8.5 Proving Triangle Similarity by SSS and SAS

Essential Question What are two ways to use corresponding sides of two triangles to determine that the triangles are similar?

EXPLORATION 1 Deciding Whether Triangles Are Similar

Work with a partner. Use dynamic geometry software.

a. Construct $\triangle ABC$ and $\triangle DEF$ with the side lengths given in column 1 of the table below.

	1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**
AB	5	5	6	15	9	24	
BC	8	8	8	20	12	18	
AC	10	10	10	10	8	16	
DE	10	15	9	12	12	8	
EF	16	24	12	16	15	6	
DF	20	30	15	8	10	8	
$m\angle A$							
$m\angle B$							
$m\angle C$							
$m\angle D$							
$m\angle E$							
$m\angle F$							

CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to analyze situations by breaking them into cases and recognize and use counterexamples.

b. Copy the table and complete column 1.

c. Are the triangles similar? Explain your reasoning.

d. Repeat parts (a)–(c) for columns 2–6 in the table.

e. How are the corresponding side lengths related in each pair of triangles that are similar? Is this true for each pair of triangles that are not similar?

f. Make a conjecture about the similarity of two triangles based on their corresponding side lengths.

g. Use your conjecture to write another set of side lengths of two similar triangles. Use the side lengths to complete column 7 of the table.

EXPLORATION 2 Deciding Whether Triangles Are Similar

Work with a partner. Use dynamic geometry software. Construct any $\triangle ABC$.

a. Find AB, AC, and $m\angle A$. Choose any positive rational number k and construct $\triangle DEF$ so that $DE = k \cdot AB$, $DF = k \cdot AC$, and $m\angle D = m\angle A$.

b. Is $\triangle DEF$ similar to $\triangle ABC$? Explain your reasoning.

c. Repeat parts (a) and (b) several times by changing $\triangle ABC$ and k. Describe your results.

Communicate Your Answer

3. What are two ways to use corresponding sides of two triangles to determine that the triangles are similar?

8.5 Lesson

Core Vocabulary

Previous
similar figures
corresponding parts
parallel lines

What You Will Learn

- Use the Side-Side-Side Similarity Theorem.
- Use the Side-Angle-Side Similarity Theorem.

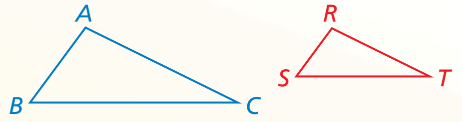
Using the Side-Side-Side Similarity Theorem

In addition to using congruent corresponding angles to show that two triangles are similar, you can use proportional corresponding side lengths.

Theorem

Side-Side-Side (SSS) Similarity Theorem

If the corresponding side lengths of two triangles are proportional, then the triangles are similar.

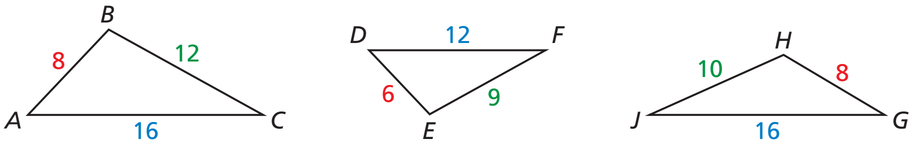


If $\frac{AB}{RS} = \frac{BC}{ST} = \frac{CA}{TR}$, then $\triangle ABC \sim \triangle RST$.

Proof p. 495

EXAMPLE 1 Using the SSS Similarity Theorem

Is either $\triangle DEF$ or $\triangle GHJ$ similar to $\triangle ABC$?



FINDING AN ENTRY POINT

When using the SSS Similarity Theorem, compare the shortest sides, the longest sides, and then the remaining sides.

SOLUTION

Compare $\triangle ABC$ and $\triangle DEF$ by finding ratios of corresponding side lengths.

Shortest sides	Longest sides	Remaining sides
$\frac{AB}{DE} = \frac{8}{6}$	$\frac{CA}{FD} = \frac{16}{12}$	$\frac{BC}{EF} = \frac{12}{9}$
$= \frac{4}{3}$	$= \frac{4}{3}$	$= \frac{4}{3}$

- All the ratios are equal, so $\triangle ABC \sim \triangle DEF$.

