7.2 Properties of Parallelograms

Essential Question

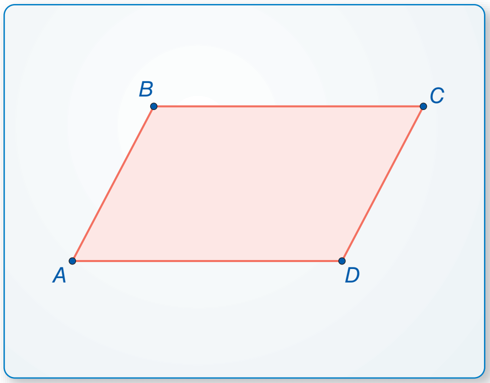
What are the properties of parallelograms?

EXPLORATION 1 Discovering Properties of Parallelograms

Work with a partner. Use dynamic geometry software.

a. Construct any parallelogram and label it *ABCD*. Explain your process.

Sample



b. Find the angle measures of the parallelogram. What do you observe?

c. Find the side lengths of the parallelogram. What do you observe?

d. Repeat parts (a)–(c) for several other parallelograms. Use your results to write conjectures about the angle measures and side lengths of a parallelogram.

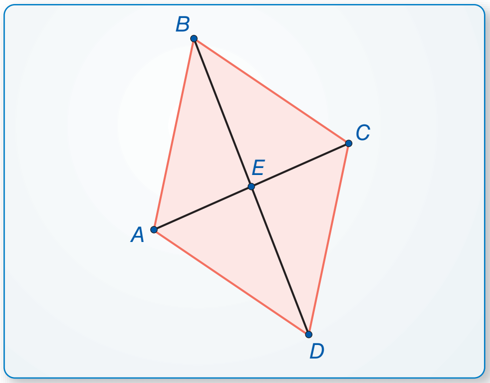
EXPLORATION 2 Discovering a Property of Parallelograms

Work with a partner. Use dynamic geometry software.

a. Construct any parallelogram and label it *ABCD*.

b. Draw the two diagonals of the parallelogram. Label the point of intersection *E*.

Sample



c. Find the segment lengths *AE*, *BE*, *CE*, and *DE*. What do you observe?

d. Repeat parts (a)–(c) for several other parallelograms. Use your results to write a conjecture about the diagonals of a parallelogram.

MAKING SENSE OF PROBLEMS

To be proficient in math, you need to analyze givens, constraints, relationships, and goals.

Communicate Your Answer

3. What are the properties of parallelograms?

7.2 Lesson

Core Vocabulary

parallelogram, *p. 412*

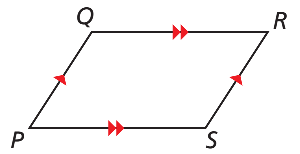
Previous
quadrilateral
diagonal
interior angles
segment bisector

What You Will Learn

- Use properties to find side lengths and angles of parallelograms.
- Use parallelograms in the coordinate plane.

Using Properties of Parallelograms

A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel. In $\square PQRS$, $\overline{PQ} \parallel \overline{RS}$ and $\overline{QR} \parallel \overline{PS}$ by definition. The theorems below describe other properties of parallelograms.



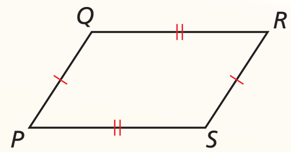
Theorems

Parallelogram Opposite Sides Theorem

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

If $PQRS$ is a parallelogram, then $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{SP}$.

Proof p. 412

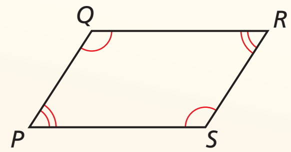


Parallelogram Opposite Angles Theorem

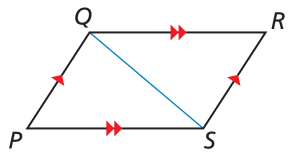
If a quadrilateral is a parallelogram, then its opposite angles are congruent.

If $PQRS$ is a parallelogram, then $\angle P \cong \angle R$ and $\angle Q \cong \angle S$.

Proof Ex. 37, p. 417



PROOF Parallelogram Opposite Sides Theorem



Given $PQRS$ is a parallelogram.

Prove $\overline{PQ} \cong \overline{RS}$, $\overline{QR} \cong \overline{SP}$

Plan for Proof

a. Draw diagonal $\overline{QS}$ to form $\triangle PQS$ and $\triangle RSQ$.

b. Use the ASA Congruence Theorem to show that $\triangle PQS \cong \triangle RSQ$.

