1 Functions and Exponents

Duckweed *(p. 49)*

Jellyfish *(p. 31)*

Miniature Golf *(p. 18)*

SEE the Big Idea

Karaoke Machine *(p. 14)*

Pool *(p. 9)*

Maintaining Mathematical Proficiency

Transforming Graphs of Linear Functions

Example 1 Graph $f(x) = x$ and $g(x) = -3x - 2$. Describe the transformations from the graph of f to the graph of g.
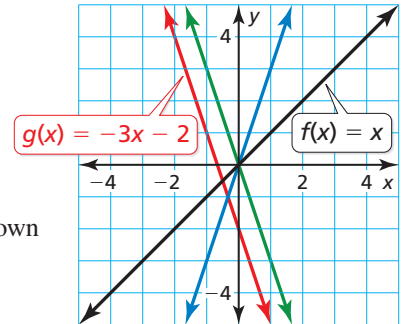
Note that you can rewrite g as $g(x) = -3f(x) - 2$.

Step 1 There is no horizontal translation from the graph of f to the graph of g.

Step 2 Stretch the graph of f vertically by a factor of 3 to get the graph of $h(x) = 3x$.

Step 3 Reflect the graph of h in the x-axis to get the graph of $r(x) = -3x$.

Step 4 Translate the graph of r vertically 2 units down to get the graph of $g(x) = -3x - 2$.

Let $f(x) = x + 1$. Graph f and g. Describe the transformation from the graph of f to the graph of g.

1. $g(x) = f(x - 4)$ **2.** $g(x) = f(x) + 5$ **3.** $g(x) = f(2x)$ **4.** $g(x) = -\dfrac{1}{2}f(x)$
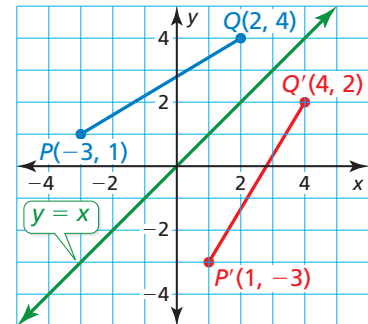
Reflecting Figures in the Line $y = x$

Example 2 Graph $\overline{PQ}$ with endpoints $P(-3, 1)$ and $Q(2, 4)$ and its image after a reflection in the line $y = x$.

Graph $\overline{PQ}$ and the line $y = x$. Use the coordinate rule for reflecting in the line $y = x$ to find the coordinates of the endpoints of the image. Then graph the image $\overline{P'Q'}$.

$$(a, b) \rightarrow (b, a)$$
$$P(-3, 1) \rightarrow P'(1, -3)$$
$$Q(2, 4) \rightarrow Q'(4, 2)$$

Graph the figure and its image after a reflection in the line $y = x$.

5. $\overline{AB}$ with endpoints $A(0, 0)$ and $B(0, -3.5)$

6. $\overline{CD}$ with endpoints $C(1, -2)$ and $D(-4, -1)$

7. $\triangle LMN$ with vertices $L(5, 3)$, $M(5, -3)$, and $N(0, -3)$

8. $\square STUV$ with vertices $S(-4, 6)$, $T(-1, 7)$, $U(-1, -4)$, and $V(-4, -5)$

9. Give an example of a line segment that when reflected in the line $y = x$, has the same endpoints as the original figure.

10. **ABSTRACT REASONING** Describe the relationship between the possible input and output values of the graph of $f(x) = a$ and its reflection in the line $y = x$.

Mathematical Practices

Mathematically proficient students use technological tools to explore concepts.

Using a Graphing Calculator

⑤ Core Concept

Restricting the Domain of a Function

You can use a graphing calculator to graph a function with a restricted domain.

1. Enter the function, in parentheses, into a graphing calculator.
2. Press the division symbol key.
3. Enter the specified domain, in parentheses.
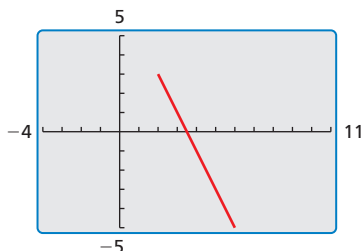4. Graph the function in an appropriate viewing window.

EXAMPLE 1   **Using a Graphing Calculator**

Use a graphing calculator to graph (a) $y = -2x + 7$, $x \geq 2$, and (b) $y = \frac{3}{4}x - 1$, $-3 < x < 4$.

SOLUTION

a. Enter the function $y = -2x + 7$ and the domain $x \geq 2$ into a graphing calculator, as shown. Then graph the function in an appropriate viewing window.

```
\Y1◼(-2X+7)/(X≥2)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

b. Enter the function $y = \frac{3}{4}x - 1$ and the domain $-3 < x < 4$ into a graphing calculator. In this case, you must enter the inequality using "and," as shown. Then graph the function in an appropriate viewing window.

```
\Y1◼(3/4X-1)/(X>-3
 and X<4)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```
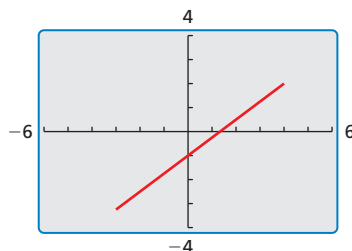
Notice that the graphing calculator does not distinguish between domains such as $x \geq 2$ and $x > 2$. You must distinguish between these yourself, based on the inequality symbol in the problem statement.

Monitoring Progress

Use a graphing calculator to graph the function with the specified domain. Describe the range of the function.

1. $y = 3x + 2$, $x \leq -1$ **2.** $y = -x - 4$, $x > 0$ **3.** $-\frac{1}{2}x + y = 1$, $-4 \leq x \leq 1$

1.1 Absolute Value Functions

Essential Question
How do the values of a, h, and k affect the graph of the absolute value function $g(x) = a|x - h| + k$?
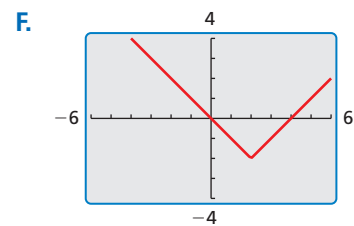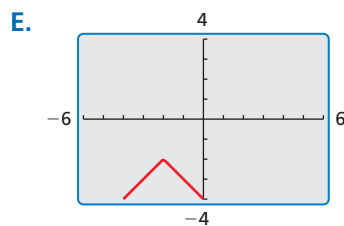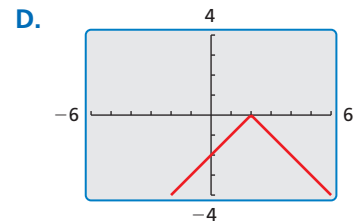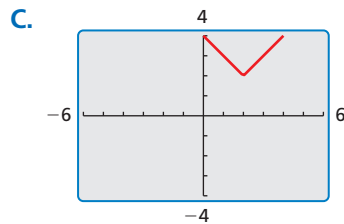
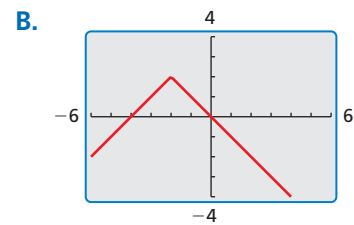The parent absolute value function is

$$f(x) = |x|. \qquad \text{Parent absolute value function}$$

The graph of f is V-shaped.

EXPLORATION 1   Identifying Graphs of Absolute Value Functions

Work with a partner. Match each absolute value function with its graph. Then use a graphing calculator to verify your answers.

a. $g(x) = -|x - 2|$ **b.** $g(x) = |x - 2| + 2$ **c.** $g(x) = -|x + 2| - 2$

d. $g(x) = |x - 2| - 2$ **e.** $g(x) = 2|x - 2|$ **f.** $g(x) = -|x + 2| + 2$

A.

B.

C.

D.

E.

F.
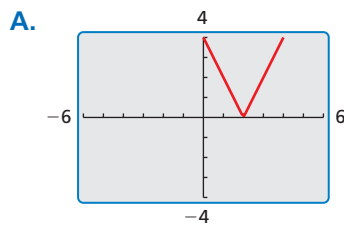
LOOKING FOR STRUCTURE

To be proficient in math, you need to look closely to discern a pattern or structure.

Communicate Your Answer

2. How do the values of a, h, and k affect the graph of the absolute value function

$$g(x) = a|x - h| + k?$$

3. Write the equation of the absolute value function whose graph is shown. Use a graphing calculator to verify your equation.

What You Will Learn

▶ Translate graphs of absolute value functions.
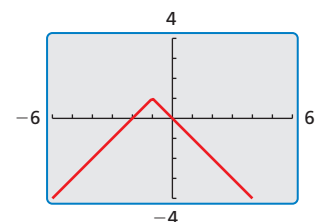
▶ Stretch, shrink, and reflect graphs of absolute value functions.

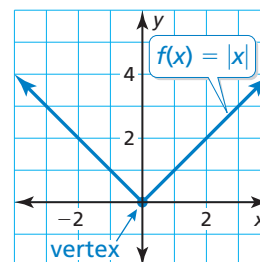▶ Combine transformations of graphs of absolute value functions.

Core Vocabulary

absolute value function, *p. 4*
vertex, *p. 4*
vertex form, *p. 6*

Previous
domain
range

Translating Graphs of Absolute Value Functions

🅖 Core Concept

Absolute Value Function

An **absolute value function** is a function that contains an absolute value expression. The parent absolute value function is $f(x) = |x|$. The graph of $f(x) = |x|$ is V-shaped and symmetric about the y-axis. The **vertex** is the point where the graph changes direction. The vertex of the graph of $f(x) = |x|$ is $(0, 0)$.

