1.6 Exponential Functions

Essential Question

Essential Question What are some of the characteristics of the graph of an exponential function?

You can use a graphing calculator to evaluate an exponential function. For example, consider the exponential function $f(x) = 2^x$.

Function Value	Graphing Calculator Keystrokes	Display
$f(-3.1) = 2^{-3.1}$	2 [^] [(–)] 3.1 [ENTER]	0.1166291
$f\left(\frac{2}{3}\right) = 2^{2/3}$	2 [^] [(] 2 [÷] 3 [)] [ENTER]	1.5874011

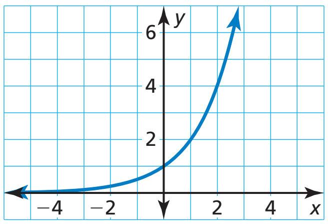
EXPLORATION 1 Identifying Graphs of Exponential Functions

Work with a partner. Match each exponential function with its graph. Use a table of values to sketch the graph of the function, if necessary.

a. $f(x) = 2^x$ **b.** $f(x) = 3^x$ **c.** $f(x) = 4^x$

d. $f(x) = \left(\frac{1}{2}\right)^x$ **e.** $f(x) = \left(\frac{1}{3}\right)^x$ **f.** $f(x) = \left(\frac{1}{4}\right)^x$

A.



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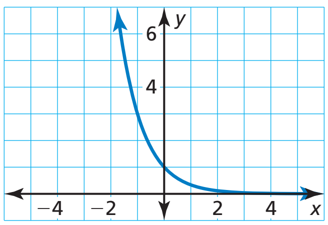

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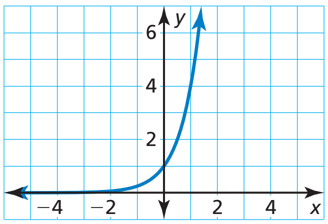

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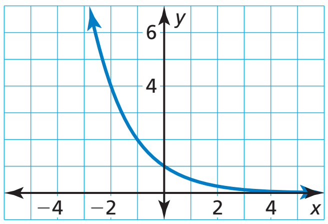

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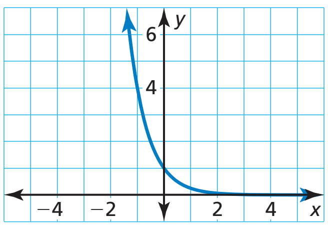

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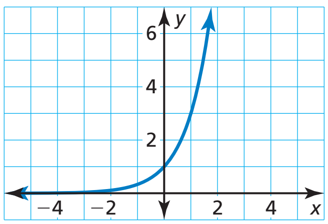


CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to justify your conclusions and communicate them to others.

EXPLORATION 2 Characteristics of Graphs of Exponential Functions

Work with a partner. Use the graphs in Exploration 1 to determine the domain, range, and y-intercept of the graph of $f(x) = b^x$, where b is a positive real number other than 1. Explain your reasoning.

Communicate Your Answer

3. What are some of the characteristics of the graph of an exponential function?

4. In Exploration 2, is it possible for the graph of $f(x) = b^x$ to have an x-intercept? Explain your reasoning.

1.6 Lesson

Core Vocabulary

exponential function, *p. 42*
exponential growth function, *p. 42*
growth factor, *p. 42*
asymptote, *p. 42*
exponential decay function, *p. 42*
decay factor, *p. 42*
recursive rule for an exponential function, *p. 44*

Previous
sequences
properties of exponents

What You Will Learn

- Graph exponential growth and decay functions.
- Write exponential models and recursive rules.
- Rewrite exponential functions.

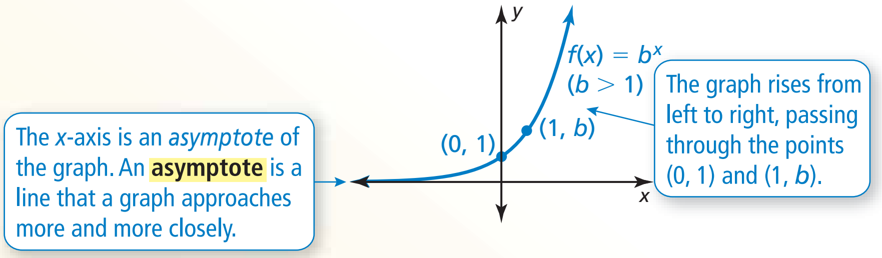
Exponential Growth and Decay Functions

An **exponential function** has the form $y = ab^x$, where $a \neq 0$ and the base b is a positive real number other than 1. If $a > 0$ and $b > 1$, then $y = ab^x$ is an **exponential growth function**, and b is called the **growth factor**. The simplest type of exponential growth function has the form $y = b^x$.

