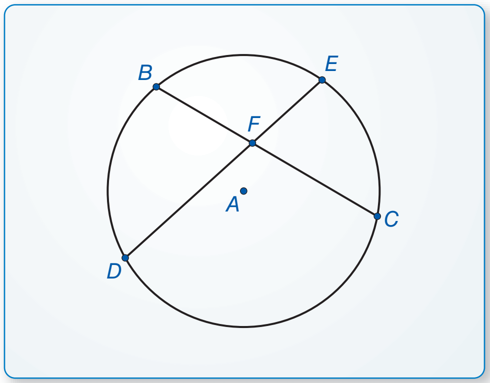
What relationships exist among the segments formed by two intersecting chords or among segments of two secants that intersect outside a circle?

EXPLORATION 1 Segments Formed by Two Intersecting Chords

Work with a partner. Use dynamic geometry software.

a. Construct two chords $\overline{BC}$ and $\overline{DE}$ that intersect in the interior of a circle at a point F.

Sample



b. Find the segment lengths BF, CF, DF, and EF and complete the table. What do you observe?

BF	CF	$BF \cdot CF$
DF	EF	$DF \cdot EF$

REASONING ABSTRACTLY

To be proficient in math, you need to make sense of quantities and their relationships in problem situations.

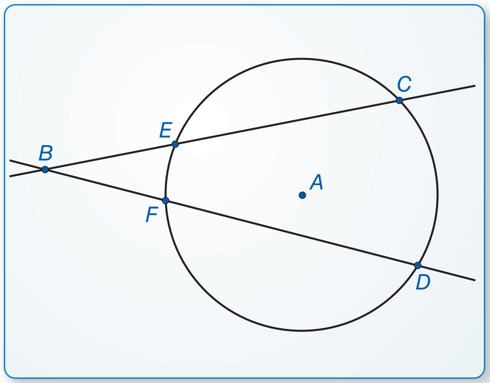
c. Repeat parts (a) and (b) several times. Write a conjecture about your results.

EXPLORATION 2 Secants Intersecting Outside a Circle

Work with a partner. Use dynamic geometry software.

a. Construct two secants $\overleftrightarrow{BC}$ and $\overleftrightarrow{BD}$ that intersect at a point B outside a circle, as shown.

Sample



b. Find the segment lengths BE, BC, BF, and BD, and complete the table. What do you observe?

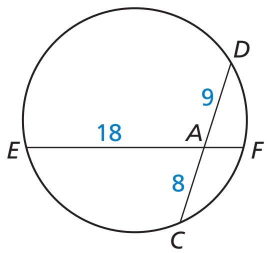
BE	BC	$BE \cdot BC$
BF	BD	$BF \cdot BD$

c. Repeat parts (a) and (b) several times. Write a conjecture about your results.

Communicate Your Answer

3. What relationships exist among the segments formed by two intersecting chords or among segments of two secants that intersect outside a circle?

4. Find the segment length AF in the figure at the left.



10.6 Lesson

Core Vocabulary

segments of a chord, *p. 614*
tangent segment, *p. 615*
secant segment, *p. 615*
external segment, *p. 615*

What You Will Learn

- Use segments of chords, tangents, and secants.

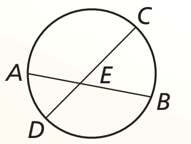
Using Segments of Chords, Tangents, and Secants

When two chords intersect in the interior of a circle, each chord is divided into two segments that are called **segments of the chord**.

Theorem

Segments of Chords Theorem

If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

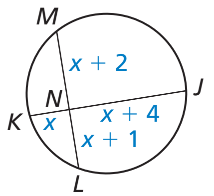


$EA \cdot EB = EC \cdot ED$

Proof Ex. 19, p. 618

EXAMPLE 1 Using Segments of Chords

Find ML and JK.



SOLUTION

$NK \cdot NJ = NL \cdot NM$	Segments of Chords Theorem
$x \cdot (x + 4) = (x + 1) \cdot (x + 2)$	Substitute.
$x^2 + 4x = x^2 + 3x + 2$	Simplify.
$4x = 3x + 2$	Subtract x^2 from each side.
$x = 2$	Subtract $3x$ from each side.

Find ML and JK by substitution.