Compare $\triangle ABC$ and $\triangle GHJ$ by finding ratios of corresponding side lengths.

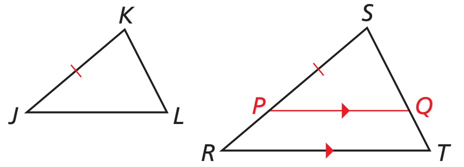
Shortest sides	Longest sides	Remaining sides
$\frac{AB}{GH} = \frac{8}{8}$	$\frac{CA}{JG} = \frac{16}{16}$	$\frac{BC}{HJ} = \frac{12}{10}$
$= 1$	$= 1$	$= \frac{6}{5}$

- The ratios are not all equal, so $\triangle ABC$ and $\triangle GHJ$ are not similar.

PROOF SSS Similarity Theorem

Given $\dfrac{RS}{JK} = \dfrac{ST}{KL} = \dfrac{TR}{LJ}$

Prove $\triangle RST \sim \triangle JKL$



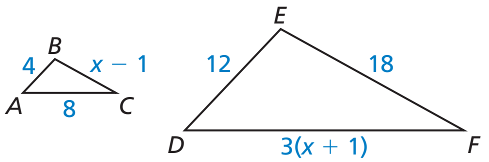
JUSTIFYING STEPS

The Parallel Postulate allows you to draw an auxiliary line $\overleftrightarrow{PQ}$ in $\triangle RST$. There is only one line through point P parallel to $\overleftrightarrow{RT}$, so you are able to draw it.

Locate P on $\overline{RS}$ so that $PS = JK$. Draw $\overline{PQ}$ so that $\overline{PQ} \parallel \overline{RT}$. Then $\triangle RST \sim \triangle PSQ$ by the AA Similarity Theorem, and $\dfrac{RS}{PS} = \dfrac{ST}{SQ} = \dfrac{TR}{QP}$. You can use the given proportion and the fact that $PS = JK$ to deduce that $SQ = KL$ and $QP = LJ$. By the SSS Congruence Theorem, it follows that $\triangle PSQ \cong \triangle JKL$. Finally, use the definition of congruent triangles and the AA Similarity Theorem to conclude that $\triangle RST \sim \triangle JKL$.

EXAMPLE 2 Using the SSS Similarity Theorem

Find the value of x that makes $\triangle ABC \sim \triangle DEF$.



FINDING AN ENTRY POINT

You can use either $\dfrac{AB}{DE} = \dfrac{BC}{EF}$ or $\dfrac{AB}{DE} = \dfrac{AC}{DF}$ in Step 1.

SOLUTION

Step 1 Find the value of x that makes corresponding side lengths proportional.

$$\frac{AB}{DE} = \frac{BC}{EF} \qquad \text{Write proportion.}$$

$$\frac{4}{12} = \frac{x-1}{18} \qquad \text{Substitute.}$$

$$4 \cdot 18 = 12(x-1) \qquad \text{Cross Products Property}$$

$$72 = 12x - 12 \qquad \text{Simplify.}$$

$$7 = x \qquad \text{Solve for } x.$$

Step 2 Check that the side lengths are proportional when $x = 7$.

$BC = x - 1 = 6$ $\qquad$ $DF = 3(x + 1) = 24$

$\dfrac{AB}{DE} \stackrel{?}{=} \dfrac{BC}{EF} \rightarrow \dfrac{4}{12} = \dfrac{6}{18}$ ✓ $\qquad$ $\dfrac{AB}{DE} \stackrel{?}{=} \dfrac{AC}{DF} \rightarrow \dfrac{4}{12} = \dfrac{8}{24}$ ✓

▶ When $x = 7$, the triangles are similar by the SSS Similarity Theorem.

Monitoring Progress

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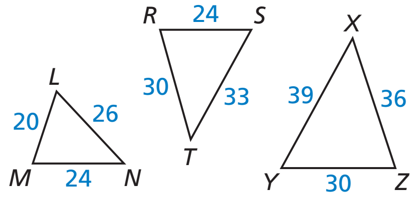
Use the diagram.