Core Concept

Parent Function for Exponential Growth Functions

The function $f(x) = b^x$, where $b > 1$, is the parent function for the family of exponential growth functions with base b. The graph shows the general shape of an exponential growth function.



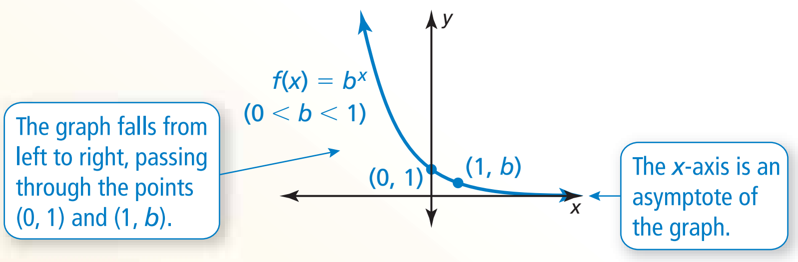
The domain of $f(x) = b^x$ is all real numbers. The range is $y > 0$.

If $a > 0$ and $0 < b < 1$, then $y = ab^x$ is an **exponential decay function**, and b is called the **decay factor**.

Core Concept

Parent Function for Exponential Decay Functions

The function $f(x) = b^x$, where $0 < b < 1$, is the parent function for the family of exponential decay functions with base b. The graph shows the general shape of an exponential decay function.



The domain of $f(x) = b^x$ is all real numbers. The range is $y > 0$.

EXAMPLE 1 Graphing Exponential Growth and Decay Functions

Determine whether each function represents *exponential growth* or *exponential decay*. Then graph the function.

a. $y = 2^x$ **b.** $y = \left(\frac{1}{2}\right)^x$

STUDY TIP

When graphing exponential functions, you connect the points with a smooth curve because $y = ab^x$ is defined for rational x-values. This should make sense from your study of rational exponents. For instance, in Example 1(a), when $x = \frac{1}{2}$,

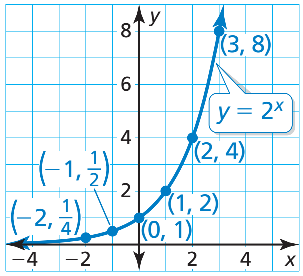
$$y = 2^{1/2} = \sqrt{2} \approx 1.4.$$

SOLUTION

a. Step 1 Identify the value of the base. The base, 2, is greater than 1, so the function represents exponential growth.

Step 2 Make a table of values.

x	-2	-1	0	1	2	3
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8



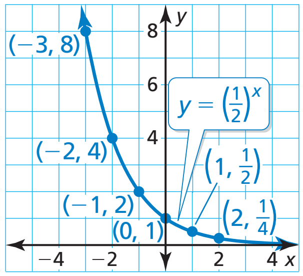
Step 3 Plot the points from the table.

Step 4 Draw, from *left to right*, a smooth curve that begins just above the x-axis, passes through the plotted points, and moves up to the right.

b. Step 1 Identify the value of the base. The base, $\frac{1}{2}$, is greater than 0 and less than 1, so the function represents exponential decay.

Step 2 Make a table of values.

x	-3	-2	-1	0	1	2
y	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$



Step 3 Plot the points from the table.

Step 4 Draw, from *right to left*, a smooth curve that begins just above the x-axis, passes through the plotted points, and moves up to the left.

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Determine whether the function represents *exponential growth* or *exponential decay*. Then graph the function.

1. $y = 4^x$

2. $y = \left(\frac{2}{3}\right)^x$

3. $f(x) = (0.25)^x$

4. $f(x) = (1.5)^x$

Writing Exponential Models and Recursive Rules

Some real-life quantities increase or decrease by a fixed percent each year (or some other time period). The amount y of such a quantity after t years can be modeled by one of these equations.