$ML = (x + 2) + (x + 1)$ $\quad\quad JK = x + (x + 4)$

$= 2 + 2 + 2 + 1$ $\quad\quad = 2 + 2 + 4$

$= 7$ $\quad\quad = 8$

- So, $ML = 7$ and $JK = 8$.

Monitoring Progress

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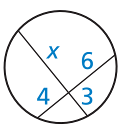
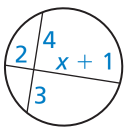
Find the value of x.

1.



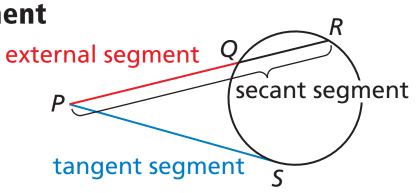
2.



Core Concept

Tangent Segment and Secant Segment

A **tangent segment** is a segment that is tangent to a circle at an endpoint. A **secant segment** is a segment that contains a chord of a circle and has exactly one endpoint outside the circle. The part of a secant segment that is outside the circle is called an **external segment**.

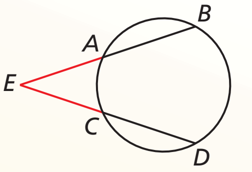


$\overline{PS}$ is a tangent segment.
$\overline{PR}$ is a secant segment.
$\overline{PQ}$ is the external segment of $\overline{PR}$.

Theorem

Segments of Secants Theorem

If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.

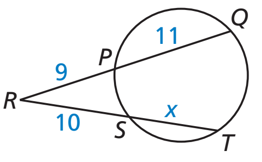


$EA \cdot EB = EC \cdot ED$

Proof Ex. 20, p. 618

EXAMPLE 2 Using Segments of Secants

Find the value of x.



SOLUTION

$RP \cdot RQ = RS \cdot RT$	Segments of Secants Theorem
$9 \cdot (11 + 9) = 10 \cdot (x + 10)$	Substitute.
$180 = 10x + 100$	Simplify.
$80 = 10x$	Subtract 100 from each side.
$8 = x$	Divide each side by 10.

▶ The value of x is 8.

Monitoring Progress

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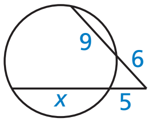
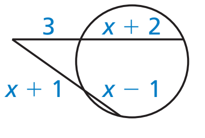
Find the value of x.

3.



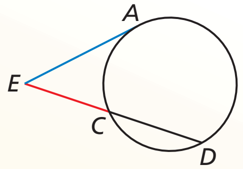
4.



Theorem

Segments of Secants and Tangents Theorem

If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.

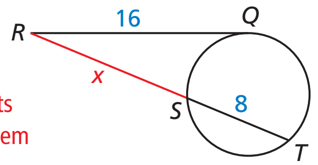


$EA^2 = EC \cdot ED$

Proof Exs. 21 and 22, p. 618

EXAMPLE 3 Using Segments of Secants and Tangents

Find RS.



SOLUTION

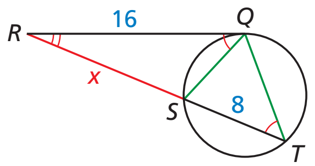
$RQ^2 = RS \cdot RT$	Segments of Secants and Tangents Theorem
$16^2 = x \cdot (x + 8)$	Substitute.
$256 = x^2 + 8x$	Simplify.
$0 = x^2 + 8x - 256$	Write in standard form.
$x = \dfrac{-8 \pm \sqrt{8^2 - 4(1)(-256)}}{2(1)}$	Use Quadratic Formula.
$x = -4 \pm 4\sqrt{17}$	Simplify.

Use the positive solution because lengths cannot be negative.

▶ So, $x = -4 + 4\sqrt{17} \approx 12.49$, and $RS \approx 12.49$.

ANOTHER WAY

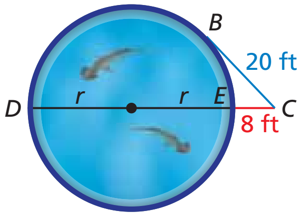
In Example 3, you can draw segments $\overline{QS}$ and $\overline{QT}$.



Because $\angle RQS$ and $\angle RTQ$ intercept the same arc, they are congruent. By the Reflexive Property of Congruence, $\angle QRS \cong \angle TRQ$. So, $\triangle RSQ \sim \triangle RQT$ by the AA Similarity Theorem. You can use this fact to write and solve a proportion to find x.

EXAMPLE 4 Finding the Radius of a Circle

Find the radius of the aquarium tank.