1. Which of the three triangles are similar? Write a similarity statement.

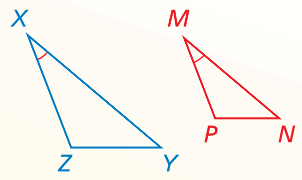
2. The shortest side of a triangle similar to $\triangle RST$ is 12 units long. Find the other side lengths of the triangle.

Using the Side-Angle-Side Similarity Theorem

Theorem

Side-Angle-Side (SAS) Similarity Theorem

If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.

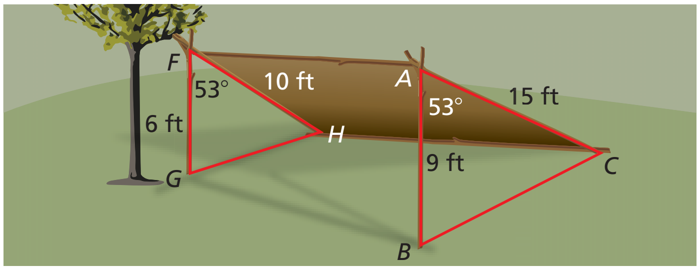


If $\angle X \cong \angle M$ and $\frac{ZX}{PM} = \frac{XY}{MN}$, then $\triangle XYZ \sim \triangle MNP$.

Proof Ex. 19, p. 498

EXAMPLE 3 Using the SAS Similarity Theorem

You are building a lean-to shelter starting from a tree branch, as shown. Can you construct the right end so it is similar to the left end using the angle measure and lengths shown?



SOLUTION

Both $m\angle A$ and $m\angle F$ equal 53°, so $\angle A \cong \angle F$. Next, compare the ratios of the lengths of the sides that include $\angle A$ and $\angle F$.

Shorter sides

$$\frac{AB}{FG} = \frac{9}{6} = \frac{3}{2}$$

Longer sides

$$\frac{AC}{FH} = \frac{15}{10} = \frac{3}{2}$$

The lengths of the sides that include $\angle A$ and $\angle F$ are proportional. So, by the SAS Similarity Theorem, $\triangle ABC \sim \triangle FGH$.

▶ Yes, you can make the right end similar to the left end of the shelter.

Monitoring Progress

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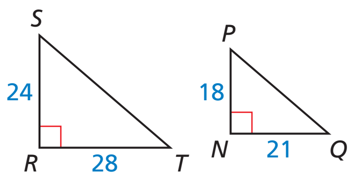
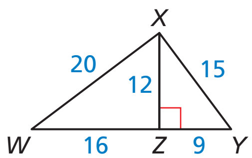
Explain how to show that the indicated triangles are similar.

3. $\triangle SRT \sim \triangle PNQ$



4. $\triangle XZW \sim \triangle YZX$



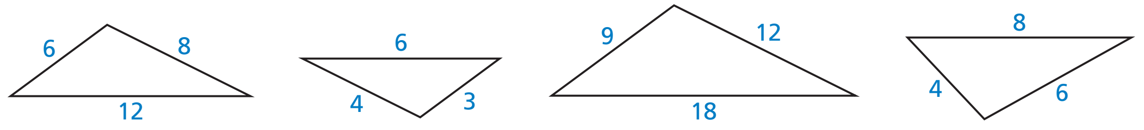
8.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** You plan to show that $\triangle QRS$ is similar to $\triangle XYZ$ by the SSS Similarity Theorem. Copy and complete the proportion that you will use: $\frac{QR}{\boxed{}} = \frac{\boxed{}}{YZ} = \frac{QS}{\boxed{}}$.

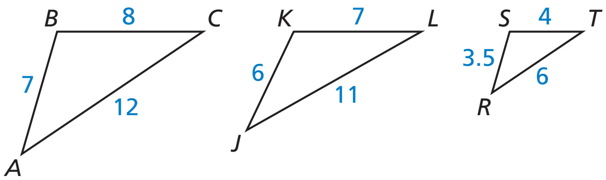
2. **WHICH ONE DOESN'T BELONG?** Which triangle does *not* belong with the other three? Explain your reasoning.



Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, determine whether $\triangle JKL$ or $\triangle RST$ is similar to $\triangle ABC$. *(See Example 1.)*

3.



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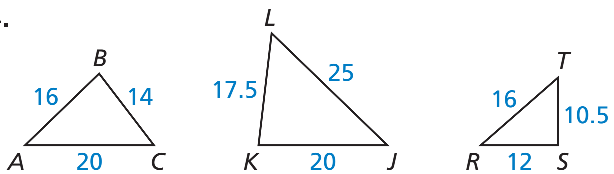
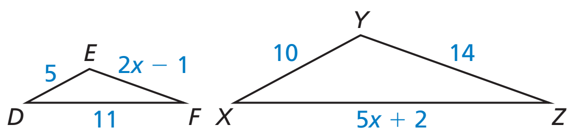


In Exercises 5 and 6, find the value of x that makes $\triangle DEF \sim \triangle XYZ$. *(See Example 2.)*

5.



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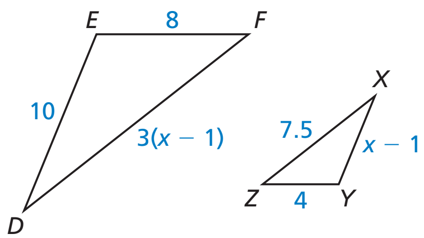


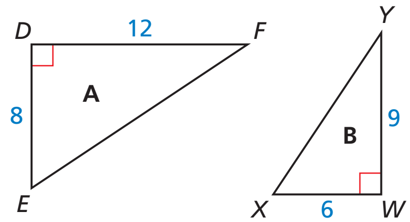
In Exercises 7 and 8, verify that $\triangle ABC \sim \triangle DEF$. Find the scale factor of $\triangle ABC$ to $\triangle DEF$.

7. $\triangle ABC$: $BC = 18$, $AB = 15$, $AC = 12$
$\triangle DEF$: $EF = 12$, $DE = 10$, $DF = 8$

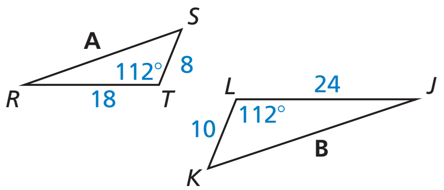
8. $\triangle ABC$: $AB = 10$, $BC = 16$, $CA = 20$
$\triangle DEF$: $DE = 25$, $EF = 40$, $FD = 50$

In Exercises 9 and 10, determine whether the two triangles are similar. If they are similar, write a similarity statement and find the scale factor of triangle B to triangle A. *(See Example 3.)*

9.



10.

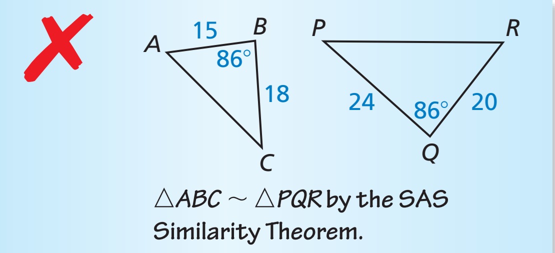


In Exercises 11 and 12, sketch the triangles using the given description. Then determine whether the two triangles can be similar.

11. In $\triangle RST$, $RS = 20$, $ST = 32$, and $m\angle S = 16°$. In $\triangle FGH$, $GH = 30$, $HF = 48$, and $m\angle H = 24°$.

12. The side lengths of $\triangle ABC$ are 24, $8x$, and 48, and the side lengths of $\triangle DEF$ are 15, 25, and $6x$.