The domain of $f(x) = |x|$ is all real numbers. The range is $y \geq 0$.

REMEMBER

The graph of $y = f(x - h)$ is a horizontal translation and $y = f(x) + k$ is a vertical translation of the graph of $y = f(x)$, where $h, k \neq 0$.

The graphs of all other absolute value functions are transformations of the graph of the parent function $f(x) = |x|$. The transformations of graphs of linear functions that you learned in a previous course also apply to absolute value functions.

EXAMPLE 1   Graphing $g(x) = |x| + k$ and $g(x) = |x - h|$

Graph each function. Compare each graph to the graph of $f(x) = |x|$. Describe the domain and range.

a. $g(x) = |x| + 3$

b. $m(x) = |x - 2|$

SOLUTION

a. Step 1 Make a table of values.

x	−2	−1	0	1	2
g(x)	5	4	3	4	5

Step 2 Plot the ordered pairs.

Step 3 Draw the V-shaped graph.

▶ The function g is of the form $y = f(x) + k$, where $k = 3$. So, the graph of g is a vertical translation 3 units up of the graph of f. The domain is all real numbers. The range is $y \geq 3$.

b. Step 1 Make a table of values.

x	0	1	2	3	4
m(x)	2	1	0	1	2

Step 2 Plot the ordered pairs.

Step 3 Draw the V-shaped graph.

▶ The function m is of the form $y = f(x - h)$, where $h = 2$. So, the graph of m is a horizontal translation 2 units right of the graph of f. The domain is all real numbers. The range is $y \geq 0$.
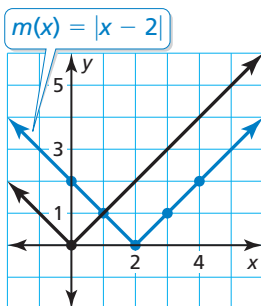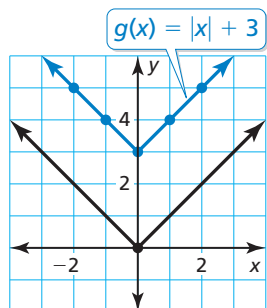

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Graph the function. Compare the graph to the graph of $f(x) = |x|$. Describe the domain and range.

1. $h(x) = |x| - 1$

2. $n(x) = |x + 4|$

Stretching, Shrinking, and Reflecting

EXAMPLE 2 **Graphing $g(x) = a|x|$**

Graph each function. Compare each graph to the graph of $f(x) = |x|$. Describe the domain and range.

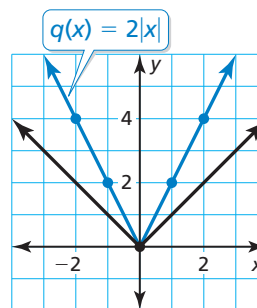**a.** $q(x) = 2|x|$ **b.** $p(x) = -\frac{1}{2}|x|$

SOLUTION

a. Step 1 Make a table of values.

x	−2	−1	0	1	2
q(x)	4	2	0	2	4

Step 2 Plot the ordered pairs.

Step 3 Draw the V-shaped graph.

▶ The function q is of the form $y = a \cdot f(x)$, where $a = 2$. So, the graph of q is a vertical stretch of the graph of f by a factor of 2. The domain is all real numbers. The range is $y \geq 0$.

b. Step 1 Make a table of values.

x	−2	−1	0	1	2
p(x)	−1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	−1

Step 2 Plot the ordered pairs.

Step 3 Draw the V-shaped graph.
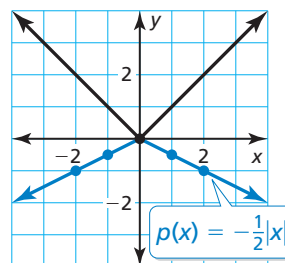
▶ The function p is of the form $y = -a \cdot f(x)$, where $a = \frac{1}{2}$. So, the graph of p is a vertical shrink of the graph of f by a factor of $\frac{1}{2}$ and a reflection in the x-axis. The domain is all real numbers. The range is $y \leq 0$.

REMEMBER

Recall the following transformations of the graph of $y = f(x)$.

$y = -f(x)$: reflection in the x-axis

$y = f(-x)$: reflection in the y-axis

$y = f(ax)$: horizontal stretch or shrink by a factor of $\frac{1}{a}$, where $a > 0$ and $a \neq 1$

$y = a \cdot f(x)$: vertical stretch or shrink by a factor of a, where $a > 0$ and $a \neq 1$

STUDY TIP

A vertical stretch of the graph of $f(x) = |x|$ is *narrower* than the graph of $f(x) = |x|$.

STUDY TIP

A vertical shrink of the graph of $f(x) = |x|$ is *wider* than the graph of $f(x) = |x|$.

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Graph the function. Compare the graph to the graph of $f(x) = |x|$. Describe the domain and range.

3. $t(x) = -3|x|$ **4.** $v(x) = \frac{1}{4}|x|$

🍄 Core Concept

Vertex Form of an Absolute Value Function

An absolute value function written in the form $g(x) = a|x - h| + k$, where $a \neq 0$, is in **vertex form**. The vertex of the graph of g is (h, k).

Any absolute value function can be written in vertex form, and its graph is symmetric about the line $x = h$.

EXAMPLE 3   Graphing $f(x) = |x - h| + k$ and $g(x) = f(ax)$

Graph $f(x) = |x + 2| - 3$ and $g(x) = |2x + 2| - 3$. Compare the graph of g to the graph of f.

SOLUTION

Step 1 Make a table of values for each function.

x	−4	−3	−2	−1	0	1	2
f(x)	−1	−2	−3	−2	−1	0	1

x	−2	−1.5	−1	−0.5	0	0.5	1
g(x)	−1	−2	−3	−2	−1	0	1

Step 2 Plot the ordered pairs.

Step 3 Draw the V-shaped graph of each function. Notice that the vertex of the graph of f is $(-2, -3)$ and the graph is symmetric about $x = -2$.

$g(x) = |2x + 2| - 3$

$f(x) = |x + 2| - 3$
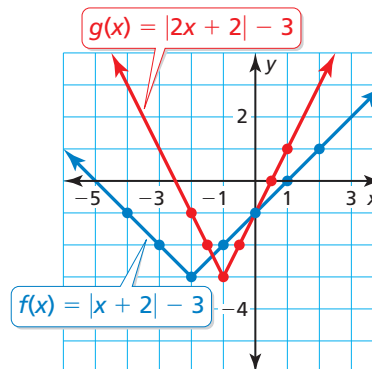
▶ Note that you can rewrite g as $g(x) = f(2x)$, which is of the form $y = f(ax)$, where $a = 2$. So, the graph of g is a horizontal shrink of the graph of f by a factor of $\frac{1}{2}$. The y-intercept is the same for both graphs. The points on the graph of f move halfway closer to the y-axis, resulting in the graph of g. When the input values of f are 2 times the input values of g, the output values of f and g are the same.

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5. Graph $f(x) = |x - 1|$ and $g(x) = \left|\frac{1}{2}x - 1\right|$. Compare the graph of g to the graph of f.

6. Graph $f(x) = |x + 2| + 2$ and $g(x) = |-4x + 2| + 2$. Compare the graph of g to the graph of f.

Combining Transformations

EXAMPLE 4   **Graphing $g(x) = a|x - h| + k$**

Let $g(x) = -2|x - 1| + 3$. (a) Describe the transformations from the graph of $f(x) = |x|$ to the graph of g. (b) Graph g.

SOLUTION

a. Step 1 Translate the graph of f horizontally 1 unit right to get the graph of $t(x) = |x - 1|$.

Step 2 Stretch the graph of t vertically by a factor of 2 to get the graph of $h(x) = 2|x - 1|$.

Step 3 Reflect the graph of h in the x-axis to get the graph of $r(x) = -2|x - 1|$.

Step 4 Translate the graph of r vertically 3 units up to get the graph of $g(x) = -2|x - 1| + 3$.

b. Method 1

Step 1 Make a table of values.

x	-1	0	1	2	3
$g(x)$	-1	1	3	1	-1

Step 2 Plot the ordered pairs.

Step 3 Draw the V-shaped graph.

$g(x) = -2|x - 1| + 3$
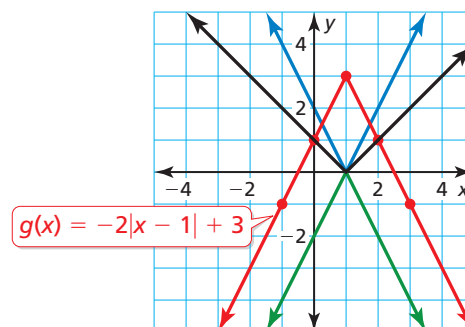
Method 2

Step 1 Identify and plot the vertex. $(h, k) = (1, 3)$

Step 2 Plot another point on the graph, such as (2, 1). Because the graph is symmetric about the line $x = 1$, you can use symmetry to plot a third point, (0, 1).

Step 3 Draw the V-shaped graph.

$g(x) = -2|x - 1| + 3$
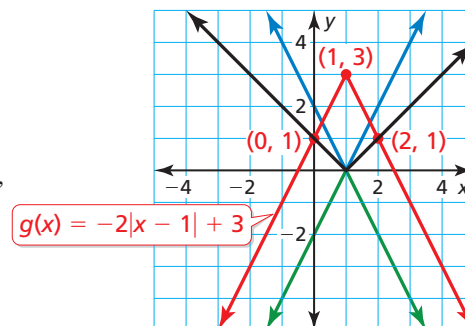
Monitoring Progress   🔊 Help in English and Spanish at *BigIdeasMath.com*

7. Let $g(x) = \left|-\frac{1}{2}x + 2\right| + 1$. (a) Describe the transformations from the graph of $f(x) = |x|$ to the graph of g. (b) Graph g.