SOLUTION

$CB^2 = CE \cdot CD$	Segments of Secants and Tangents Theorem
$20^2 = 8 \cdot (2r + 8)$	Substitute.
$400 = 16r + 64$	Simplify.
$336 = 16r$	Subtract 64 from each side.
$21 = r$	Divide each side by 16.

▶ So, the radius of the tank is 21 feet.

Monitoring Progress

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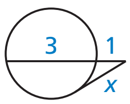
Find the value of x.

5.



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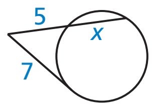

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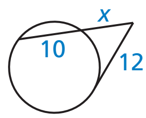


8. **WHAT IF?** In Example 4, $CB = 35$ feet and $CE = 14$ feet. Find the radius of the tank.

10.6 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

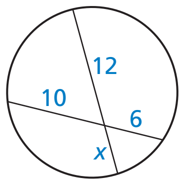
Vocabulary and Core Concept Check

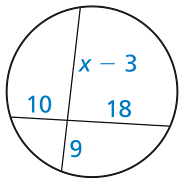
1. **VOCABULARY** The part of the secant segment that is outside the circle is called a(n) ______________.

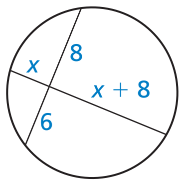
2. **WRITING** Explain the difference between a tangent segment and a secant segment.

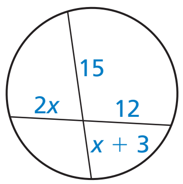
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the value of *x*. *(See Example 1.)*

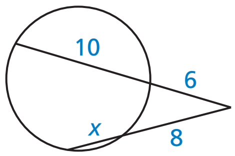
3. 

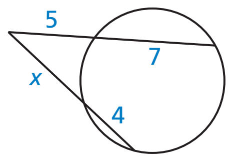
4. 

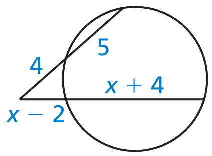
5. 

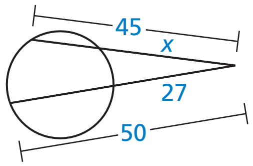
6. 

In Exercises 7–10, find the value of *x*. *(See Example 2.)*

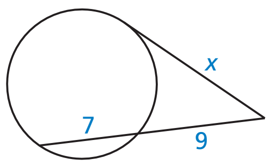
7. 

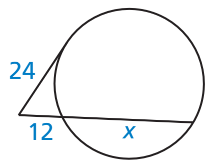
8. 

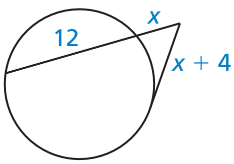
9. 

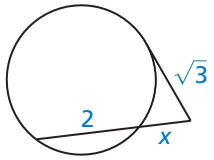
10. 

In Exercises 11–14, find the value of *x*. *(See Example 3.)*

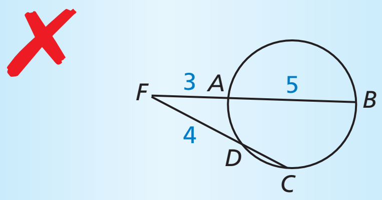
11. 

12. 

13. 

14. 

15. **ERROR ANALYSIS** Describe and correct the error in finding *CD*.



