

# 3 Graphing Quadratic Functions

- 3.1 Graphing  $f(x) = ax^2$
- 3.2 Graphing  $f(x) = ax^2 + c$
- 3.3 Graphing  $f(x) = ax^2 + bx + c$
- 3.4 Graphing  $f(x) = a(x - h)^2 + k$
- 3.5 Graphing  $f(x) = a(x - p)(x - q)$
- 3.6 Focus of a Parabola
- 3.7 Comparing Linear, Exponential, and Quadratic Functions



Breathing Rates (p. 177)



Roller Coaster (p. 152)



Garden Waterfalls (p. 134)



Electricity-Generating Dish (p. 165)



Firework Explosion (p. 141)

# Maintaining Mathematical Proficiency

## Graphing Linear Equations

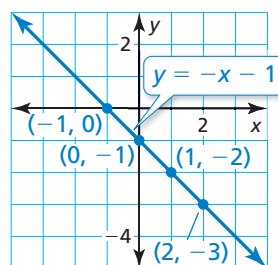
**Example 1** Graph  $y = -x - 1$ .

**Step 1** Make a table of values.

$x$	$y = -x - 1$	$y$	$(x, y)$
-1	$y = -(-1) - 1$	0	$(-1, 0)$
0	$y = -(0) - 1$	-1	$(0, -1)$
1	$y = -(1) - 1$	-2	$(1, -2)$
2	$y = -(2) - 1$	-3	$(2, -3)$

**Step 2** Plot the ordered pairs.

**Step 3** Draw a line through the points.



**Graph the linear equation.**

1.  $y = 2x - 3$

2.  $y = -3x + 4$

3.  $y = -\frac{1}{2}x - 2$

4.  $y = x + 5$

## Evaluating Expressions

**Example 2** Evaluate  $2x^2 + 3x - 5$  when  $x = -1$ .

$$\begin{aligned} 2x^2 + 3x - 5 &= 2(-1)^2 + 3(-1) - 5 \\ &= 2(1) + 3(-1) - 5 \\ &= 2 - 3 - 5 \\ &= -6 \end{aligned}$$

Substitute  $-1$  for  $x$ .

Evaluate the power.

Multiply.

Subtract.

**Evaluate the expression when  $x = -2$ .**

5.  $5x^2 - 9$

6.  $3x^2 + x - 2$

7.  $-x^2 + 4x + 1$

8.  $x^2 + 8x + 5$

9.  $-2x^2 - 4x + 3$

10.  $-4x^2 + 2x - 6$

**11. ABSTRACT REASONING** Complete the table. Find a pattern in the differences of consecutive  $y$ -values. Use the pattern to write an expression for  $y$  when  $x = 6$ .

$x$	1	2	3	4	5
$y = ax^2$					

## Problem-Solving Strategies

### Core Concept

#### Trying Special Cases

When solving a problem in mathematics, it can be helpful to try special cases of the original problem. For instance, in this chapter, you will learn to graph a quadratic function of the form  $f(x) = ax^2 + bx + c$ . The problem-solving strategy used is to first graph quadratic functions of the form  $f(x) = ax^2$ . From there, you progress to other forms of quadratic functions.

$$f(x) = ax^2 \quad \text{Section 3.1}$$

$$f(x) = ax^2 + c \quad \text{Section 3.2}$$

$$f(x) = ax^2 + bx + c \quad \text{Section 3.3}$$

$$f(x) = a(x - h)^2 + k \quad \text{Section 3.4}$$

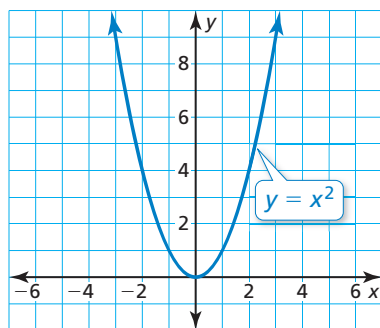
$$f(x) = a(x - p)(x - q) \quad \text{Section 3.5}$$

#### EXAMPLE 1 Graphing the Parent Quadratic Function

Graph the parent quadratic function  $y = x^2$ . Then describe its graph.

#### SOLUTION

The function is of the form  $y = ax^2$ , where  $a = 1$ . By plotting several points, you can see that the graph is U-shaped, as shown.



► The graph opens up, and the lowest point is at the origin.

### Monitoring Progress

Graph the quadratic function. Then describe its graph.

1.  $y = -x^2$

2.  $y = 2x^2$

3.  $f(x) = 2x^2 + 1$

4.  $f(x) = 2x^2 - 1$

5.  $f(x) = \frac{1}{2}x^2 + 4x + 3$

6.  $f(x) = \frac{1}{2}x^2 - 4x + 3$

7.  $y = -2(x + 1)^2 + 1$

8.  $y = -2(x - 1)^2 + 1$

9. How are the graphs in Monitoring Progress Questions 1–8 similar? How are they different?

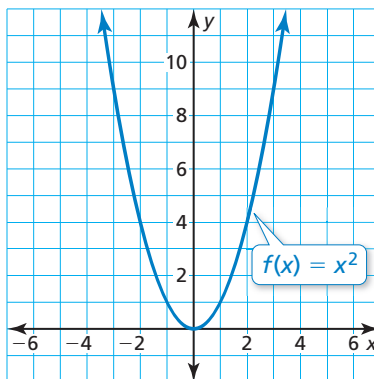
# 3.1 Graphing $f(x) = ax^2$

**Essential Question** What are some of the characteristics of the graph of a quadratic function of the form  $f(x) = ax^2$ ?

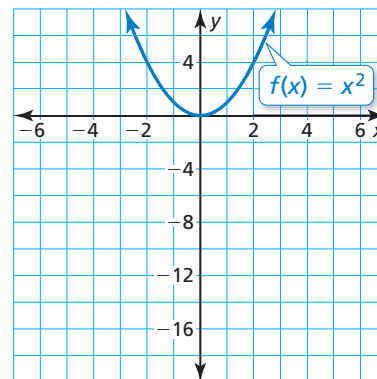
## EXPLORATION 1 Graphing Quadratic Functions

**Work with a partner.** Graph each quadratic function. Compare each graph to the graph of  $f(x) = x^2$ .

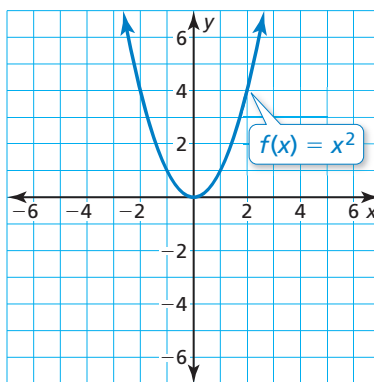
a.  $g(x) = 3x^2$



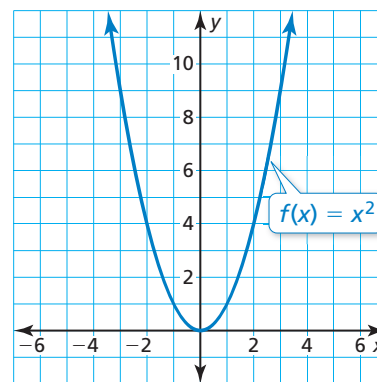
b.  $g(x) = -5x^2$



c.  $g(x) = -0.2x^2$



d.  $g(x) = \frac{1}{10}x^2$

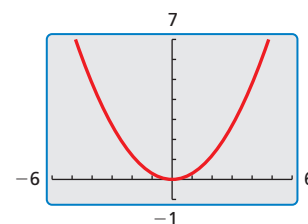


### REASONING QUANTITATIVELY

To be proficient in math, you need to make sense of quantities and their relationships in problem situations.

### Communicate Your Answer

- What are some of the characteristics of the graph of a quadratic function of the form  $f(x) = ax^2$ ?
- How does the value of  $a$  affect the graph of  $f(x) = ax^2$ ? Consider  $0 < a < 1$ ,  $a > 1$ ,  $-1 < a < 0$ , and  $a < -1$ . Use a graphing calculator to verify your answers.
- The figure shows the graph of a quadratic function of the form  $y = ax^2$ . Which of the intervals in Question 3 describes the value of  $a$ ? Explain your reasoning.



# 3.1 Lesson

## Core Vocabulary

quadratic function, p. 124  
parabola, p. 124  
vertex, p. 124  
axis of symmetry, p. 124

### Previous

domain  
range  
vertical shrink  
vertical stretch  
reflection

## REMEMBER

The notation  $f(x)$  is another name for  $y$ .

## What You Will Learn

- ▶ Identify characteristics of quadratic functions.
- ▶ Graph and use quadratic functions of the form  $f(x) = ax^2$ .

## Identifying Characteristics of Quadratic Functions

A **quadratic function** is a nonlinear function that can be written in the standard form  $y = ax^2 + bx + c$ , where  $a \neq 0$ . The U-shaped graph of a quadratic function is called a **parabola**. In this lesson, you will graph quadratic functions, where  $b$  and  $c$  equal 0.

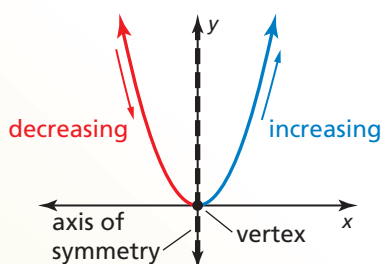
## Core Concept

### Characteristics of Quadratic Functions

The *parent quadratic function* is  $f(x) = x^2$ . The graphs of all other quadratic functions are *transformations* of the graph of the parent quadratic function.

The lowest point on a parabola that opens up or the highest point on a parabola that opens down is the **vertex**.

The vertex of the graph of  $f(x) = x^2$  is  $(0, 0)$ .

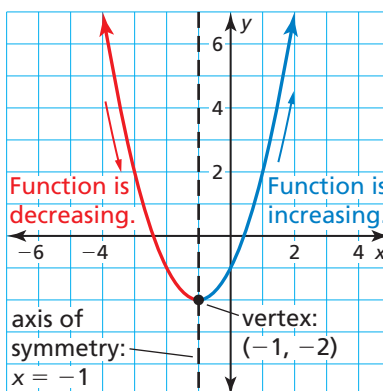


The vertical line that divides the parabola into two symmetric parts is the **axis of symmetry**. The axis of symmetry passes through the vertex. For the graph of  $f(x) = x^2$ , the axis of symmetry is the  $y$ -axis, or  $x = 0$ .

## EXAMPLE 1 Identifying Characteristics of a Quadratic Function

Consider the graph of the quadratic function.

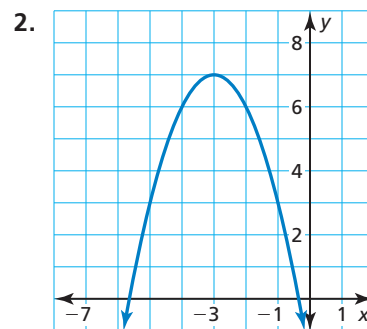
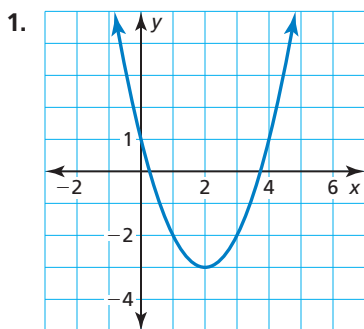
Using the graph, you can identify characteristics such as the vertex, axis of symmetry, and the behavior of the graph, as shown.



You can also determine the following:

- The domain is all real numbers.
- The range is all real numbers greater than or equal to  $-2$ .
- When  $x < -1$ ,  $y$  increases as  $x$  decreases.
- When  $x > -1$ ,  $y$  increases as  $x$  increases.

Identify characteristics of the quadratic function and its graph.



**REMEMBER**

The graph of  $y = a \cdot f(x)$  is a vertical stretch or shrink by a factor of  $a$  of the graph of  $y = f(x)$ .

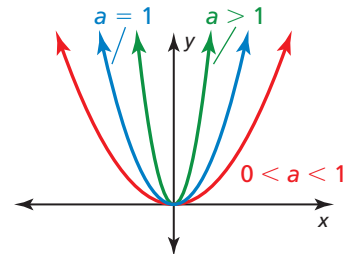
The graph of  $y = -f(x)$  is a reflection in the  $x$ -axis of the graph of  $y = f(x)$ .

**Graphing and Using  $f(x) = ax^2$**

 **Core Concept**

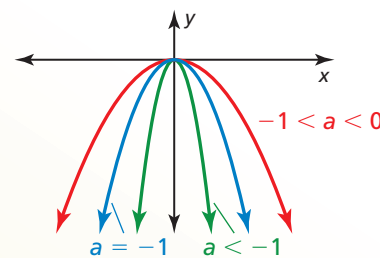
**Graphing  $f(x) = ax^2$  When  $a > 0$**

- When  $0 < a < 1$ , the graph of  $f(x) = ax^2$  is a vertical shrink of the graph of  $f(x) = x^2$ .
- When  $a > 1$ , the graph of  $f(x) = ax^2$  is a vertical stretch of the graph of  $f(x) = x^2$ .



**Graphing  $f(x) = ax^2$  When  $a < 0$**

- When  $-1 < a < 0$ , the graph of  $f(x) = ax^2$  is a vertical shrink with a reflection in the  $x$ -axis of the graph of  $f(x) = x^2$ .
- When  $a < -1$ , the graph of  $f(x) = ax^2$  is a vertical stretch with a reflection in the  $x$ -axis of the graph of  $f(x) = x^2$ .



**EXAMPLE 2** Graphing  $y = ax^2$  When  $a > 0$

Graph  $g(x) = 2x^2$ . Compare the graph to the graph of  $f(x) = x^2$ .

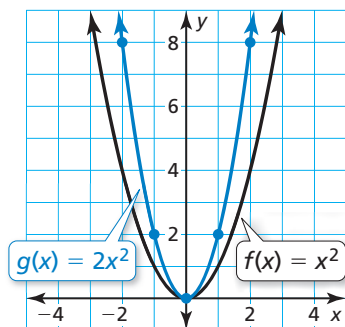
**SOLUTION**

**Step 1** Make a table of values.

$x$	-2	-1	0	1	2
$g(x)$	8	2	0	2	8

**Step 2** Plot the ordered pairs.

**Step 3** Draw a smooth curve through the points.



► Both graphs open up and have the same vertex,  $(0, 0)$ , and the same axis of symmetry,  $x = 0$ . The graph of  $g$  is narrower than the graph of  $f$  because the graph of  $g$  is a vertical stretch by a factor of 2 of the graph of  $f$ .

## STUDY TIP

To make the calculations easier, choose  $x$ -values that are multiples of 3.

### EXAMPLE 3 Graphing $y = ax^2$ When $a < 0$

Graph  $h(x) = -\frac{1}{3}x^2$ . Compare the graph to the graph of  $f(x) = x^2$ .

#### SOLUTION

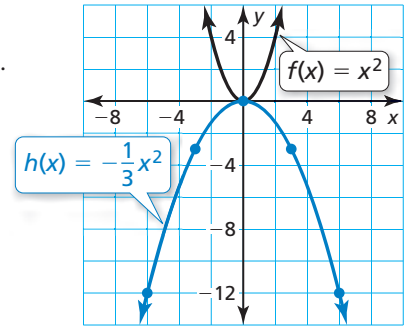
**Step 1** Make a table of values.

$x$	-6	-3	0	3	6
$h(x)$	-12	-3	0	-3	-12

**Step 2** Plot the ordered pairs.

**Step 3** Draw a smooth curve through the points.

- ▶ The graphs have the same vertex,  $(0, 0)$ , and the same axis of symmetry,  $x = 0$ , but the graph of  $h$  opens down and is wider than the graph of  $f$ . So, the graph of  $h$  is a vertical shrink by a factor of  $\frac{1}{3}$  and a reflection in the  $x$ -axis of the graph of  $f$ .

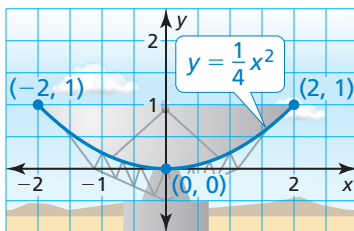


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Graph the function. Compare the graph to the graph of  $f(x) = x^2$ .

- $g(x) = 5x^2$
- $h(x) = \frac{1}{3}x^2$
- $n(x) = \frac{3}{2}x^2$
- $p(x) = -3x^2$
- $q(x) = -0.1x^2$
- $g(x) = -\frac{1}{4}x^2$

### EXAMPLE 4 Solving a Real-Life Problem



The diagram at the left shows the cross section of a satellite dish, where  $x$  and  $y$  are measured in meters. Find the width and depth of the dish.

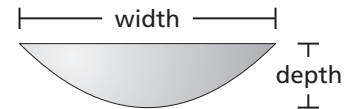
#### SOLUTION

Use the domain of the function to find the width of the dish. Use the range to find the depth.

The leftmost point on the graph is  $(-2, 1)$ , and the rightmost point is  $(2, 1)$ . So, the domain is  $-2 \leq x \leq 2$ , which represents 4 meters.

The lowest point on the graph is  $(0, 0)$ , and the highest points on the graph are  $(-2, 1)$  and  $(2, 1)$ . So, the range is  $0 \leq y \leq 1$ , which represents 1 meter.

- ▶ So, the satellite dish is 4 meters wide and 1 meter deep.



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- The cross section of a spotlight can be modeled by the graph of  $y = 0.5x^2$ , where  $x$  and  $y$  are measured in inches and  $-2 \leq x \leq 2$ . Find the width and depth of the spotlight.

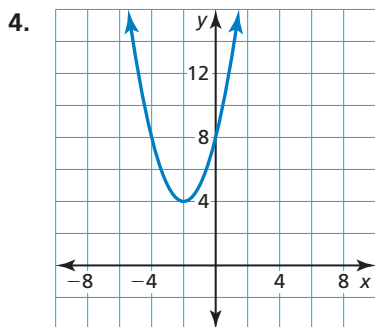
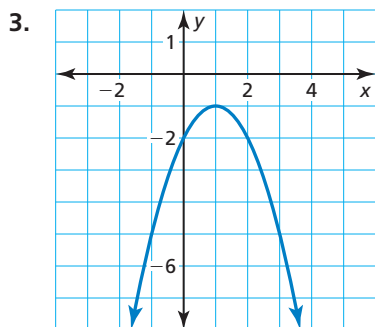
# 3.1 Exercises

## Vocabulary and Core Concept Check

- VOCABULARY** What is the U-shaped graph of a quadratic function called?
- WRITING** When does the graph of a quadratic function open up? open down?

## Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, identify characteristics of the quadratic function and its graph. (See Example 1.)



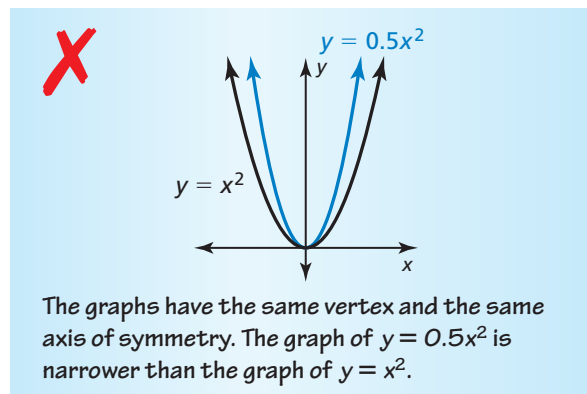
In Exercises 5–12, graph the function. Compare the graph to the graph of  $f(x) = x^2$ . (See Examples 2 and 3.)

- |                            |                              |
|----------------------------|------------------------------|
| 5. $g(x) = 6x^2$           | 6. $b(x) = 2.5x^2$           |
| 7. $h(x) = \frac{1}{4}x^2$ | 8. $j(x) = 0.75x^2$          |
| 9. $m(x) = -2x^2$          | 10. $q(x) = -\frac{9}{2}x^2$ |
| 11. $k(x) = -0.2x^2$       | 12. $p(x) = -\frac{2}{3}x^2$ |

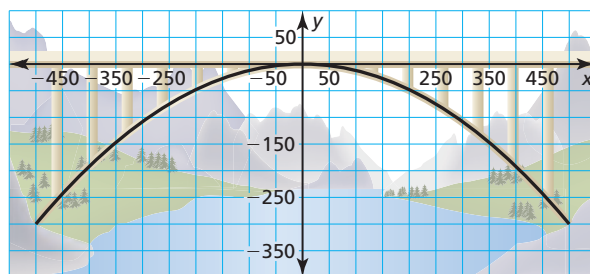
In Exercises 13–16, use a graphing calculator to graph the function. Compare the graph to the graph of  $y = -4x^2$ .

- |                    |                     |
|--------------------|---------------------|
| 13. $y = 4x^2$     | 14. $y = -0.4x^2$   |
| 15. $y = -0.04x^2$ | 16. $y = -0.004x^2$ |

17. **ERROR ANALYSIS** Describe and correct the error in graphing and comparing  $y = x^2$  and  $y = 0.5x^2$ .



18. **MODELING WITH MATHEMATICS** The arch support of a bridge can be modeled by  $y = -0.0012x^2$ , where  $x$  and  $y$  are measured in feet. Find the height and width of the arch. (See Example 4.)



19. **PROBLEM SOLVING** The breaking strength  $z$  (in pounds) of a manila rope can be modeled by  $z = 8900d^2$ , where  $d$  is the diameter (in inches) of the rope.

- Describe the domain and range of the function.
- Graph the function using the domain in part (a).

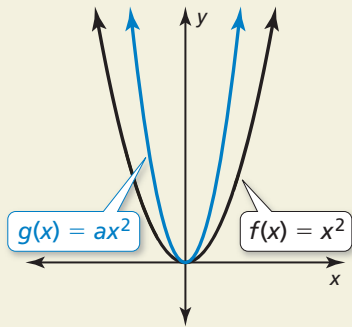


- A manila rope has four times the breaking strength of another manila rope. Does the stronger rope have four times the diameter? Explain.

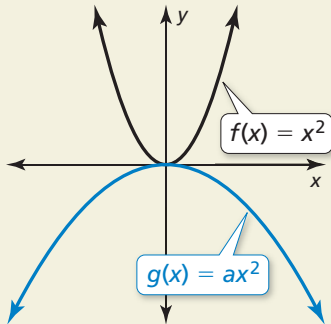


20. **HOW DO YOU SEE IT?** Describe the possible values of  $a$ .

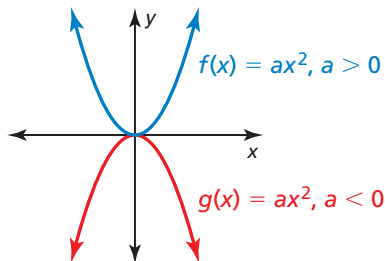
a.



b.



**ANALYZING GRAPHS** In Exercises 21–23, use the graph.

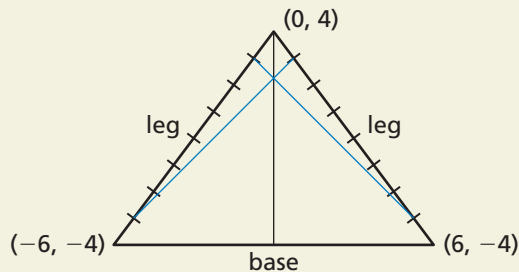


21. When is each function increasing?
22. When is each function decreasing?
23. Which function could include the point  $(-2, 3)$ ? Find the value of  $a$  when the graph passes through  $(-2, 3)$ .
24. **REASONING** Is the  $x$ -intercept of the graph of  $y = ax^2$  always 0? Justify your answer.
25. **REASONING** A parabola opens up and passes through  $(-4, 2)$  and  $(6, -3)$ . How do you know that  $(-4, 2)$  is not the vertex?

**ABSTRACT REASONING** In Exercises 26–29, determine whether the statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

26. The graph of  $f(x) = ax^2$  is narrower than the graph of  $g(x) = x^2$  when  $a > 0$ .
27. The graph of  $f(x) = ax^2$  is narrower than the graph of  $g(x) = x^2$  when  $|a| > 1$ .
28. The graph of  $f(x) = ax^2$  is wider than the graph of  $g(x) = x^2$  when  $0 < |a| < 1$ .
29. The graph of  $f(x) = ax^2$  is wider than the graph of  $g(x) = dx^2$  when  $|a| > |d|$ .

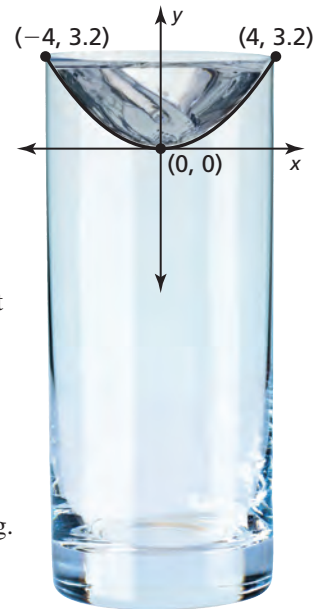
30. **THOUGHT PROVOKING** Draw the isosceles triangle shown. Divide each leg into eight congruent segments. Connect the highest point of one leg with the lowest point of the other leg. Then connect the second highest point of one leg to the second lowest point of the other leg. Continue this process. Write a quadratic function whose graph models the shape that appears.



31. **MAKING AN ARGUMENT**

The diagram shows the parabolic cross section of a swirling glass of water, where  $x$  and  $y$  are measured in centimeters.

- a. About how wide is the mouth of the glass?
- b. Your friend claims that the rotational speed of the water would have to increase for the cross section to be modeled by  $y = 0.1x^2$ . Is your friend correct? Explain your reasoning.



## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Evaluate the expression when  $n = 3$  and  $x = -2$ . (*Skills Review Handbook*)

32.  $n^2 + 5$

33.  $3x^2 - 9$

34.  $-4n^2 + 11$

35.  $n + 2x^2$

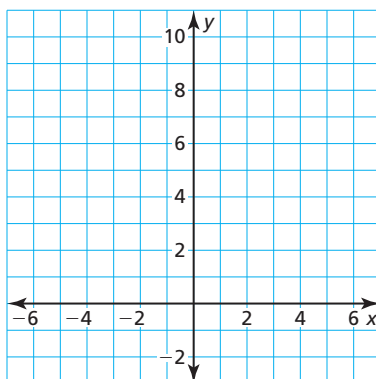
## 3.2 Graphing $f(x) = ax^2 + c$

**Essential Question** How does the value of  $c$  affect the graph of  $f(x) = ax^2 + c$ ?

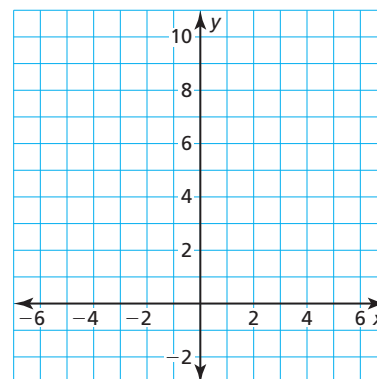
### EXPLORATION 1 Graphing $y = ax^2 + c$

**Work with a partner.** Sketch the graphs of the functions in the same coordinate plane. What do you notice?

a.  $f(x) = x^2$  and  $g(x) = x^2 + 2$



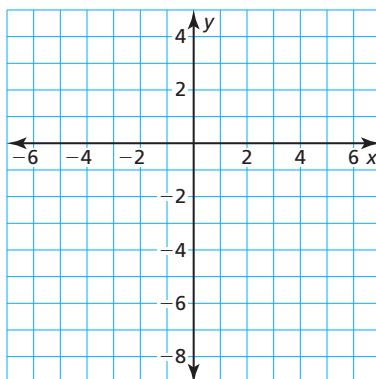
b.  $f(x) = 2x^2$  and  $g(x) = 2x^2 - 2$



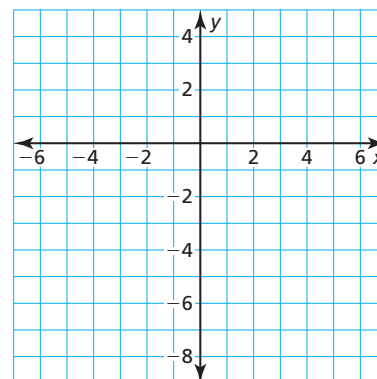
### EXPLORATION 2 Finding $x$ -Intercepts of Graphs

**Work with a partner.** Graph each function. Find the  $x$ -intercepts of the graph. Explain how you found the  $x$ -intercepts.

a.  $y = x^2 - 7$



b.  $y = -x^2 + 1$

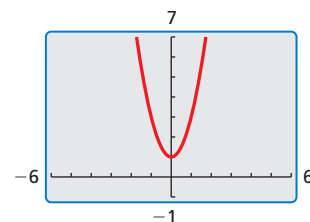


### USING TOOLS STRATEGICALLY

To be proficient in math, you need to consider the available tools, such as a graphing calculator, when solving a mathematical problem.