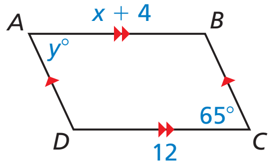
c. Use congruent triangles to show that $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{SP}$.

Plan in Action

	STATEMENTS	REASONS
	1. $PQRS$ is a parallelogram.	**1.** Given
a.	**2.** Draw $\overline{QS}$.	**2.** Through any two points, there exists exactly one line.
	3. $\overline{PQ} \parallel \overline{RS}$, $\overline{QR} \parallel \overline{PS}$	**3.** Definition of parallelogram
b.	**4.** $\angle PQS \cong \angle RSQ$, $\angle PSQ \cong \angle RQS$	**4.** Alternate Interior Angles Theorem
	5. $\overline{QS} \cong \overline{SQ}$	**5.** Reflexive Property of Congruence
	6. $\triangle PQS \cong \triangle RSQ$	**6.** ASA Congruence Theorem
c.	**7.** $\overline{PQ} \cong \overline{RS}$, $\overline{QR} \cong \overline{SP}$	**7.** Corresponding parts of congruent triangles are congruent.

EXAMPLE 1 Using Properties of Parallelograms

Find the values of x and y.



SOLUTION

ABCD is a parallelogram by the definition of a parallelogram. Use the Parallelogram Opposite Sides Theorem to find the value of x.

$AB = CD$	Opposite sides of a parallelogram are congruent.
$x + 4 = 12$	Substitute $x + 4$ for AB and 12 for CD.
$x = 8$	Subtract 4 from each side.

By the Parallelogram Opposite Angles Theorem, $\angle A \cong \angle C$, or $m\angle A = m\angle C$. So, $y° = 65°$.

▶ In ▱$ABCD$, $x = 8$ and $y = 65$.

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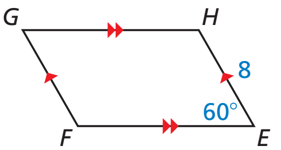
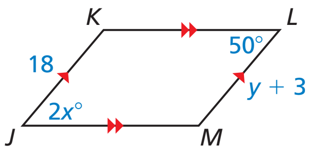
1. Find FG and $m\angle G$.

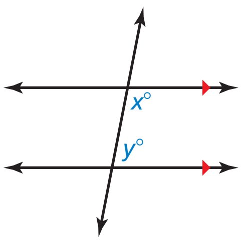


2. Find the values of x and y.



The Consecutive Interior Angles Theorem states that if two parallel lines are cut by a transversal, then the pairs of consecutive interior angles formed are supplementary.

A pair of consecutive angles in a parallelogram is like a pair of consecutive interior angles between parallel lines. This similarity suggests the Parallelogram Consecutive Angles Theorem.



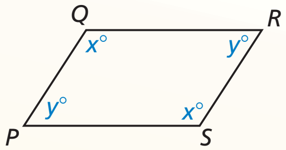
Theorems

Parallelogram Consecutive Angles Theorem

If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

If $PQRS$ is a parallelogram, then $x° + y° = 180°$.

Proof Ex. 38, p. 417

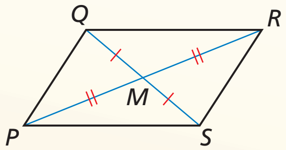


Parallelogram Diagonals Theorem

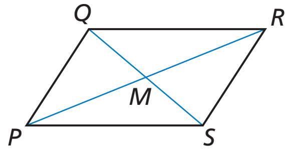
If a quadrilateral is a parallelogram, then its diagonals bisect each other.

If $PQRS$ is a parallelogram, then $\overline{QM} \cong \overline{SM}$ and $\overline{PM} \cong \overline{RM}$.

Proof p. 414



PROOF Parallelogram Diagonals Theorem

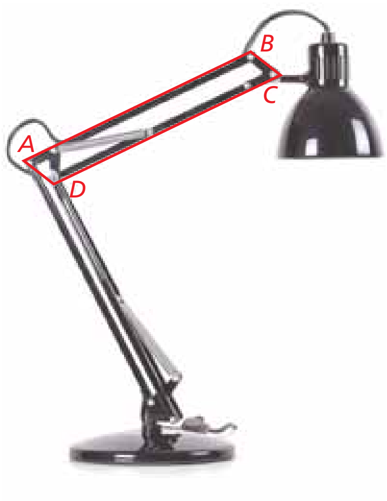


Given *PQRS* is a parallelogram. Diagonals $\overline{PR}$ and $\overline{QS}$ intersect at point *M*.