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The point $(1, -4)$ is the _____ of the graph of $f(x) = -3|x - 1| - 4$.

2. **USING STRUCTURE** How do you know whether the graph of $f(x) = a|x - h| + k$ is a vertical stretch or a vertical shrink of the graph of $f(x) = |x|$?

3. **WRITING** Describe three different types of transformations of the graph of an absolute value function.

4. **REASONING** The graph of which function has the same *y*-intercept as the graph of $f(x) = |x - 2| + 5$? Explain.

$$g(x) = |3x - 2| + 5 \qquad h(x) = 3|x - 2| + 5$$

Monitoring Progress and Modeling with Mathematics

In Exercises 5–12, graph the function. Compare the graph to the graph of $f(x) = |x|$. Describe the domain and range. *(See Examples 1 and 2.)*

5. $d(x) = |x| - 4$

6. $r(x) = |x| + 5$

7. $m(x) = |x + 1|$

8. $v(x) = |x - 3|$

9. $p(x) = \frac{1}{3}|x|$

10. $j(x) = 3|x|$

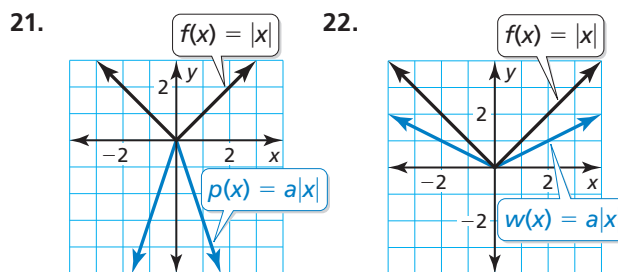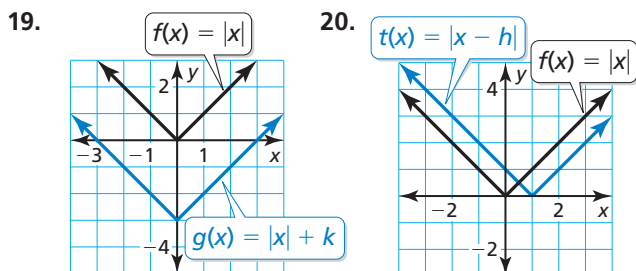11. $a(x) = -5|x|$

12. $q(x) = -\frac{3}{2}|x|$

In Exercises 13–16, graph the function. Compare the graph to the graph of $f(x) = |x - 6|$.

13. $h(x) = |x - 6| + 2$

14. $n(x) = \frac{1}{2}|x - 6|$

15. $k(x) = -3|x - 6|$

16. $g(x) = |x - 1|$

In Exercises 17 and 18, graph the function. Compare the graph to the graph of $f(x) = |x + 3| - 2$.

17. $y(x) = |x + 4| - 2$

18. $b(x) = |x + 3| + 3$

In Exercises 19–22, compare the graphs. Find the value of h, k, or a.

19.

20.

21.

22.

In Exercises 23–26, write an equation that represents the given transformation(s) of the graph of $g(x) = |x|$.

23. vertical translation 7 units down

24. horizontal translation 10 units left

25. vertical shrink by a factor of $\frac{1}{4}$

26. vertical stretch by a factor of 3 and a reflection in the *x*-axis

In Exercises 27–32, graph and compare the two functions. *(See Example 3.)*

27. $f(x) = |x - 4|$; $g(x) = |3x - 4|$

28. $h(x) = |x + 5|$; $t(x) = |2x + 5|$

29. $p(x) = |x + 1| - 2$; $q(x) = \left|\frac{1}{4}x + 1\right| - 2$

30. $w(x) = |x - 3| + 4$; $y(x) = |5x - 3| + 4$

31. $a(x) = |x + 2| + 3$; $b(x) = |-4x + 2| + 3$

32. $u(x) = |x - 1| + 2$; $v(x) = \left|-\frac{1}{2}x - 1\right| + 2$

In Exercises 33–40, describe the transformations from the graph of $f(x) = |x|$ to the graph of the given function. Then graph the given function. *(See Example 4.)*

33. $r(x) = |x + 2| - 6$ **34.** $c(x) = |x + 4| + 4$

35. $d(x) = -|x - 3| + 5$ **36.** $v(x) = -3|x + 1| + 4$

37. $m(x) = \frac{1}{2}|x + 4| - 1$ **38.** $s(x) = |2x - 2| - 3$

39. $j(x) = |-x + 1| - 5$ **40.** $n(x) = \left|-\frac{1}{3}x + 1\right| + 2$

41. MODELING WITH MATHEMATICS The number of pairs of shoes sold s (in thousands) increases and then decreases as described by the function $s(t) = -2|t - 15| + 50$, where t is the time (in weeks).

   a. Graph the function.

   b. What is the greatest number of pairs of shoes sold in 1 week?

42. MODELING WITH MATHEMATICS On the pool table shown, you bank the five ball off the side represented by the x-axis. The path of the ball is described by the function $p(x) = \frac{4}{3}\left|x - \frac{5}{4}\right|$.
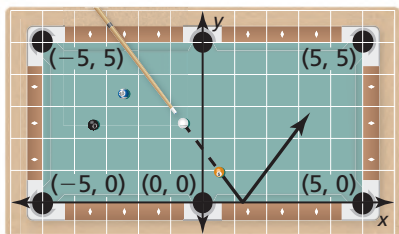

   a. At what point does the five ball bank off the side?

   b. Do you make the shot? Explain your reasoning.

43. USING TRANSFORMATIONS The points $A\left(-\frac{1}{2}, 3\right)$, $B(1, 0)$, and $C(-4, -2)$ lie on the graph of the absolute value function f. Find the coordinates of the points corresponding to A, B, and C on the graph of each function.

   a. $g(x) = f(x) - 5$ **b.** $h(x) = f(x - 3)$

   c. $j(x) = -f(x)$ **d.** $k(x) = 4f(x)$

44. USING STRUCTURE Explain how the graph of each function compares to the graph of $y = |x|$ for positive and negative values of k, h, and a.

   a. $y = |x| + k$

   b. $y = |x - h|$

   c. $y = a|x|$

   d. $y = |ax|$

ERROR ANALYSIS In Exercises 45 and 46, describe and correct the error in graphing the function.

45.

$y = |x - 1| - 3$

46.

$y = -3|x|$
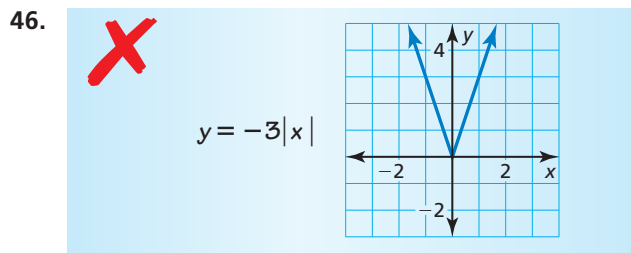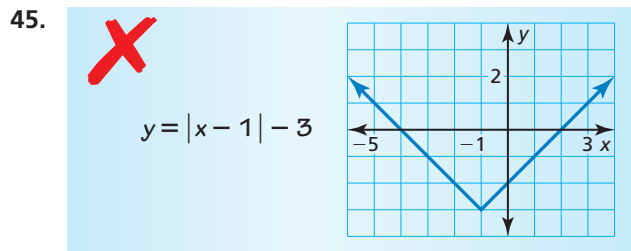
MATHEMATICAL CONNECTIONS In Exercises 47 and 48, write an absolute value function whose graph forms a square with the given graph.

47.

$y = |x| - 2$

48.

$y = |x - 3| + 1$
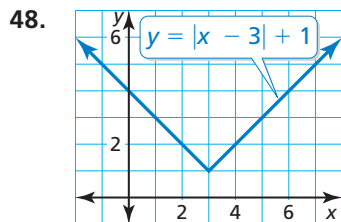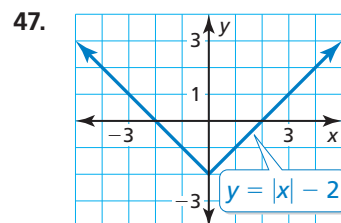
49. WRITING Compare the graphs of $p(x) = |x - 6|$ and $q(x) = |x| - 6$.

50. HOW DO YOU SEE IT? The object of a computer game is to break bricks by deflecting a ball toward them using a paddle. The graph shows the current path of the ball and the location of the last brick.

a. You can move the paddle up, down, left, and right. At what coordinates should you place the paddle to break the last brick? Assume the ball deflects at a right angle.

b. You move the paddle to the coordinates in part (a), and the ball is deflected. How can you write an absolute value function that describes the path of the ball?

In Exercises 51–54, graph the function. Then rewrite the absolute value function as two linear functions, one that has the domain $x < 0$ and one that has the domain $x \geq 0$.

51. $y = |x|$

52. $y = |x| - 3$

53. $y = -|x| + 9$

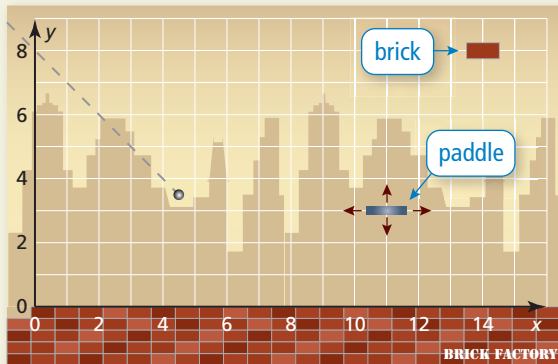**54.** $y = -4|x|$

In Exercises 55–58, graph and compare the two functions.