### Communicate Your Answer

- How does the value of  $c$  affect the graph of  $f(x) = ax^2 + c$ ?
- Use a graphing calculator to verify your answers to Question 3.
- The figure shows the graph of a quadratic function of the form  $y = ax^2 + c$ . Describe possible values of  $a$  and  $c$ . Explain your reasoning.



## 3.2 Lesson

### Core Vocabulary

zero of a function, *p.* 132

#### Previous

translation  
vertex of a parabola  
axis of symmetry  
vertical stretch  
vertical shrink

## What You Will Learn

- ▶ Graph quadratic functions of the form  $f(x) = ax^2 + c$ .
- ▶ Solve real-life problems involving functions of the form  $f(x) = ax^2 + c$ .

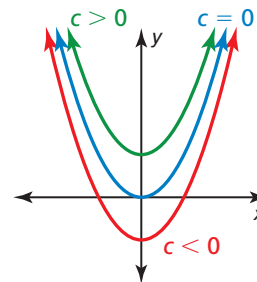
## Graphing $f(x) = ax^2 + c$

### Core Concept

#### Graphing $f(x) = ax^2 + c$

- When  $c > 0$ , the graph of  $f(x) = ax^2 + c$  is a vertical translation  $c$  units up of the graph of  $f(x) = ax^2$ .
- When  $c < 0$ , the graph of  $f(x) = ax^2 + c$  is a vertical translation  $|c|$  units down of the graph of  $f(x) = ax^2$ .

The vertex of the graph of  $f(x) = ax^2 + c$  is  $(0, c)$ , and the axis of symmetry is  $x = 0$ .



#### EXAMPLE 1 Graphing $y = x^2 + c$

Graph  $g(x) = x^2 - 2$ . Compare the graph to the graph of  $f(x) = x^2$ .

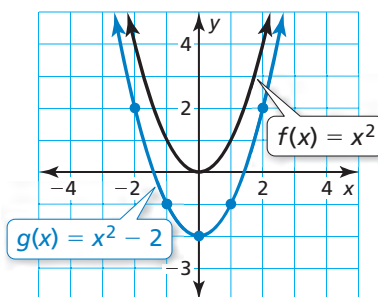
#### SOLUTION

**Step 1** Make a table of values.

$x$	-2	-1	0	1	2
$g(x)$	2	-1	-2	-1	2

**Step 2** Plot the ordered pairs.

**Step 3** Draw a smooth curve through the points.



### REMEMBER

The graph of  $y = f(x) + k$  is a vertical translation, and the graph of  $y = f(x - h)$  is a horizontal translation of the graph of  $f$ .

- ▶ Both graphs open up and have the same axis of symmetry,  $x = 0$ . The vertex of the graph of  $g$ ,  $(0, -2)$ , is below the vertex of the graph of  $f$ ,  $(0, 0)$ , because the graph of  $g$  is a vertical translation 2 units down of the graph of  $f$ .

### Monitoring Progress



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Graph the function. Compare the graph to the graph of  $f(x) = x^2$ .

- $g(x) = x^2 - 5$
- $h(x) = x^2 + 3$

### EXAMPLE 2 Graphing $y = ax^2 + c$

Graph  $g(x) = 4x^2 + 1$ . Compare the graph to the graph of  $f(x) = x^2$ .

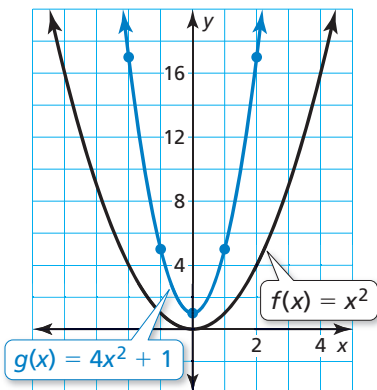
#### SOLUTION

**Step 1** Make a table of values.

$x$	-2	-1	0	1	2
$g(x)$	17	5	1	5	17

**Step 2** Plot the ordered pairs.

**Step 3** Draw a smooth curve through the points.



► Both graphs open up and have the same axis of symmetry,  $x = 0$ . The graph of  $g$  is narrower, and its vertex,  $(0, 1)$ , is above the vertex of the graph of  $f$ ,  $(0, 0)$ . So, the graph of  $g$  is a vertical stretch by a factor of 4 and a vertical translation 1 unit up of the graph of  $f$ .

### EXAMPLE 3 Translating the Graph of $y = ax^2 + c$

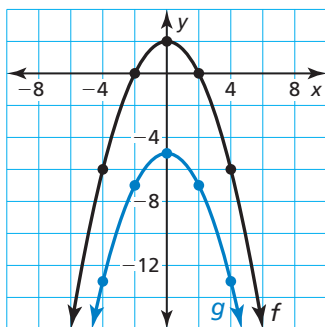
Let  $f(x) = -0.5x^2 + 2$  and  $g(x) = f(x) - 7$ .

**a.** Describe the transformation from the graph of  $f$  to the graph of  $g$ . Then graph  $f$  and  $g$  in the same coordinate plane.

**b.** Write an equation that represents  $g$  in terms of  $x$ .

#### SOLUTION

**a.** The function  $g$  is of the form  $y = f(x) + k$ , where  $k = -7$ . So, the graph of  $g$  is a vertical translation 7 units down of the graph of  $f$ .



$x$	-4	-2	0	2	4
$f(x)$	-6	0	2	0	-6
$g(x)$	-13	-7	-5	-7	-13

$-0.5x^2 + 2$   
 $f(x) - 7$

**b.**  $g(x) = f(x) - 7$  Write the function  $g$ .

$$= -0.5x^2 + 2 - 7$$
Substitute for  $f(x)$ .

$$= -0.5x^2 - 5$$
Subtract.

► So, the equation  $g(x) = -0.5x^2 - 5$  represents  $g$  in terms of  $x$ .

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Graph the function. Compare the graph to the graph of  $f(x) = x^2$ .

3.  $g(x) = 2x^2 - 5$

4.  $h(x) = -\frac{1}{4}x^2 + 4$

5. Let  $f(x) = 3x^2 - 1$  and  $g(x) = f(x) + 3$ .

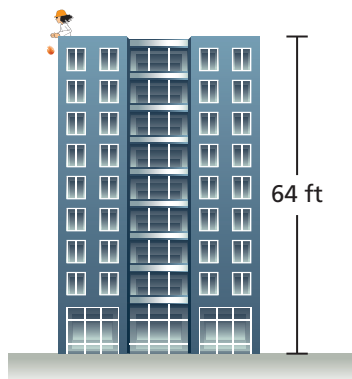
**a.** Describe the transformation from the graph of  $f$  to the graph of  $g$ . Then graph  $f$  and  $g$  in the same coordinate plane.

**b.** Write an equation that represents  $g$  in terms of  $x$ .

## Solving Real-Life Problems

A **zero of a function**  $f$  is an  $x$ -value for which  $f(x) = 0$ . A zero of a function is an  $x$ -intercept of the graph of the function.

### EXAMPLE 4 Solving a Real-Life Problem



The function  $f(t) = 16t^2$  represents the distance (in feet) a dropped object falls in  $t$  seconds. The function  $g(t) = s_0$  represents the initial height (in feet) of the object.

- Find and interpret  $h(t) = -f(t) + g(t)$ .
- An egg is dropped from a height of 64 feet. After how many seconds does the egg hit the ground?
- Suppose the initial height is adjusted by  $k$  feet. How will this affect part (b)?

### SOLUTION

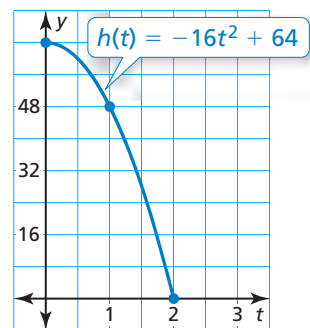
$$\begin{aligned} \text{a. } h(t) &= -f(t) + g(t) && \text{Write the function } h. \\ &= -(16t^2) + s_0 && \text{Substitute for } f(t) \text{ and } g(t). \\ &= -16t^2 + s_0 && \text{Simplify.} \end{aligned}$$

▶ The function  $h(t) = -16t^2 + s_0$  represents the height (in feet) of a falling object  $t$  seconds after it is dropped from an initial height  $s_0$  (in feet).

- The initial height is 64 feet. So, the function  $h(t) = -16t^2 + 64$  represents the height of the egg  $t$  seconds after it is dropped. The egg hits the ground when  $h(t) = 0$ .

**Step 1** Make a table of values and sketch the graph.

$t$	0	1	2
$h(t)$	64	48	0



**Step 2** Find the positive zero of the function. When  $t = 2$ ,  $h(t) = 0$ . So, the zero is 2.

- The egg hits the ground 2 seconds after it is dropped.
- When the initial height is adjusted by  $k$  feet, the graph of  $h$  is translated up  $k$  units when  $k > 0$  or down  $|k|$  units when  $k < 0$ . So, the  $x$ -intercept of the graph of  $h$  will move right when  $k > 0$  or left when  $k < 0$ .
  - ▶ When  $k > 0$ , the egg will take more than 2 seconds to hit the ground.
  - ▶ When  $k < 0$ , the egg will take less than 2 seconds to hit the ground.

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- Explain why only nonnegative values of  $t$  are used in Example 4.
- WHAT IF?** The egg is dropped from a height of 100 feet. After how many seconds does the egg hit the ground?

### COMMON ERROR

The graph in Step 1 shows the height of the object over time, not the path of the object.



## 3.2 Exercises

Dynamic Solutions available at [BigIdeasMath.com](http://BigIdeasMath.com)

### Vocabulary and Core Concept Check

- VOCABULARY** State the vertex and axis of symmetry of the graph of  $y = ax^2 + c$ .
- WRITING** How does the graph of  $y = ax^2 + c$  compare to the graph of  $y = ax^2$ ?

### Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, graph the function. Compare the graph to the graph of  $f(x) = x^2$ . (See Example 1.)

- $g(x) = x^2 + 6$
- $h(x) = x^2 + 8$
- $p(x) = x^2 - 3$
- $q(x) = x^2 - 1$

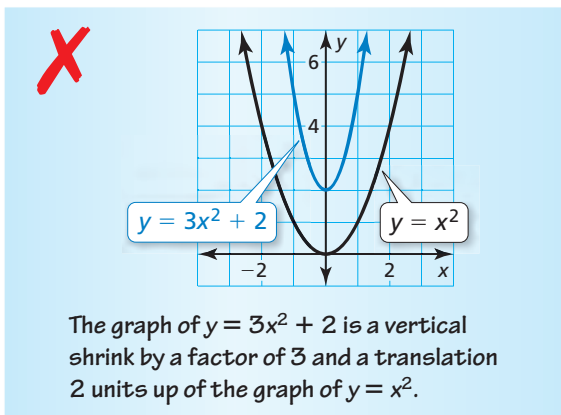
In Exercises 7–12, graph the function. Compare the graph to the graph of  $f(x) = x^2$ . (See Example 2.)

- $g(x) = -x^2 + 3$
- $h(x) = -x^2 - 7$
- $s(x) = 2x^2 - 4$
- $t(x) = -3x^2 + 1$
- $p(x) = -\frac{1}{3}x^2 - 2$
- $q(x) = \frac{1}{2}x^2 + 6$

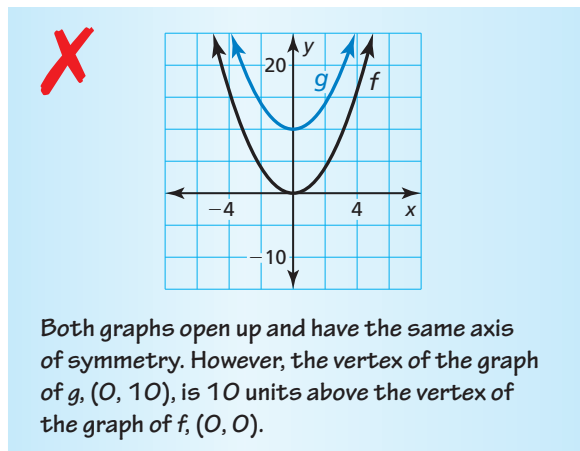
In Exercises 13–16, describe the transformation from the graph of  $f$  to the graph of  $g$ . Then graph  $f$  and  $g$  in the same coordinate plane. Write an equation that represents  $g$  in terms of  $x$ . (See Example 3.)

- $f(x) = 3x^2 + 4$   
 $g(x) = f(x) + 2$
- $f(x) = \frac{1}{2}x^2 + 1$   
 $g(x) = f(x) - 4$
- $f(x) = -\frac{1}{4}x^2 - 6$   
 $g(x) = f(x) - 3$
- $f(x) = 4x^2 - 5$   
 $g(x) = f(x) + 7$

- ERROR ANALYSIS** Describe and correct the error in comparing the graphs.



- ERROR ANALYSIS** Describe and correct the error in graphing and comparing  $f(x) = x^2$  and  $g(x) = x^2 - 10$ .



In Exercises 19–26, find the zeros of the function.

- $y = x^2 - 1$
- $y = x^2 - 36$
- $f(x) = -x^2 + 25$
- $f(x) = -x^2 + 49$
- $f(x) = 4x^2 - 16$
- $f(x) = 3x^2 - 27$
- $f(x) = -12x^2 + 3$
- $f(x) = -8x^2 + 98$

- MODELING WITH MATHEMATICS** A water balloon is dropped from a height of 144 feet. (See Example 4.)

- What does the function  $h(t) = -16t^2 + 144$  represent in this situation?
- After how many seconds does the water balloon hit the ground?
- Suppose the initial height is adjusted by  $k$  feet. How does this affect part (b)?

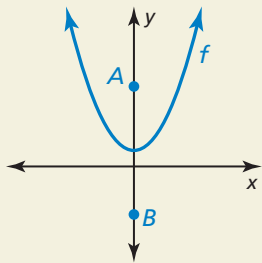
- MODELING WITH MATHEMATICS** The function  $y = -16x^2 + 36$  represents the height  $y$  (in feet) of an apple  $x$  seconds after falling from a tree. Find and interpret the  $x$ - and  $y$ -intercepts.

**In Exercises 29–32, sketch a parabola with the given characteristics.**

29. The parabola opens up, and the vertex is  $(0, 3)$ .
30. The vertex is  $(0, 4)$ , and one of the  $x$ -intercepts is 2.
31. The related function is increasing when  $x < 0$ , and the zeros are  $-1$  and 1.
32. The highest point on the parabola is  $(0, -5)$ .
33. **DRAWING CONCLUSIONS** You and your friend both drop a ball at the same time. The function  $h(x) = -16x^2 + 256$  represents the height (in feet) of your ball after  $x$  seconds. The function  $g(x) = -16x^2 + 300$  represents the height (in feet) of your friend's ball after  $x$  seconds.
- Write the function  $T(x) = h(x) - g(x)$ . What does  $T(x)$  represent?
  - When your ball hits the ground, what is the height of your friend's ball? Use a graph to justify your answer.
34. **MAKING AN ARGUMENT** Your friend claims that in the equation  $y = ax^2 + c$ , the vertex changes when the value of  $a$  changes. Is your friend correct? Explain your reasoning.

35. **MATHEMATICAL CONNECTIONS** The area  $A$  (in square feet) of a square patio is represented by  $A = x^2$ , where  $x$  is the length of one side of the patio. You add 48 square feet to the patio, resulting in a total area of 192 square feet. What are the dimensions of the original patio? Use a graph to justify your answer.

36. **HOW DO YOU SEE IT?** The graph of  $f(x) = ax^2 + c$  is shown. Points  $A$  and  $B$  are the same distance from the vertex of the graph of  $f$ . Which point is closer to the vertex of the graph of  $f$  as  $c$  increases?

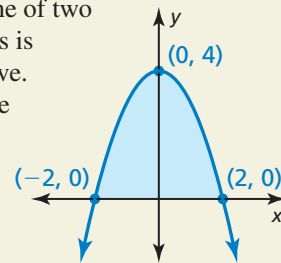


37. **REASONING** Describe two methods you can use to find the zeros of the function  $f(t) = -16t^2 + 400$ . Check your answer by graphing.
38. **PROBLEM SOLVING** The paths of water from three different garden waterfalls are given below. Each function gives the height  $h$  (in feet) and the horizontal distance  $d$  (in feet) of the water.
- Waterfall 1**  $h = -3.1d^2 + 4.8$
- Waterfall 2**  $h = -3.5d^2 + 1.9$
- Waterfall 3**  $h = -1.1d^2 + 1.6$
- Which waterfall drops water from the highest point?
  - Which waterfall follows the narrowest path?
  - Which waterfall sends water the farthest?

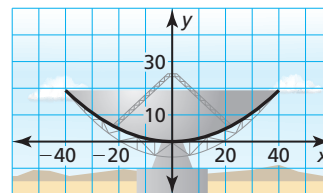


39. **WRITING EQUATIONS** Two acorns fall to the ground from an oak tree. One falls 45 feet, while the other falls 32 feet.
- For each acorn, write an equation that represents the height  $h$  (in feet) as a function of the time  $t$  (in seconds).
  - Describe how the graphs of the two equations are related.

40. **THOUGHT PROVOKING** One of two classic problems in calculus is to find the area under a curve. Approximate the area of the region bounded by the parabola and the  $x$ -axis. Show your work.



41. **CRITICAL THINKING** A cross section of the parabolic surface of the antenna shown can be modeled by  $y = 0.012x^2$ , where  $x$  and  $y$  are measured



in feet. The antenna is moved up so that the outer edges of the dish are 25 feet above the  $x$ -axis. Where is the vertex of the cross section located? Explain.

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Evaluate the expression when  $a = 4$  and  $b = -3$ . (*Skills Review Handbook*)

42.  $\frac{a}{4b}$

43.  $-\frac{b}{2a}$

44.  $\frac{a-b}{3a+b}$

45.  $-\frac{b+2a}{ab}$

## 3.3 Graphing $f(x) = ax^2 + bx + c$

**Essential Question** How can you find the vertex of the graph of  $f(x) = ax^2 + bx + c$ ?

### EXPLORATION 1 Comparing $x$ -Intercepts with the Vertex

Work with a partner.

- Sketch the graphs of  $y = 2x^2 - 8x$  and  $y = 2x^2 - 8x + 6$ .
- What do you notice about the  $x$ -coordinate of the vertex of each graph?
- Use the graph of  $y = 2x^2 - 8x$  to find its  $x$ -intercepts. Verify your answer by solving  $0 = 2x^2 - 8x$ .
- Compare the value of the  $x$ -coordinate of the vertex with the values of the  $x$ -intercepts.

### EXPLORATION 2 Finding $x$ -Intercepts

Work with a partner.

- Solve  $0 = ax^2 + bx$  for  $x$  by factoring.
- What are the  $x$ -intercepts of the graph of  $y = ax^2 + bx$ ?
- Copy and complete the table to verify your answer.

$x$	$y = ax^2 + bx$
0	
$-\frac{b}{a}$	

### CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to make conjectures and build a logical progression of statements.

### EXPLORATION 3 Deductive Reasoning

Work with a partner. Complete the following logical argument.

The  $x$ -intercepts of the graph of  $y = ax^2 + bx$  are 0 and  $-\frac{b}{a}$ .

The vertex of the graph of  $y = ax^2 + bx$  occurs when  $x =$  .

The vertices of the graphs of  $y = ax^2 + bx$  and  $y = ax^2 + bx + c$  have the same  $x$ -coordinate.

The vertex of the graph of  $y = ax^2 + bx + c$  occurs when  $x =$  .

### Communicate Your Answer

- How can you find the vertex of the graph of  $f(x) = ax^2 + bx + c$ ?
- Without graphing, find the vertex of the graph of  $f(x) = x^2 - 4x + 3$ . Check your result by graphing.



# 3.3 Lesson

## Core Vocabulary

maximum value, p. 137  
 minimum value, p. 137

### Previous

independent variable  
 dependent variable

## What You Will Learn

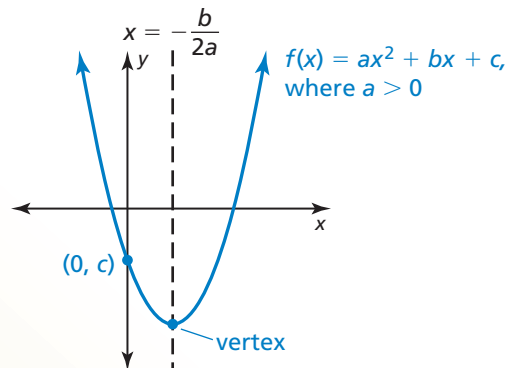
- ▶ Graph quadratic functions of the form  $f(x) = ax^2 + bx + c$ .
- ▶ Find maximum and minimum values of quadratic functions.

## Graphing $f(x) = ax^2 + bx + c$

### Core Concept

#### Graphing $f(x) = ax^2 + bx + c$

- The graph opens up when  $a > 0$ , and the graph opens down when  $a < 0$ .
- The  $y$ -intercept is  $c$ .
- The  $x$ -coordinate of the vertex is  $-\frac{b}{2a}$ .
- The axis of symmetry is  $x = -\frac{b}{2a}$ .



### EXAMPLE 1 Finding the Axis of Symmetry and the Vertex

Find (a) the axis of symmetry and (b) the vertex of the graph of  $f(x) = 2x^2 + 8x - 1$ .

#### SOLUTION

- a. Find the axis of symmetry when  $a = 2$  and  $b = 8$ .

$$x = -\frac{b}{2a}$$

Write the equation for the axis of symmetry.

$$x = -\frac{8}{2(2)}$$

Substitute 2 for  $a$  and 8 for  $b$ .

$$x = -2$$

Simplify.

- ▶ The axis of symmetry is  $x = -2$ .

- b. The axis of symmetry is  $x = -2$ , so the  $x$ -coordinate of the vertex is  $-2$ . Use the function to find the  $y$ -coordinate of the vertex.

$$f(x) = 2x^2 + 8x - 1$$

Write the function.

$$\begin{aligned} f(-2) &= 2(-2)^2 + 8(-2) - 1 \\ &= -9 \end{aligned}$$

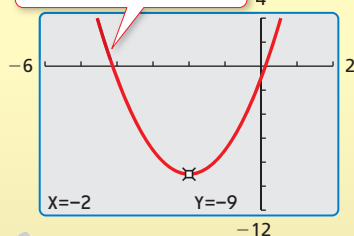
Substitute  $-2$  for  $x$ .

Simplify.

- ▶ The vertex is  $(-2, -9)$ .

#### Check

$$f(x) = 2x^2 + 8x - 1$$



### Monitoring Progress



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Find (a) the axis of symmetry and (b) the vertex of the graph of the function.

1.  $f(x) = 3x^2 - 2x$

2.  $g(x) = x^2 + 6x + 5$

3.  $h(x) = -\frac{1}{2}x^2 + 7x - 4$

**EXAMPLE 2****Graphing  $f(x) = ax^2 + bx + c$** **COMMON ERROR**

Be sure to include the negative sign before the fraction when finding the axis of symmetry.

Graph  $f(x) = 3x^2 - 6x + 5$ . Describe the domain and range.

**SOLUTION**

**Step 1** Find and graph the axis of symmetry.

$$x = -\frac{b}{2a} = -\frac{(-6)}{2(3)} = 1 \quad \text{Substitute and simplify.}$$

**Step 2** Find and plot the vertex.

The axis of symmetry is  $x = 1$ , so the  $x$ -coordinate of the vertex is 1. Use the function to find the  $y$ -coordinate of the vertex.

$$f(x) = 3x^2 - 6x + 5 \quad \text{Write the function.}$$

$$f(1) = 3(1)^2 - 6(1) + 5 \quad \text{Substitute 1 for } x.$$

$$= 2 \quad \text{Simplify.}$$

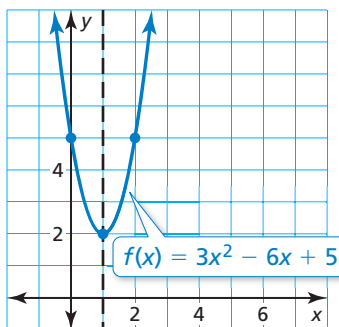
So, the vertex is  $(1, 2)$ .

**Step 3** Use the  $y$ -intercept to find two more points on the graph.

Because  $c = 5$ , the  $y$ -intercept is 5. So,  $(0, 5)$  lies on the graph. Because the axis of symmetry is  $x = 1$ , the point  $(2, 5)$  also lies on the graph.

**Step 4** Draw a smooth curve through the points.

▶ The domain is all real numbers. The range is  $y \geq 2$ .

**REMEMBER**

The domain is the set of all possible input values of the independent variable  $x$ . The range is the set of all possible output values of the dependent variable  $y$ .

**Monitoring Progress**

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Graph the function. Describe the domain and range.

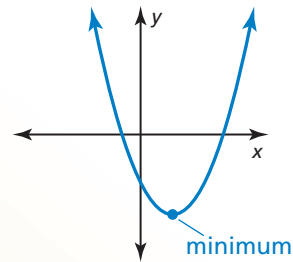
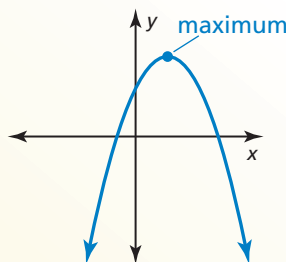
4.  $h(x) = 2x^2 + 4x + 1$     5.  $k(x) = x^2 - 8x + 7$     6.  $p(x) = -5x^2 - 10x - 2$

**Finding Maximum and Minimum Values****Core Concept****Maximum and Minimum Values**

The  $y$ -coordinate of the vertex of the graph of  $f(x) = ax^2 + bx + c$  is the **maximum value** of the function when  $a < 0$  or the **minimum value** of the function when  $a > 0$ .

$$f(x) = ax^2 + bx + c, a < 0$$

$$f(x) = ax^2 + bx + c, a > 0$$



### EXAMPLE 3 Finding a Maximum or Minimum Value

Tell whether the function  $f(x) = -4x^2 - 24x - 19$  has a minimum value or a maximum value. Then find the value.

#### SOLUTION

For  $f(x) = -4x^2 - 24x - 19$ ,  $a = -4$  and  $-4 < 0$ . So, the parabola opens down and the function has a maximum value. To find the maximum value, find the  $y$ -coordinate of the vertex.