Exponential Growth Model

$$y = a(1 + r)^t$$

Exponential Decay Model

$$y = a(1 - r)^t$$

Note that a is the initial amount and r is the percent increase or decrease written as a decimal. The quantity $1 + r$ is the growth factor, and $1 - r$ is the decay factor.

EXAMPLE 2 Writing an Exponential Model

The population of a country was about 6.09 million on January 1, 2000. The population at the beginning of each subsequent year increased by about 1.18%.

a. Write an exponential growth model that represents the population y (in millions) t years after January 1, 2000. Find and interpret the y-value when $t = 7.5$.

b. Estimate when the population was 7 million.

SOLUTION

a. The initial amount is $a = 6.09$, and the percent increase is $r = 0.0118$. So, the exponential growth model is

$$y = a(1 + r)^t \qquad \text{Write exponential growth model.}$$

$$= 6.09(1 + 0.0118)^t \qquad \text{Substitute 6.09 for } a \text{ and 0.0118 for } r.$$

$$= 6.09(1.0118)^t. \qquad \text{Simplify.}$$

Using this model, you can estimate the *midyear* population in 2007 ($t = 7.5$) to be $y = 6.09(1.0118)^{7.5} \approx 6.65$ million.

X	Y1
11.4	6.9614
11.5	6.9696
11.6	6.9778
11.7	6.9859
11.8	6.9941
11.9	7.0024
12	7.0106

X=11.9

b. Use the *table* feature of a graphing calculator to determine that $y \approx 7$ when $t \approx 11.9$. So, the population was about 7 million near the end of 2011.

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5. **WHAT IF?** Assume the population increased by 1.5% each year. Write an equation to model this situation. Estimate when the population was 7 million.

In real-life situations, you can also show exponential relationships using *recursive rules.*

REMEMBER

Recall that for a sequence, a recursive rule gives the beginning term(s) of the sequence and a recursive equation that tells how a_n is related to one or more preceding terms.

A **recursive rule** for an exponential function gives the initial value of the function $f(0)$, and a recursive equation that tells how a value $f(n)$ is related to a preceding value $f(n - 1)$.

Core Concept

Writing Recursive Rules for Exponential Functions

An exponential function of the form $f(x) = ab^x$ is written using a recursive rule as follows.

Recursive Rule $f(0) = a,\ f(n) = r \cdot f(n - 1)$

where $a \neq 0$, r is the common ratio, and n is a natural number

Example $y = 6(3)^x$ can be written as $f(0) = 6,\ f(n) = 3 \cdot f(n - 1)$

initial value

common ratio

Notice that the base b of the exponential function is the common ratio r in the recursive equation. Also, notice the value of a in the exponential function is the initial value of the recursive rule.

EXAMPLE 3 Writing a Recursive Rule for an Exponential Function

Write a recursive rule for the function you wrote in Example 2.

SOLUTION

The function $y = 6.09(1.0118)^t$ is exponential with initial value $f(0) = 6.09$ and common ratio $r = 1.0118$. So, a recursive equation is

$f(n) = r \cdot f(n-1)$ — Recursive equation for exponential functions

$= 1.0118 \cdot f(n-1)$. — Substitute 1.0118 for r.

▶ A recursive rule for the exponential function is $f(0) = 6.09$, $f(n) = 1.0118 \cdot f(n-1)$.

STUDY TIP

Notice that the domain consists of the natural numbers when written recursively.

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Write an recursive rule for the exponential function.

6. $f(x) = 4(7)^x$

7. $y = 9\left(\frac{1}{3}\right)^t$

Rewriting Exponential Functions

EXAMPLE 4 Rewriting Exponential Functions

Rewrite each function to determine whether it represents *exponential growth* or *exponential decay*. Then identify the percent rate of change.

a. $y = 100(0.96)^{t/4}$

b. $f(t) = (1.1)^{t-3}$

STUDY TIP

You can rewrite exponential expressions and functions using the properties of exponents. Changing the form of an exponential function can reveal important attributes of the function.

SOLUTION

a. $y = 100(0.96)^{t/4}$ — Write the function.

$= 100(0.96^{1/4})^t$ — Power of a Power Property

$\approx 100(0.99)^t$ — Evaluate the power.

▶ So, the function represents exponential decay. Use the decay factor $1 - r \approx 0.99$ to find the rate of decay $r \approx 0.01$, or about 1%.

b. $f(t) = (1.1)^{t-3}$ — Write the function.