55. $f(x) = |x - 1| + 2$; $g(x) = 4|x - 1| + 8$

56. $s(x) = |2x - 5| - 6$; $t(x) = \frac{1}{2}|2x - 5| - 3$

57. $v(x) = -2|3x + 1| + 4$; $w(x) = 3|3x + 1| - 6$

58. $c(x) = 4|x + 3| - 1$; $d(x) = -\frac{4}{3}|x + 3| + \frac{1}{3}$

59. REASONING Describe the transformations from the graph of $g(x) = -2|x + 1| + 4$ to the graph of $h(x) = |x|$. Explain your reasoning.

60. THOUGHT PROVOKING Graph an absolute value function f that represents the route a wide receiver runs in a football game. Let the x-axis represent distance (in yards) across the field horizontally. Let the y-axis represent distance (in yards) down the field. Be sure to limit the domain so the route is realistic.

61. SOLVING BY GRAPHING Graph $y = 2|x + 2| - 6$ and $y = -2$ in the same coordinate plane. Use the graph to solve the equation $2|x + 2| - 6 = -2$. Check your solutions.

62. MAKING AN ARGUMENT Let p be a positive constant. Your friend says that because the graph of $y = |x| + p$ is a *positive* vertical translation of the graph of $y = |x|$, the graph of $y = |x + p|$ is a *positive* horizontal translation of the graph of $y = |x|$. Is your friend correct? Explain.

63. ABSTRACT REASONING Write the vertex of the absolute value function $f(x) = |ax - h| + k$ in terms of a, h, and k.

Maintaining Mathematical Proficiency
Reviewing what you learned in previous grades and lessons

Solve the formula for the indicated variable. *(Skills Review Handbook)*

64. Solve for h.
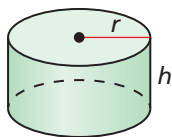
$V = \pi r^2 h$

65. Solve for w.

$P = 2\ell + 2w$

Write an equation of the line that passes through the points. *(Skills Review Handbook)*

66. $(1, -4), (2, -1)$

67. $(-3, 12), (0, 6)$

68. $(3, 1), (-5, 5)$

69. $(2, -1), (0, 1)$

Essential Question How can you describe a function that is represented by more than one equation?

EXPLORATION 1 Writing Equations for a Function

Work with a partner.

a. Does the graph represent y as a function of x? Justify your conclusion.

b. What is the value of the function when $x = 0$? How can you tell?

c. Write an equation that represents the values of the function when $x \leq 0$.

$$f(x) = \boxed{}, \text{ if } x \leq 0$$

d. Write an equation that represents the values of the function when $x > 0$.

$$f(x) = \boxed{}, \text{ if } x > 0$$

e. Combine the results of parts (c) and (d) to write a single description of the function.

$$f(x) = \begin{cases} \boxed{}, & \text{if } x \leq 0 \\ \boxed{}, & \text{if } x > 0 \end{cases}$$
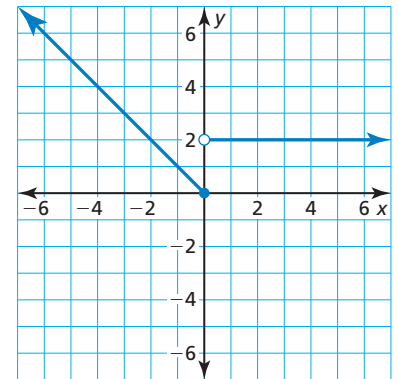
EXPLORATION 2 Writing Equations for a Function

Work with a partner.

a. Does the graph represent y as a function of x? Justify your conclusion.

b. Describe the values of the function for the following intervals.

$$f(x) = \begin{cases} \boxed{}, & \text{if } -6 \leq x < -3 \\ \boxed{}, & \text{if } -3 \leq x < 0 \\ \boxed{}, & \text{if } 0 \leq x < 3 \\ \boxed{}, & \text{if } 3 \leq x < 6 \end{cases}$$
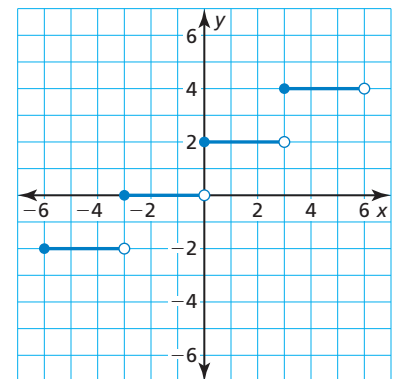
Communicate Your Answer

3. How can you describe a function that is represented by more than one equation?

4. Use two equations to describe the function represented by the graph.
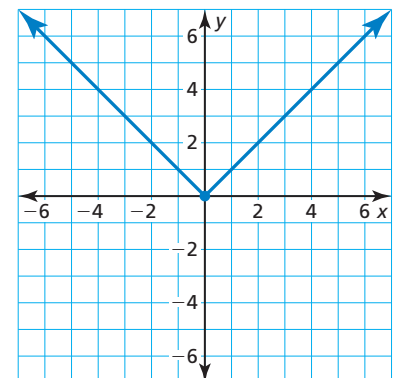
Core Vocabulary

piecewise function, *p. 12*
step function, *p. 14*

Previous
absolute value function
vertex form
vertex

What You Will Learn

▶ Evaluate piecewise functions.
▶ Graph and write piecewise functions.
▶ Graph and write step functions.
▶ Write absolute value functions.

Evaluating Piecewise Functions

Core Concept

Piecewise Function

A **piecewise function** is a function defined by two or more equations. Each "piece" of the function applies to a different part of its domain. An example is shown below.

$$f(x) = \begin{cases} x - 2, & \text{if } x \le 0 \\ 2x + 1, & \text{if } x > 0 \end{cases}$$

- The expression $x - 2$ represents the value of f when x is less than or equal to 0.

- The expression $2x + 1$ represents the value of f when x is greater than 0.

$f(x) = 2x + 1, x > 0$
$f(x) = x - 2, x \le 0$
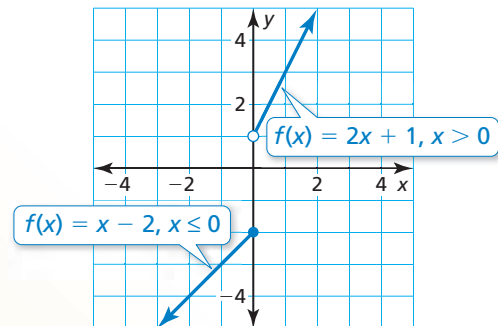
EXAMPLE 1 **Evaluating a Piecewise Function**

Evaluate the function f above when (a) $x = 0$ and (b) $x = 4$.

SOLUTION

a. $f(x) = x - 2$ Because $0 \le 0$, use the first equation.

   $f(0) = 0 - 2$ Substitute 0 for *x*.

   $f(0) = -2$ Simplify.

   ▶ The value of f is -2 when $x = 0$.

b. $f(x) = 2x + 1$ Because $4 > 0$, use the second equation.

   $f(4) = 2(4) + 1$ Substitute 4 for *x*.

   $f(4) = 9$ Simplify.

   ▶ The value of f is 9 when $x = 4$.

Monitoring Progress

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Evaluate the function.

$$f(x) = \begin{cases} 3, & \text{if } x < -2 \\ x + 2, & \text{if } -2 \le x \le 5 \\ 4x, & \text{if } x > 5 \end{cases}$$

1. $f(-8)$ 2. $f(-2)$
3. $f(0)$ 4. $f(3)$
5. $f(5)$ 6. $f(10)$

Graphing and Writing Piecewise Functions

EXAMPLE 2 **Graphing a Piecewise Function**

Graph $y = \begin{cases} -x - 4, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$. Describe the domain and range.

SOLUTION

Step 1 Graph $y = -x - 4$ for $x < 0$. Because x is not equal to 0, use an open circle at $(0, -4)$.

Step 2 Graph $y = x$ for $x \geq 0$. Because x is greater than or equal to 0, use a closed circle at $(0, 0)$.

▶ The domain is all real numbers. The range is $y > -4$.
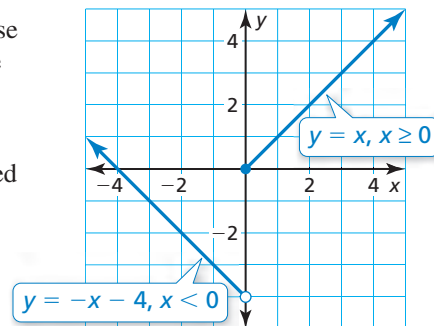
$y = x, x \geq 0$

$y = -x - 4, x < 0$

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Graph the function. Describe the domain and range.

7. $y = \begin{cases} x + 1, & \text{if } x \leq 0 \\ -x, & \text{if } x > 0 \end{cases}$

8. $y = \begin{cases} x - 2, & \text{if } x < 0 \\ 4x, & \text{if } x \geq 0 \end{cases}$

EXAMPLE 3 **Writing a Piecewise Function**

Write a piecewise function for the graph.

SOLUTION

Each "piece" of the function is linear.

Left Piece When $x < 0$, the graph is the line given by $y = x + 3$.

Right Piece When $x \geq 0$, the graph is the line given by $y = 2x - 1$.