First, find the  $x$ -coordinate of the vertex. Use  $a = -4$  and  $b = -24$ .

$$x = -\frac{b}{2a} = -\frac{(-24)}{2(-4)} = -3 \quad \text{Substitute and simplify.}$$

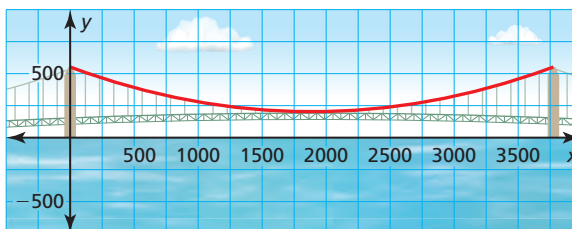
Then evaluate the function when  $x = -3$  to find the  $y$ -coordinate of the vertex.

$$\begin{aligned} f(-3) &= -4(-3)^2 - 24(-3) - 19 && \text{Substitute } -3 \text{ for } x. \\ &= 17 && \text{Simplify.} \end{aligned}$$

► The maximum value is 17.

### EXAMPLE 4 Finding a Minimum Value

The suspension cables between the two towers of the Mackinac Bridge in Michigan form a parabola that can be modeled by  $y = 0.000098x^2 - 0.37x + 552$ , where  $x$  and  $y$  are measured in feet. What is the height of the cable above the water at its lowest point?



#### SOLUTION

The lowest point of the cable is at the vertex of the parabola. Find the  $x$ -coordinate of the vertex. Use  $a = 0.000098$  and  $b = -0.37$ .

$$x = -\frac{b}{2a} = -\frac{(-0.37)}{2(0.000098)} \approx 1888 \quad \text{Substitute and use a calculator.}$$

Substitute 1888 for  $x$  in the equation to find the  $y$ -coordinate of the vertex.

$$y = 0.000098(1888)^2 - 0.37(1888) + 552 \approx 203$$

► The cable is about 203 feet above the water at its lowest point.

### Monitoring Progress Help in English and Spanish at [BigIdeasMath.com](http://BigIdeasMath.com)

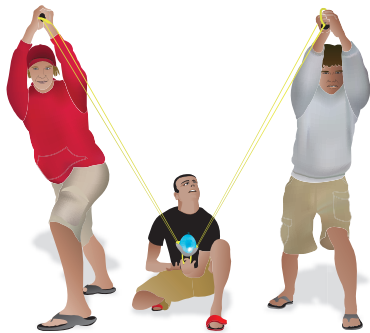
Tell whether the function has a minimum value or a maximum value. Then find the value.

7.  $g(x) = 8x^2 - 8x + 6$

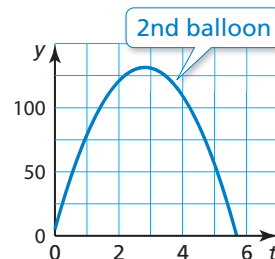
8.  $h(x) = -\frac{1}{4}x^2 + 3x + 1$

9. The cables between the two towers of the Tacoma Narrows Bridge in Washington form a parabola that can be modeled by  $y = 0.00016x^2 - 0.46x + 507$ , where  $x$  and  $y$  are measured in feet. What is the height of the cable above the water at its lowest point?

## EXAMPLE 5 Modeling with Mathematics



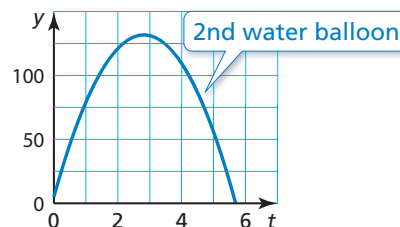
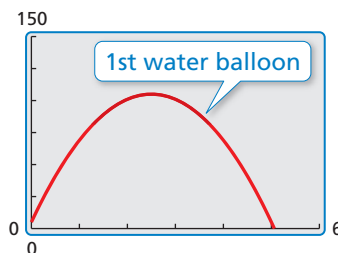
A group of friends is launching water balloons. The function  $f(t) = -16t^2 + 80t + 5$  represents the height (in feet) of the first water balloon  $t$  seconds after it is launched. The height of the second water balloon  $t$  seconds after it is launched is shown in the graph. Which water balloon went higher?



### MODELING WITH MATHEMATICS

Because time cannot be negative, use only nonnegative values of  $t$ .

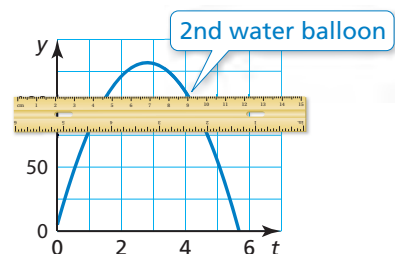
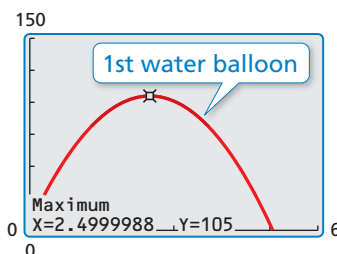
- Understand the Problem** You are given a function that represents the height of the first water balloon. The height of the second water balloon is represented graphically. You need to find and compare the maximum heights of the water balloons.
- Make a Plan** To compare the maximum heights, represent both functions graphically. Use a graphing calculator to graph  $f(t) = -16t^2 + 80t + 5$  in an appropriate viewing window. Then visually compare the heights of the water balloons.
- Solve the Problem** Enter the function  $f(t) = -16t^2 + 80t + 5$  into your calculator and graph it. Compare the graphs to determine which function has a greater maximum value.



You can see that the second water balloon reaches a height of about 125 feet, while the first water balloon reaches a height of only about 105 feet.

► So, the second water balloon went higher.

- Look Back** Use the *maximum* feature to determine that the maximum value of  $f(t) = -16t^2 + 80t + 5$  is 105. Use a straightedge to represent a height of 105 feet on the graph that represents the second water balloon to clearly see that the second water balloon went higher.



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- Which balloon is in the air longer? Explain your reasoning.
- Which balloon reaches its maximum height faster? Explain your reasoning.

## Vocabulary and Core Concept Check

- VOCABULARY** Explain how you can tell whether a quadratic function has a maximum value or a minimum value without graphing the function.
- DIFFERENT WORDS, SAME QUESTION** Consider the quadratic function  $f(x) = -2x^2 + 8x + 24$ . Which is different? Find “both” answers.

What is the maximum value of the function?

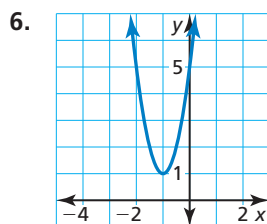
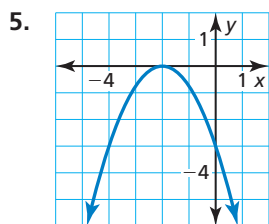
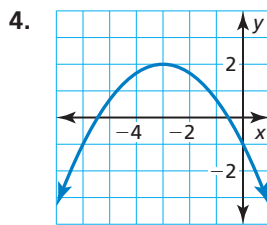
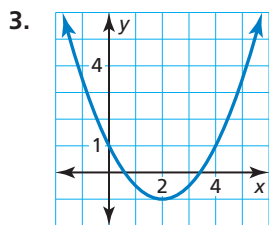
What is the greatest number in the range of the function?

What is the y-coordinate of the vertex of the graph of the function?

What is the axis of symmetry of the graph of the function?

## Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the vertex, the axis of symmetry, and the y-intercept of the graph.



In Exercises 7–12, find (a) the axis of symmetry and (b) the vertex of the graph of the function. (See Example 1.)

- $f(x) = 2x^2 - 4x$
- $y = 3x^2 + 2x$
- $y = -9x^2 - 18x - 1$
- $f(x) = -6x^2 + 24x - 20$
- $f(x) = \frac{2}{5}x^2 - 4x + 14$
- $y = -\frac{3}{4}x^2 + 9x - 18$

In Exercises 13–18, graph the function. Describe the domain and range. (See Example 2.)

- $f(x) = 2x^2 + 12x + 4$
- $y = 4x^2 + 24x + 13$
- $y = -8x^2 - 16x - 9$
- $f(x) = -5x^2 + 20x - 7$

17.  $y = \frac{2}{3}x^2 - 6x + 5$       18.  $f(x) = -\frac{1}{2}x^2 - 3x - 4$

19. **ERROR ANALYSIS** Describe and correct the error in finding the axis of symmetry of the graph of  $y = 3x^2 - 12x + 11$ .



$$x = -\frac{b}{2a} = \frac{-12}{2(3)} = -2$$

The axis of symmetry is  $x = -2$ .

20. **ERROR ANALYSIS** Describe and correct the error in graphing the function  $f(x) = x^2 + 4x + 3$ .

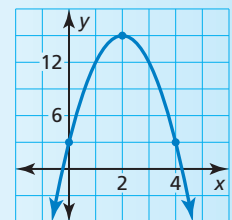


The axis of symmetry is  $x = \frac{b}{2a} = \frac{4}{2(1)} = 2$ .

$$f(2) = 2^2 + 4(2) + 3 = 15$$

So, the vertex is  $(2, 15)$ .

The y-intercept is 3. So, the points  $(0, 3)$  and  $(4, 3)$  lie on the graph.



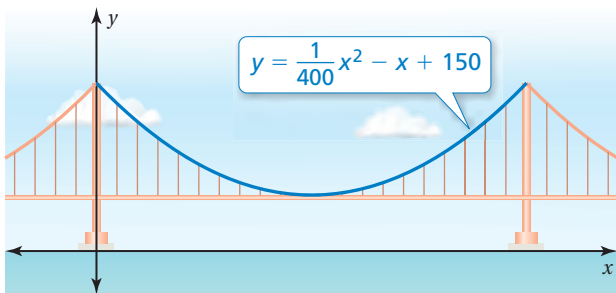
In Exercises 21–26, tell whether the function has a minimum value or a maximum value. Then find the value. (See Example 3.)

- $y = 3x^2 - 18x + 15$
- $f(x) = -5x^2 + 10x + 7$
- $f(x) = -4x^2 + 4x - 2$
- $y = 2x^2 - 10x + 13$
- $y = -\frac{1}{2}x^2 - 11x + 6$
- $f(x) = \frac{1}{5}x^2 - 5x + 27$

27. **MODELING WITH MATHEMATICS** The function shown represents the height  $h$  (in feet) of a firework  $t$  seconds after it is launched. The firework explodes at its highest point. (See Example 4.)



- When does the firework explode?
  - At what height does the firework explode?
28. **MODELING WITH MATHEMATICS** The function  $h(t) = -16t^2 + 16t$  represents the height (in feet) of a horse  $t$  seconds after it jumps during a steeplechase.
- When does the horse reach its maximum height?
  - Can the horse clear a fence that is 3.5 feet tall? If so, by how much?
  - How long is the horse in the air?
29. **MODELING WITH MATHEMATICS** The cable between two towers of a suspension bridge can be modeled by the function shown, where  $x$  and  $y$  are measured in feet. The cable is at road level midway between the towers.



- How far from each tower shown is the lowest point of the cable?
  - How high is the road above the water?
  - Describe the domain and range of the function shown.
30. **REASONING** Find the axis of symmetry of the graph of the equation  $y = ax^2 + bx + c$  when  $b = 0$ . Can you find the axis of symmetry when  $a = 0$ ? Explain.

31. **ATTENDING TO PRECISION** The vertex of a parabola is  $(3, -1)$ . One point on the parabola is  $(6, 8)$ . Find another point on the parabola. Justify your answer.
32. **MAKING AN ARGUMENT** Your friend claims that it is possible to draw a parabola through any two points with different  $x$ -coordinates. Is your friend correct? Explain.

**USING TOOLS** In Exercises 33–36, use the *minimum* or *maximum* feature of a graphing calculator to approximate the vertex of the graph of the function.

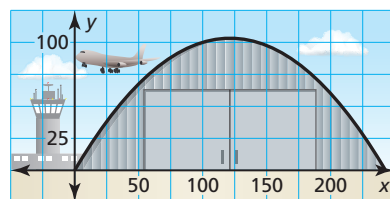
33.  $y = 0.5x^2 + \sqrt{2}x - 3$

34.  $y = -6.2x^2 + 4.8x - 1$

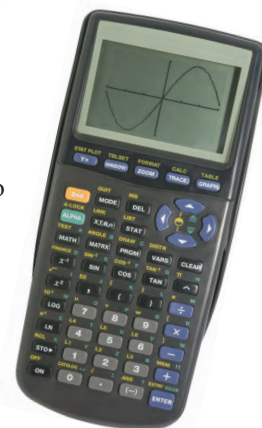
35.  $y = -\pi x^2 + 3x$

36.  $y = 0.25x^2 - 5^{2/3}x + 2$

37. **MODELING WITH MATHEMATICS** The opening of one aircraft hangar is a parabolic arch that can be modeled by the equation  $y = -0.006x^2 + 1.5x$ , where  $x$  and  $y$  are measured in feet. The opening of a second aircraft hangar is shown in the graph. (See Example 5.)

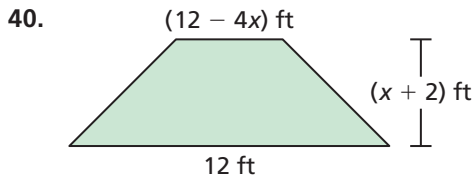
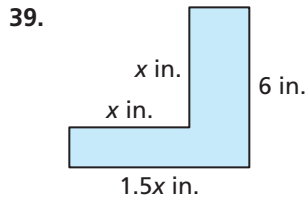


- Which aircraft hangar is taller?
  - Which aircraft hangar is wider?
38. **MODELING WITH MATHEMATICS** An office supply store sells about 80 graphing calculators per month for \$120 each. For each \$6 decrease in price, the store expects to sell eight more calculators. The revenue from calculator sales is given by the function  $R(n) = (\text{unit price})(\text{units sold})$ , or  $R(n) = (120 - 6n)(80 + 8n)$ , where  $n$  is the number of \$6 price decreases.
- How much should the store charge to maximize monthly revenue?
  - Using a different revenue model, the store expects to sell five more calculators for each \$4 decrease in price. Which revenue model results in a greater maximum monthly revenue? Explain.



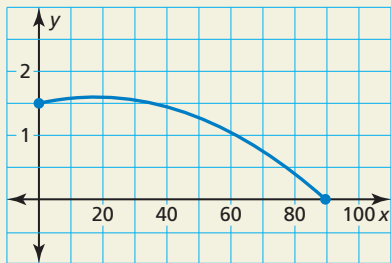
**MATHEMATICAL CONNECTIONS** In Exercises 39 and 40,

(a) find the value of  $x$  that maximizes the area of the figure and (b) find the maximum area.



41. **WRITING** Compare the graph of  $g(x) = x^2 + 4x + 1$  with the graph of  $h(x) = x^2 - 4x + 1$ .

42. **HOW DO YOU SEE IT?** During an archery competition, an archer shoots an arrow. The arrow follows the parabolic path shown, where  $x$  and  $y$  are measured in meters.



- What is the initial height of the arrow?
- Estimate the maximum height of the arrow.
- How far does the arrow travel?

43. **USING TOOLS** The graph of a quadratic function passes through  $(3, 2)$ ,  $(4, 7)$ , and  $(9, 2)$ . Does the graph open up or down? Explain your reasoning.

44. **REASONING** For a quadratic function  $f$ , what does  $f\left(-\frac{b}{2a}\right)$  represent? Explain your reasoning.

45. **PROBLEM SOLVING** Write a function of the form  $y = ax^2 + bx$  whose graph contains the points  $(1, 6)$  and  $(3, 6)$ .

46. **CRITICAL THINKING** Parabolas A and B contain the points shown. Identify characteristics of each parabola, if possible. Explain your reasoning.

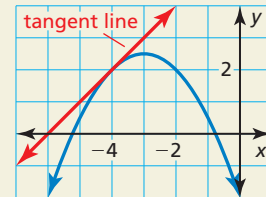
Parabola A	
$x$	$y$
2	3
6	4

Parabola B	
$x$	$y$
1	4
3	-4
5	4

47. **MODELING WITH MATHEMATICS** At a basketball game, an air cannon launches T-shirts into the crowd. The function  $y = -\frac{1}{8}x^2 + 4x$  represents the path of a T-shirt. The function  $3y = 2x - 14$  represents the height of the bleachers. In both functions,  $y$  represents vertical height (in feet) and  $x$  represents horizontal distance (in feet). At what height does the T-shirt land in the bleachers?

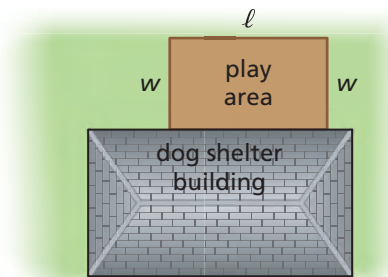
48. **THOUGHT PROVOKING**

One of two classic problems in calculus is finding the slope of a *tangent line* to a curve. An example of a tangent line, which just touches the parabola at one point, is shown.



Approximate the slope of the tangent line to the graph of  $y = x^2$  at the point  $(1, 1)$ . Explain your reasoning.

49. **PROBLEM SOLVING** The owners of a dog shelter want to enclose a rectangular play area on the side of their building. They have  $k$  feet of fencing. What is the maximum area of the outside enclosure in terms of  $k$ ? (*Hint:* Find the  $y$ -coordinate of the vertex of the graph of the area function.)



## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Describe the transformation(s) from the graph of  $f(x) = |x|$  to the graph of the given function.

(Section 1.1)

50.  $q(x) = |x + 6|$       51.  $h(x) = -0.5|x|$       52.  $g(x) = |x - 2| + 5$       53.  $p(x) = 3|x + 1|$

## 3.1–3.3 What Did You Learn?

### Core Vocabulary

quadratic function, *p. 124*  
parabola, *p. 124*  
vertex, *p. 124*

axis of symmetry, *p. 124*  
zero of a function, *p. 132*

maximum value, *p. 137*  
minimum value, *p. 137*

### Core Concepts

#### Section 3.1

Characteristics of Quadratic Functions, *p. 124*  
Graphing  $f(x) = ax^2$  When  $a > 0$ , *p. 125*  
Graphing  $f(x) = ax^2$  When  $a < 0$ , *p. 125*

#### Section 3.2

Graphing  $f(x) = ax^2 + c$ , *p. 130*

#### Section 3.3

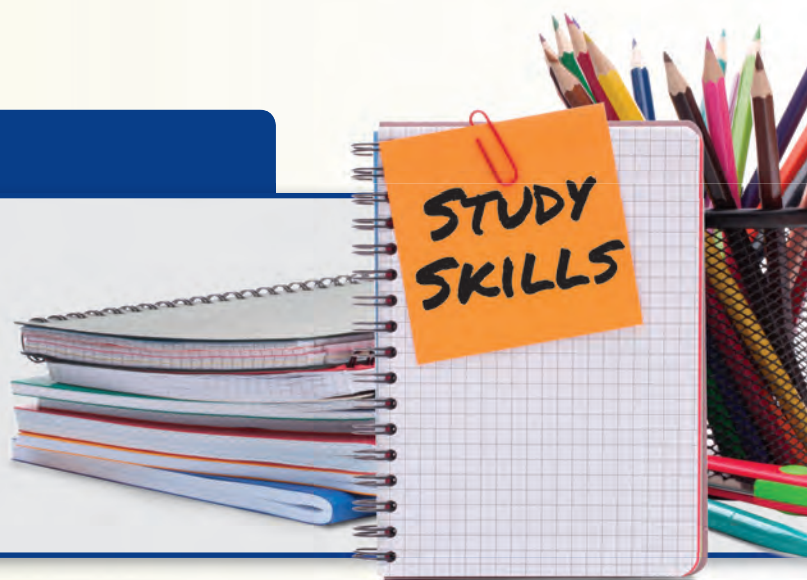
Graphing  $f(x) = ax^2 + bx + c$ , *p. 136*  
Maximum and Minimum Values, *p. 137*

### Mathematical Practices

1. Explain your plan for solving Exercise 18 on page 127.
2. How does graphing the function in Exercise 27 on page 133 help you answer the questions?
3. What definition and characteristics of the graph of a quadratic function did you use to answer Exercise 44 on page 142?

## Learning Visually

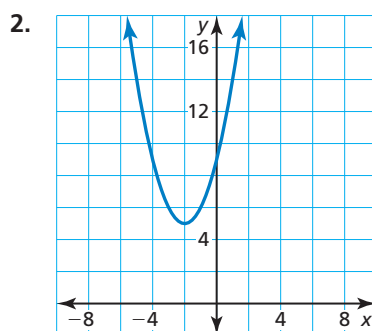
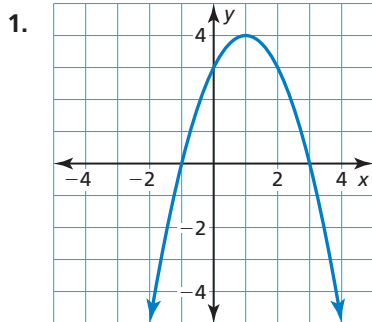
- Draw a picture of a word problem before writing a verbal model. You do not have to be an artist.
- When making a review card for a word problem, include a picture. This will help you recall the information while taking a test.
- Make sure your notes are visually neat for easy recall.





# 3.1–3.3 Quiz

Identify characteristics of the quadratic function and its graph. (Section 3.1)



Graph the function. Compare the graph to the graph of  $f(x) = x^2$ . (Section 3.1 and Section 3.2)

- |                            |                                 |
|----------------------------|---------------------------------|
| 3. $h(x) = -x^2$           | 4. $p(x) = 2x^2 + 2$            |
| 5. $r(x) = 4x^2 - 16$      | 6. $b(x) = 8x^2$                |
| 7. $g(x) = \frac{2}{5}x^2$ | 8. $m(x) = -\frac{1}{2}x^2 - 4$ |

Describe the transformation from the graph of  $f$  to the graph of  $g$ . Then graph  $f$  and  $g$  in the same coordinate plane. Write an equation that represents  $g$  in terms of  $x$ . (Section 3.2)

- |   |   |
|---|---|
| 9. $f(x) = 2x^2 + 1$ ; $g(x) = f(x) + 2$            | 10. $f(x) = -3x^2 + 12$ ; $g(x) = f(x) - 9$ |
| 11. $f(x) = \frac{1}{2}x^2 - 2$ ; $g(x) = f(x) - 6$ | 12. $f(x) = 5x^2 - 3$ ; $g(x) = f(x) + 1$   |

Graph the function. Describe the domain and range. (Section 3.3)

- |                             |                             |
|-----------------------------|-----------------------------|
| 13. $f(x) = -4x^2 - 4x + 7$ | 14. $f(x) = 2x^2 + 12x + 5$ |
| 15. $y = x^2 + 4x - 5$      | 16. $y = -3x^2 + 6x + 9$    |

Tell whether the function has a minimum value or a maximum value. Then find the value. (Section 3.3)

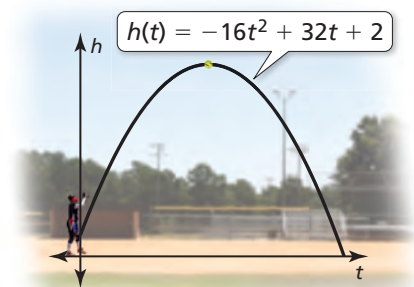
- |                             |  |
|-----------------------------|--|
| 17. $f(x) = 5x^2 + 10x - 3$ | 18. $f(x) = -\frac{1}{2}x^2 + 2x + 16$ |
| 19. $y = -x^2 + 4x + 12$    | 20. $y = 2x^2 + 8x + 3$                |

21. The distance  $y$  (in feet) that a coconut falls after  $t$  seconds is given by the function  $y = 16t^2$ . Use a graph to determine how many seconds it takes for the coconut to fall 64 feet. (Section 3.1)

22. The function  $y = -16t^2 + 25$  represents the height  $y$  (in feet) of a pinecone  $t$  seconds after falling from a tree. (Section 3.2)

- After how many seconds does the pinecone hit the ground?
- A second pinecone falls from a height of 36 feet. Which pinecone hits the ground in the least amount of time? Explain.

23. The function shown models the height (in feet) of a softball  $t$  seconds after it is pitched in an underhand motion. Describe the domain and range. Find the maximum height of the softball. (Section 3.3)



## 3.4 Graphing $f(x) = a(x - h)^2 + k$

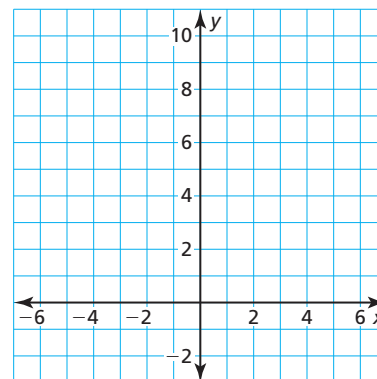
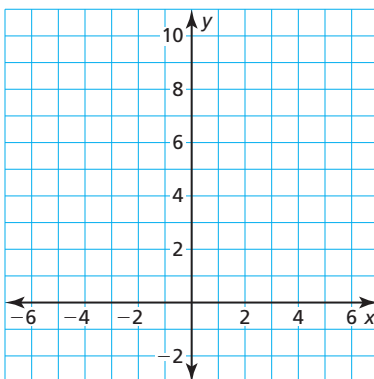
**Essential Question** How can you describe the graph of  $f(x) = a(x - h)^2$ ?

### EXPLORATION 1 Graphing $y = a(x - h)^2$ When $h > 0$

**Work with a partner.** Sketch the graphs of the functions in the same coordinate plane. How does the value of  $h$  affect the graph of  $y = a(x - h)^2$ ?

a.  $f(x) = x^2$  and  $g(x) = (x - 2)^2$

b.  $f(x) = 2x^2$  and  $g(x) = 2(x - 2)^2$

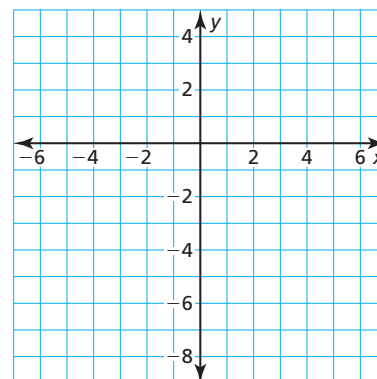
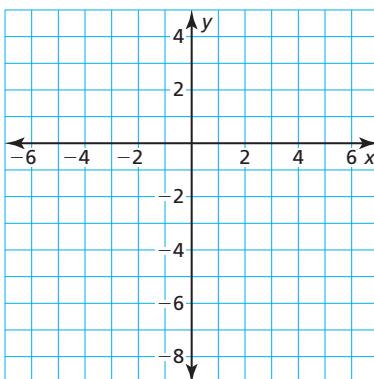


### EXPLORATION 2 Graphing $y = a(x - h)^2$ When $h < 0$

**Work with a partner.** Sketch the graphs of the functions in the same coordinate plane. How does the value of  $h$  affect the graph of  $y = a(x - h)^2$ ?

a.  $f(x) = -x^2$  and  $g(x) = -(x + 2)^2$

b.  $f(x) = -2x^2$  and  $g(x) = -2(x + 2)^2$



### USING TOOLS STRATEGICALLY

To be proficient in math, you need to consider the available tools, such as a graphing calculator, when solving a mathematical problem.