$= \dfrac{(1.1)^t}{(1.1)^3}$ — Quotient of Powers Property

$\approx 0.75(1.1)^t$ — Evaluate the power and simplify.

▶ So, the function represents exponential growth. Use the growth factor $1 + r = 1.1$ to find the rate of growth $r = 0.1$, or 10%.

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Rewrite the function to determine whether it represents *exponential growth* or *exponential decay*. Then identify the percent rate of change.

8. $f(t) = 3(1.02)^{10t}$

9. $y = (0.95)^{t+2}$

EXAMPLE 5 Solving a Real-Life Problem

The value of a car is \$21,500. It loses 12% of its value every year. (a) Write a function that represents the value y (in dollars) of the car after t years. (b) Find the approximate monthly percent decrease in value. (c) Graph the function from part (a). Use the graph to estimate the value of the car after 6 years.

SOLUTION

1. **Understand the Problem** You know the value of the car and its annual percent decrease in value. You are asked to write a function that represents the value of the car over time and approximate the monthly percent decrease in value. Then graph the function and use the graph to estimate the value of the car in the future.
2. **Make a Plan** Use the initial amount and the annual percent decrease in value to write an exponential decay function. Note that the annual percent decrease represents the rate of decay. Rewrite the function using the properties of exponents to approximate the monthly percent decrease (rate of decay). Then graph the original function and use the graph to estimate the y-value when the t-value is 6.
3. **Solve the Problem**

a. The initial value is \$21,500, and the rate of decay is 12%, or 0.12.

$$y = a(1 - r)^t \qquad \text{Write exponential decay model.}$$
$$= 21{,}500(1 - 0.12)^t \qquad \text{Substitute 21,500 for } a \text{ and 0.12 for } r.$$
$$= 21{,}500(0.88)^t \qquad \text{Simplify.}$$

▶ The value of the car can be represented by $y = 21{,}500(0.88)^t$.

b. Use the fact that $t = \frac{1}{12}(12t)$ and the properties of exponents to rewrite the function in a form that reveals the monthly rate of decay.

$$y = 21{,}500(0.88)^t \qquad \text{Write the original function.}$$
$$= 21{,}500(0.88)^{(1/12)(12t)} \qquad \text{Rewrite the exponent.}$$
$$= 21{,}500(0.88^{1/12})^{12t} \qquad \text{Power of a Power Property}$$
$$\approx 21{,}500(0.989)^{12t} \qquad \text{Evaluate the power.}$$

Use the decay factor $1 - r \approx 0.989$ to find the rate of decay $r \approx 0.011$.

▶ So, the monthly percent decrease is about 1.1%.

c. From the graph, you can see that the y-value is about 10,000 when $t = 6$.

▶ So, the value of the car is about \$10,000 after 6 years.

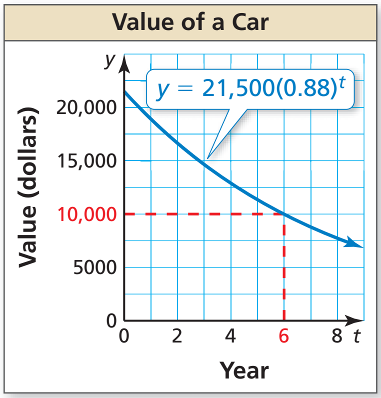
4. **Look Back** To check that the monthly percent decrease is reasonable, multiply it by 12 to see if it is close in value to the annual percent decrease of 12%.

$$1.1\% \times 12 = 13.2\% \qquad \text{13.2\% is close to 12\%, so 1.1\% is reasonable.}$$

When you evaluate $y = 21{,}500(0.88)^t$ for $t = 6$, you get about \$9985. So, \$10,000 is a reasonable estimation.

REASONING QUANTITATIVELY

The decay factor, 0.88, tells you what fraction of the car's value *remains* each year. The rate of decay, 12%, tells you how much value the car *loses* each year. In real life, the percent decrease in value of an asset is the *depreciation rate*.



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10. **WHAT IF?** The car loses 9% of its value every year. (a) Write a function that represents the value y (in dollars) of the car after t years. (b) Find the approximate monthly percent decrease in value. (c) Graph the function from part (a). Use the graph to estimate the value of the car after 12 years. Round your answer to the nearest thousand.