▶ So, a piecewise function for the graph is

$$f(x) = \begin{cases} x + 3, & \text{if } x < 0 \\ 2x - 1, & \text{if } x \geq 0 \end{cases}.$$
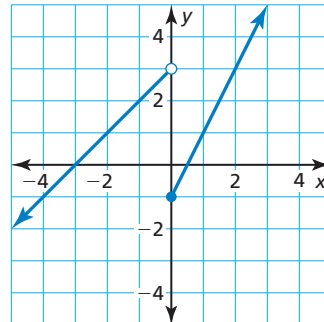
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Write a piecewise function for the graph.

9.

10.
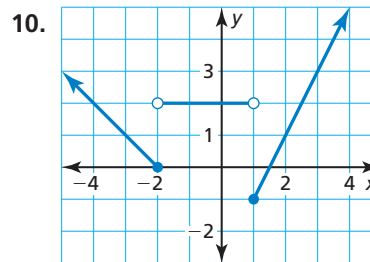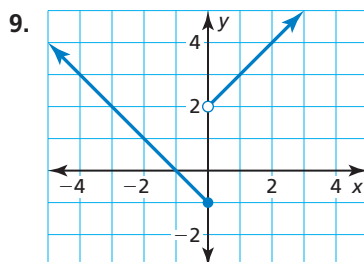
Graphing and Writing Step Functions

A **step function** is a piecewise function defined by a constant value over each part of its domain. The graph of a step function consists of a series of line segments.
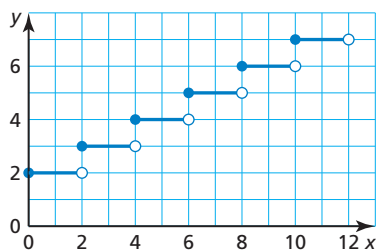
$$f(x) = \begin{cases} 2, & \text{if } 0 \le x < 2 \\ 3, & \text{if } 2 \le x < 4 \\ 4, & \text{if } 4 \le x < 6 \\ 5, & \text{if } 6 \le x < 8 \\ 6, & \text{if } 8 \le x < 10 \\ 7, & \text{if } 10 \le x < 12 \end{cases}$$

EXAMPLE 4 Graphing and Writing a Step Function

You rent a karaoke machine for 5 days. The rental company charges $50 for the first day and $25 for each additional day. Write and graph a step function that represents the relationship between the number x of days and the total cost y (in dollars) of renting the karaoke machine.

SOLUTION

Step 1 Use a table to organize the information.

Number of days	Total cost (dollars)
$0 < x \le 1$	50
$1 < x \le 2$	75
$2 < x \le 3$	100
$3 < x \le 4$	125
$4 < x \le 5$	150

Step 2 Write the step function.

$$f(x) = \begin{cases} 50, & \text{if } 0 < x \le 1 \\ 75, & \text{if } 1 < x \le 2 \\ 100, & \text{if } 2 < x \le 3 \\ 125, & \text{if } 3 < x \le 4 \\ 150, & \text{if } 4 < x \le 5 \end{cases}$$

Step 3 Graph the step function.

Karaoke Machine Rental
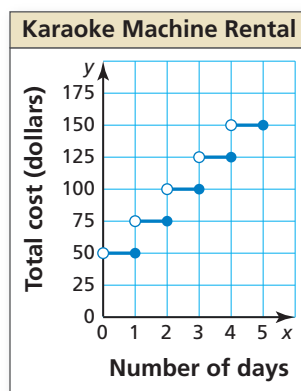
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11. A landscaper rents a wood chipper for 4 days. The rental company charges $100 for the first day and $50 for each additional day. Write and graph a step function that represents the relationship between the number x of days and the total cost y (in dollars) of renting the chipper.

Writing Absolute Value Functions

The absolute value function $f(x) = |x|$ can be written as a piecewise function.

$$f(x) = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

Similarly, the vertex form of an absolute value function $g(x) = a|x - h| + k$ can be written as a piecewise function.

$$g(x) = \begin{cases} a[-(x - h)] + k, & \text{if } x - h < 0 \\ a(x - h) + k, & \text{if } x - h \geq 0 \end{cases}$$

EXAMPLE 5   Writing an Absolute Value Function

In holography, light from a laser beam is split into two beams, a reference beam and an object beam. Light from the object beam reflects off an object and is recombined with the reference beam to form images on film that can be used to create three-dimensional images.

a. Write an absolute value function that represents the path of the reference beam.

b. Write the function in part (a) as a piecewise function.
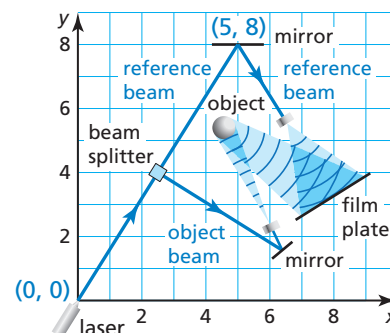
SOLUTION

a. The vertex of the path of the reference beam is (5, 8). So, the function has the form $g(x) = a|x - 5| + 8$. Substitute the coordinates of the point (0, 0) into the equation and solve for a.

$g(x) = a\|x - 5\| + 8$	Vertex form of the function
$0 = a\|0 - 5\| + 8$	Substitute 0 for x and 0 for $g(x)$.
$-1.6 = a$	Solve for a.

▶ So, the function $g(x) = -1.6|x - 5| + 8$ represents the path of the reference beam.

b. Write $g(x) = -1.6|x - 5| + 8$ as a piecewise function.

$$g(x) = \begin{cases} -1.6[-(x - 5)] + 8, & \text{if } x - 5 < 0 \\ -1.6(x - 5) + 8, & \text{if } x - 5 \geq 0 \end{cases}$$

Simplify each expression and solve the inequalities.

▶ So, a piecewise function for $g(x) = -1.6|x - 5| + 8$ is

$$g(x) = \begin{cases} 1.6x, & \text{if } x < 5 \\ -1.6x + 16, & \text{if } x \geq 5 \end{cases}.$$

STUDY TIP

The graph of an absolute value function is symmetric about the line $x = h$. So, it makes sense that the piecewise definition "splits" the function at $x = 5$.

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12. **WHAT IF?** The reference beam originates at (3, 0) and reflects off a mirror at (5, 4).

   a. Write an absolute value function that represents the path of the reference beam.

   b. Write the function in part (a) as a piecewise function.

Vocabulary and Core Concept Check

1. **VOCABULARY** Compare piecewise functions and step functions.

2. **WRITING** Use a graph to explain why you can write the absolute value function $y = |x|$ as a piecewise function.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–12, evaluate the function. *(See Example 1.)*

$$f(x) = \begin{cases} 5x - 1, & \text{if } x < -2 \\ x + 3, & \text{if } x \geq -2 \end{cases}$$

$$g(x) = \begin{cases} -x + 4, & \text{if } x \leq -1 \\ 3, & \text{if } -1 < x < 2 \\ 2x - 5, & \text{if } x \geq 2 \end{cases}$$

3. $f(-3)$ 4. $f(-2)$

5. $f(0)$ 6. $f(5)$

7. $g(-4)$ 8. $g(-1)$

9. $g(0)$ 10. $g(1)$

11. $g(2)$ 12. $g(5)$

13. **MODELING WITH MATHEMATICS** On a trip, the total distance (in miles) you travel in x hours is represented by the piecewise function

$$d(x) = \begin{cases} 55x, & \text{if } 0 \leq x \leq 2 \\ 65x - 20, & \text{if } 2 < x \leq 5 \end{cases}.$$

How far do you travel in 4 hours?

14. **MODELING WITH MATHEMATICS** The total cost (in dollars) of ordering x custom shirts is represented by the piecewise function

$$c(x) = \begin{cases} 17x + 20, & \text{if } 0 \leq x < 25 \\ 15.80x + 20, & \text{if } 25 \leq x < 50. \\ 14x + 20, & \text{if } x \geq 50 \end{cases}$$

Determine the total cost of ordering 26 shirts.

CUSTOM SHIRTS

THINKING...

Quantity	Price/Shirt
0-24	17^{00}
25-49	15^{80}
50+	14^{00}

plus a $20 processing fee on all orders

In Exercises 15–20, graph the function. Describe the domain and range. *(See Example 2.)*

15. $y = \begin{cases} -x, & \text{if } x < 2 \\ x - 6, & \text{if } x \geq 2 \end{cases}$

16. $y = \begin{cases} 2x, & \text{if } x \leq -3 \\ -2x, & \text{if } x > -3 \end{cases}$

17. $y = \begin{cases} -3x - 2, & \text{if } x \leq -1 \\ x + 2, & \text{if } x > -1 \end{cases}$

18. $y = \begin{cases} x + 8, & \text{if } x < 4 \\ 4x - 4, & \text{if } x \geq 4 \end{cases}$

19. $y = \begin{cases} 1, & \text{if } x < -3 \\ x - 1, & \text{if } -3 \leq x \leq 3 \\ -2x + 4, & \text{if } x > 3 \end{cases}$

20. $y = \begin{cases} 2x + 1, & \text{if } x \leq -1 \\ -x + 2, & \text{if } -1 < x < 2 \\ -3, & \text{if } x \geq 2 \end{cases}$

21. **ERROR ANALYSIS** Describe and correct the error in finding $f(5)$ when $f(x) = \begin{cases} 2x - 3, & \text{if } x < 5 \\ x + 8, & \text{if } x \geq 5 \end{cases}$.