### Communicate Your Answer

- How can you describe the graph of  $f(x) = a(x - h)^2$ ?
- Without graphing, describe the graph of each function. Use a graphing calculator to check your answer.
  - $y = (x - 3)^2$
  - $y = (x + 3)^2$
  - $y = -(x - 3)^2$

## 3.4 Lesson

### Core Vocabulary

even function, p. 146  
odd function, p. 146  
vertex form (of a quadratic function), p. 148

Previous  
reflection

### STUDY TIP

The graph of an odd function looks the same after a  $180^\circ$  rotation about the origin.

### STUDY TIP

Most functions are neither even nor odd.

## What You Will Learn

- ▶ Identify even and odd functions.
- ▶ Graph quadratic functions of the form  $f(x) = a(x - h)^2$ .
- ▶ Graph quadratic functions of the form  $f(x) = a(x - h)^2 + k$ .
- ▶ Model real-life problems using  $f(x) = a(x - h)^2 + k$ .

## Identifying Even and Odd Functions

### Core Concept

#### Even and Odd Functions

A function  $y = f(x)$  is **even** when  $f(-x) = f(x)$  for each  $x$  in the domain of  $f$ . The graph of an even function is symmetric about the  $y$ -axis.

A function  $y = f(x)$  is **odd** when  $f(-x) = -f(x)$  for each  $x$  in the domain of  $f$ . The graph of an odd function is symmetric about the origin. A graph is *symmetric about the origin* when it looks the same after reflections in the  $x$ -axis and then in the  $y$ -axis.

### EXAMPLE 1 Identifying Even and Odd Functions

Determine whether each function is *even*, *odd*, or *neither*.

- a.  $f(x) = 2x$                       b.  $g(x) = x^2 - 2$                       c.  $h(x) = 2x^2 + x - 2$

#### SOLUTION

- a.  $f(x) = 2x$                       Write the original function.  
 $f(-x) = 2(-x)$                       Substitute  $-x$  for  $x$ .  
 $= -2x$                       Simplify.  
 $= -f(x)$                       Substitute  $f(x)$  for  $2x$ .

▶ Because  $f(-x) = -f(x)$ , the function is odd.

- b.  $g(x) = x^2 - 2$                       Write the original function.  
 $g(-x) = (-x)^2 - 2$                       Substitute  $-x$  for  $x$ .  
 $= x^2 - 2$                       Simplify.  
 $= g(x)$                       Substitute  $g(x)$  for  $x^2 - 2$ .

▶ Because  $g(-x) = g(x)$ , the function is even.

- c.  $h(x) = 2x^2 + x - 2$                       Write the original function.  
 $h(-x) = 2(-x)^2 + (-x) - 2$                       Substitute  $-x$  for  $x$ .  
 $= 2x^2 - x - 2$                       Simplify.

▶ Because  $h(x) = 2x^2 + x - 2$  and  $-h(x) = -2x^2 - x + 2$ , you can conclude that  $h(-x) \neq h(x)$  and  $h(-x) \neq -h(x)$ . So, the function is neither even nor odd.

### Monitoring Progress



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Determine whether the function is *even*, *odd*, or *neither*.

1.  $f(x) = 5x$                       2.  $g(x) = 2^x$                       3.  $h(x) = 2x^2 + 3$

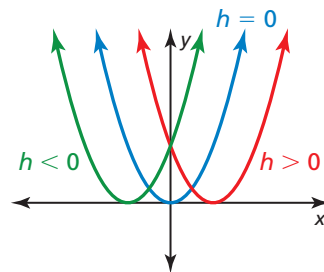
## Graphing $f(x) = a(x - h)^2$

### Core Concept

#### Graphing $f(x) = a(x - h)^2$

- When  $h > 0$ , the graph of  $f(x) = a(x - h)^2$  is a horizontal translation  $h$  units right of the graph of  $f(x) = ax^2$ .
- When  $h < 0$ , the graph of  $f(x) = a(x - h)^2$  is a horizontal translation  $|h|$  units left of the graph of  $f(x) = ax^2$ .

The vertex of the graph of  $f(x) = a(x - h)^2$  is  $(h, 0)$ , and the axis of symmetry is  $x = h$ .



#### EXAMPLE 2 Graphing $y = a(x - h)^2$

Graph  $g(x) = \frac{1}{2}(x - 4)^2$ . Compare the graph to the graph of  $f(x) = x^2$ .

#### SOLUTION

**Step 1** Graph the axis of symmetry. Because  $h = 4$ , graph  $x = 4$ .

**Step 2** Plot the vertex. Because  $h = 4$ , plot  $(4, 0)$ .

**Step 3** Find and plot two more points on the graph. Choose two  $x$ -values less than the  $x$ -coordinate of the vertex. Then find  $g(x)$  for each  $x$ -value.

When  $x = 0$ :

$$\begin{aligned} g(0) &= \frac{1}{2}(0 - 4)^2 \\ &= 8 \end{aligned}$$

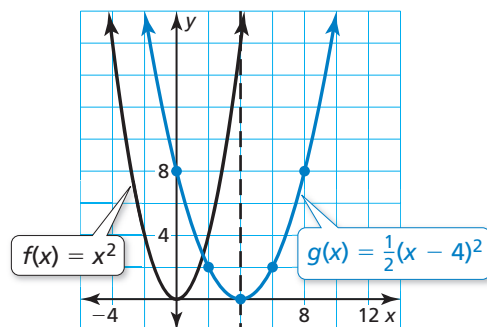
When  $x = 2$ :

$$\begin{aligned} g(2) &= \frac{1}{2}(2 - 4)^2 \\ &= 2 \end{aligned}$$

So, plot  $(0, 8)$  and  $(2, 2)$ .

**Step 4** Reflect the points plotted in Step 3 in the axis of symmetry. So, plot  $(8, 8)$  and  $(6, 2)$ .

**Step 5** Draw a smooth curve through the points.



- Both graphs open up. The graph of  $g$  is wider than the graph of  $f$ . The axis of symmetry  $x = 4$  and the vertex  $(4, 0)$  of the graph of  $g$  are 4 units right of the axis of symmetry  $x = 0$  and the vertex  $(0, 0)$  of the graph of  $f$ . So, the graph of  $g$  is a translation 4 units right and a vertical shrink by a factor of  $\frac{1}{2}$  of the graph of  $f$ .

#### ANOTHER WAY

In Step 3, you could instead choose two  $x$ -values greater than the  $x$ -coordinate of the vertex.

#### STUDY TIP

From the graph, you can see that  $f(x) = x^2$  is an even function. However,  $g(x) = \frac{1}{2}(x - 4)^2$  is neither even nor odd.

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Graph the function. Compare the graph to the graph of  $f(x) = x^2$ .

4.  $g(x) = 2(x + 5)^2$

5.  $h(x) = -(x - 2)^2$

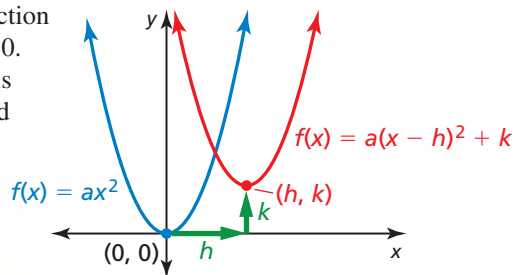
## Graphing $f(x) = a(x - h)^2 + k$

### Core Concept

#### Graphing $f(x) = a(x - h)^2 + k$

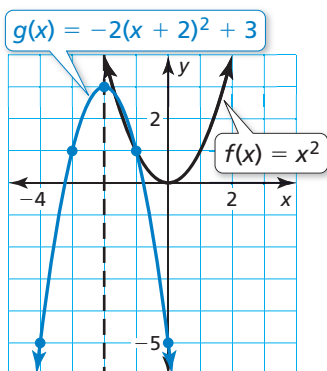
The **vertex form** of a quadratic function is  $f(x) = a(x - h)^2 + k$ , where  $a \neq 0$ . The graph of  $f(x) = a(x - h)^2 + k$  is a translation  $h$  units horizontally and  $k$  units vertically of the graph of  $f(x) = ax^2$ .

The vertex of the graph of  $f(x) = a(x - h)^2 + k$  is  $(h, k)$ , and the axis of symmetry is  $x = h$ .



#### EXAMPLE 3 Graphing $y = a(x - h)^2 + k$

Graph  $g(x) = -2(x + 2)^2 + 3$ . Compare the graph to the graph of  $f(x) = x^2$ .



#### SOLUTION

**Step 1** Graph the axis of symmetry. Because  $h = -2$ , graph  $x = -2$ .

**Step 2** Plot the vertex. Because  $h = -2$  and  $k = 3$ , plot  $(-2, 3)$ .

**Step 3** Find and plot two more points on the graph. Choose two  $x$ -values less than the  $x$ -coordinate of the vertex. Then find  $g(x)$  for each  $x$ -value. So, plot  $(-4, -5)$  and  $(-3, 1)$ .

$x$	$-4$	$-3$
$g(x)$	$-5$	$1$

**Step 4** Reflect the points plotted in Step 3 in the axis of symmetry. So, plot  $(-1, 1)$  and  $(0, -5)$ .

**Step 5** Draw a smooth curve through the points.

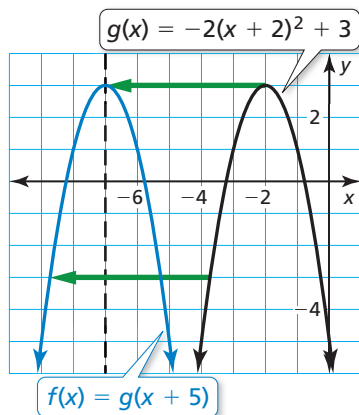
► The graph of  $g$  opens down and is narrower than the graph of  $f$ . The vertex of the graph of  $g$ ,  $(-2, 3)$ , is 2 units left and 3 units up of the vertex of the graph of  $f$ ,  $(0, 0)$ . So, the graph of  $g$  is a vertical stretch by a factor of 2, a reflection in the  $x$ -axis, and a translation 2 units left and 3 units up of the graph of  $f$ .

#### EXAMPLE 4 Transforming the Graph of $y = a(x - h)^2 + k$

Consider function  $g$  in Example 3. Graph  $f(x) = g(x + 5)$ .

#### SOLUTION

The function  $f$  is of the form  $y = g(x - h)$ , where  $h = -5$ . So, the graph of  $f$  is a horizontal translation 5 units left of the graph of  $g$ . To graph  $f$ , subtract 5 from the  $x$ -coordinates of the points on the graph of  $g$ .



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Graph the function. Compare the graph to the graph of  $f(x) = x^2$ .

6.  $g(x) = 3(x - 1)^2 + 6$

7.  $h(x) = \frac{1}{2}(x + 4)^2 - 2$

8. Consider function  $g$  in Example 3. Graph  $f(x) = g(x) - 3$ .

## Modeling Real-Life Problems

### EXAMPLE 5 Modeling with Mathematics



Water fountains are usually designed to give a specific visual effect. For example, the water fountain shown consists of streams of water that are shaped like parabolas. Notice how the streams are designed to land on the underwater spotlights. Write and graph a quadratic function that models the path of a stream of water with a maximum height of 5 feet, represented by a vertex of  $(3, 5)$ , landing on a spotlight 6 feet from the water jet, represented by  $(6, 0)$ .

#### SOLUTION

- 1. Understand the Problem** You know the vertex and another point on the graph that represents the parabolic path. You are asked to write and graph a quadratic function that models the path.
- 2. Make a Plan** Use the given points and the vertex form to write a quadratic function. Then graph the function.

#### 3. Solve the Problem

Use the vertex form, vertex  $(3, 5)$ , and point  $(6, 0)$  to find the value of  $a$ .

$$f(x) = a(x - h)^2 + k \quad \text{Write the vertex form of a quadratic function.}$$

$$f(x) = a(x - 3)^2 + 5 \quad \text{Substitute 3 for } h \text{ and 5 for } k.$$

$$0 = a(6 - 3)^2 + 5 \quad \text{Substitute 6 for } x \text{ and 0 for } f(x).$$

$$0 = 9a + 5 \quad \text{Simplify.}$$

$$-\frac{5}{9} = a \quad \text{Solve for } a.$$

So,  $f(x) = -\frac{5}{9}(x - 3)^2 + 5$  models the path of a stream of water. Now graph the function.

**Step 1** Graph the axis of symmetry. Because  $h = 3$ , graph  $x = 3$ .

**Step 2** Plot the vertex,  $(3, 5)$ .

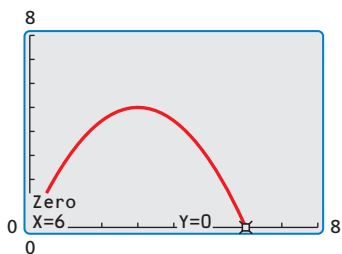
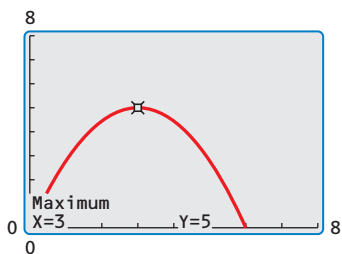
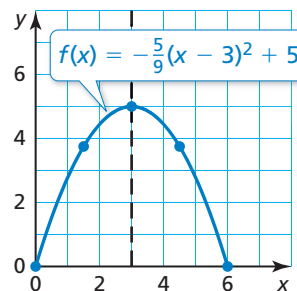
**Step 3** Find and plot two more points on the graph. Because the  $x$ -axis represents the water surface, the graph should only contain points with nonnegative values of  $f(x)$ . You know that  $(6, 0)$  is on the graph. To find another point, choose an  $x$ -value between  $x = 3$  and  $x = 6$ . Then find the corresponding value of  $f(x)$ .

$$f(4.5) = -\frac{5}{9}(4.5 - 3)^2 + 5 = 3.75$$

So, plot  $(6, 0)$  and  $(4.5, 3.75)$ .

**Step 4** Reflect the points plotted in Step 3 in the axis of symmetry. So, plot  $(0, 0)$  and  $(1.5, 3.75)$ .

**Step 5** Draw a smooth curve through the points.



- 4. Look Back** Use a graphing calculator to graph  $f(x) = -\frac{5}{9}(x - 3)^2 + 5$ . Use the *maximum* feature to verify that the maximum value is 5. Then use the *zero* feature to verify that  $x = 6$  is a zero of the function.

### Monitoring Progress



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- 9. WHAT IF?** The vertex is  $(3, 6)$ . Write and graph a quadratic function that models the path.

# 3.4 Exercises

## Vocabulary and Core Concept Check

- VOCABULARY** Compare the graph of an even function with the graph of an odd function.
- OPEN-ENDED** Write a quadratic function whose graph has a vertex of  $(1, 2)$ .
- WRITING** Describe the transformation from the graph of  $f(x) = ax^2$  to the graph of  $g(x) = a(x - h)^2 + k$ .
- WHICH ONE DOESN'T BELONG?** Which function does *not* belong with the other three? Explain your reasoning.

$$f(x) = 8(x + 4)^2$$

$$f(x) = (x - 2)^2 + 4$$

$$f(x) = 2(x + 0)^2$$

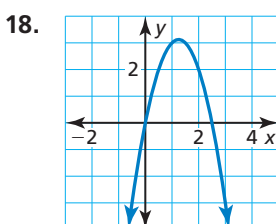
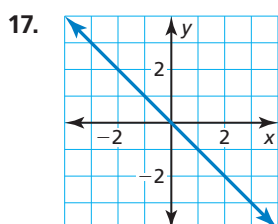
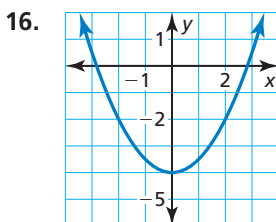
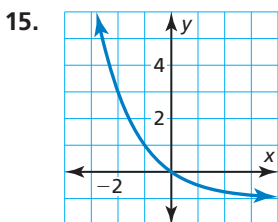
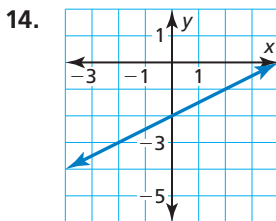
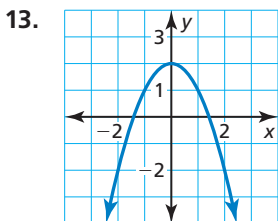
$$f(x) = 3(x + 1)^2 + 1$$

## Monitoring Progress and Modeling with Mathematics

In Exercises 5–12, determine whether the function is *even*, *odd*, or *neither*. (See Example 1.)

- $f(x) = 4x + 3$
- $g(x) = 3x^2$
- $h(x) = 5^x + 2$
- $m(x) = 2x^2 - 7x$
- $p(x) = -x^2 + 8$
- $f(x) = -\frac{1}{2}x$
- $n(x) = 2x^2 - 7x + 3$
- $r(x) = -6x^2 + 5$

In Exercises 13–18, determine whether the function represented by the graph is *even*, *odd*, or *neither*.



In Exercises 19–22, find the vertex and the axis of symmetry of the graph of the function.

- $f(x) = 3(x + 1)^2$
- $f(x) = \frac{1}{4}(x - 6)^2$
- $y = -\frac{1}{8}(x - 4)^2$
- $y = -5(x + 9)^2$

In Exercises 23–28, graph the function. Compare the graph to the graph of  $f(x) = x^2$ . (See Example 2.)

- $g(x) = 2(x + 3)^2$
- $p(x) = 3(x - 1)^2$
- $r(x) = \frac{1}{4}(x + 10)^2$
- $n(x) = \frac{1}{3}(x - 6)^2$
- $d(x) = \frac{1}{5}(x - 5)^2$
- $q(x) = 6(x + 2)^2$

29. **ERROR ANALYSIS** Describe and correct the error in determining whether the function  $f(x) = x^2 + 3$  is even, odd, or neither.



$$\begin{aligned} f(x) &= x^2 + 3 \\ f(-x) &= (-x)^2 + 3 \\ &= x^2 + 3 \\ &= f(x) \end{aligned}$$

So,  $f(x)$  is an odd function.

30. **ERROR ANALYSIS** Describe and correct the error in finding the vertex of the graph of the function.



$$y = -(x + 8)^2$$

Because  $h = -8$ , the vertex is  $(0, -8)$ .

In Exercises 31–34, find the vertex and the axis of symmetry of the graph of the function.

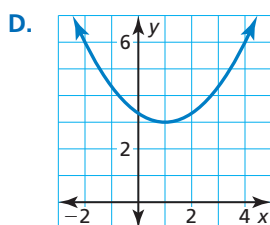
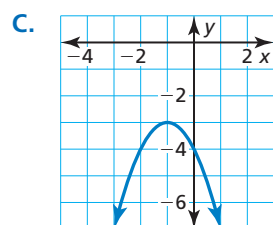
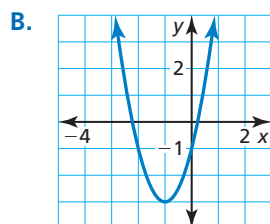
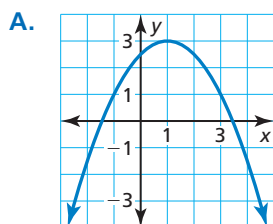
31.  $y = -6(x + 4)^2 - 3$     32.  $f(x) = 3(x - 3)^2 + 6$

33.  $f(x) = -4(x + 3)^2 + 1$     34.  $y = -(x - 6)^2 - 5$

In Exercises 35–38, match the function with its graph.

35.  $y = -(x + 1)^2 - 3$     36.  $y = -\frac{1}{2}(x - 1)^2 + 3$

37.  $y = \frac{1}{3}(x - 1)^2 + 3$     38.  $y = 2(x + 1)^2 - 3$



In Exercises 39–44, graph the function. Compare the graph to the graph of  $f(x) = x^2$ . (See Example 3.)

39.  $h(x) = (x - 2)^2 + 4$     40.  $g(x) = (x + 1)^2 - 7$

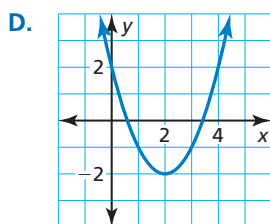
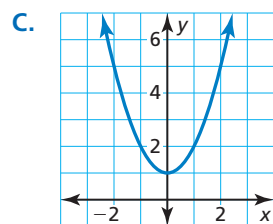
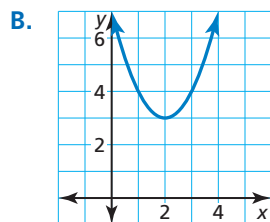
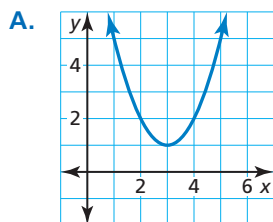
41.  $r(x) = 4(x - 1)^2 - 5$     42.  $n(x) = -(x + 4)^2 + 2$

43.  $g(x) = -\frac{1}{3}(x + 3)^2 - 2$     44.  $r(x) = \frac{1}{2}(x - 2)^2 - 4$

In Exercises 45–48, let  $f(x) = (x - 2)^2 + 1$ . Match the function with its graph.

45.  $g(x) = f(x - 1)$     46.  $r(x) = f(x + 2)$

47.  $h(x) = f(x) + 2$     48.  $p(x) = f(x) - 3$



In Exercises 49–54, graph  $g$ . (See Example 4.)

49.  $f(x) = 2(x - 1)^2 + 1$ ;  $g(x) = f(x + 3)$

50.  $f(x) = -(x + 1)^2 + 2$ ;  $g(x) = \frac{1}{2}f(x)$

51.  $f(x) = -3(x + 5)^2 - 6$ ;  $g(x) = 2f(x)$

52.  $f(x) = 5(x - 3)^2 - 1$ ;  $g(x) = f(x) - 6$

53.  $f(x) = (x + 3)^2 + 5$ ;  $g(x) = f(x - 4)$

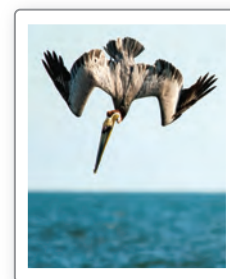
54.  $f(x) = -2(x - 4)^2 - 8$ ;  $g(x) = -f(x)$

55. **MODELING WITH MATHEMATICS** The height (in meters) of a bird diving to catch a fish is represented by  $h(t) = 5(t - 2.5)^2$ , where  $t$  is the number of seconds after beginning the dive.

a. Graph  $h$ .

b. Another bird's dive is represented by  $r(t) = 2h(t)$ . Graph  $r$ .

c. Compare the graphs. Which bird starts its dive from a greater height? Explain.



56. **MODELING WITH MATHEMATICS** A kicker punts a football. The height (in yards) of the football is represented by  $f(x) = -\frac{1}{9}(x - 30)^2 + 25$ , where  $x$  is the horizontal distance (in yards) from the kicker's goal line.

a. Graph  $f$ . Describe the domain and range.

b. On the next possession, the kicker punts the football. The height of the football is represented by  $g(x) = f(x + 5)$ . Graph  $g$ . Describe the domain and range.

c. Compare the graphs. On which possession does the kicker punt closer to his goal line? Explain.

In Exercises 57–62, write a quadratic function in vertex form whose graph has the given vertex and passes through the given point.

57. vertex: (1, 2); passes through (3, 10)

58. vertex: (-3, 5); passes through (0, -14)

59. vertex: (-2, -4); passes through (-1, -6)

60. vertex: (1, 8); passes through (3, 12)

61. vertex: (5, -2); passes through (7, 0)

62. vertex: (-5, -1); passes through (-2, 2)



- 63. MODELING WITH MATHEMATICS** A portion of a roller coaster track is in the shape of a parabola. Write and graph a quadratic function that models this portion of the roller coaster with a maximum height of 90 feet, represented by a vertex of (25, 90), passing through the point (50, 0). (See Example 5.)



- 64. MODELING WITH MATHEMATICS** A flare is launched from a boat and travels in a parabolic path until reaching the water. Write and graph a quadratic function that models the path of the flare with a maximum height of 300 meters, represented by a vertex of (59, 300), landing in the water at the point (119, 0).

**In Exercises 65–68, rewrite the quadratic function in vertex form.**

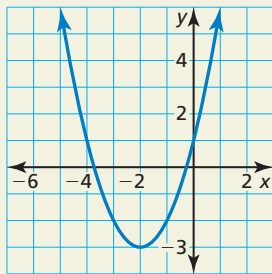
65.  $y = 2x^2 - 8x + 4$       66.  $y = 3x^2 + 6x - 1$

67.  $f(x) = -5x^2 + 10x + 3$

68.  $f(x) = -x^2 - 4x + 2$

- 69. REASONING** Can a function be symmetric about the  $x$ -axis? Explain.

- 70. HOW DO YOU SEE IT?** The graph of a quadratic function is shown. Determine which symbols to use to complete the vertex form of the quadratic function. Explain your reasoning.



$y = a(x \quad 2)^2 \quad 3$

**In Exercises 71–74, describe the transformation from the graph of  $f$  to the graph of  $h$ . Write an equation that represents  $h$  in terms of  $x$ .**

71.  $f(x) = -(x + 1)^2 - 2$       72.  $f(x) = 2(x - 1)^2 + 1$   
 $h(x) = f(x) + 4$                        $h(x) = f(x - 5)$

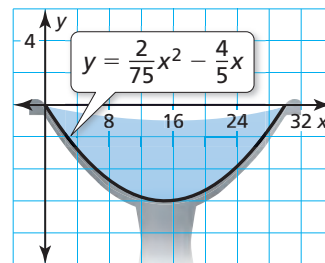
73.  $f(x) = 4(x - 2)^2 + 3$       74.  $f(x) = -(x + 5)^2 - 6$   
 $h(x) = 2f(x)$                        $h(x) = \frac{1}{3}f(x)$

- 75. REASONING** The graph of  $y = x^2$  is translated 2 units right and 5 units down. Write an equation for the function in vertex form and in standard form. Describe advantages of writing the function in each form.

- 76. THOUGHT PROVOKING** Which of the following are true? Justify your answers.

- Any constant multiple of an even function is even.
- Any constant multiple of an odd function is odd.
- The sum or difference of two even functions is even.
- The sum or difference of two odd functions is odd.
- The sum or difference of an even function and an odd function is odd.

- 77. COMPARING FUNCTIONS** A cross section of a birdbath can be modeled by  $y = \frac{1}{81}(x - 18)^2 - 4$ , where  $x$  and  $y$  are measured in inches. The graph shows the cross section of another birdbath.



- Which birdbath is deeper? Explain.
- Which birdbath is wider? Explain.

- 78. REASONING** Compare the graphs of  $y = 2x^2 + 8x + 8$  and  $y = x^2$  without graphing the functions. How can factoring help you compare the parabolas? Explain.

- 79. MAKING AN ARGUMENT** Your friend says all absolute value functions are even because of their symmetry. Is your friend correct? Explain.