1.6 Exercises

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Vocabulary and Core Concept Check

1. **VOCABULARY** In the exponential growth model $y = 2.4(1.5)^x$, identify the initial amount, the growth factor, and the percent increase.

2. **WHICH ONE DOESN'T BELONG?** Which characteristic of an exponential decay function does *not* belong with the other three? Explain your reasoning.

base of 0.8	decay factor of 0.8
decay rate of 20%	80% decrease

Monitoring Progress and Modeling with Mathematics

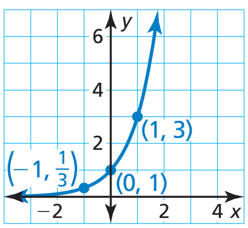
In Exercises 3–8, evaluate the expression for (a) $x = -2$ and (b) $x = 3$.

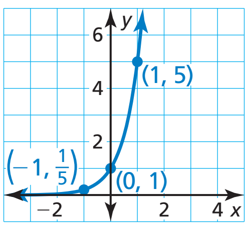
3. 2^x
4. 4^x
5. $8 \cdot 3^x$
6. $6 \cdot 2^x$
7. $5 + 3^x$
8. $2^x - 2$

In Exercises 9–18, determine whether the function represents *exponential growth* or *exponential decay*. Then graph the function. *(See Example 1.)*

9. $y = 6^x$
10. $y = 7^x$
11. $y = \left(\frac{1}{6}\right)^x$
12. $y = \left(\frac{1}{8}\right)^x$
13. $y = \left(\frac{4}{3}\right)^x$
14. $y = \left(\frac{2}{5}\right)^x$
15. $y = (1.2)^x$
16. $y = (0.75)^x$
17. $y = (0.6)^x$
18. $y = (1.8)^x$

ANALYZING RELATIONSHIPS **In Exercises 19 and 20, use the graph of $f(x) = b^x$ to identify the value of the base b.**

19. 

20. 

21. **MODELING WITH MATHEMATICS** The population of Austin, Texas, was about 494,000 at the beginning of a decade. The population increased by about 3% each year. *(See Example 2.)*
    a. Write an exponential growth model that represents the population y (in thousands) t years after the beginning of the decade. Find and interpret the y-value when $t = 10$.
    b. Estimate when the population was about 590,000.

22. **MODELING WITH MATHEMATICS** You take a 325 milligram dosage of ibuprofen. During each subsequent hour, the amount of medication in your bloodstream decreases by about 29% each hour.
    a. Write an exponential decay model giving the amount y (in milligrams) of ibuprofen in your bloodstream t hours after the initial dose. Find and interpret the y-value when $t = 1.5$.
    b. Estimate how long it takes for you to have 100 milligrams of ibuprofen in your bloodstream.

23. **ERROR ANALYSIS** You invest \$500 in the stock of a company. The value of the stock decreases 2% each year. Describe and correct the error in writing a model for the value of the stock after t years.

$$y = \begin{pmatrix}\text{Initial}\\\text{amount}\end{pmatrix}\begin{pmatrix}\text{Decay}\\\text{factor}\end{pmatrix}^t$$

$$y = 500(0.02)^t$$

24. **USING EQUATIONS** Complete a table of values for $0 \le n \le 5$ using the given recursive rule of an exponential function.

$$f(0) = 4,\ f(n) = 3 \cdot f(n-1)$$

In Exercises 25–32, write a recursive rule for the exponential function. *(See Example 3.)*

25. $y = 3(7)^x$

26. $y = 25(0.2)^x$

27. $y = 12(0.5)^t$

28. $y = 19(4)^t$

29. $g(x) = 0.5(3)^x$

30. $m(t) = \frac{1}{3}(2)^t$

31. $f(x) = 4\left(\frac{1}{6}\right)^x$

32. $f(t) = 0.25\left(\frac{2}{3}\right)^t$

In Exercises 33–36, show that an exponential model fits the data. Then write a recursive rule that models the data.

33.

n	0	1	2	3	4	5
$f(n)$	0.75	1.5	3	6	12	24

34.

n	0	1	2	3	4
$f(n)$	2	8	32	128	512

35.

n	0	1	2	3	4	5
$f(n)$	96	48	24	12	6	3

36.