> ✗ $f(5) = 2(5) - 3$
> $= 7$

22. **ERROR ANALYSIS** Describe and correct the error in graphing $y = \begin{cases} x + 6, & \text{if } x \leq -2 \\ 1, & \text{if } x > -2 \end{cases}$.
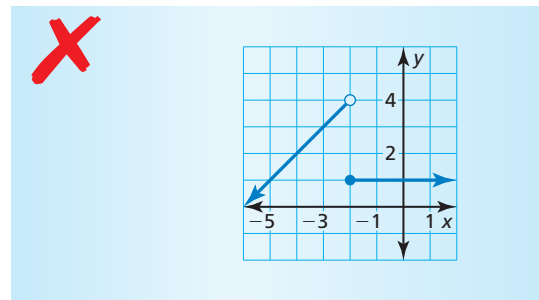
In Exercises 23–30, write a piecewise function for the graph. *(See Example 3.)*

23.

24.

25.

26.

27.

28.

29.

30.
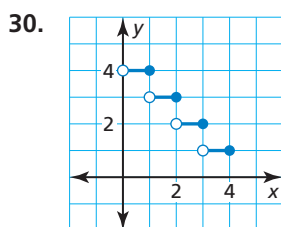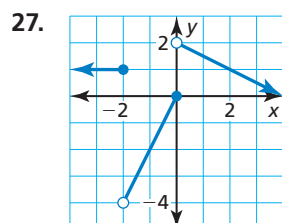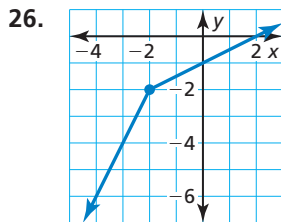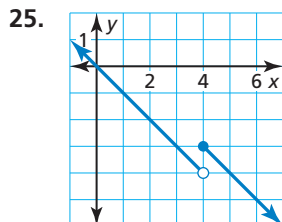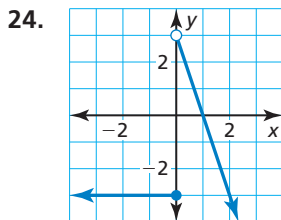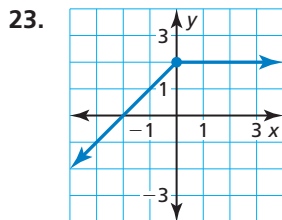
In Exercises 31–34, graph the step function. Describe the domain and range.

31. $f(x) = \begin{cases} 3, & \text{if } 0 \le x < 2 \\ 4, & \text{if } 2 \le x < 4 \\ 5, & \text{if } 4 \le x < 6 \\ 6, & \text{if } 6 \le x < 8 \end{cases}$

32. $f(x) = \begin{cases} -4, & \text{if } 1 < x \le 2 \\ -6, & \text{if } 2 < x \le 3 \\ -8, & \text{if } 3 < x \le 4 \\ -10, & \text{if } 4 < x \le 5 \end{cases}$

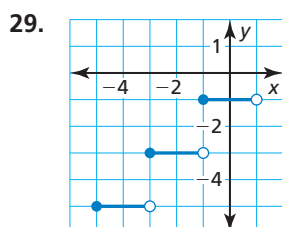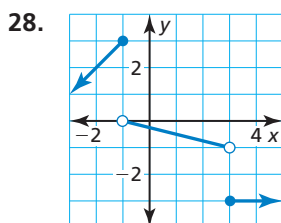**33.** $f(x) = \begin{cases} 9, & \text{if } 1 < x \le 2 \\ 6, & \text{if } 2 < x \le 4 \\ 5, & \text{if } 4 < x \le 9 \\ 1, & \text{if } 9 < x \le 12 \end{cases}$

34. $f(x) = \begin{cases} -2, & \text{if } -6 \le x < -5 \\ -1, & \text{if } -5 \le x < -3 \\ 0, & \text{if } -3 \le x < -2 \\ 1, & \text{if } -2 \le x < 0 \end{cases}$

35. MODELING WITH MATHEMATICS The cost to join an intramural sports league is $180 per team and includes the first five team members. For each additional team member, there is a $30 fee. You plan to have nine people on your team. Write and graph a step function that represents the relationship between the number p of people on your team and the total cost of joining the league. *(See Example 4.)*

36. MODELING WITH MATHEMATICS The rates for a parking garage are shown. Write and graph a step function that represents the relationship between the number x of hours a car is parked in the garage and the total cost of parking in the garage for 1 day.

Daily Parking Garage Rates
$4 per hour
$15 daily maximum

In Exercises 37–46, write the absolute value function as a piecewise function.

37. $y = |x| + 1$

38. $y = |x| - 3$

39. $y = |x - 2|$

40. $y = |x + 5|$

41. $y = 2|x + 3|$

42. $y = 4|x - 1|$

43. $y = -5|x - 8|$

44. $y = -3|x + 6|$

45. $y = -|x - 3| + 2$

46. $y = 7|x + 1| - 5$

47. MODELING WITH MATHEMATICS You are sitting on a boat on a lake. You can get a sunburn from the sunlight that hits you directly and also from the sunlight that reflects off the water. *(See Example 5.)*
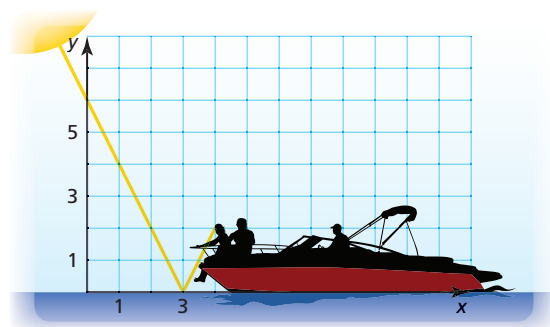
a. Write an absolute value function that represents the path of the sunlight that reflects off the water.

b. Write the function in part (a) as a piecewise function.

48. MODELING WITH MATHEMATICS You are trying to make a hole in one on the miniature golf green.
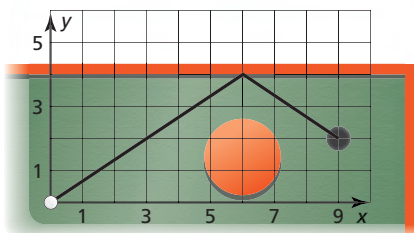
a. Write an absolute value function that represents the path of the golf ball.

b. Write the function in part (a) as a piecewise function.

49. REASONING The piecewise function f consists of two linear "pieces." The graph of f is shown.
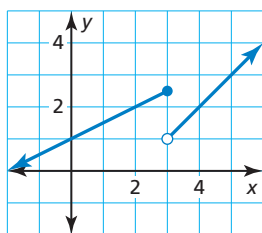
a. What is the value of $f(-10)$?

b. What is the value of $f(8)$?

50. CRITICAL THINKING Describe how the graph of each piecewise function changes when $<$ is replaced with $\leq$ and $\geq$ is replaced with $>$. Do the domain and range change? Explain.

a. $f(x) = \begin{cases} x + 2, & \text{if } x < 2 \\ -x - 1, & \text{if } x \geq 2 \end{cases}$

b. $f(x) = \begin{cases} \frac{1}{2}x + \frac{3}{2}, & \text{if } x < 1 \\ -x + 3, & \text{if } x \geq 1 \end{cases}$

51. USING STRUCTURE Graph

$$y = \begin{cases} -x + 2, & \text{if } x \leq -2 \\ |x|, & \text{if } x > -2 \end{cases}.$$

Describe the domain and range.

52. HOW DO YOU SEE IT? The graph shows the total cost C of making x photocopies at a copy shop.
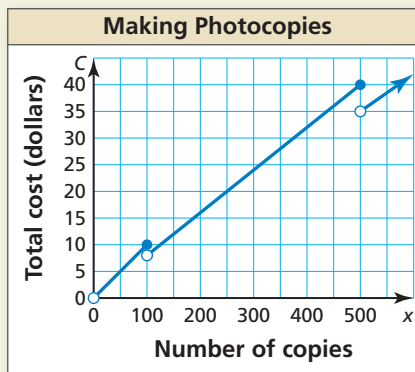
a. Does it cost more money to make 100 photocopies or 101 photocopies? Explain.

b. You have \$40 to make photocopies. Can you buy more than 500 photocopies? Explain.

53. USING STRUCTURE The output y of the *greatest integer function* is the greatest integer less than or equal to the input value x. This function is written as $f(x) = [\![x]\!]$. Graph the function for $-4 \leq x < 4$. Is it a piecewise function? a step function? Explain.

54. THOUGHT PROVOKING Explain why

$$y = \begin{cases} 2x - 2, & \text{if } x \leq 3 \\ -3, & \text{if } x \geq 3 \end{cases}$$

does not represent a function. How can you redefine y so that it does represent a function?

55. MAKING AN ARGUMENT During a 9-hour snowstorm, it snows at a rate of 1 inch per hour for the first 2 hours, 2 inches per hour for the next 6 hours, and 1 inch per hour for the final hour.

a. Write and graph a piecewise function that represents the depth of the snow during the snowstorm.

b. Your friend says 12 inches of snow accumulated during the storm. Is your friend correct? Explain.

Maintaining Mathematical Proficiency

Write the sentence as an inequality. Graph the inequality. *(Skills Review Handbook)*

56. A number r is greater than -12 and no more than 13.

57. A number t is less than or equal to 4 or no less than 18.

Describe the transformation(s) from the graph of $f(x) = |x|$ to the graph of the given function. *(Section 1.1)*

58. $g(x) = |x - 5|$

59. $h(x) = -2.5|x|$

60. $p(x) = |x + 1| + 6$

61. $v(x) = 4|x + 3|$

Essential Question   How are a function and its inverse related?