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. (Section 2.4)

80.  $x(x - 1) = 0$

81.  $(x + 3)(x - 8) = 0$

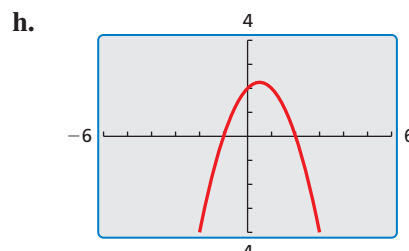
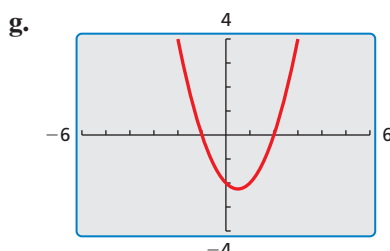
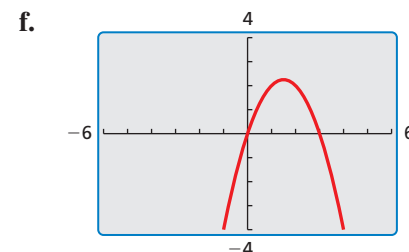
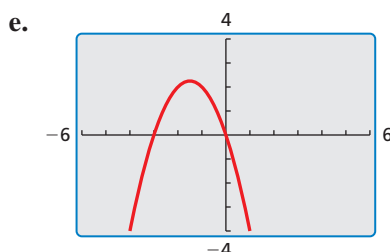
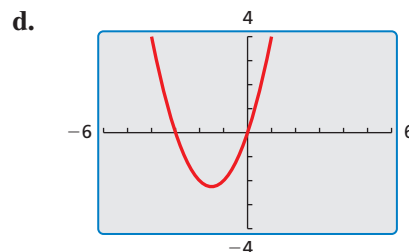
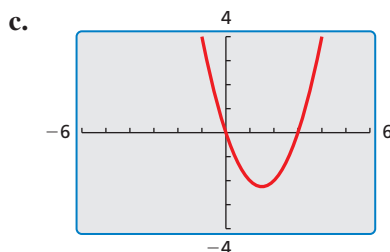
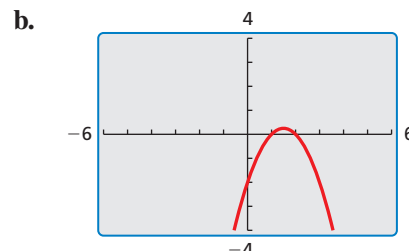
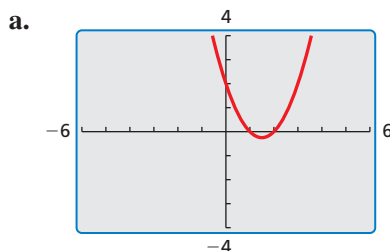
82.  $(3x - 9)(4x + 12) = 0$

## 3.5 Graphing $f(x) = a(x - p)(x - q)$

**Essential Question** What are some of the characteristics of the graph of  $f(x) = a(x - p)(x - q)$ ?

### EXPLORATION 1 Using Zeros to Write Functions

**Work with a partner.** Each graph represents a function of the form  $f(x) = (x - p)(x - q)$  or  $f(x) = -(x - p)(x - q)$ . Write the function represented by each graph. Explain your reasoning.



### CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to justify your conclusions and communicate them to others.

### Communicate Your Answer

- What are some of the characteristics of the graph of  $f(x) = a(x - p)(x - q)$ ?
- Consider the graph of  $f(x) = a(x - p)(x - q)$ .
  - Does changing the sign of  $a$  change the  $x$ -intercepts? Does changing the sign of  $a$  change the  $y$ -intercept? Explain your reasoning.
  - Does changing the value of  $p$  change the  $x$ -intercepts? Does changing the value of  $p$  change the  $y$ -intercept? Explain your reasoning.

## 3.5 Lesson

### Core Vocabulary

intercept form, p. 154

## What You Will Learn

- ▶ Graph quadratic functions of the form  $f(x) = a(x - p)(x - q)$ .
- ▶ Use intercept form to find zeros of functions.
- ▶ Use characteristics to graph and write quadratic functions.

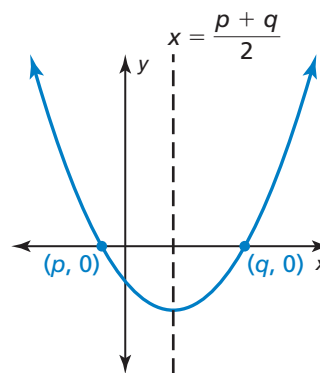
### Graphing $f(x) = a(x - p)(x - q)$

You have already graphed quadratic functions written in several different forms, such as  $f(x) = ax^2 + bx + c$  (standard form) and  $g(x) = a(x - h)^2 + k$  (vertex form). Quadratic functions can also be written in **intercept form**,  $f(x) = a(x - p)(x - q)$ , where  $a \neq 0$ . In this form, the polynomial that defines a function is in factored form and the  $x$ -intercepts of the graph can be easily determined.

### Core Concept

#### Graphing $f(x) = a(x - p)(x - q)$

- The  $x$ -intercepts are  $p$  and  $q$ .
- The axis of symmetry is halfway between  $(p, 0)$  and  $(q, 0)$ . So, the axis of symmetry is  $x = \frac{p + q}{2}$ .
- The graph opens up when  $a > 0$ , and the graph opens down when  $a < 0$ .



#### EXAMPLE 1 Graphing $f(x) = a(x - p)(x - q)$

Graph  $f(x) = -(x + 1)(x - 5)$ . Describe the domain and range.

#### SOLUTION

**Step 1** Identify the  $x$ -intercepts. Because the  $x$ -intercepts are  $p = -1$  and  $q = 5$ , plot  $(-1, 0)$  and  $(5, 0)$ .

**Step 2** Find and graph the axis of symmetry.

$$x = \frac{p + q}{2} = \frac{-1 + 5}{2} = 2$$

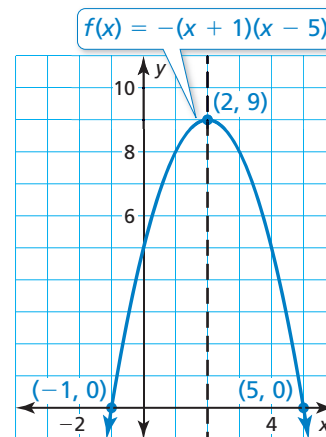
**Step 3** Find and plot the vertex.

The  $x$ -coordinate of the vertex is 2.  
To find the  $y$ -coordinate of the vertex, substitute 2 for  $x$  and simplify.

$$f(2) = -(2 + 1)(2 - 5) = 9$$

So, the vertex is  $(2, 9)$ .

**Step 4** Draw a parabola through the vertex and the points where the  $x$ -intercepts occur.



- ▶ The domain is all real numbers. The range is  $y \leq 9$ .

## EXAMPLE 2 Graphing a Quadratic Function

Graph  $f(x) = 2x^2 - 8$ . Describe the domain and range.

### SOLUTION

**Step 1** Rewrite the quadratic function in intercept form.

$$\begin{aligned} f(x) &= 2x^2 - 8 && \text{Write the function.} \\ &= 2(x^2 - 4) && \text{Factor out common factor.} \\ &= 2(x + 2)(x - 2) && \text{Difference of two squares pattern} \end{aligned}$$

**Step 2** Identify the  $x$ -intercepts. Because the  $x$ -intercepts are  $p = -2$  and  $q = 2$ , plot  $(-2, 0)$  and  $(2, 0)$ .

**Step 3** Find and graph the axis of symmetry.

$$x = \frac{p + q}{2} = \frac{-2 + 2}{2} = 0$$

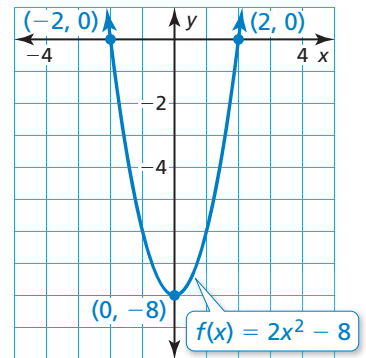
**Step 4** Find and plot the vertex.

The  $x$ -coordinate of the vertex is 0.  
The  $y$ -coordinate of the vertex is

$$f(0) = 2(0)^2 - 8 = -8.$$

So, the vertex is  $(0, -8)$ .

**Step 5** Draw a parabola through the vertex and the points where the  $x$ -intercepts occur.



► The domain is all real numbers. The range is  $y \geq -8$ .

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Graph the quadratic function. Label the vertex, axis of symmetry, and  $x$ -intercepts. Describe the domain and range of the function.

1.  $f(x) = (x + 2)(x - 3)$
2.  $g(x) = -2(x - 4)(x + 1)$
3.  $h(x) = 4x^2 - 36$

## REMEMBER

Functions have zeros, and graphs have  $x$ -intercepts.

## Using Intercept Form to Find Zeros of Functions

In Section 3.2, you learned that a zero of a function is an  $x$ -value for which  $f(x) = 0$ . You can use the intercept form of a function to find the zeros of the function.

## EXAMPLE 3 Finding Zeros of a Function

Find the zeros of  $f(x) = (x - 1)(x + 2)$ .

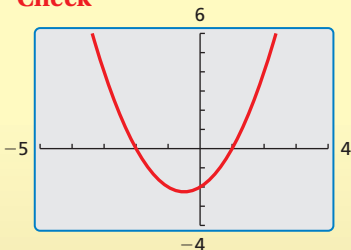
### SOLUTION

To find the zeros, determine the  $x$ -values for which  $f(x)$  is 0.

$$\begin{aligned} f(x) &= (x - 1)(x + 2) && \text{Write the function.} \\ 0 &= (x - 1)(x + 2) && \text{Substitute 0 for } f(x). \\ x - 1 = 0 & \text{ or } & x + 2 = 0 && \text{Zero-Product Property} \\ x = 1 & \text{ or } & x = -2 && \text{Solve for } x. \end{aligned}$$

► So, the zeros of the function are  $-2$  and  $1$ .

### Check



## Core Concept

### Factors and Zeros

For any factor  $x - n$  of a polynomial,  $n$  is a zero of the function defined by the polynomial.

### EXAMPLE 4 Finding Zeros of Functions

Find the zeros of each function.

a.  $h(x) = x^2 - 16$

b.  $f(x) = -2x^2 - 10x - 12$

### SOLUTION

Write each function in intercept form to identify the zeros.

a.  $h(x) = x^2 - 16$

Write the function.

$$= (x + 4)(x - 4)$$

Difference of two squares pattern

▶ So, the zeros of the function are  $-4$  and  $4$ .

b.  $f(x) = -2x^2 - 10x - 12$

Write the function.

$$= -2(x^2 + 5x + 6)$$

Factor out common factor.

$$= -2(x + 3)(x + 2)$$

Factor the trinomial.

▶ So, the zeros of the function are  $-3$  and  $-2$ .

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Find the zero(s) of the function.

4.  $f(x) = (x - 6)(x - 1)$

5.  $h(x) = x^2 - 1$

6.  $g(x) = 3x^2 - 12x + 12$

## Using Characteristics to Graph and Write Quadratic Functions

### EXAMPLE 5 Graphing a Quadratic Function Using Zeros

Use zeros to graph  $h(x) = x^2 - 2x - 3$ .

### SOLUTION

The function is in standard form. You know that the parabola opens up ( $a > 0$ ) and the  $y$ -intercept is  $-3$ . So, begin by plotting  $(0, -3)$ .

Notice that the polynomial that defines the function is factorable. So, write the function in intercept form and identify the zeros.

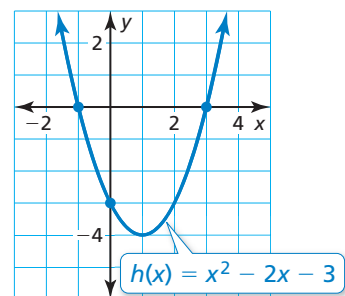
$$h(x) = x^2 - 2x - 3$$

Write the function.

$$= (x + 1)(x - 3)$$

Factor the trinomial.

The zeros of the function are  $-1$  and  $3$ . So, plot  $(-1, 0)$  and  $(3, 0)$ . Draw a parabola through the points.



### ATTENDING TO PRECISION

To sketch a more precise graph, make a table of values and plot other points on the graph.



### EXAMPLE 6 Writing Quadratic Functions

#### STUDY TIP

In part (a), many possible functions satisfy the given condition. The value  $a$  can be *any* nonzero number. To allow easier calculations, let  $a = 1$ . By letting  $a = 2$ , the resulting function would be  $f(x) = 2x^2 + 12x + 22$ .



Write a quadratic function in standard form whose graph satisfies the given condition(s).

- vertex:  $(-3, 4)$
- passes through  $(-9, 0)$ ,  $(-2, 0)$ , and  $(-4, 20)$

#### SOLUTION

- Because you know the vertex, use vertex form to write a function.

$$\begin{aligned} f(x) &= a(x - h)^2 + k \\ &= 1(x + 3)^2 + 4 \\ &= x^2 + 6x + 9 + 4 \\ &= x^2 + 6x + 13 \end{aligned}$$

Vertex form

Substitute for  $a$ ,  $h$ , and  $k$ .

Find the product  $(x + 3)^2$ .

Combine like terms.

- The given points indicate that the  $x$ -intercepts are  $-9$  and  $-2$ . So, use intercept form to write a function.

$$\begin{aligned} f(x) &= a(x - p)(x - q) \\ &= a(x + 9)(x + 2) \end{aligned}$$

Intercept form

Substitute for  $p$  and  $q$ .

Use the other given point,  $(-4, 20)$ , to find the value of  $a$ .

$$20 = a(-4 + 9)(-4 + 2)$$

Substitute  $-4$  for  $x$  and  $20$  for  $f(x)$ .

$$20 = a(5)(-2)$$

Simplify.

$$-2 = a$$

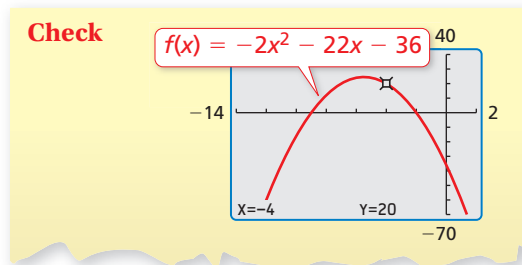
Solve for  $a$ .

Use the value of  $a$  to write the function.

$$\begin{aligned} f(x) &= -2(x + 9)(x + 2) \\ &= -2x^2 - 22x - 36 \end{aligned}$$

Substitute  $-2$  for  $a$ .

Simplify.



### Monitoring Progress



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Use zeros to graph the function.

7.  $f(x) = (x - 1)(x - 4)$

8.  $g(x) = x^2 + x - 12$

Write a quadratic function in standard form whose graph satisfies the given condition(s).

9.  $x$ -intercepts:  $-1$  and  $1$

10. vertex:  $(8, 8)$

11. passes through  $(0, 0)$ ,  $(10, 0)$ , and  $(4, 12)$

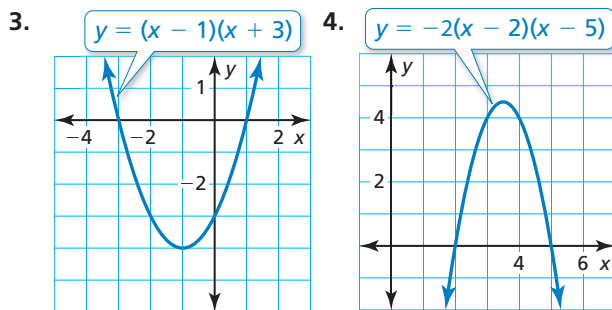
12. passes through  $(-5, 0)$ ,  $(4, 0)$ , and  $(3, -16)$

## Vocabulary and Core Concept Check

- COMPLETE THE SENTENCE** The values  $p$  and  $q$  are \_\_\_\_\_ of the graph of the function  $f(x) = a(x - p)(x - q)$ .
- WRITING** Explain how to find the maximum value or minimum value of a quadratic function when the function is given in intercept form.

## Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the  $x$ -intercepts and axis of symmetry of the graph of the function.



5.  $f(x) = -5(x + 7)(x - 5)$  6.  $g(x) = \frac{2}{3}x(x + 8)$

In Exercises 7–12, graph the quadratic function. Label the vertex, axis of symmetry, and  $x$ -intercepts. Describe the domain and range of the function. (See Example 1.)

7.  $f(x) = (x + 4)(x + 1)$  8.  $y = (x - 2)(x + 2)$   
 9.  $y = -(x + 6)(x - 4)$  10.  $h(x) = -4(x - 7)(x - 3)$   
 11.  $g(x) = 5(x + 1)(x + 2)$  12.  $y = -2(x - 3)(x + 4)$

In Exercises 13–20, graph the quadratic function. Label the vertex, axis of symmetry, and  $x$ -intercepts. Describe the domain and range of the function. (See Example 2.)

13.  $y = x^2 - 9$  14.  $f(x) = x^2 - 8x$   
 15.  $h(x) = -5x^2 + 5x$  16.  $y = 3x^2 - 48$   
 17.  $q(x) = x^2 + 9x + 14$  18.  $p(x) = x^2 + 6x - 27$   
 19.  $y = 4x^2 - 36x + 32$  20.  $y = -2x^2 - 4x + 30$

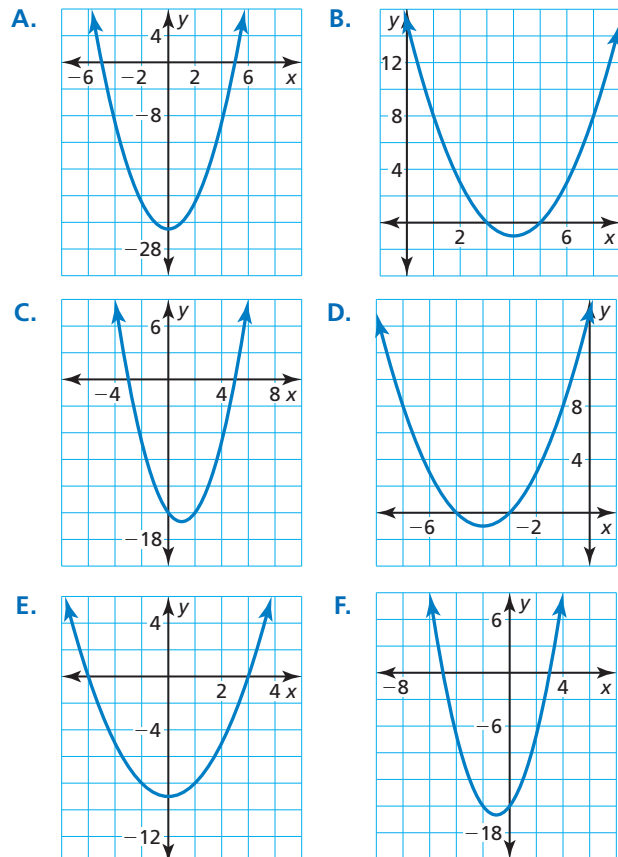
In Exercises 21–28, find the zero(s) of the function. (See Examples 3 and 4.)

21.  $y = -2(x - 2)(x - 10)$  22.  $f(x) = \frac{1}{3}(x + 5)(x - 1)$   
 23.  $f(x) = x^2 - 4$  24.  $h(x) = x^2 - 36$

25.  $g(x) = x^2 + 5x - 24$  26.  $y = x^2 - 17x + 52$   
 27.  $y = 3x^2 - 15x - 42$  28.  $g(x) = -4x^2 - 8x - 4$

In Exercises 29–34, match the function with its graph.

29.  $y = (x + 5)(x + 3)$  30.  $y = (x + 5)(x - 3)$   
 31.  $y = (x - 5)(x + 3)$  32.  $y = (x - 5)(x - 3)$   
 33.  $y = (x + 5)(x - 5)$  34.  $y = (x + 3)(x - 3)$



In Exercises 35–40, use zeros to graph the function.

(See Example 5.)

35.  $f(x) = (x + 2)(x - 6)$     36.  $g(x) = -3(x + 1)(x + 7)$

37.  $y = x^2 - 11x + 18$     38.  $y = x^2 - x - 30$

39.  $y = -5x^2 - 10x + 40$     40.  $h(x) = 8x^2 - 8$

**ERROR ANALYSIS** In Exercises 41 and 42, describe and correct the error in finding the zeros of the function.

41.

**X**  $y = 5(x + 3)(x - 2)$   
The zeros of the function are 3 and  $-2$ .

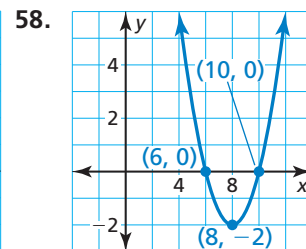
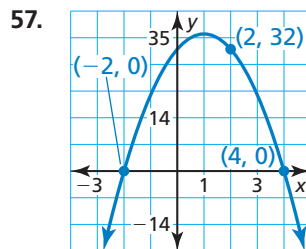
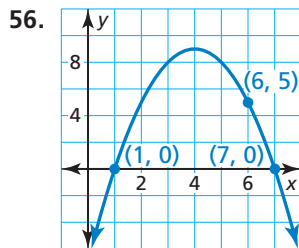
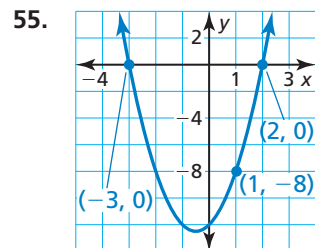
42.

**X**  $y = x^2 - 9$   
The zero of the function is 9.

In Exercises 43–54, write a quadratic function in standard form whose graph satisfies the given condition(s). (See Example 6.)

43. vertex:  $(7, -3)$     44. vertex:  $(4, 8)$   
45.  $x$ -intercepts: 1 and 9    46.  $x$ -intercepts:  $-2$  and  $-5$   
47. passes through  $(-4, 0)$ ,  $(3, 0)$ , and  $(2, -18)$   
48. passes through  $(-5, 0)$ ,  $(-1, 0)$ , and  $(-4, 3)$   
49. passes through  $(7, 0)$   
50. passes through  $(0, 0)$  and  $(6, 0)$   
51. axis of symmetry:  $x = -5$   
52.  $y$  increases as  $x$  increases when  $x < 4$ ;  $y$  decreases as  $x$  increases when  $x > 4$ .  
53. range:  $y \geq -3$     54. range:  $y \leq 10$

In Exercises 55–58, write the quadratic function represented by the graph.



In Exercises 59 and 60, all the zeros of a function are given. Use the zeros and the other point given to write a quadratic function represented by the table.

59.

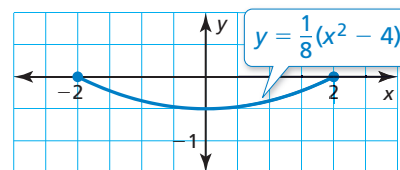
$x$	$y$
0	0
2	30
7	0

60.

$x$	$y$
$-3$	0
1	$-72$
4	0

In Exercises 61–64, sketch a parabola that satisfies the given conditions.

61.  $x$ -intercepts:  $-4$  and  $2$ ; range:  $y \geq -3$   
62. axis of symmetry:  $x = 6$ ; passes through  $(4, 15)$   
63. range:  $y \leq 5$ ; passes through  $(0, 2)$   
64.  $x$ -intercept:  $6$ ;  $y$ -intercept:  $1$ ; range:  $y \geq -4$   
65. **MODELING WITH MATHEMATICS** Satellite dishes are shaped like parabolas to optimally receive signals. The cross section of a satellite dish can be modeled by the function shown, where  $x$  and  $y$  are measured in feet. The  $x$ -axis represents the top of the opening of the dish.

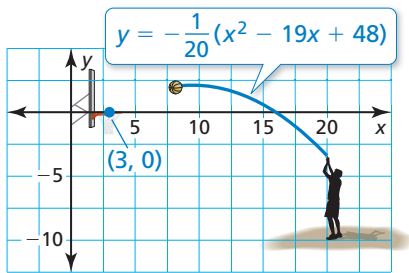


- How wide is the satellite dish?
- How deep is the satellite dish?
- Write a quadratic function in standard form that models the cross section of a satellite dish that is 6 feet wide and 1.5 feet deep.





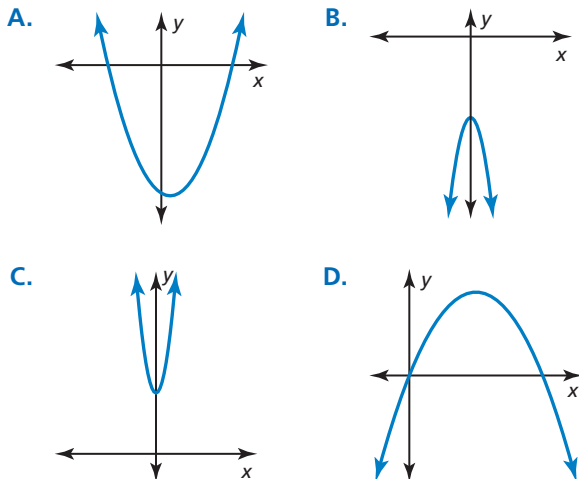
66. **MODELING WITH MATHEMATICS** A professional basketball player's shot is modeled by the function shown, where  $x$  and  $y$  are measured in feet.



- a. Does the player make the shot? Explain.
- b. The basketball player releases another shot from the point  $(13, 0)$  and makes the shot. The shot also passes through the point  $(10, 1.4)$ . Write a quadratic function in standard form that models the path of the shot.

**USING STRUCTURE** In Exercises 67–70, match the function with its graph.

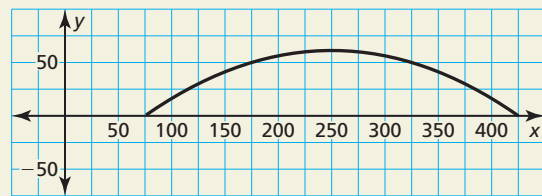
67.  $y = -x^2 + 5x$       68.  $y = x^2 - x - 12$
69.  $y = -2x^2 - 8$       70.  $y = 3x^2 + 6$



71. **CRITICAL THINKING** Write a quadratic function represented by the table, if possible. If not, explain why.

$x$	-5	-3	-1	1
$y$	0	12	4	0

72. **HOW DO YOU SEE IT?** The graph shows the parabolic arch that supports the roof of a convention center, where  $x$  and  $y$  are measured in feet.



- a. The arch can be represented by a function of the form  $f(x) = a(x - p)(x - q)$ . Estimate the values of  $p$  and  $q$ .
- b. Estimate the width and height of the arch. Explain how you can use your height estimate to calculate  $a$ .

**ANALYZING EQUATIONS** In Exercises 73 and 74,

- (a) rewrite the quadratic function in intercept form and
- (b) graph the function using any method. Explain the method you used.

73.  $f(x) = -3(x + 1)^2 + 27$

74.  $g(x) = 2(x - 1)^2 - 2$

75. **WRITING** Can a quadratic function with exactly one real zero be written in intercept form? Explain.

76. **MAKING AN ARGUMENT** Your friend claims that any quadratic function can be written in standard form and in vertex form. Is your friend correct? Explain.

77. **REASONING** Let  $k$  be a constant. Find the zeros of the function  $f(x) = kx^2 - k^2x - 2k^3$  in terms of  $k$ .

78. **THOUGHT PROVOKING** Sketch the graph of each function. Explain your procedure.

a.  $f(x) = (x^2 - 1)(x^2 - 4)$

b.  $g(x) = x(x^2 - 1)(x^2 - 4)$

**PROBLEM SOLVING** In Exercises 79 and 80, write two quadratic functions whose graphs intersect at the given points. Explain your reasoning.