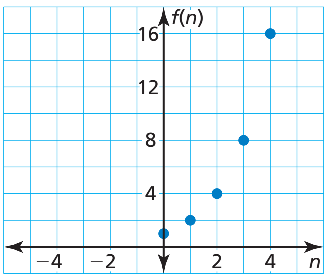
n	0	1	2	3	4	5
$f(n)$	162	54	18	6	2	$\frac{2}{3}$

In Exercises 37 and 38, write an exponential function for the recursive rule.

37. $f(0) = 24, f(n) = 0.1 \cdot f(n-1)$

38. $f(0) = \frac{1}{2}, f(n) = \frac{5}{2} \cdot f(n-1)$

39. **PROBLEM SOLVING** Describe a real-life situation that can be represented by the graph. Write a recursive rule that models the data. Then find and interpret $f(6)$.



40. **PROBLEM SOLVING** The cross-sectional area of a tree 4.5 feet from the ground is called its *basal area*. The table shows the basal areas (in square inches) of Tree A over time.

Year	0	1	2	3	4
Basal area	120	132	145.2	159.7	175.7

a. Write recursive rules that represent the basal areas of the trees after t years.

b. Which tree has a greater initial basal area? a greater basal area growth rate? Use the recursive rules you wrote in part (a) to justify your answers.

c. Use a graph to represent the growth of the trees over the interval $0 \le t \le 10$. Does the graph support your answers in part (b)? Explain. Then make an additional observation from the graph.

In Exercises 41–44, determine whether the function represents *exponential growth* or *exponential decay*. Then identify the percent rate of change.

41. $y = 5(0.6)^t$

42. $y = 10(1.07)^t$

43. $f(t) = 200\left(\frac{4}{3}\right)^t$

44. $f(t) = 0.8\left(\frac{1}{4}\right)^t$

45. **PROBLEM SOLVING** A website recorded the number y of referrals it received from social media websites over a 10-year period. The results can be modeled by $y = 2500(1.50)^t$, where t is the year and $0 \le t \le 9$.

a. Describe the real-life meaning of 2500 and 1.50 in the model.

b. What is the annual percent increase? Explain.

46. **PROBLEM SOLVING** The population p of a small town after x years can be modeled by the function $p = 6850(0.97)^x$.

a. What is the annual percent decrease? Explain.

b. What is the average rate of change in the population over the first 6 years? Justify your answer.

JUSTIFYING STEPS **In Exercises 47 and 48, justify each step in rewriting the exponential function.**

47. $y = a(3)^{t/14}$ Write original function.

$= a[(3)^{1/14}]^t$

$\approx a(1.0816)^t$

$= a(1 + 0.0816)^t$

48. $y = a(0.1)^{t/3}$ Write original function.

$= a[(0.1)^{1/3}]^t$

$\approx a(0.4642)^t$

$= a(1 - 0.5358)^t$

In Exercises 49–56, rewrite the function to determine whether it represents *exponential growth* or *exponential decay.* Then identify the percent rate of change. *(See Example 4.)*

49. $y = (0.9)^{t-4}$

50. $y = (1.4)^{t+8}$

51. $y = 2(1.06)^{9t}$

52. $y = 5(0.82)^{t/5}$

53. $x(t) = (1.45)^{t/2}$

54. $f(t) = 0.4(1.16)^{t-1}$

55. $b(t) = 4(0.55)^{t+3}$

56. $r(t) = (0.88)^{4t}$

In Exercises 57–62, rewrite the function in the form $y = a(1 + r)^t$ or $y = a(1 - r)^t$. Then state the growth or decay rate.

57. $y = a(2)^{t/3}$

58. $y = a(0.5)^{t/12}$

59. $y = a\left(\frac{2}{3}\right)^{t/10}$

60. $y = a\left(\frac{5}{4}\right)^{t/22}$

61. $y = a(2)^{8t}$

62. $y = a\left(\frac{1}{3}\right)^{3t}$

63. **PROBLEM SOLVING** When a plant or animal dies, it stops acquiring carbon-14 from the atmosphere. The amount y (in grams) of carbon-14 in the body of an organism after t years is $y = a(0.5)^{t/5730}$, where a is the initial amount (in grams). What percent of the carbon-14 is released each year?

64. **PROBLEM SOLVING** The number y of duckweed fronds in a pond after t days is $y = a(1230.25)^{t/16}$, where a is the initial number of fronds. By what percent does the duckweed increase each day?