EXPLORATION 1   Exploring Inverse Functions

Work with a partner. The functions f and g are *inverses* of each other. Compare the tables of values of the two functions. How are the functions related?

x	0	0.5	1	1.5	2	2.5	3	3.5
f(x)	−2	−0.75	0.5	1.75	3	4.25	5.5	6.75

x	−2	−0.75	0.5	1.75	3	4.25	5.5	6.75
g(x)	0	0.5	1	1.5	2	2.5	3	3.5

EXPLORATION 2   Exploring Inverse Functions

Work with a partner.

a. Use the coordinate plane below to plot the two sets of points represented by the tables in Exploration 1. Then draw a line through each set of points.

b. Describe the relationship between the two graphs.

c. Write an equation for each function.
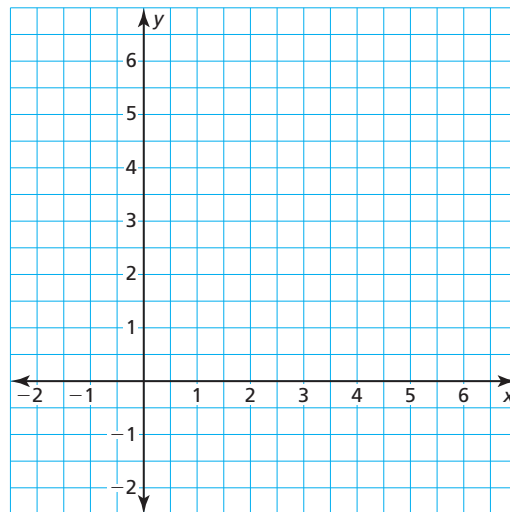
Communicate Your Answer

3. How are a function and its inverse related?

4. A table of values for a function f is given. Create a table of values for a function g, the inverse of f.

x	0	1	2	3	4	5	6	7
f(x)	1	2	3	4	5	6	7	8

5. Sketch the graphs of $f(x) = x + 4$ and its inverse in the same coordinate plane. Then write an equation of the inverse of f. Explain your reasoning.

ATTENDING TO PRECISION

To be proficient in math, you need to communicate precisely with others.

What You Will Learn

▶ Find inverses of relations.
▶ Explore inverses of functions.
▶ Find inverses of functions algebraically.

Finding Inverses of Relations

Recall that a relation pairs inputs with outputs. An **inverse relation** switches the input and output values of the original relation.

🅖 Core Concept

Inverse Relation

When a relation contains (a, b), the inverse relation contains (b, a).

EXAMPLE 1   **Finding Inverses of Relations**

Find the inverse of each relation.
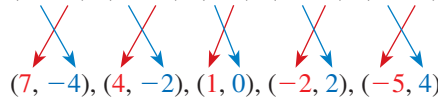
a. $(-4, 7), (-2, 4), (0, 1), (2, -2), (4, -5)$ Switch the coordinates of each ordered pair.

$(7, -4), (4, -2), (1, 0), (-2, 2), (-5, 4)$ Inverse relation

b.

Input	−1	0	1	2	3	4
Output	5	10	15	20	25	30

Inverse relation:

Input	5	10	15	20	25	30
Output	−1	0	1	2	3	4

Switch the inputs and outputs.

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Find the inverse of the relation.

1. $(-3, -4), (-2, 0), (-1, 4), (0, 8), (1, 12), (2, 16), (3, 20)$

2.

Input	−2	−1	0	1	2
Output	4	1	0	1	4

Exploring Inverses of Functions

You have previously used given inputs to find corresponding outputs of $y = f(x)$ for various types of functions. You have also used given outputs to find corresponding inputs. Now you will solve equations of the form $y = f(x)$ for x to obtain a formula for finding the input given a specific output of the function f.

EXAMPLE 2   **Writing a Formula for the Input of a Function**

Let $f(x) = 2x + 1$. Solve $y = f(x)$ for x. Then find the input when the output is -3.

SOLUTION

$$y = 2x + 1 \qquad \text{Set } y \text{ equal to } f(x).$$

$$y - 1 = 2x \qquad \text{Subtract 1 from each side.}$$

$$\frac{y - 1}{2} = x \qquad \text{Divide each side by 2.}$$

Find the input when $y = -3$.

$$x = \frac{-3 - 1}{2} \qquad \text{Substitute } -3 \text{ for } y.$$

$$= \frac{-4}{2} \qquad \text{Subtract.}$$

$$= -2 \qquad \text{Divide.}$$

> **Check**
> $$f(-2) = 2(-2) + 1$$
> $$= -4 + 1$$
> $$= -3 \checkmark$$

▶ So, the input is -2 when the output is -3.

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Solve $y = f(x)$ for x. Then find the input when the output is 4.

3. $f(x) = x - 6$ **4.** $f(x) = \frac{1}{2}x + 3$ **5.** $f(x) = 10 - x$

UNDERSTANDING MATHEMATICAL TERMS

The term *inverse functions* does not refer to a new type of function. Rather, it describes any pair of functions that are inverses.

In Example 2, notice the steps involved after substituting for x in $y = 2x + 1$ and after substituting for y in $x = \dfrac{y - 1}{2}$.

$$y = 2x + 1 \qquad\qquad\qquad x = \frac{y - 1}{2}$$

Step 1 Multiply by 2.   **Step 1** Subtract 1.

Step 2 Add 1.   *inverse operations in*   **Step 2** Divide by 2.
                        the reverse order

Notice that these steps *undo* each other. **Inverse functions** are functions that undo each other. In Example 2, you can use the equation solved for x to write the inverse of f by switching the roles of x and y.

$$f(x) = 2x + 1 \quad \text{original function} \qquad\qquad g(x) = \frac{x - 1}{2} \quad \text{inverse function}$$

Because an inverse function interchanges the input and output values of the original function, the domain and range are also interchanged.

LOOKING FOR A PATTERN

Notice that the graph of the inverse function g is a reflection of the graph of the original function f. The line of reflection is $y = x$.

Original function: $f(x) = 2x + 1$

x	−2	−1	0	1	2
y	−3	−1	1	3	5

Inverse function: $g(x) = \dfrac{x - 1}{2}$

x	−3	−1	1	3	5
y	−2	−1	0	1	2
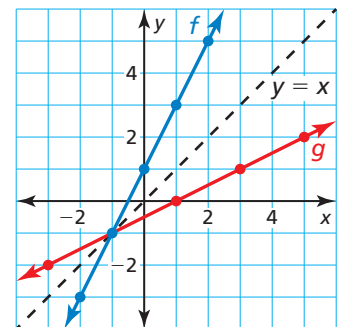
Finding Inverses of Functions Algebraically

🅖 Core Concept

Finding Inverses of Functions Algebraically

Step 1 Set y equal to $f(x)$.

Step 2 Switch x and y in the equation.

Step 3 Solve the equation for y.

STUDY TIP

On the previous page, you solved a function for x and switched the roles of x and y to find the inverse function. You can also find the inverse function by switching x and y first, and then solving for y.

EXAMPLE 3   Finding the Inverse of a Function

Find the inverse of $f(x) = 4x - 9$.

SOLUTION

Method 1 Use the method above.

Step 1	$f(x) = 4x - 9$	Write the function.
	$y = 4x - 9$	Set y equal to $f(x)$.
Step 2	$x = 4y - 9$	Switch x and y in the equation.
Step 3	$x + 9 = 4y$	Add 9 to each side.
	$\dfrac{x + 9}{4} = y$	Divide each side by 4.

▶ The inverse of f is $g(x) = \dfrac{x + 9}{4}$, or $g(x) = \dfrac{1}{4}x + \dfrac{9}{4}$.

Method 2 Use inverse operations in the reverse order.

$$f(x) = 4x - 9 \qquad \text{Multiply the input } x \text{ by 4 and then subtract 9.}$$

To find the inverse, apply inverse operations in the reverse order.

$$g(x) = \dfrac{x + 9}{4} \qquad \text{Add 9 to the input } x \text{ and then divide by 4.}$$

▶ The inverse of f is $g(x) = \dfrac{x + 9}{4}$, or $g(x) = \dfrac{1}{4}x + \dfrac{9}{4}$.

Check
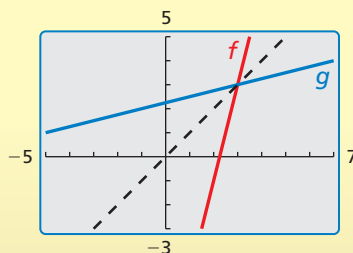
The graph of g appears to be a reflection of the graph of f in the line $y = x$. ✔

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Find the inverse of the function. Then graph the function and its inverse.

6. $f(x) = 6x$ **7.** $f(x) = -x + 5$ **8.** $f(x) = \frac{1}{4}x - 1$

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** A relation contains the point $(-3, 10)$. The _____ contains the point $(10, -3)$.

2. **DIFFERENT WORDS, SAME QUESTION** Consider the function f represented by the graph. Which is different? Find "both" answers.
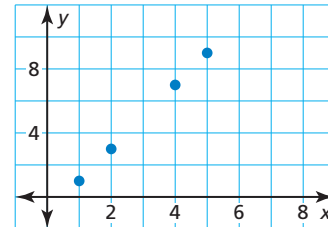
   Graph the inverse of the function.

   Reflect the graph of the function in the x-axis.

   Reflect the graph of the function in the line $y = x$.

   Switch the inputs and outputs of the function and graph the resulting function.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, find the inverse of the relation. *(See Example 1.)*

3. $(1, 0), (3, -8), (4, -3), (7, -5), (9, -1)$

4. $(2, 1), (4, -3), (6, 7), (8, 1), (10, -4)$

5.

Input	−5	−5	0	5	10
Output	8	6	0	6	8

6.