79.  $(-4, 0)$  and  $(2, 0)$       80.  $(3, 6)$  and  $(7, 6)$

## Maintaining Mathematical Proficiency Reviewing what you learned in previous grades and lessons

Find the distance between the two points. (*Skills Review Handbook*)

81.  $(2, 1)$  and  $(3, 0)$

82.  $(-1, 8)$  and  $(-9, 2)$

83.  $(-1, -6)$  and  $(-3, -5)$

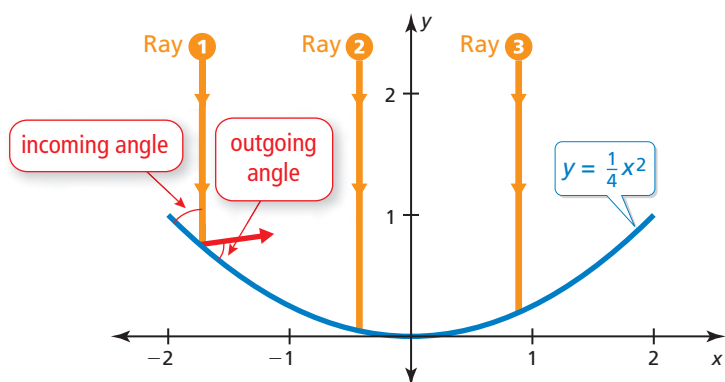
## 3.6 Focus of a Parabola

**Essential Question** What is the focus of a parabola?

### EXPLORATION 1 Analyzing Satellite Dishes

**Work with a partner.** Vertical rays enter a satellite dish whose cross section is a parabola. When the rays hit the parabola, they reflect at the same angle at which they entered. (See Ray 1 in the figure.)

- Draw the reflected rays so that they intersect the  $y$ -axis.
- What do the reflected rays have in common?
- The optimal location for the receiver of the satellite dish is at a point called the *focus* of the parabola. Determine the location of the focus. Explain why this makes sense in this situation.

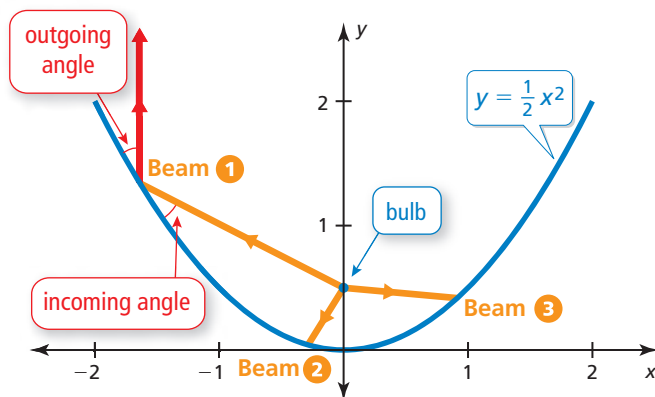


### CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to make conjectures and build logical progressions of statements to explore the truth of your conjectures.

### EXPLORATION 2 Analyzing Spotlights

**Work with a partner.** Beams of light are coming from the bulb in a spotlight, located at the focus of the parabola. When the beams hit the parabola, they reflect at the same angle at which they hit. (See Beam 1 in the figure.) Draw the reflected beams. What do they have in common? Would you consider this to be the optimal result? Explain.



### Communicate Your Answer

- What is the focus of a parabola?
- Describe some of the properties of the focus of a parabola.

# 3.6 Lesson

## Core Vocabulary

focus, p. 162  
directrix, p. 162

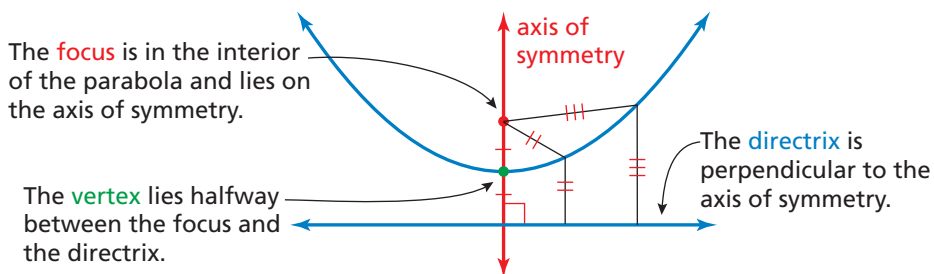
**Previous**  
perpendicular  
Distance Formula  
congruent

## What You Will Learn

- ▶ Explore the focus and the directrix of a parabola.
- ▶ Write equations of parabolas.
- ▶ Solve real-life problems.

## Exploring the Focus and Directrix

Previously, you learned that the graph of a quadratic function is a parabola that opens up or down. A parabola can also be defined as the set of all points  $(x, y)$  in a plane that are equidistant from a fixed point called the **focus** and a fixed line called the **directrix**.



## REMEMBER

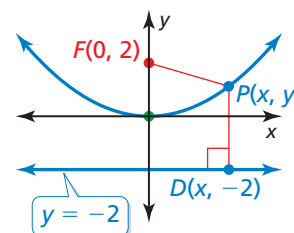
The distance from a point to a line is defined as the length of the perpendicular segment from the point to the line.

### EXAMPLE 1 Using the Distance Formula to Write an Equation

Use the Distance Formula to write an equation of the parabola with focus  $F(0, 2)$  and directrix  $y = -2$ .

#### SOLUTION

Notice the line segments drawn from point  $F$  to point  $P$  and from point  $P$  to point  $D$ . By the definition of a parabola, these line segments must be congruent.



Definition of a parabola

Distance Formula

Substitute for  $x_1, y_1, x_2,$  and  $y_2$ .

Simplify.

Square each side.

Expand.

Combine like terms.

Divide each side by 8.

$$PD = PF$$

$$\sqrt{(x - x_1)^2 + (y - y_1)^2} = \sqrt{(x - x_2)^2 + (y - y_2)^2}$$

$$\sqrt{(x - 0)^2 + (y - (-2))^2} = \sqrt{(x - 0)^2 + (y - 2)^2}$$

$$\sqrt{(y + 2)^2} = \sqrt{x^2 + (y - 2)^2}$$

$$(y + 2)^2 = x^2 + (y - 2)^2$$

$$y^2 + 4y + 4 = x^2 + y^2 - 4y + 4$$

$$8y = x^2$$

$$y = \frac{1}{8}x^2$$

## REMEMBER

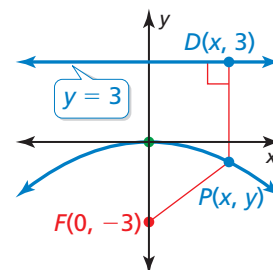
Squaring a positive number and finding a square root are inverse operations. So, you can square each side of the equation to eliminate the radicals.

## Monitoring Progress

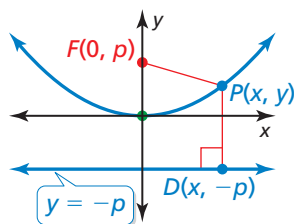


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1. Use the Distance Formula to write an equation of the parabola with focus  $F(0, -3)$  and directrix  $y = 3$ .



You can derive the equation of a parabola that opens up or down with vertex  $(0, 0)$ , focus  $(0, p)$ , and directrix  $y = -p$  using the procedure in Example 1.



$$\begin{aligned}\sqrt{(x - 0)^2 + (y - (-p))^2} &= \sqrt{(x - 0)^2 + (y - p)^2} \\ (y + p)^2 &= x^2 + (y - p)^2 \\ y^2 + 2py + p^2 &= x^2 + y^2 - 2py + p^2 \\ 4py &= x^2 \\ y &= \frac{1}{4p}x^2\end{aligned}$$

## LOOKING FOR STRUCTURE

Notice that  $y = \frac{1}{4p}x^2$  is of the form  $y = ax^2$ . So, changing the value of  $p$  vertically stretches or shrinks the parabola.

The focus and directrix each lie  $|p|$  units from the vertex. Parabolas can also open left or right, in which case the equation has the form  $x = \frac{1}{4p}y^2$  when the vertex is  $(0, 0)$ .

## Core Concept

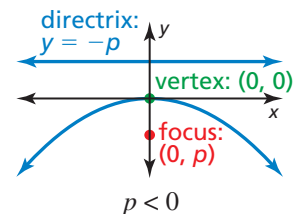
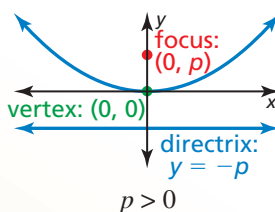
### Standard Equations of a Parabola with Vertex at the Origin

#### Vertical axis of symmetry ( $x = 0$ )

Equation:  $y = \frac{1}{4p}x^2$

Focus:  $(0, p)$

Directrix:  $y = -p$

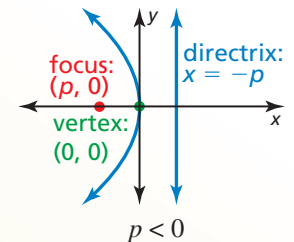
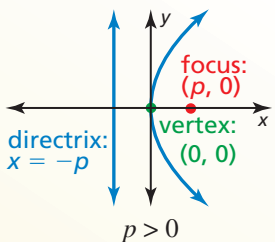


#### Horizontal axis of symmetry ( $y = 0$ )

Equation:  $x = \frac{1}{4p}y^2$

Focus:  $(p, 0)$

Directrix:  $x = -p$



## STUDY TIP

Notice that parabolas opening left or right do *not* represent functions.

### EXAMPLE 2 Graphing an Equation of a Parabola

Identify the focus, directrix, and axis of symmetry of  $-4x = y^2$ . Graph the equation.

#### SOLUTION

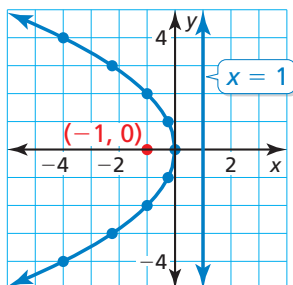
**Step 1** Rewrite the equation in standard form.

$$\begin{aligned}-4x &= y^2 && \text{Write the original equation.} \\ x &= -\frac{1}{4}y^2 && \text{Divide each side by } -4.\end{aligned}$$

**Step 2** Identify the focus, directrix, and axis of symmetry. The equation has the form  $x = \frac{1}{4p}y^2$ , where  $p = -1$ . The focus is  $(p, 0)$ , or  $(-1, 0)$ . The directrix is  $x = -p$ , or  $x = 1$ . Because  $y$  is squared, the axis of symmetry is the  $x$ -axis.

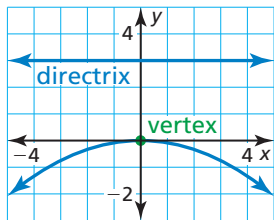
**Step 3** Use a table of values to graph the equation. Notice that it is easier to substitute  $y$ -values and solve for  $x$ . Opposite  $y$ -values result in the same  $x$ -value.

$y$	0	$\pm 1$	$\pm 2$	$\pm 3$	$\pm 4$
$x$	0	-0.25	-1	-2.25	-4



## Writing Equations of Parabolas

### EXAMPLE 3 Writing an Equation of a Parabola



Write an equation of the parabola shown.

#### SOLUTION

Because the vertex is at the origin and the axis of symmetry is vertical, the equation has the form  $y = \frac{1}{4p}x^2$ . The directrix is  $y = -p = 3$ , so  $p = -3$ . Substitute  $-3$  for  $p$  to write an equation of the parabola.

$$y = \frac{1}{4(-3)}x^2 = -\frac{1}{12}x^2$$

▶ So, an equation of the parabola is  $y = -\frac{1}{12}x^2$ .

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Identify the focus, directrix, and axis of symmetry of the parabola. Then graph the equation.

2.  $y = 0.5x^2$

3.  $-y = x^2$

4.  $y^2 = 6x$

Write an equation of the parabola with vertex at  $(0, 0)$  and the given directrix or focus.

5. directrix:  $x = -3$

6. focus:  $(-2, 0)$

7. focus:  $(0, \frac{3}{2})$

The vertex of a parabola is not always at the origin. As in previous transformations, adding a value to the input or output of a function translates its graph.

## Core Concept

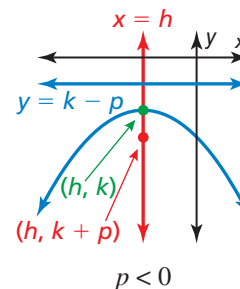
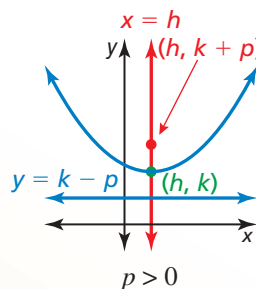
### Standard Equations of a Parabola with Vertex at $(h, k)$

#### Vertical axis of symmetry ( $x = h$ )

Equation:  $y = \frac{1}{4p}(x - h)^2 + k$

Focus:  $(h, k + p)$

Directrix:  $y = k - p$

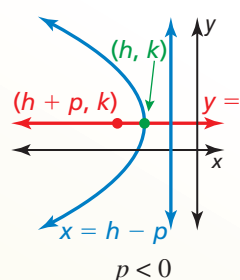
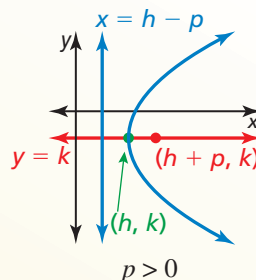


#### Horizontal axis of symmetry ( $y = k$ )

Equation:  $x = \frac{1}{4p}(y - k)^2 + h$

Focus:  $(h + p, k)$

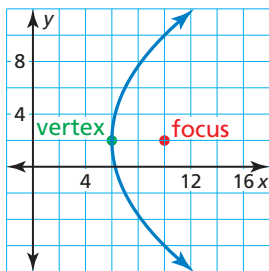
Directrix:  $x = h - p$



#### STUDY TIP

The standard form for a vertical axis of symmetry looks like vertex form. To remember the standard form for a horizontal axis of symmetry, switch  $x$  and  $y$ , and  $h$  and  $k$ .

### EXAMPLE 4 Writing an Equation of a Translated Parabola



Write an equation of the parabola shown.

#### SOLUTION

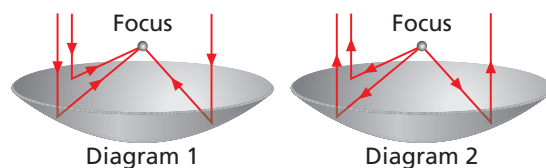
Because the vertex is not at the origin and the axis of symmetry is horizontal, the equation has the form  $x = \frac{1}{4p}(y - k)^2 + h$ . The vertex  $(h, k)$  is  $(6, 2)$  and the focus  $(h + p, k)$  is  $(10, 2)$ , so  $h = 6$ ,  $k = 2$ , and  $p = 4$ . Substitute these values to write an equation of the parabola.

$$x = \frac{1}{4(4)}(y - 2)^2 + 6 = \frac{1}{16}(y - 2)^2 + 6$$

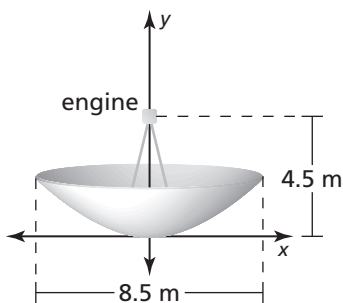
► So, an equation of the parabola is  $x = \frac{1}{16}(y - 2)^2 + 6$ .

### Solving Real-Life Problems

*Parabolic reflectors* have cross sections that are parabolas. Incoming sound, light, or other energy that arrives at a parabolic reflector parallel to the axis of symmetry is directed to the focus (Diagram 1). Similarly, energy that is emitted from the focus of a parabolic reflector and then strikes the reflector is directed parallel to the axis of symmetry (Diagram 2).



### EXAMPLE 5 Solving a Real-Life Problem



An electricity-generating dish uses a parabolic reflector to concentrate sunlight onto a high-frequency engine located at the focus of the reflector. The sunlight heats helium to  $650^\circ\text{C}$  to power the engine. Write an equation that represents the cross section of the dish shown with its vertex at  $(0, 0)$ . What is the depth of the dish?

#### SOLUTION

Because the vertex is at the origin, and the axis of symmetry is vertical, the equation has the form  $y = \frac{1}{4p}x^2$ . The engine is at the focus, which is 4.5 meters above the vertex. So,  $p = 4.5$ . Substitute 4.5 for  $p$  to write the equation.

$$y = \frac{1}{4(4.5)}x^2 = \frac{1}{18}x^2$$

The depth of the dish is the  $y$ -value at the dish's outside edge. The dish extends  $\frac{8.5}{2} = 4.25$  meters to either side of the vertex  $(0, 0)$ , so find  $y$  when  $x = 4.25$ .

$$y = \frac{1}{18}(4.25)^2 \approx 1$$

► The depth of the dish is about 1 meter.

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- Write an equation of a parabola with vertex  $(-1, 4)$  and focus  $(-1, 2)$ .
- A parabolic microwave antenna is 16 feet in diameter. Write an equation that represents the cross section of the antenna with its vertex at  $(0, 0)$  and its focus 10 feet to the right of the vertex. What is the depth of the antenna?

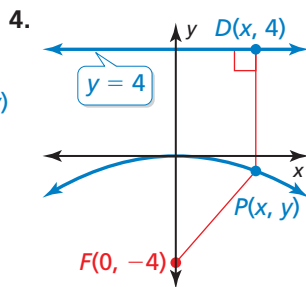
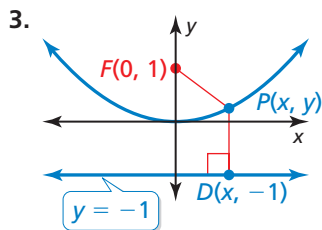
# 3.6 Exercises

## Vocabulary and Core Concept Check

- COMPLETE THE SENTENCE** A parabola is the set of all points in a plane equidistant from a fixed point called the \_\_\_\_\_ and a fixed line called the \_\_\_\_\_.
- WRITING** Explain how to find the coordinates of the focus of a parabola with vertex  $(0, 0)$  and directrix  $y = 5$ .

## Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, use the Distance Formula to write an equation of the parabola. (See Example 1.)

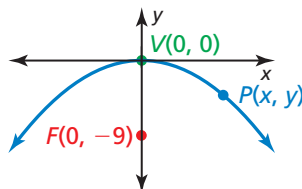


- |  |   |
|--|---|
| 5. focus: $(0, -2)$<br>directrix: $y = 2$  | 6. directrix: $y = 7$<br>focus: $(0, -7)$   |
| 7. vertex: $(0, 0)$<br>directrix: $y = -6$ | 8. vertex: $(0, 0)$<br>focus: $(0, 5)$      |
| 9. vertex: $(0, 0)$<br>focus: $(0, -10)$   | 10. vertex: $(0, 0)$<br>directrix: $y = -9$ |

11. **ANALYZING RELATIONSHIPS** Which of the given characteristics describe parabolas that open down? Explain your reasoning.

- |  |  |
|--|--|
| (A) focus: $(0, -6)$<br>directrix: $y = 6$ | (B) focus: $(0, -2)$<br>directrix: $y = 2$ |
| (C) focus: $(0, 6)$<br>directrix: $y = -6$ | (D) focus: $(0, -1)$<br>directrix: $y = 1$ |

12. **REASONING** Which of the following are possible coordinates of the point  $P$  in the graph shown? Explain.

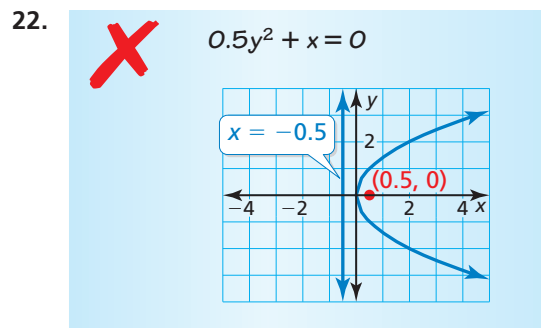
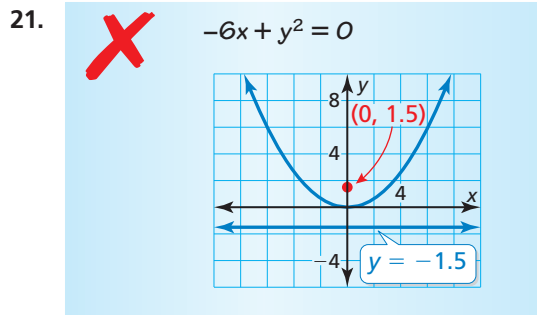


- |                         |                         |                          |
|-------------------------|-------------------------|--------------------------|
| (A) $(-6, -1)$          | (B) $(3, -\frac{1}{4})$ | (C) $(4, -\frac{4}{9})$  |
| (D) $(1, \frac{1}{36})$ | (E) $(6, -1)$           | (F) $(2, -\frac{1}{18})$ |

In Exercises 13–20, identify the focus, directrix, and axis of symmetry of the parabola. Graph the equation. (See Example 2.)

- |                            |                            |
|----------------------------|----------------------------|
| 13. $y = \frac{1}{8}x^2$   | 14. $y = -\frac{1}{12}x^2$ |
| 15. $x = -\frac{1}{20}y^2$ | 16. $x = \frac{1}{24}y^2$  |
| 17. $y^2 = 16x$            | 18. $-x^2 = 48y$           |
| 19. $6x^2 + 3y = 0$        | 20. $8x^2 - y = 0$         |

**ERROR ANALYSIS** In Exercises 21 and 22, describe and correct the error in graphing the parabola.

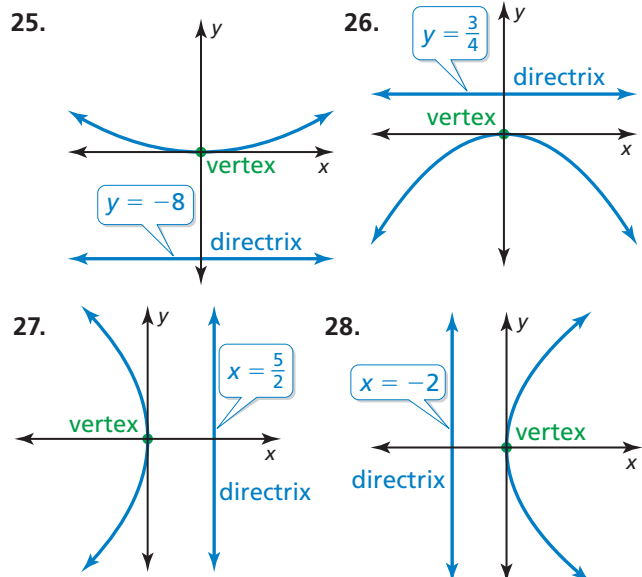


23. **ANALYZING EQUATIONS** The cross section (with units in inches) of a parabolic satellite dish can be modeled by the equation  $y = \frac{1}{38}x^2$ . How far is the receiver from the vertex of the cross section? Explain.

- 24. ANALYZING EQUATIONS** The cross section (with units in inches) of a parabolic spotlight can be modeled by the equation  $x = \frac{1}{20}y^2$ . How far is the bulb from the vertex of the cross section? Explain.



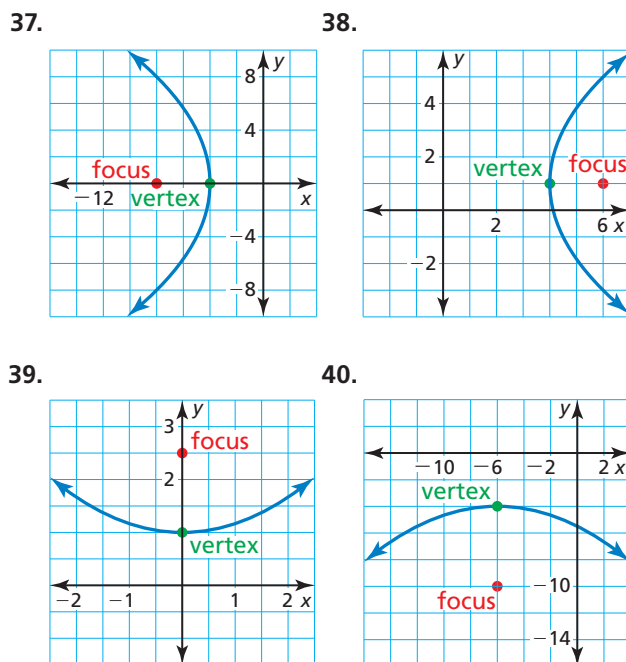
In Exercises 25–28, write an equation of the parabola shown. (See Example 3.)



In Exercises 29–36, write an equation of the parabola with the given characteristics.

29. focus:  $(3, 0)$   
directrix:  $x = -3$
30. focus:  $(\frac{2}{3}, 0)$   
directrix:  $x = -\frac{2}{3}$
31. directrix:  $x = -10$   
vertex:  $(0, 0)$
32. directrix:  $y = \frac{8}{3}$   
vertex:  $(0, 0)$
33. focus:  $(0, -\frac{5}{3})$   
directrix:  $y = \frac{5}{3}$
34. focus:  $(0, \frac{5}{4})$   
directrix:  $y = -\frac{5}{4}$
35. focus:  $(0, \frac{6}{7})$   
vertex:  $(0, 0)$
36. focus:  $(-\frac{4}{5}, 0)$   
vertex:  $(0, 0)$

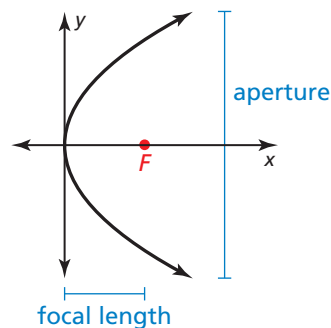
In Exercises 37–40, write an equation of the parabola shown. (See Example 4.)



In Exercises 41–46, identify the vertex, focus, directrix, and axis of symmetry of the parabola. Describe the transformations of the graph of the standard equation with  $p = 1$  and vertex  $(0, 0)$ .

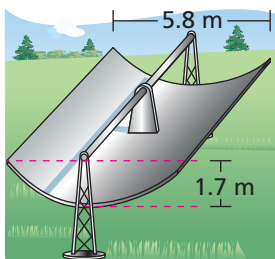
41.  $y = \frac{1}{8}(x - 3)^2 + 2$  42.  $y = -\frac{1}{4}(x + 2)^2 + 1$
43.  $x = \frac{1}{16}(y - 3)^2 + 1$  44.  $y = (x + 3)^2 - 5$
45.  $x = -3(y + 4)^2 + 2$  46.  $x = 4(y + 5)^2 - 1$

- 47. MODELING WITH MATHEMATICS** Scientists studying dolphin echolocation simulate the projection of a bottlenose dolphin's clicking sounds using computer models. The models originate the sounds at the focus of a parabolic reflector. The parabola in the graph shows the cross section of the reflector with focal length of 1.3 inches and aperture width of 8 inches. Write an equation to represent the cross section of the reflector. What is the depth of the reflector? (See Example 5.)



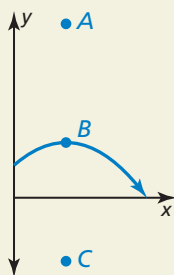


48. **MODELING WITH MATHEMATICS** Solar energy can be concentrated using long troughs that have a parabolic cross section as shown in the figure. Write an equation to represent the cross section of the trough. What are the domain and range in this situation? What do they represent?



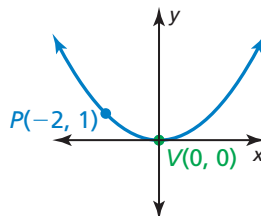
49. **ABSTRACT REASONING** As  $|p|$  increases, how does the width of the graph of the equation  $y = \frac{1}{4p}x^2$  change? Explain your reasoning.

50. **HOW DO YOU SEE IT?** The graph shows the path of a volleyball served from an initial height of 6 feet as it travels over a net.



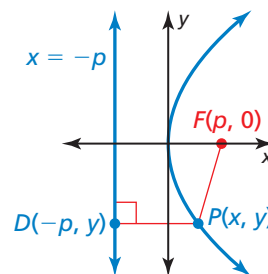
- Label the vertex, focus, and a point on the directrix.
- An underhand serve follows the same parabolic path but is hit from a height of 3 feet. How does this affect the focus? the directrix?