65. **PROBLEM SOLVING** A city has a population of 25,000. The population is expected to increase by 5.5% annually for the next decade. *(See Example 5.)*

a. Write a function that represents the population y after t years.

b. Find the approximate monthly percent increase in population.

c. Graph the function from part (a). Use the graph to estimate the population after 4 years.

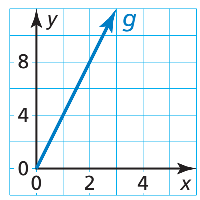
66. **PROBLEM SOLVING** Plutonium-238 is a material that generates steady heat due to decay and is used in power systems for some spacecraft. The function $y = a(0.5)^{t/x}$ represents the amount y of a substance remaining after t years, where a is the initial amount and x is the length of the half-life (in years).

a. A scientist is studying a 3-gram sample. Write a function that represents the amount y of plutonium-238 after t years.

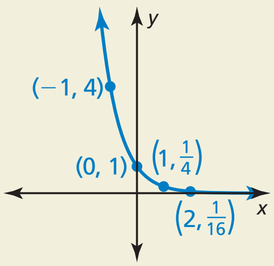
b. What is the yearly percent decrease of plutonium-238?

c. Graph the function from part (a). Use the graph to estimate the amount remaining after 12 years.

67. **MAKING AN ARGUMENT** Your friend says the graph of $f(x) = 2^x$ increases at a faster rate than the graph of $g(x) = 4x$ when $x \geq 0$. Is your friend correct? Explain your reasoning.



68. HOW DO YOU SEE IT? Consider the graph of an exponential function of the form $f(x) = ab^x$.



a. Determine whether the graph of f represents exponential growth or exponential decay.

b. What are the domain and range of the function? Explain.

69. COMPARING FUNCTIONS The two given functions describe the amount y of ibuprofen (in milligrams) in a person's bloodstream t hours after taking the dosage.

$$y \approx 325(0.9943)^{60t} \qquad y \approx 325(0.843)^{2t}$$

a. Show that these models are approximately equivalent to the model you wrote in Exercise 22.

b. Describe the information given by each of the models above.

70. DRAWING CONCLUSIONS The amount A in an account after t years with principal P, annual interest rate r (expressed as a decimal), and compounded n times per year is given by

$$A = P\left(1 + \frac{r}{n}\right)^{nt}.$$

You deposit $1000 into three separate bank accounts that each pay 3% annual interest. For each account, evaluate $\left(1 + \frac{r}{n}\right)^n$. Interpret this quantity in the context of the problem. Then complete the table.

Account	Compounding	Balance after 1 year
1	quarterly	
2	monthly	
3	daily	

71. REASONING Consider the exponential function $f(x) = ab^x$.

a. Show that $\dfrac{f(x+1)}{f(x)} = b$.

b. Use the equation in part (a) to explain why there is no exponential function of the form $f(x) = ab^x$ whose graph passes through the points in the table below.

x	0	1	2	3	4
y	4	4	8	24	72

72. THOUGHT PROVOKING The function $f(x) = b^x$ represents an exponential decay function. Write a second exponential decay function in terms of b and x.

73. PROBLEM SOLVING The number E of eggs a Leghorn chicken produces per year can be modeled by the equation $E = 179.2(0.89)^{w/52}$, where w is the age (in weeks) of the chicken and $w \geq 22$.

a. Identify the decay factor and the percent decrease.

b. Graph the model.

c. Estimate the egg production of a chicken that is 2.5 years old.

d. Explain how you can rewrite the model so that time is measured in years rather than in weeks.

74. CRITICAL THINKING You buy a new stereo for $1300 and are able to sell it 4 years later for $275. Assume that the resale value of the stereo decays exponentially with time. Write an equation giving the resale value V (in dollars) of the stereo as a function of the time t (in years) since you bought it.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Simplify the expression. *(Skills Review Handbook)*

75. $x + 3x$ **76.** $8y - 21y$ **77.** $13z + 9 - 8z$ **78.** $-9w + w - 5$

Simplify the expression. Write your answer using only positive exponents. *(Section 1.4)*

79. $x^{-9} \cdot x^2$ **80.** $\dfrac{x^4}{x^3}$ **81.** $(-6x)^2$ **82.** $\left(\dfrac{4x^8}{2x^6}\right)^4$