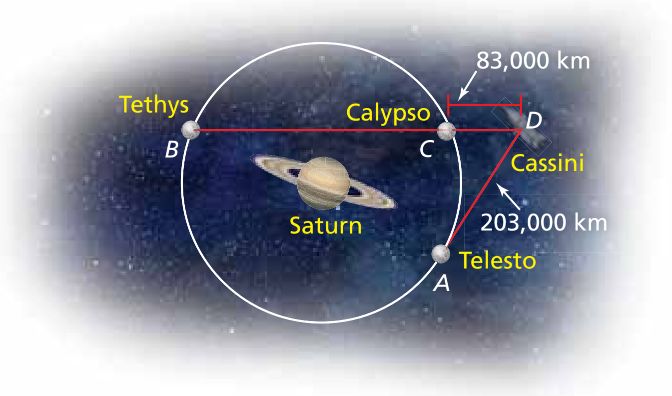
$CD \cdot DF = AB \cdot AF$

$CD \cdot 4 = 5 \cdot 3$

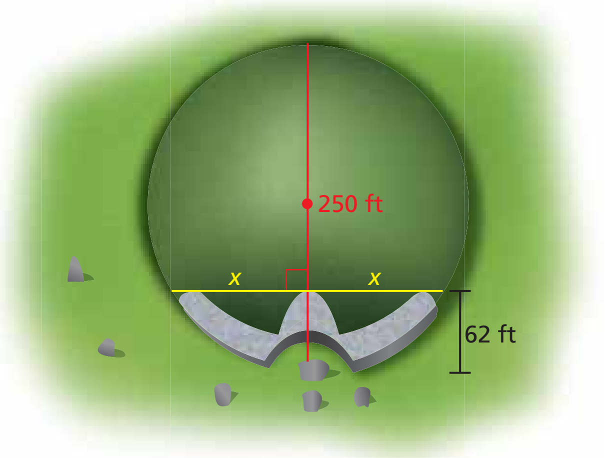
$CD \cdot 4 = 15$

$CD = 3.75$

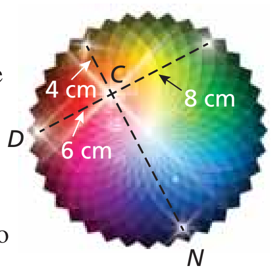
16. **MODELING WITH MATHEMATICS** The Cassini spacecraft is on a mission in orbit around Saturn until September 2017. Three of Saturn's moons, Tethys, Calypso, and Telesto, have nearly circular orbits of radius 295,000 kilometers. The diagram shows the positions of the moons and the spacecraft on one of Cassini's missions. Find the distance *DB* from Cassini to Tethys when $\overline{AD}$ is tangent to the circular orbit. *(See Example 4.)*



17. MODELING WITH MATHEMATICS The circular stone mound in Ireland called Newgrange has a diameter of 250 feet. A passage 62 feet long leads toward the center of the mound. Find the perpendicular distance x from the end of the passage to either side of the mound.



18. MODELING WITH MATHEMATICS You are designing an animated logo for your website. Sparkles leave point C and move to the outer circle along the segments shown so that all of the sparkles reach the outer circle at the same time. Sparkles travel from point C to point D at 2 centimeters per second. How fast should sparkles move from point C to point N? Explain.



19. PROVING A THEOREM Write a two-column proof of the Segments of Chords Theorem.

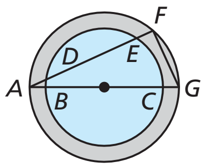
Plan for Proof Use the diagram from page 614. Draw $\overline{AC}$ and $\overline{DB}$. Show that $\triangle EAC$ and $\triangle EDB$ are similar. Use the fact that corresponding side lengths in similar triangles are proportional.

20. PROVING A THEOREM Prove the Segments of Secants Theorem. (*Hint*: Draw a diagram and add auxiliary line segments to form similar triangles.)

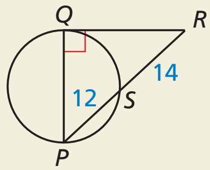
21. PROVING A THEOREM Use the Tangent Line to Circle Theorem to prove the Segments of Secants and Tangents Theorem for the special case when the secant segment contains the center of the circle.

22. PROVING A THEOREM Prove the Segments of Secants and Tangents Theorem. (*Hint*: Draw a diagram and add auxiliary line segments to form similar triangles.)

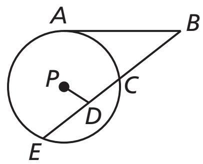
23. WRITING EQUATIONS In the diagram of the water well, AB, AD, and DE are known. Write an equation for BC using these three measurements.



24. HOW DO YOU SEE IT? Which two theorems would you need to use to find PQ? Explain your reasoning.



25. CRITICAL THINKING In the figure, $AB = 12$, $BC = 8$, $DE = 6$, $PD = 4$, and A is a point of tangency. Find the radius of $\odot P$.



26. THOUGHT PROVOKING Circumscribe a triangle about a circle. Then, using the points of tangency, inscribe a triangle in the circle. Must it be true that the two triangles are similar? Explain your reasoning.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation by completing the square. *(Section 4.4)*

27. $x^2 + 4x = 45$

28. $x^2 - 2x - 1 = 8$

29. $2x^2 + 12x + 20 = 34$

30. $-4x^2 + 8x + 44 = 16$