51. **CRITICAL THINKING** The distance from point  $P$  to the directrix is 2 units. Write an equation of the parabola.

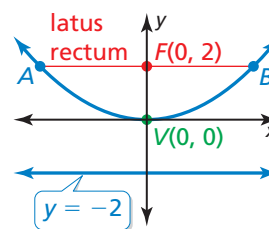


52. **THOUGHT PROVOKING** Two parabolas have the same focus  $(a, b)$  and focal length of 2 units. Write an equation of each parabola. Identify the directrix of each parabola.

53. **REPEATED REASONING** Use the Distance Formula to derive the equation of a parabola that opens to the right with vertex  $(0, 0)$ , focus  $(p, 0)$ , and directrix  $x = -p$ .



54. **PROBLEM SOLVING** The *latus rectum* of a parabola is the line segment that is parallel to the directrix, passes through the focus, and has endpoints that lie on the parabola. Find the length of the latus rectum of the parabola shown.



## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Show that an exponential model fits the data. Then write a recursive rule that models the data.

(Section 1.6)

55.

$n$	0	1	2	3	4	5
$f(n)$	0.25	0.5	1	2	4	8

56.

$n$	0	1	2	3	4	5
$f(n)$	2	6	18	54	162	486

57.

$n$	0	1	2	3	4	5
$f(n)$	144	72	36	18	9	4.5

58.

$n$	0	1	2	3	4	5
$f(n)$	1250	250	50	10	2	$\frac{2}{5}$

# 3.7 Comparing Linear, Exponential, and Quadratic Functions

**Essential Question** How can you compare the growth rates of linear, exponential, and quadratic functions?

## EXPLORATION 1 Comparing Speeds

**Work with a partner.** Three cars start traveling at the same time. The distance traveled in  $t$  minutes is  $y$  miles. Complete each table and sketch all three graphs in the same coordinate plane. Compare the speeds of the three cars. Which car has a constant speed? Which car is accelerating the most? Explain your reasoning.

$t$	$y = t$
0	
0.2	
0.4	
0.6	
0.8	
1.0	

$t$	$y = 2^t - 1$
0	
0.2	
0.4	
0.6	
0.8	
1.0	

$t$	$y = t^2$
0	
0.2	
0.4	
0.6	
0.8	
1.0	

### COMPARING PREDICTIONS

To be proficient in math, you need to visualize the results of varying assumptions, explore consequences, and compare predictions with data.

## EXPLORATION 2 Comparing Speeds

**Work with a partner.** Analyze the speeds of the three cars over the given time periods. The distance traveled in  $t$  minutes is  $y$  miles. Which car eventually overtakes the others?

$t$	$y = t$
1.0	
1.5	
2.0	
2.5	
3.0	
3.5	
4.0	
4.5	
5.0	

$t$	$y = 2^t - 1$
1.0	
1.5	
2.0	
2.5	
3.0	
3.5	
4.0	
4.5	
5.0	

$t$	$y = t^2$
1.0	
1.5	
2.0	
2.5	
3.0	
3.5	
4.0	
4.5	
5.0	

## Communicate Your Answer

- How can you compare the growth rates of linear, exponential, and quadratic functions?
- Which function has a growth rate that is eventually much greater than the growth rates of the other two functions? Explain your reasoning.

# 3.7 Lesson

## Core Vocabulary

### Previous

average rate of change  
slope

## What You Will Learn

- ▶ Choose functions to model data.
- ▶ Write functions to model data.
- ▶ Compare functions using average rates of change.

## Choosing Functions to Model Data

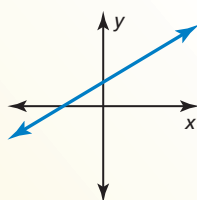
So far, you have studied linear functions, exponential functions, and quadratic functions. You can use these functions to model data.

## Core Concept

### Linear, Exponential, and Quadratic Functions

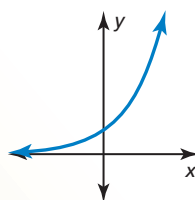
Linear Function

$$y = mx + b$$



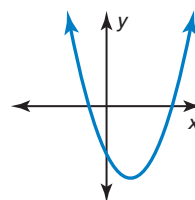
Exponential Function

$$y = ab^x$$



Quadratic Function

$$y = ax^2 + bx + c$$

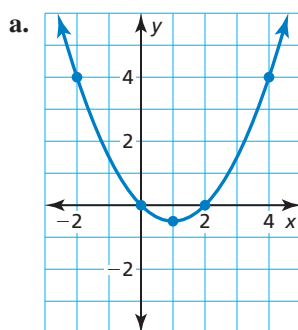


### EXAMPLE 1 Using Graphs to Identify Functions

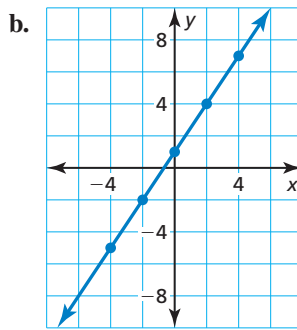
Plot the points. Tell whether the points appear to represent a *linear*, an *exponential*, or a *quadratic* function.

- a.  $(4, 4), (2, 0), (0, 0), (1, -\frac{1}{2}), (-2, 4)$       b.  $(0, 1), (2, 4), (4, 7), (-2, -2), (-4, -5)$       c.  $(0, 2), (2, 8), (1, 4), (-1, 1), (-2, \frac{1}{2})$

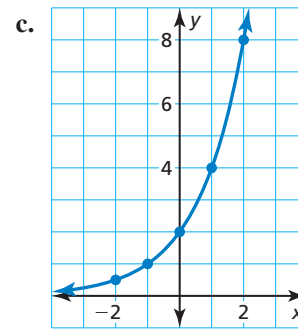
### SOLUTION



▶ quadratic



▶ linear



▶ exponential

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Plot the points. Tell whether the points appear to represent a *linear*, an *exponential*, or a *quadratic* function.

1.  $(-1, 5), (2, -1), (0, -1), (3, 5), (1, -3)$
2.  $(-1, 2), (-2, 8), (-3, 32), (0, \frac{1}{2}), (1, \frac{1}{8})$
3.  $(-3, 5), (0, -1), (2, -5), (-4, 7), (1, -3)$

## Core Concept

### Differences and Ratios of Functions

You can use patterns between consecutive data pairs to determine which type of function models the data. The differences of consecutive  $y$ -values are called *first differences*. The differences of consecutive first differences are called *second differences*.

- **Linear Function** The first differences are constant.
- **Exponential Function** Consecutive  $y$ -values have a common *ratio*.
- **Quadratic Function** The second differences are constant.

In all cases, the differences of consecutive  $x$ -values need to be constant.

### STUDY TIP

The first differences for exponential and quadratic functions are *not* constant.

### STUDY TIP

First determine that the differences of consecutive  $x$ -values are constant. Then check whether the first differences are constant or consecutive  $y$ -values have a common ratio. If neither of these is true, check whether the second differences are constant.

### EXAMPLE 2 Using Differences or Ratios to Identify Functions

Tell whether each table of values represents a *linear*, an *exponential*, or a *quadratic* function.

a.

<b>x</b>	-3	-2	-1	0	1
<b>y</b>	11	8	5	2	-1

b.

<b>x</b>	-2	-1	0	1	2
<b>y</b>	1	2	4	8	16

c.

<b>x</b>	-2	-1	0	1	2
<b>y</b>	-1	-2	-1	2	7

### SOLUTION

a.

<b>x</b>	-3	-2	-1	0	1
<b>y</b>	11	8	5	2	-1

$+1$   $+1$   $+1$   $+1$   
 $-3$   $-3$   $-3$   $-3$

b.

<b>x</b>	-2	-1	0	1	2
<b>y</b>	1	2	4	8	16

$+1$   $+1$   $+1$   $+1$   
 $\times 2$   $\times 2$   $\times 2$   $\times 2$

▶ The first differences are constant. So, the table represents a linear function.

▶ Consecutive  $y$ -values have a common ratio. So, the table represents an exponential function.

c.

<b>x</b>	-2	-1	0	1	2
<b>y</b>	-1	-2	-1	2	7

$+1$   $+1$   $+1$   $+1$

first differences  $\rightarrow$   $-1$   $+1$   $+3$   $+5$   
 second differences  $\rightarrow$   $+2$   $+2$   $+2$

▶ The second differences are constant. So, the table represents a quadratic function.

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<b>x</b>	-1	0	1	2	3
<b>y</b>	1	3	9	27	81

4. Tell whether the table of values represents a *linear*, an *exponential*, or a *quadratic* function.

## Writing Functions to Model Data

### EXAMPLE 3 Writing a Function to Model Data

<b>x</b>	0	1	2	3	4
<b>f(x)</b>	12	0	-4	0	12

Tell whether the table of values represents a *linear*, an *exponential*, or a *quadratic* function. Then write the function.

#### SOLUTION

**Step 1** Determine which type of function the table of values represents.

The second differences are constant. So, the table represents a quadratic function.

<b>x</b>	0	1	2	3	4
<b>f(x)</b>	12	0	-4	0	12

**Step 2** Write an equation of the quadratic function. Using the table, notice that the  $x$ -intercepts are 1 and 3. So, use intercept form to write a function.

$$f(x) = a(x - 1)(x - 3) \quad \text{Substitute for } p \text{ and } q \text{ in intercept form.}$$

Use another point from the table, such as (0, 12), to find  $a$ .

$$12 = a(0 - 1)(0 - 3) \quad \text{Substitute 0 for } x \text{ and 12 for } f(x).$$

$$4 = a \quad \text{Solve for } a.$$

Use the value of  $a$  to write the function.

$$f(x) = 4(x - 1)(x - 3) \quad \text{Substitute 4 for } a.$$

$$= 4x^2 - 16x + 12 \quad \text{Use the FOIL Method and combine like terms.}$$

#### STUDY TIP

To check your function in Example 3, substitute the other points from the table to verify that they satisfy the function.

► So, the quadratic function is  $f(x) = 4x^2 - 16x + 12$ .

Previously, you wrote recursive rules for linear and exponential functions. You can use the pattern in the first differences to write recursive rules for quadratic functions.

### EXAMPLE 4 Writing a Recursive Rule

Write a recursive rule for the quadratic function  $f$  in Example 3.

#### SOLUTION

An expression for the  $n$ th term of the sequence of first differences is  $-12 + 8(n - 1)$ , or  $8n - 20$ . Notice that  $f(n) - f(n - 1) = 8n - 20$ .

► So, a recursive rule for the quadratic function is  $f(0) = 12$ ,  
 $f(n) = f(n - 1) + 8n - 20$ .

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5. Tell whether the table of values represents a *linear*, an *exponential*, or a *quadratic* function. Then write the function.

<b>x</b>	-1	0	1	2	3
<b>y</b>	16	8	4	2	1

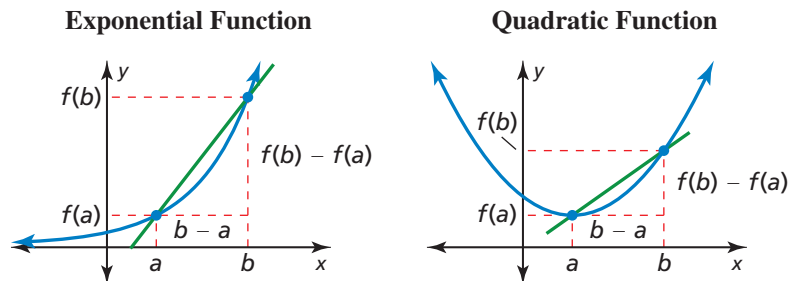
6. Write a recursive rule for the function represented by the table of values.

<b>x</b>	0	1	2	3	4
<b>f(x)</b>	-8	-1	10	25	44

## Comparing Functions Using Average Rates of Change

For nonlinear functions, the rate of change is not constant. You can compare two nonlinear functions over the same interval using their average rates of change. Recall that the average rate of change of a function  $y = f(x)$  between  $x = a$  and  $x = b$  is the slope of the **line** through  $(a, f(a))$  and  $(b, f(b))$ .

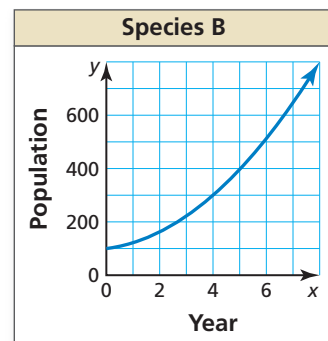
$$\text{average rate of change} = \frac{\text{change in } y}{\text{change in } x} = \frac{f(b) - f(a)}{b - a}$$



### EXAMPLE 5 Using and Interpreting Average Rates of Change

The table and graph show the populations of two species of fish introduced into a lake. Compare the populations by calculating and interpreting the average rates of change from Year 0 to Year 4.

Species A	
Year, $x$	Population, $y$
0	100
2	264
4	428
6	592
8	756
10	920



### SOLUTION

Calculate the average rates of change by using the points whose  $x$ -coordinates are 0 and 4.

Species A: Use  $(0, 100)$  and  $(4, 428)$ .

$$\text{average rate of change} = \frac{f(b) - f(a)}{b - a} = \frac{428 - 100}{4 - 0} = 82$$

Species B: Use the graph to estimate the points when  $x = 0$  and  $x = 4$ . Use  $(0, 100)$  and  $(4, 300)$ .

$$\text{average rate of change} = \frac{f(b) - f(a)}{b - a} \approx \frac{300 - 100}{4 - 0} = 50$$

- From Year 0 to Year 4, the population of Species A increases at an average rate of 82 fish per year, and the population of Species B increases at an average rate of about 50 fish per year. So, the population of Species A is growing faster.

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7. Compare the populations by calculating and interpreting the average rates of change from Year 4 to Year 8.

## Core Concept

### Comparing Functions Using Average Rates of Change

- As  $a$  and  $b$  increase, the average rate of change between  $x = a$  and  $x = b$  of an increasing exponential function  $y = f(x)$  will eventually exceed the average rate of change between  $x = a$  and  $x = b$  of an increasing quadratic function  $y = g(x)$  or an increasing linear function  $y = h(x)$ . So, as  $x$  increases,  $f(x)$  will eventually exceed  $g(x)$  or  $h(x)$ .
- As  $a$  and  $b$  increase, the average rate of change between  $x = a$  and  $x = b$  of an increasing quadratic function  $y = g(x)$  will eventually exceed the average rate of change between  $x = a$  and  $x = b$  of an increasing linear function  $y = h(x)$ . So, as  $x$  increases,  $g(x)$  will eventually exceed  $h(x)$ .

### STUDY TIP

You can explore these concepts using a graphing calculator.

### EXAMPLE 6 Comparing Different Function Types

Let  $x$  represent the number of years since 1900. The function  $C(x) = 12x^2 + 12x + 100$  represents the population of Cedar Ridge. In 1900, Pine Valley had a population of 50 people. Pine Valley's population increased by 9% each year.

- From 1900 to 1950, which town's population had a greater average rate of change?
- Which town will eventually have a greater population? Explain.

### SOLUTION

- Write the function that represents the population of Cedar Ridge and a function to model the population of Pine Valley.

Cedar Ridge:  $C(x) = 12x^2 + 12x + 100$       Quadratic function

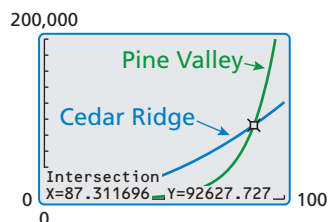
Pine Valley:  $P(x) = 50(1.09)^x$       Exponential function

Find the average rate of change from 1900 to 1950 for the population of each town.

$$\begin{array}{l} \text{Cedar Ridge} \\ \frac{C(50) - C(0)}{50 - 0} = 612 \end{array} \qquad \begin{array}{l} \text{Pine Valley} \\ \frac{P(50) - P(0)}{50 - 0} \approx 73 \end{array}$$

- From 1900 to 1950, the average rate of change of Cedar Ridge's population was greater.

- Because Pine Valley's population is given by an increasing exponential function and Cedar Ridge's population is given by an increasing quadratic function, Pine Valley will eventually have a greater population. Using a graphing calculator, you can see that Pine Valley's population caught up to and exceeded Cedar Ridge's population between  $x = 87$  and  $x = 88$ , which corresponds to 1987.



X	Y1	Y2
87	90173	91972
87.1	90953	92182
87.2	91740	92392
87.3	92534	92603
87.4	93335	92814
87.5	94143	93025
87.6	94958	93236

X=87

### Monitoring Progress

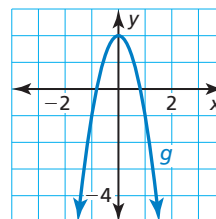
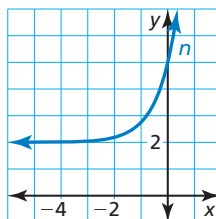
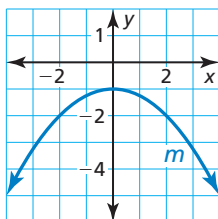
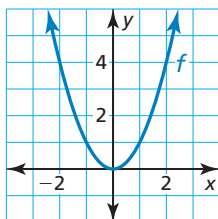


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- From 1950 to 2000, which town's population had a greater average rate of change?

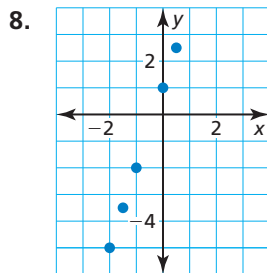
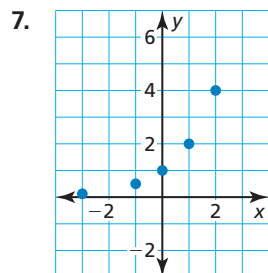
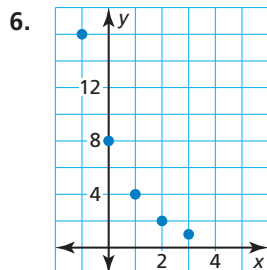
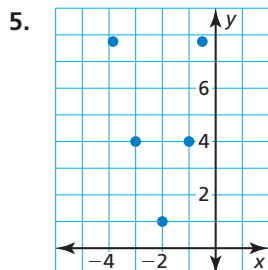
## Vocabulary and Core Concept Check

- WRITING** Name three types of functions that you can use to model data. Describe the equation and graph of each type of function.
- WRITING** How can you decide whether to use a linear, an exponential, or a quadratic function to model a data set?
- VOCABULARY** Describe how to find the average rate of change of a function  $y = f(x)$  between  $x = a$  and  $x = b$ .
- WHICH ONE DOESN'T BELONG?** Which graph does *not* belong with the other three? Explain your reasoning.



## Monitoring Progress and Modeling with Mathematics

In Exercises 5–8, tell whether the points appear to represent a *linear*, an *exponential*, or a *quadratic* function.



In Exercises 9–14, plot the points. Tell whether the points appear to represent a *linear*, an *exponential*, or a *quadratic* function. (See Example 1.)

- $(-2, -1), (-1, 0), (1, 2), (2, 3), (0, 1)$
- $(0, \frac{1}{4}), (1, 1), (2, 4), (3, 16), (-1, \frac{1}{16})$

- $(0, -3), (1, 0), (2, 9), (-2, 9), (-1, 0)$
- $(-1, -3), (-3, 5), (0, -1), (1, 5), (2, 15)$
- $(-4, -4), (-2, -3.4), (0, -3), (2, -2.6), (4, -2)$
- $(0, 8), (-4, 0.25), (-3, 0.4), (-2, 1), (-1, 3)$

In Exercises 15–18, tell whether the table of values represents a *linear*, an *exponential*, or a *quadratic* function. (See Example 2.)

15. 

<b>x</b>	-2	-1	0	1	2
<b>y</b>	0	0.5	1	1.5	2

16. 

<b>x</b>	-1	0	1	2	3
<b>y</b>	0.2	1	5	25	125

17. 

<b>x</b>	2	3	4	5	6
<b>y</b>	2	6	18	54	162

18. 

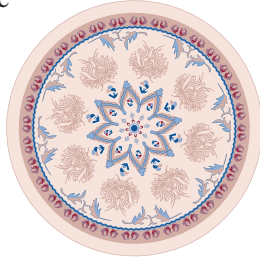
<b>x</b>	-3	-2	-1	0	1
<b>y</b>	2	4.5	8	12.5	18



- 19. MODELING WITH MATHEMATICS** A student takes a subway to a public library. The table shows the distances  $d$  (in miles) the student travels in  $t$  minutes. Let the time  $t$  represent the independent variable. Tell whether the data can be modeled by a *linear*, an *exponential*, or a *quadratic* function. Explain.

Time, $t$	0.5	1	3	5
Distance, $d$	0.335	0.67	2.01	3.35

- 20. MODELING WITH MATHEMATICS** A store sells custom circular rugs. The table shows the costs  $c$  (in dollars) of rugs that have diameters of  $d$  feet. Let the diameter  $d$  represent the independent variable. Tell whether the data can be modeled by a *linear*, an *exponential*, or a *quadratic* function. Explain.



Diameter, $d$	3	4	5	6
Cost, $c$	63.90	113.60	177.50	255.60

In Exercises 21–26, tell whether the data represent a *linear*, an *exponential*, or a *quadratic* function. Then write the function. (See Example 3.)

21.  $(-2, 8), (-1, 0), (0, -4), (1, -4), (2, 0), (3, 8)$   
 22.  $(-3, 8), (-2, 4), (-1, 2), (0, 1), (1, 0.5)$

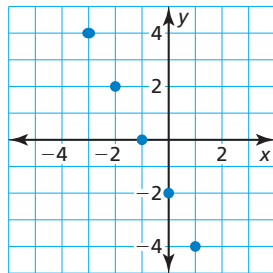
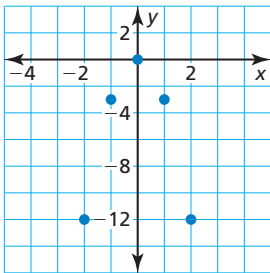
23. 

$x$	-2	-1	0	1	2
$y$	4	1	-2	-5	-8

24. 

$x$	-1	0	1	2	3
$y$	2.5	5	10	20	40

25. 26.



In Exercises 27 and 28, write a recursive rule for the function represented by the table of values. (See Example 4.)

27. 

$x$	0	1	2	3	4
$f(x)$	2	1	2	5	10

28. 

$x$	0	1	2	3	4
$f(x)$	3	6	12	24	48

- 29. ERROR ANALYSIS** Describe and correct the error in determining whether the table represents a linear, an exponential, or a quadratic function.

**X**

$x$	1	2	3	4	5
$y$	3	9	27	81	243

Consecutive  $y$ -values change by a constant amount. So, the table represents a linear function.

- 30. ERROR ANALYSIS** Describe and correct the error in writing the function represented by the table.

**X**

$x$	-3	-2	-1	0	1
$y$	4	0	-2	-2	0

first differences:  $-4, -2, +0, +2$   
 second differences:  $+2, +2, +2$

The table represents a quadratic function.

$$f(x) = a(x - 2)(x - 1)$$

$$4 = a(-3 - 2)(-3 - 1)$$

$$\frac{1}{5} = a$$

$$f(x) = \frac{1}{5}(x - 2)(x - 1)$$

$$= \frac{1}{5}x^2 - \frac{3}{5}x + \frac{2}{5}$$

So, the function is  $f(x) = \frac{1}{5}x^2 - \frac{3}{5}x + \frac{2}{5}$ .

- 31. REASONING** The table shows the numbers of people attending the first five football games at a high school.

Game, $g$	1	2	3	4	5
People, $p$	252	325	270	249	310

- Plot the points. Let the game  $g$  represent the independent variable.
- Can a linear, an exponential, or a quadratic function represent this situation? Explain.

- 32. MODELING WITH MATHEMATICS** The table shows the breathing rates  $y$  (in liters of air per minute) of a cyclist traveling at different speeds  $x$  (in miles per hour).

Speed, $x$	20	21	22	23	24
Breathing rate, $y$	51.4	57.1	63.3	70.3	78.0

- Plot the points. Let the speed  $x$  represent the independent variable. Then determine the type of function that best represents this situation.
- Write a function that models the data.
- Find the breathing rate of a cyclist traveling 18 miles per hour. Round your answer to the nearest tenth.



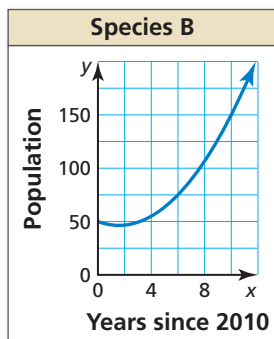
- 33. ANALYZING RATES OF CHANGE** The function  $f(t) = -16t^2 + 48t + 3$  represents the height (in feet) of a volleyball  $t$  seconds after it is hit into the air.

- a. Copy and complete the table.

$t$	0	0.5	1	1.5	2	2.5	3
$f(t)$							

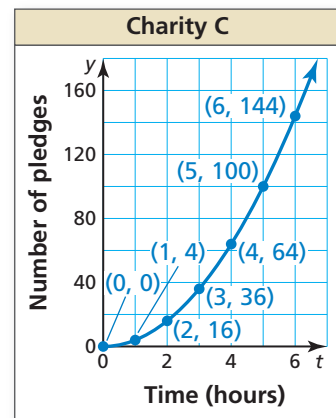
- Plot the ordered pairs and draw a smooth curve through the points.
- Describe where the function is increasing and decreasing.
- Find the average rate of change for each 0.5-second interval in the table. What do you notice about the average rates of change when the function is increasing? decreasing?

- 34. ANALYZING RELATIONSHIPS** The population of Species A in 2010 was 100. The population of Species A increased by 6% every year. Let  $x$  represent the number of years since 2010. The graph shows the population of Species B. Compare the populations of the species by calculating and interpreting the average rates of change from 2010 to 2016. (See Example 5.)



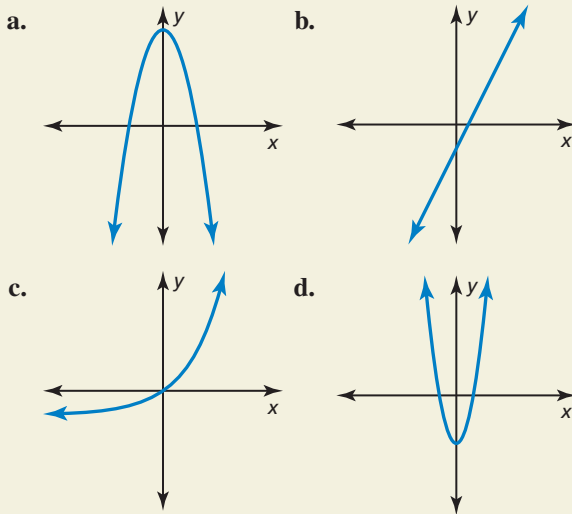
- 35. ANALYZING RELATIONSHIPS** Three charities are holding telethons. Charity A begins with one pledge, and the number of pledges triples each hour. The table shows the numbers of pledges received by Charity B. The graph shows the numbers of pledges received by Charity C.