Prove *M* bisects $\overline{QS}$ and $\overline{PR}$.

STATEMENTS	REASONS
1. *PQRS* is a parallelogram.	**1.** Given
2. $\overline{PQ} \parallel \overline{RS}$	**2.** Definition of a parallelogram
3. $\angle QPR \cong \angle SRP$, $\angle PQS \cong \angle RSQ$	**3.** Alternate Interior Angles Theorem
4. $\overline{PQ} \cong \overline{RS}$	**4.** Parallelogram Opposite Sides Theorem
5. $\triangle PMQ \cong \triangle RMS$	**5.** ASA Congruence Theorem
6. $\overline{QM} \cong \overline{SM}$, $\overline{PM} \cong \overline{RM}$	**6.** Corresponding parts of congruent triangles are congruent.
7. *M* bisects $\overline{QS}$ and $\overline{PR}$.	**7.** Definition of segment bisector

EXAMPLE 2 Using Properties of a Parallelogram

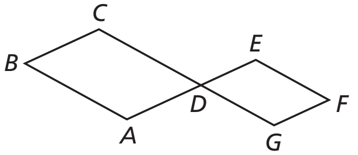


As shown, part of the extending arm of a desk lamp is a parallelogram. The angles of the parallelogram change as the lamp is raised and lowered. Find $m\angle BCD$ when $m\angle ADC = 110°$.

SOLUTION

By the Parallelogram Consecutive Angles Theorem, the consecutive angle pairs in $\square ABCD$ are supplementary. So, $m\angle ADC + m\angle BCD = 180°$. Because $m\angle ADC = 110°$, $m\angle BCD = 180° - 110° = 70°$.

EXAMPLE 3 Writing a Two-Column Proof



Write a two-column proof.

Given *ABCD* and *GDEF* are parallelograms.

Prove $\angle B \cong \angle F$

STATEMENTS	REASONS
1. *ABCD* and *GDEF* are parallelograms.	**1.** Given
2. $\angle CDA \cong \angle B$, $\angle EDG \cong \angle F$	**2.** If a quadrilateral is a parallelogram, then its opposite angles are congruent.
3. $\angle CDA \cong \angle EDG$	**3.** Vertical Angles Congruence Theorem
4. $\angle B \cong \angle F$	**4.** Transitive Property of Congruence

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3. **WHAT IF?** In Example 2, find $m\angle BCD$ when $m\angle ADC$ is twice the measure of $\angle BCD$.

4. Using the figure and the given statement in Example 3, prove that $\angle C$ and $\angle F$ are supplementary angles.

Using Parallelograms in the Coordinate Plane

EXAMPLE 4 Using Parallelograms in the Coordinate Plane

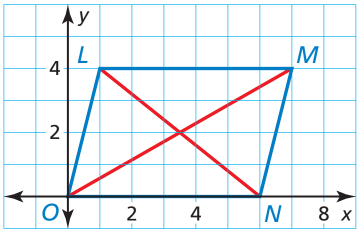
Find the coordinates of the intersection of the diagonals of ▱*LMNO* with vertices $L(1, 4)$, $M(7, 4)$, $N(6, 0)$, and $O(0, 0)$.

SOLUTION

By the Parallelogram Diagonals Theorem, the diagonals of a parallelogram bisect each other. So, the coordinates of the intersection are the midpoints of diagonals $\overline{LN}$ and $\overline{OM}$.

$$\text{coordinates of midpoint of } \overline{OM} = \left(\frac{7+0}{2}, \frac{4+0}{2}\right) = \left(\frac{7}{2}, 2\right) \qquad \text{Midpoint Formula}$$

▶ The coordinates of the intersection of the diagonals are $\left(\frac{7}{2}, 2\right)$. You can check your answer by graphing ▱*LMNO* and drawing the diagonals. The point of intersection appears to be correct.



JUSTIFYING STEPS

In Example 4, you can use either diagonal to find the coordinates of the intersection. Using diagonal $\overline{OM}$ helps simplify the calculation because one endpoint is (0, 0).