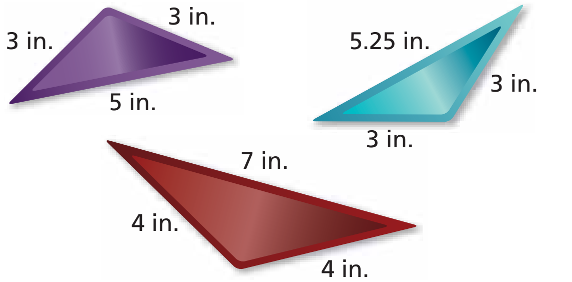
13. **ERROR ANALYSIS** Describe and correct the error in writing a similarity statement.



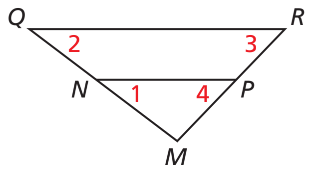
14. **MATHEMATICAL CONNECTIONS** Find the value of n that makes $\triangle DEF \sim \triangle XYZ$ when $DE = 4$, $EF = 5$, $XY = 4(n + 1)$, $YZ = 7n - 1$, and $\angle E \cong \angle Y$. Include a sketch.

15. **MAKING AN ARGUMENT** Your friend claims that $\triangle JKL \sim \triangle MNO$ by the SAS Similarity Theorem when $JK = 18$, $m\angle K = 130°$, $KL = 16$, $MN = 9$, $m\angle N = 65°$, and $NO = 8$. Do you support your friend's claim? Explain your reasoning.

16. **ANALYZING RELATIONSHIPS** Certain sections of stained glass are sold in triangular, beveled pieces. Which of the three beveled pieces, if any, are similar?



17. **ATTENDING TO PRECISION** In the diagram, $\frac{MN}{MR} = \frac{MP}{MQ}$. Which of the statements must be true? Select all that apply. Explain your reasoning.



Ⓐ $\angle 1 \cong \angle 2$ Ⓑ $\overline{QR} \parallel \overline{NP}$

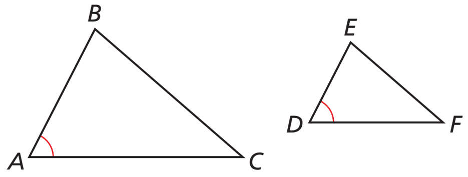
Ⓒ $\angle 1 \cong \angle 4$ Ⓓ $\triangle MNP \sim \triangle MRQ$

18. **WRITING** Are any two right triangles similar? Explain.

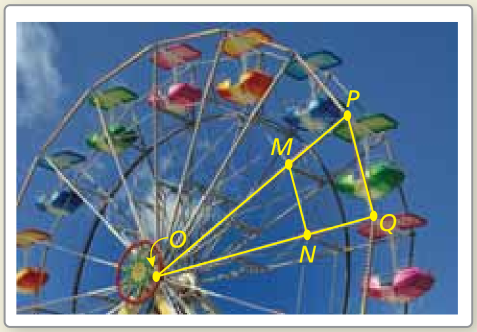
19. **PROVING A THEOREM** Write a two-column proof of the SAS Similarity Theorem.

Given $\angle A \cong \angle D$, $\frac{AB}{DE} = \frac{AC}{DF}$

Prove $\triangle ABC \sim \triangle DEF$



20. **HOW DO YOU SEE IT?** Which theorem could you use to show that $\triangle OPQ \sim \triangle OMN$ in the portion of the Ferris wheel shown when $PM = QN = 5$ feet and $MO = NO = 10$ feet?



21. **DRAWING CONCLUSIONS** Explain why it is not necessary to have an Angle-Side-Angle Similarity Theorem.

22. **THOUGHT PROVOKING** Decide whether each is a valid method of showing that two quadrilaterals are similar. Justify your answer.

a. SASA **b.** SASAS **c.** SSSS **d.** SASSS

23. **WRITING** Can two triangles have all three ratios of corresponding angle measures equal to a value greater than 1? less than 1? Explain.

24. **MULTIPLE REPRESENTATIONS** Use a diagram to show why there is no Side-Side-Angle Similarity Theorem.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the slope of the line that passes through the given points. *(Skills Review Handbook)*

25. $(-3, 6), (2, 1)$ **26.** $(-3, -5), (9, -1)$ **27.** $(1, -2), (8, 12)$