Time (hours), $t$	Number of pledges, $y$
0	12
1	24
2	36
3	48
4	60
5	72
6	84



- What type of function represents the numbers of pledges received by Charity A? B? C?
  - Find the average rates of change of each function for each 1-hour interval from  $t = 0$  to  $t = 6$ .
  - For which function does the average rate of change increase most quickly? What does this tell you about the numbers of pledges received by the three charities?
- 36. COMPARING FUNCTIONS** Let  $x$  represent the number of years since 1900. The function  $H(x) = 10x^2 + 10x + 500$  represents the population of Oak Hill. In 1900, Poplar Grove had a population of 200 people. Poplar Grove's population increased by 8% each year. (See Example 6.)
- From 1900 to 1950, which town's population had a greater average rate of change?
  - Which town will eventually have a greater population? Explain.
- 37. COMPARING FUNCTIONS** Let  $x$  represent the number of years since 2000. The function  $R(x) = 0.01x^2 + 0.22x + 1.08$  represents the revenue (in millions of dollars) of Company A. In 2000, Company B had a revenue of \$2.12 million. Company B's revenue increased by \$0.32 million each year.
- From 2000 to 2015, which company's revenue had a greater average rate of change?
  - Which company will eventually have a greater revenue? Explain.

38. **HOW DO YOU SEE IT?** Match each graph with its function. Explain your reasoning.



- A.  $y = 2x^2 - 4$       B.  $y = 2x - 1$   
 C.  $y = 2x - 1$       D.  $y = -2x^2 + 4$

39. **REASONING** Explain why the average rate of change of a linear function is constant and the average rate of change of a quadratic or exponential function is not constant.

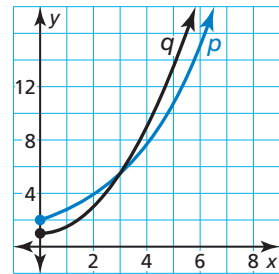
40. **CRITICAL THINKING** In the ordered pairs below, the  $y$ -values are given in terms of  $n$ . Tell whether the ordered pairs represent a *linear*, an *exponential*, or a *quadratic* function. Explain.

- $(1, 3n - 1)$ ,  $(2, 10n + 2)$ ,  $(3, 26n)$ ,  
 $(4, 51n - 7)$ ,  $(5, 85n - 19)$

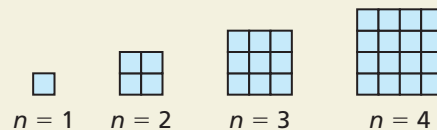
41. **USING STRUCTURE** Write a function that has constant second differences of 3.

42. **CRITICAL THINKING** Is the graph of a set of points enough to determine whether the points represent a linear, an exponential, or a quadratic function? Justify your answer.

43. **MAKING AN ARGUMENT** Function  $p$  is an exponential function and function  $q$  is a quadratic function. Your friend says that after about  $x = 3$ , function  $q$  will always have a greater  $y$ -value than function  $p$ . Is your friend correct? Explain.



44. **THOUGHT PROVOKING** Find three different patterns in the figure. Determine whether each pattern represents a *linear*, an *exponential*, or a *quadratic* function. Write a model for each pattern.



45. **USING TOOLS** The table shows the amount  $a$  (in billions of dollars) United States residents spent on pets or pet-related products and services each year for a 5-year period. Let the year  $x$  represent the independent variable. Using technology, find a function that models the data. How did you choose the model? Predict how much residents will spend on pets or pet-related products and services in Year 7.

Year, $x$	1	2	3	4	5
Amount, $a$	53.1	56.9	61.8	65.7	67.1

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Evaluate the expression. (Section 1.5)

46.  $\sqrt{121}$       47.  $\sqrt[3]{125}$   
 48.  $\sqrt[3]{512}$       49.  $\sqrt[5]{243}$

Find the product. (Section 2.3)

50.  $(x + 6)(x - 6)$       51.  $(2y + 5)(2y - 5)$   
 52.  $(4c - 3d)(4c + 3d)$       53.  $(-3s + 8t)(-3s - 8t)$

## 3.4–3.7 What Did You Learn?

### Core Vocabulary

even function, *p.* 146  
odd function, *p.* 146

vertex form (of a quadratic function), *p.* 148

intercept form, *p.* 154  
focus, *p.* 162  
directrix, *p.* 162

### Core Concepts

#### Section 3.4

Even and Odd Functions, *p.* 146  
Graphing  $f(x) = a(x - h)^2$ , *p.* 147  
Graphing  $f(x) = a(x - h)^2 + k$ , *p.* 148

Writing Quadratic Functions of the Form  
 $f(x) = a(x - h)^2 + k$ , *p.* 149

#### Section 3.5

Graphing  $f(x) = a(x - p)(x - q)$ , *p.* 154  
Factors and Zeros, *p.* 156

Using Characteristics to Graph and Write Quadratic Functions, *p.* 156

#### Section 3.6

Standard Equations of a Parabola with Vertex at the Origin, *p.* 163

Standard Equations of a Parabola with Vertex at  $(h, k)$ , *p.* 164

#### Section 3.7

Linear, Exponential, and Quadratic Functions, *p.* 170  
Differences and Ratios of Functions, *p.* 171

Writing Functions to Model Data, *p.* 172  
Comparing Functions Using Average Rates of Change, *p.* 173

### Mathematical Practices

1. How can you use technology to confirm your answer in Exercise 64 on page 152?
2. How did you use the structure of the equation in Exercise 65 on page 159 to solve the problem?
3. Describe why your answer makes sense considering the context of the data in Exercise 20 on page 176.

### Performance Task:

## Solar Energy

There are many factors that influence the design of a product, such as aesthetics, available space, and special properties. What are the special properties of a parabola that contribute to its use? How do parabolas help create power from solar energy?

To explore the answers to these questions and more, check out the Performance Task and Real-Life STEM video at [BigIdeasMath.com](http://BigIdeasMath.com).



### 3.1 Graphing $f(x) = ax^2$ (pp. 123–128)

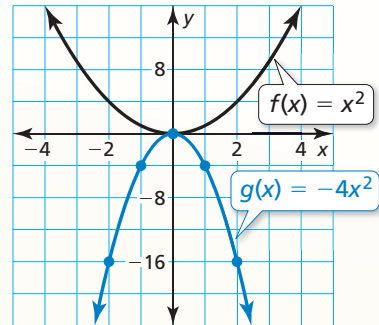
Graph  $g(x) = -4x^2$ . Compare the graph to the graph of  $f(x) = x^2$ .

**Step 1** Make a table of values.

$x$	-2	-1	0	1	2
$g(x)$	-16	-4	0	-4	-16

**Step 2** Plot the ordered pairs.

**Step 3** Draw a smooth curve through the points.

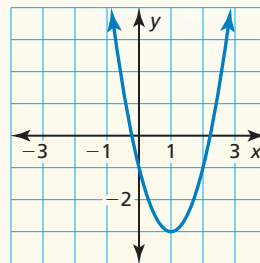


▶ The graphs have the same vertex,  $(0, 0)$ , and the same axis of symmetry,  $x = 0$ , but the graph of  $g$  opens down and is narrower than the graph of  $f$ . So, the graph of  $g$  is a vertical stretch by a factor of 4 and a reflection in the  $x$ -axis of the graph of  $f$ .

**Graph the function. Compare the graph to the graph of  $f(x) = x^2$ .**

1.  $p(x) = 7x^2$       2.  $q(x) = \frac{1}{2}x^2$       3.  $g(x) = -\frac{3}{4}x^2$       4.  $h(x) = -6x^2$

5. Identify characteristics of the quadratic function and its graph.



### 3.2 Graphing $f(x) = ax^2 + c$ (pp. 129–134)

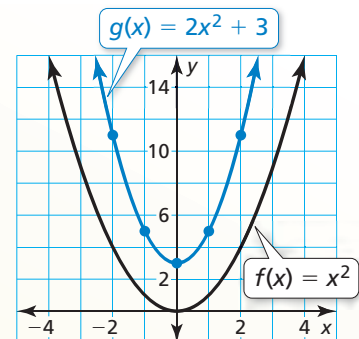
Graph  $g(x) = 2x^2 + 3$ . Compare the graph to the graph of  $f(x) = x^2$ .

**Step 1** Make a table of values.

$x$	-2	-1	0	1	2
$g(x)$	11	5	3	5	11

**Step 2** Plot the ordered pairs.

**Step 3** Draw a smooth curve through the points.



▶ Both graphs open up and have the same axis of symmetry,  $x = 0$ . The graph of  $g$  is narrower, and its vertex,  $(0, 3)$ , is above the vertex of the graph of  $f$ ,  $(0, 0)$ . So, the graph of  $g$  is a vertical stretch by a factor of 2 and a vertical translation 3 units up of the graph of  $f$ .

**Graph the function. Compare the graph to the graph of  $f(x) = x^2$ .**

6.  $g(x) = x^2 + 5$       7.  $h(x) = -x^2 - 4$       8.  $m(x) = -2x^2 + 6$       9.  $n(x) = \frac{1}{3}x^2 - 5$

### 3.3 Graphing $f(x) = ax^2 + bx + c$ (pp. 135–142)

Graph  $f(x) = 4x^2 + 8x - 1$ . Describe the domain and range.

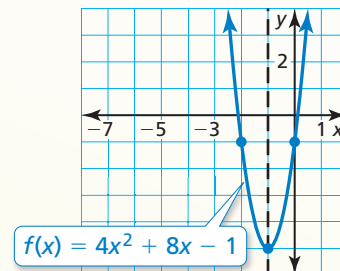
**Step 1** Find and graph the axis of symmetry:  $x = -\frac{b}{2a} = -\frac{8}{2(4)} = -1$ .

**Step 2** Find and plot the vertex. The axis of symmetry is  $x = -1$ . So, the  $x$ -coordinate of the vertex is  $-1$ . The  $y$ -coordinate of the vertex is  $f(-1) = 4(-1)^2 + 8(-1) - 1 = -5$ . So, the vertex is  $(-1, -5)$ .

**Step 3** Use the  $y$ -intercept to find two more points on the graph. Because  $c = -1$ , the  $y$ -intercept is  $-1$ . So,  $(0, -1)$  lies on the graph. Because the axis of symmetry is  $x = -1$ , the point  $(-2, -1)$  also lies on the graph.

**Step 4** Draw a smooth curve through the points.

▶ The domain is all real numbers. The range is  $y \geq -5$ .



**Graph the function. Describe the domain and range.**

10.  $y = x^2 - 2x + 7$       11.  $f(x) = -3x^2 + 3x - 4$       12.  $y = \frac{1}{2}x^2 - 6x + 10$

13. The function  $f(t) = -16t^2 + 88t + 12$  represents the height (in feet) of a pumpkin  $t$  seconds after it is launched from a catapult. When does the pumpkin reach its maximum height? What is the maximum height of the pumpkin?

### 3.4 Graphing $f(x) = a(x - h)^2 + k$ (pp. 145–152)

Determine whether  $f(x) = 2x^2 + 4$  is *even*, *odd*, or *neither*.

$f(x) = 2x^2 + 4$	Write the original function.
$f(-x) = 2(-x)^2 + 4$	Substitute $-x$ for $x$ .
$= 2x^2 + 4$	Simplify.
$= f(x)$	Substitute $f(x)$ for $2x^2 + 4$ .

▶ Because  $f(-x) = f(x)$ , the function is even.

**Determine whether the function is *even*, *odd*, or *neither*.**

14.  $w(x) = 5^x$       15.  $r(x) = -8x$       16.  $h(x) = 3x^2 - 2x$

**Graph the function. Compare the graph to the graph of  $f(x) = x^2$ .**

17.  $h(x) = 2(x - 4)^2$       18.  $g(x) = \frac{1}{2}(x - 1)^2 + 1$       19.  $q(x) = -(x + 4)^2 + 7$

20. Consider the function  $g(x) = -3(x + 2)^2 - 4$ . Graph  $h(x) = g(x - 1)$ .

21. Write a quadratic function whose graph has a vertex of  $(3, 2)$  and passes through the point  $(4, 7)$ .

### 3.5 Graphing $f(x) = a(x - p)(x - q)$ (pp. 153–160)

a. Graph  $f(x) = -(x + 4)(x - 2)$ . Describe the domain and range.

**Step 1** Identify the  $x$ -intercepts. Because the  $x$ -intercepts are  $p = -4$  and  $q = 2$ , plot  $(-4, 0)$  and  $(2, 0)$ .

**Step 2** Find and graph the axis of symmetry.

$$x = \frac{p + q}{2} = \frac{-4 + 2}{2} = -1$$

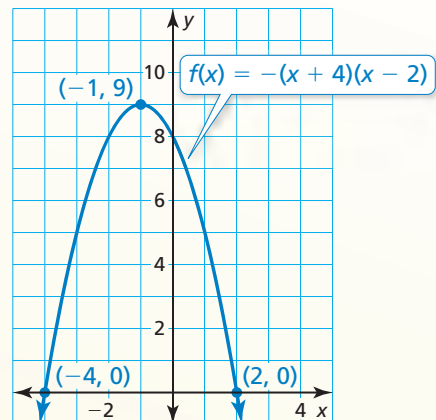
**Step 3** Find and plot the vertex.

The  $x$ -coordinate of the vertex is  $-1$ .  
To find the  $y$ -coordinate of the vertex, substitute  $-1$  for  $x$  and simplify.

$$f(-1) = -(-1 + 4)(-1 - 2) = 9$$

So, the vertex is  $(-1, 9)$ .

**Step 4** Draw a parabola through the vertex and the points where the  $x$ -intercepts occur.



► The domain is all real numbers. The range is  $y \leq 9$ .

b. Use zeros to graph  $h(x) = x^2 - 7x + 6$ .

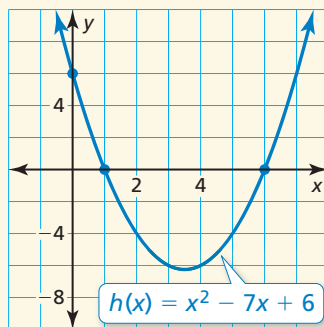
The function is in standard form. The parabola opens up ( $a > 0$ ), and the  $y$ -intercept is 6. So, plot  $(0, 6)$ .

The polynomial that defines the function is factorable. So, write the function in intercept form and identify the zeros.

$$h(x) = x^2 - 7x + 6 \quad \text{Write the function.}$$

$$= (x - 6)(x - 1) \quad \text{Factor the trinomial.}$$

The zeros of the function are 1 and 6. So, plot  $(1, 0)$  and  $(6, 0)$ . Draw a parabola through the points.



**Graph the quadratic function. Label the vertex, axis of symmetry, and  $x$ -intercepts. Describe the domain and range of the function.**

22.  $y = (x - 4)(x + 2)$       23.  $f(x) = -3(x + 3)(x + 1)$       24.  $y = x^2 - 8x + 15$

**Use zeros to graph the function.**

25.  $y = -2x^2 + 6x + 8$       26.  $f(x) = x^2 + x - 2$       27.  $f(x) = 2x^2 - 18$

28. Write a quadratic function in standard form whose graph passes through  $(4, 0)$  and  $(6, 0)$ .

### 3.6 Focus of a Parabola (pp. 161–168)

- a. Identify the focus, directrix, and axis of symmetry of  $8x = y^2$ . Graph the equation.

**Step 1** Rewrite the equation in standard form.

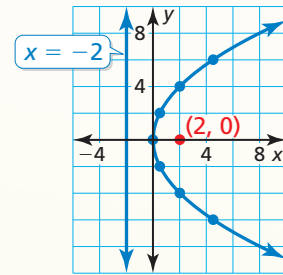
$$8x = y^2 \quad \text{Write the original equation.}$$

$$x = \frac{1}{8}y^2 \quad \text{Divide each side by 8.}$$

**Step 2** Identify the focus, directrix, and axis of symmetry. The equation has the form  $x = \frac{1}{4p}y^2$ , where  $p = 2$ . The focus is  $(p, 0)$ , or  $(2, 0)$ . The directrix is  $x = -p$ , or  $x = -2$ . Because  $y$  is squared, the axis of symmetry is the  $x$ -axis.

**Step 3** Use a table of values to graph the equation. Notice that it is easier to substitute  $y$ -values and solve for  $x$ .

<b>y</b>	0	$\pm 2$	$\pm 4$	$\pm 6$
<b>x</b>	0	0.5	2	4.5

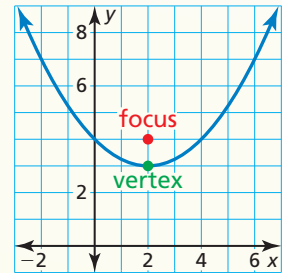


- b. Write an equation of the parabola shown.

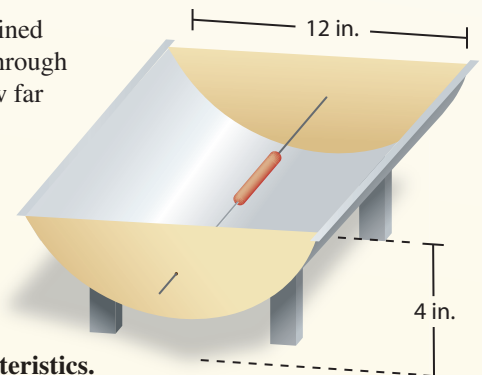
Because the vertex is not at the origin and the axis of symmetry is vertical, the equation has the form  $y = \frac{1}{4p}(x - h)^2 + k$ . The vertex  $(h, k)$  is  $(2, 3)$  and the focus  $(h, k + p)$  is  $(2, 4)$ , so  $h = 2$ ,  $k = 3$ , and  $p = 1$ . Substitute these values to write an equation of the parabola.

$$y = \frac{1}{4(1)}(x - 2)^2 + 3 = \frac{1}{4}(x - 2)^2 + 3$$

► An equation of the parabola is  $y = \frac{1}{4}(x - 2)^2 + 3$ .



29. You can make a solar hot-dog cooker by shaping foil-lined cardboard into a parabolic trough and passing a wire through the focus of each end piece. For the trough shown, how far from the bottom should the wire be placed?



30. Graph the equation  $36y = x^2$ . Identify the focus, directrix, and axis of symmetry.

**Write an equation of the parabola with the given characteristics.**

31. vertex:  $(0, 0)$   
directrix:  $x = 2$

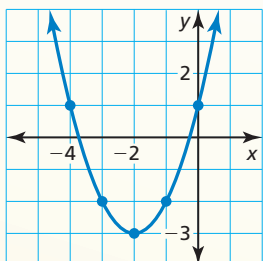
32. focus:  $(2, 2)$   
vertex:  $(2, 6)$



### 3.7 Comparing Linear, Exponential, and Quadratic Functions (pp. 169–178)

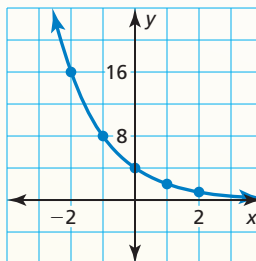
Tell whether the data represent a *linear*, an *exponential*, or a *quadratic* function.

- a.  $(-4, 1), (-3, -2), (-2, -3)$   
 $(-1, -2), (0, 1)$



▶ The points appear to represent a quadratic function.

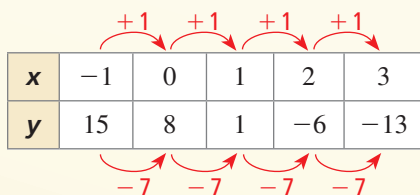
- b.  $(-2, 16), (-1, 8), (0, 4)$   
 $(1, 2), (2, 1)$



▶ The points appear to represent an exponential function.

c.

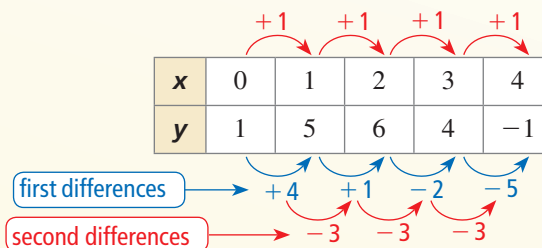
<b>x</b>	-1	0	1	2	3
<b>y</b>	15	8	1	-6	-13



▶ The first differences are constant. So, the table represents a linear function.

d.

<b>x</b>	0	1	2	3	4
<b>y</b>	1	5	6	4	-1



▶ The second differences are constant. So, the table represents a quadratic function.

33. Tell whether the table of values represents a *linear*, an *exponential*, or a *quadratic* function. Then write the function.

<b>x</b>	-1	0	1	2	3
<b>y</b>	512	128	32	8	2

34. Write a recursive rule for the function represented by the table of values.

<b>x</b>	0	1	2	3	4
<b>f(x)</b>	5	6	11	20	33

35. The balance  $y$  (in dollars) of your savings account after  $t$  years is represented by  $y = 200(1.1)^t$ . The beginning balance of your friend's account is \$250, and the balance increases by \$20 each year. Compare the account balances by calculating and interpreting the average rates of change from  $t = 2$  to  $t = 7$ .

# 3 Chapter Test

Graph the function. Compare the graph to the graph of  $f(x) = x^2$ .

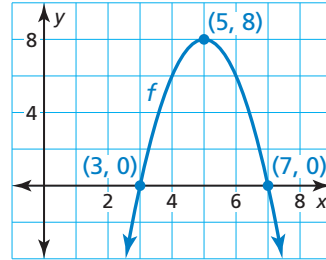
1.  $h(x) = 2x^2 - 3$

2.  $g(x) = -\frac{1}{2}x^2$

3.  $p(x) = \frac{1}{2}(x + 1)^2 - 1$

4. Consider the graph of the function  $f$ .

- Find the domain, range, and zeros of the function.
- Write the function  $f$  in standard form.
- Compare the graph of  $f$  to the graph of  $g(x) = x^2$ .
- Graph  $h(x) = f(x - 6)$ .



Use zeros to graph the function. Describe the domain and range of the function.

5.  $f(x) = 2x^2 - 8x + 8$

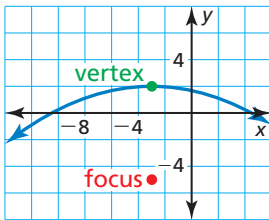
6.  $y = -(x + 5)(x - 1)$

7.  $h(x) = 16x^2 - 4$

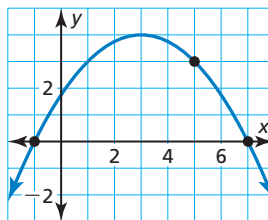
8. Identify an focus, directrix, and axis of symmetry of  $x = 2y^2$ . Graph the equation.

Write an equation of the parabola. Justify your answer.

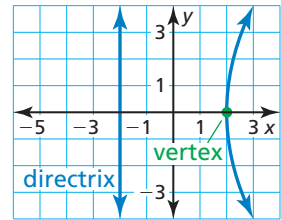
9.



10.



11.



Tell whether the table of values represents a *linear*, an *exponential*, or a *quadratic* function. Explain your reasoning. Then write the function.

12.

$x$	-1	0	1	2	3
$y$	4	8	16	32	64

13.

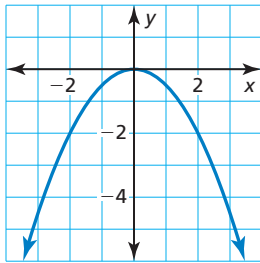
$x$	-2	-1	0	1	2
$y$	-8	-2	0	-2	-8

- You are playing tennis with a friend. The path of the tennis ball after you return a serve can be modeled by the function  $y = -0.005x^2 + 0.17x + 3$ , where  $x$  is the horizontal distance (in feet) from where you hit the ball and  $y$  is the height (in feet) of the ball.
  - What is the maximum height of the tennis ball?
  - You are standing 30 feet from the net, which is 3 feet high. Will the ball clear the net? Explain your reasoning.
- Find values of  $a$ ,  $b$ , and  $c$  so that the function  $f(x) = ax^2 + bx + c$  is (a) even, (b) odd, and (c) neither even nor odd.
- Consider the function  $f(x) = x^2 + 4$ . Find the average rate of change from  $x = 0$  to  $x = 1$ , from  $x = 1$  to  $x = 2$ , and from  $x = 2$  to  $x = 3$ . What do you notice about the average rates of change when the function is increasing?

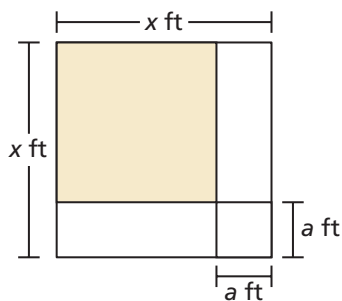
# 3

## Cumulative Assessment

1. Which function is represented by the graph?



- (A)  $y = \frac{1}{2}x^2$ 
   
  (B)  $y = 2x^2$
- (C)  $y = -\frac{1}{2}x^2$ 
   
  (D)  $y = -2x^2$
2. A linear function  $f$  has the following values at  $x = 2$  and  $x = 5$ .
- $$f(2) = 6, f(5) = 3$$
- What is the value of the inverse of  $f$  at  $x = 4$ ?
3. The function  $f(t) = -16t^2 + v_0t + s_0$  represents the height (in feet) of a ball  $t$  seconds after it is thrown from an initial height  $s_0$  (in feet) with an initial vertical velocity  $v_0$  (in feet per second). The ball reaches its maximum height after  $\frac{7}{8}$  second when it is thrown with an initial vertical velocity of \_\_\_\_\_ feet per second.
4. Order the following parabolas from widest to narrowest.
- A.** focus:  $(0, -3)$ ; directrix:  $y = 3$ 
   
 **B.**  $y = \frac{1}{16}x^2 + 4$
- C.**  $x = \frac{1}{8}y^2$ 
   
 **D.**  $y = \frac{1}{4}(x - 2)^2 + 3$
5. Which polynomial represents the area (in square feet) of the shaded region of the figure?



- (A)  $a^2 - x^2$ 
   
  (B)  $x^2 - a^2$
- (C)  $x^2 - 2ax + a^2$ 
   
  (D)  $x^2 + 2ax + a^2$

6. Consider the functions represented by the tables.

<b>x</b>	0	1	2	3
<b>p(x)</b>	-4	-16	-28	-40

<b>x</b>	1	2	3	4
<b>r(x)</b>	0	15	40	75

<b>x</b>	1	2	3	4
<b>s(x)</b>	72	36	18	9

<b>x</b>	1	3	5	7
<b>t(x)</b>	3	-5	-21	-45

- a. Classify each function as *linear*, *exponential*, or *quadratic*.
- b. Order the functions from least to greatest according to the average rates of change between  $x = 1$  and  $x = 3$ .
7. Which expressions are equivalent to  $14x - 15 + 8x^2$ ?

$$(4x + 3)(2x - 5)$$

$$(4x - 3)(2x + 5)$$

$$(4x + 3)(2x + 5)$$

$$(4x - 3)(2x - 5)$$

$$8x^2 - 14x - 15$$

$$8x^2 + 14x - 15$$

8. Complete each function using the symbols  $+$  or  $-$ , so that the graph of the quadratic function satisfies the given conditions.
- a.  $f(x) = 5(x \quad 3)^2 \quad 4$ ; vertex:  $(-3, 4)$
- b.  $g(x) = -(x \quad 2)(x \quad 8)$ ;  $x$ -intercepts:  $-8$  and  $2$
- c.  $h(x) = \quad 3x^2 \quad 6$ ; range:  $y \geq -6$
- d.  $j(x) = \quad 4(x \quad 1)(x \quad 1)$ ; range:  $y \leq 4$
9. Which expressions are equivalent to  $(b^{-5})^{-4}$ ?

$$b^{-20}$$

$$b^{-6}b^{-14}$$

$$(b^{-4})^{-5}$$

$$b^{-9}$$

$$(b^{-2})^{-7}$$

$$b^{-5}b^{-4}$$

$$(b^{10})^2$$

$$b^{20}$$

$$(b^{-12})^3$$

$$b^{12}b^8$$