EXAMPLE 5 Using Parallelograms in the Coordinate Plane

Three vertices of ▱*WXYZ* are $W(-1, -3)$, $X(-3, 2)$, and $Z(4, -4)$. Find the coordinates of vertex *Y*.

SOLUTION

Step 1 Graph the vertices *W*, *X*, and *Z*.

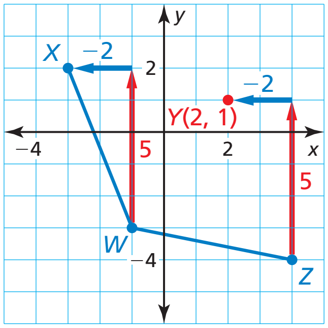
Step 2 Find the slope of $\overline{WX}$.

$$\text{slope of } \overline{WX} = \frac{2-(-3)}{-3-(-1)} = \frac{5}{-2} = -\frac{5}{2}$$

Step 3 Start at $Z(4, -4)$. Use the rise and run from Step 2 to find vertex *Y*.

A rise of 5 represents a change of 5 units up. A run of -2 represents a change of 2 units left.

So, plot the point that is 5 units up and 2 units left from $Z(4, -4)$. The point is (2, 1). Label it as vertex *Y*.



Step 4 Find the slopes of $\overline{XY}$ and $\overline{WZ}$ to verify that they are parallel.

$$\text{slope of } \overline{XY} = \frac{1-2}{2-(-3)} = \frac{-1}{5} = -\frac{1}{5} \qquad \text{slope of } \overline{WZ} = \frac{-4-(-3)}{4-(-1)} = \frac{-1}{5} = -\frac{1}{5}$$

▶ So, the coordinates of vertex *Y* are (2, 1).

REMEMBER

When graphing a polygon in the coordinate plane, the name of the polygon gives the order of the vertices.

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5. Find the coordinates of the intersection of the diagonals of ▱*STUV* with vertices $S(-2, 3)$, $T(1, 5)$, $U(6, 3)$, and $V(3, 1)$.

6. Three vertices of ▱*ABCD* are $A(2, 4)$, $B(5, 2)$, and $C(3, -1)$. Find the coordinates of vertex *D*.

7.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

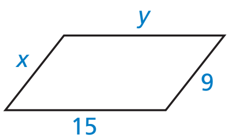
Vocabulary and Core Concept Check

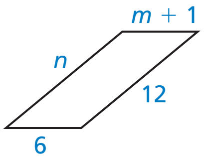
1. **VOCABULARY** Why is a parallelogram always a quadrilateral, but a quadrilateral is only sometimes a parallelogram?

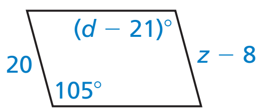
2. **WRITING** You are given one angle measure of a parallelogram. Explain how you can find the other angle measures of the parallelogram.

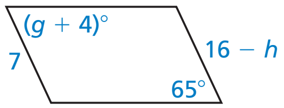
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the value of each variable in the parallelogram. *(See Example 1.)*

3. 

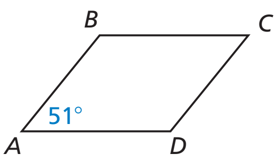
4. 

5. 

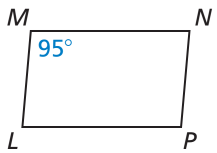
6. 

In Exercises 7 and 8, find the measure of the indicated angle in the parallelogram. *(See Example 2.)*

7. Find $m\angle B$.



8. Find $m\angle N$.



In Exercises 9–16, find the indicated measure in ▱*LMNQ*. Explain your reasoning.

9. *LM*

10. *LP*

11. *LQ*

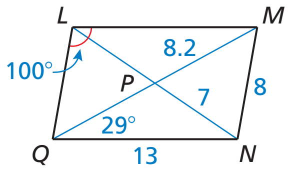
12. *MQ*

13. $m\angle LMN$

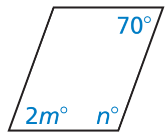
14. $m\angle NQL$

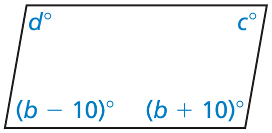
15. $m\angle MNQ$

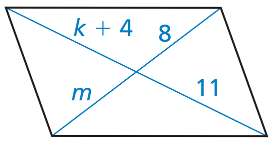
16. $m\angle LMQ$

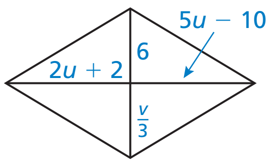


In Exercises 17–20, find the value of each variable in the parallelogram.

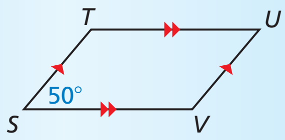
17. 

18. 

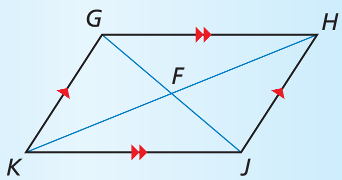
19. 

20. 

ERROR ANALYSIS **In Exercises 21 and 22, describe and correct the error in using properties of parallelograms.**

21. 

Because quadrilateral *STUV* is a parallelogram, $\angle S \cong \angle V$. So, $m\angle V = 50°$.

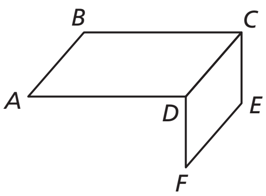
22. 

Because quadrilateral *GHJK* is a parallelogram, $\overline{GF} \cong \overline{FH}$.

PROOF In Exercises 23 and 24, write a two-column proof. *(See Example 3.)*

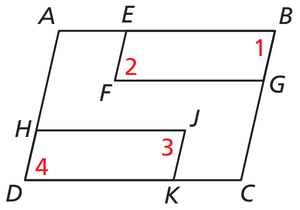
23. **Given** *ABCD* and *CEFD* are parallelograms.

Prove $\overline{AB} \cong \overline{FE}$



24. **Given** *ABCD*, *EBGF,* and *HJKD* are parallelograms.

Prove $\angle 2 \cong \angle 3$



In Exercises 25 and 26, find the coordinates of the intersection of the diagonals of the parallelogram with the given vertices. *(See Example 4.)*

25. $W(-2, 5)$, $X(2, 5)$, $Y(4, 0)$, $Z(0, 0)$

26. $Q(-1, 3)$, $R(5, 2)$, $S(1, -2)$, $T(-5, -1)$

In Exercises 27–30, three vertices of ▱*DEFG* are given. Find the coordinates of the remaining vertex. *(See Example 5.)*

27. $D(0, 2)$, $E(-1, 5)$, $G(4, 0)$

28. $D(-2, -4)$, $F(0, 7)$, $G(1, 0)$

29. $D(-4, -2)$, $E(-3, 1)$, $F(3, 3)$

30. $E(-1, 4)$, $F(5, 6)$, $G(8, 0)$

MATHEMATICAL CONNECTIONS In Exercises 31 and 32, find the measure of each angle.

31. The measure of one interior angle of a parallelogram is 0.25 times the measure of another angle.

32. The measure of one interior angle of a parallelogram is 50 degrees more than 4 times the measure of another angle.

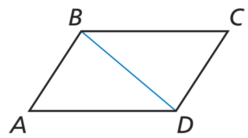
33. **MAKING AN ARGUMENT** In quadrilateral *ABCD*, $m\angle B = 124°$, $m\angle A = 56°$, and $m\angle C = 124°$. Your friend claims quadrilateral *ABCD* could be a parallelogram. Is your friend correct? Explain your reasoning.

34. **ATTENDING TO PRECISION** $\angle J$ and $\angle K$ are consecutive angles in a parallelogram, $m\angle J = (3x + 7)°$, and $m\angle K = (5x - 11)°$. Find the measure of each angle.

35. **CONSTRUCTION** Construct any parallelogram and label it *ABCD*. Draw diagonals $\overline{AC}$ and $\overline{BD}$. Explain how to use paper folding to verify the Parallelogram Diagonals Theorem for ▱*ABCD*.

36. **MODELING WITH MATHEMATICS** The feathers on an arrow form two congruent parallelograms. The parallelograms are reflections of each other over the line that contains their shared side. Show that $m\angle 2 = 2m\angle 1$.

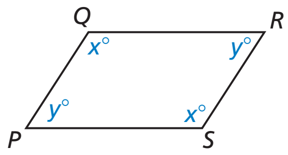
37. **PROVING A THEOREM** Use the diagram to write a two-column proof of the Parallelogram Opposite Angles Theorem.



Given *ABCD* is a parallelogram.

Prove $\angle A \cong \angle C$, $\angle B \cong \angle D$

38. **PROVING A THEOREM** Use the diagram to write a two-column proof of the Parallelogram Consecutive Angles Theorem.



Given *PQRS* is a parallelogram.

Prove $x° + y° = 180°$

39. **PROBLEM SOLVING** The sides of ▱*MNPQ* are represented by the expressions below. Sketch ▱*MNPQ* and find its perimeter.

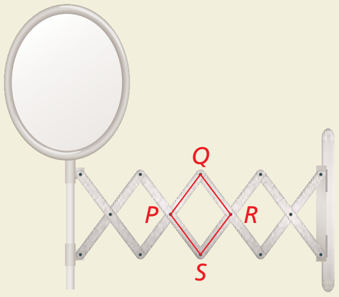
$MQ = -2x + 37$ $QP = y + 14$

$NP = x - 5$ $MN = 4y + 5$

40. **PROBLEM SOLVING** In ▱*LMNP*, the ratio of *LM* to *MN* is 4 : 3. Find *LM* when the perimeter of ▱*LMNP* is 28.

41. **ABSTRACT REASONING** Can you prove that two parallelograms are congruent by proving that all their corresponding sides are congruent? Explain your reasoning.

42. **HOW DO YOU SEE IT?** The mirror shown is attached to the wall by an arm that can extend away from the wall. In the figure, points P, Q, R, and S are the vertices of a parallelogram. This parallelogram is one of several that change shape as the mirror is extended.

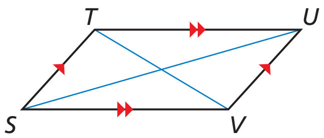


a. What happens to $m\angle P$ as $m\angle Q$ increases? Explain.

b. What happens to QS as $m\angle Q$ decreases? Explain.

c. What happens to the overall distance between the mirror and the wall when $m\angle Q$ decreases? Explain.

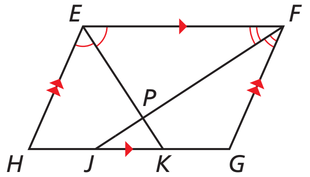
43. **MATHEMATICAL CONNECTIONS** In ▱$STUV$, $m\angle TSU = 32°$, $m\angle USV = (x^2)°$, $m\angle TUV = 12x°$, and $\angle TUV$ is an acute angle. Find $m\angle USV$.



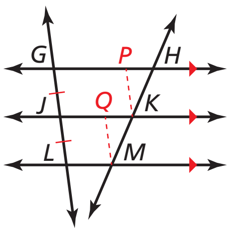
44. **THOUGHT PROVOKING** Is it possible that any triangle can be partitioned into four congruent triangles that can be rearranged to form a parallelogram? Explain your reasoning.

45. **CRITICAL THINKING** Points $W(1, 2)$, $X(3, 6)$, and $Y(6, 4)$ are three vertices of a parallelogram. How many parallelograms can be created using these three vertices? Find the coordinates of each point that could be the fourth vertex.

46. **PROOF** In the diagram, $\overline{EK}$ bisects $\angle FEH$, and $\overline{FJ}$ bisects $\angle EFG$. Prove that $\overline{EK} \perp \overline{FJ}$. (*Hint*: Write equations using the angle measures of the triangles and quadrilaterals formed.)



47. **PROOF** Prove the *Congruent Parts of Parallel Lines Corollary*: If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.



Given $\overleftrightarrow{GH} \parallel \overleftrightarrow{JK} \parallel \overleftrightarrow{LM}$, $\overline{GJ} \cong \overline{JL}$

Prove $\overline{HK} \cong \overline{KM}$

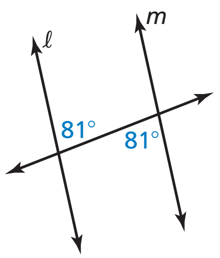
(*Hint:* Draw $\overline{KP}$ and $\overline{MQ}$ such that quadrilateral $GPKJ$ and quadrilateral $JQML$ are parallelograms.)

Maintaining Mathematical Proficiency

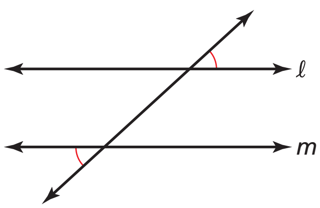
Reviewing what you learned in previous grades and lessons

Determine whether lines ℓ and m are parallel. Explain your reasoning. *(Skills Review Handbook)*

48.



49.



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