Input	−12	−8	−5	−3	−2
Output	2	5	−1	10	−2

7.

Input	Output
−3	9
−1	1
1	4
3	

8.

Input	Output
0	4
2	−5
7	0
15	−9
	11
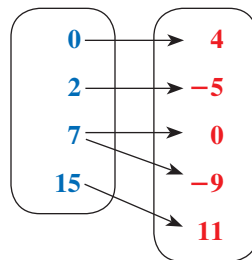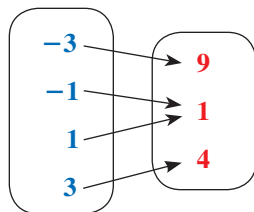
In Exercises 9–12, solve $y = f(x)$ for x. Then find the input when the output is 2. *(See Example 2.)*

9. $f(x) = x + 5$

10. $f(x) = 2x - 3$

11. $f(x) = \frac{1}{4}x - 1$

12. $f(x) = \frac{2}{3}x + 4$

In Exercises 13 and 14, graph the inverse of the function by reflecting the graph in the line $y = x$. Describe the domain and range of the inverse.

13.

14.
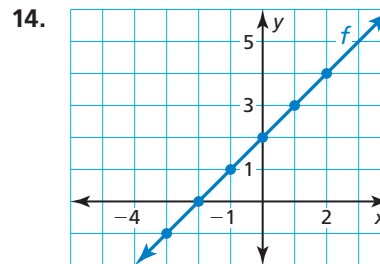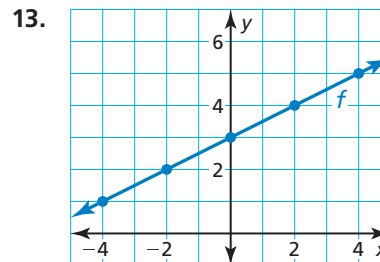
In Exercises 15–22, find the inverse of the function. Then graph the function and its inverse. *(See Example 3.)*

15. $f(x) = 3x$

16. $f(x) = -5x$

17. $f(x) = 4x - 1$

18. $f(x) = -2x + 5$

19. $f(x) = -3x - 2$

20. $f(x) = 2x + 3$

21. $f(x) = \frac{1}{3}x + 8$

22. $f(x) = -\frac{3}{2}x + \frac{7}{2}$

23. ERROR ANALYSIS Describe and correct the error in finding the inverse of the function $f(x) = 3x + 5$.

$$y = 3x + 5$$
$$y - 5 = 3x$$
$$\frac{y-5}{3} = x$$

The inverse of f is $g(x) = \dfrac{y-5}{3}$, or $g(x) = \dfrac{y}{3} - \dfrac{5}{3}$.

24. ERROR ANALYSIS Describe and correct the error in graphing the inverse g of the function f.
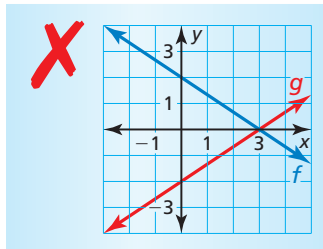
25. MODELING WITH MATHEMATICS The euro is the unit of currency for the European Union. On a certain day, the number E of euros that could be obtained for D U.S. dollars was represented by the formula shown.

$$E = 0.74683D$$

Solve the formula for D. Then find the number of U.S. dollars that could be obtained for 250 euros on that day.

26. MODELING WITH MATHEMATICS You work at a manufacturing plant and earn $11 per hour plus $0.50 for each unit you produce per hour.

a. Write a linear model that represents your hourly wage as a function of the number of units you produce per hour.

b. Find the inverse of the function you wrote in part (a). What does each variable in the inverse function represent?

c. What is your hourly wage when you produce 8 units per hour?

d. How many units must you produce per hour to double your hourly wage in part (c)?

27. OPEN-ENDED Write a function such that the graph of its inverse is a line with a slope of 4.

28. HOW DO YOU SEE IT? Pair the graph of each function with the graph of its inverse.

A.

B.

C.

D.

E.

F.
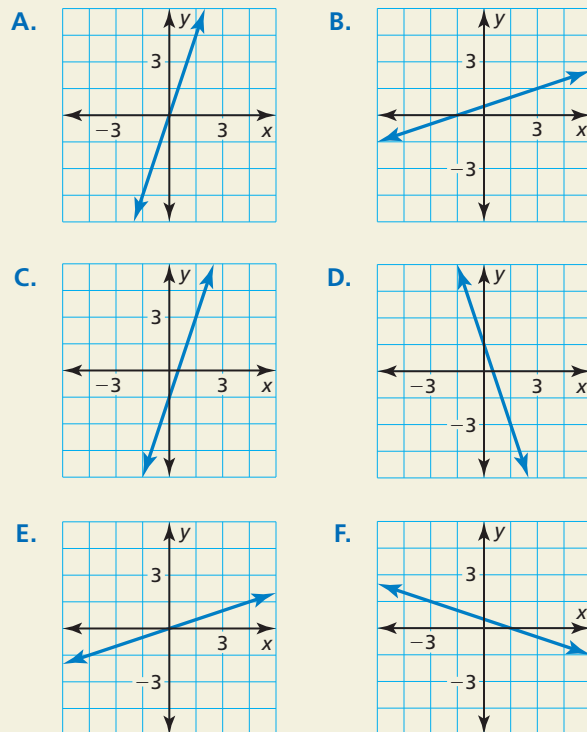
29. CRITICAL THINKING Consider the function $g(x) = -x$.

a. Graph $g(x) = -x$ and explain why it is its own inverse.

b. Graph other linear functions that are their own inverses. Write equations of the lines you graph.

c. Use your results from part (b) to write a general equation that describes the family of linear functions that are their own inverses.

30. THOUGHT PROVOKING Find a formula (for instance, from geometry or physics) that has an inverse function. Explain what the variables in your formula represent.

31. REASONING Show that the inverse of any linear function $f(x) = mx + b$, where $m \neq 0$, is also a linear function. Write the slope and y-intercept of the graph of the inverse in terms of m and b.

Maintaining Mathematical Proficiency
Reviewing what you learned in previous grades and lessons

Evaluate the expression. *(Skills Review Handbook)*

32. 12^5

33. -11^3

34. $\left(\dfrac{5}{6}\right)^4$

35. $-\left(\dfrac{1}{3}\right)^6$

Write the number in scientific notation. *(Skills Review Handbook)*

36. 84,375

37. 0.00944

38. 400,700,000

39. 0.000000082

Core Vocabulary

absolute value function, *p. 4*   piecewise function, *p. 12*   inverse relation, *p. 20*
vertex, *p. 4*   step function, *p. 14*   inverse function, *p. 21*
vertex form, *p. 6*

Core Concepts

Section 1.1

Absolute Value Function, *p. 4*   Vertex Form of an Absolute Value Function, *p. 6*

Section 1.2

Piecewise Function, *p. 12*   Writing Absolute Value Functions, *p. 15*
Step Function, *p. 14*

Section 1.3

Inverse Relation, *p. 20*   Finding Inverses of Functions Algebraically, *p. 22*

Mathematical Practices

1. Describe the definitions you used when you explained your answer in Exercise 53 on page 18.

2. What external resources could you use to check the reasonableness of your answer in Exercise 25 on page 24?

Making Note Cards

Invest in three different colors of note cards. Use one color for each of the following: vocabulary words, rules, and calculator keystrokes.

- Using the first color of note cards, write a vocabulary word on one side of a card. On the other side, write the definition and an example. If possible, put the definition in your own words.

- Using the second color of note cards, write a rule on one side of a card. On the other side, write an explanation and an example.

- Using the third color of note cards, write a calculation on one side of a card. On the other side, write the keystrokes required to perform the calculation.

Use the note cards as references while completing your homework. Quiz yourself once a day.
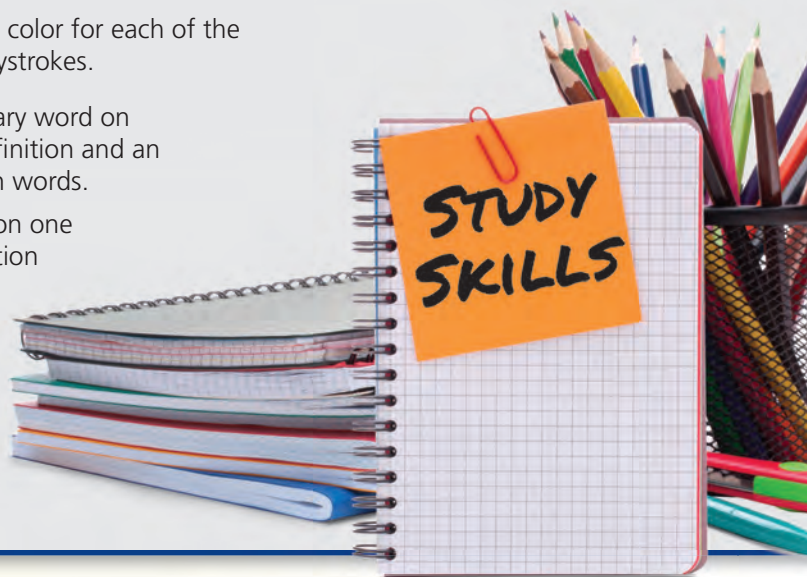
STUDY SKILLS

Describe the transformations from the graph of $f(x) = |x|$ to the graph of the given function. Then graph the given function. *(Section 1.1)*

1. $f(x) = |x + 5| + 4$

2. $g(x) = -\frac{1}{2}|x - 3| - 6$

3. $h(x) = |-x + 2| - 4$

Write an equation that represents the given transformation(s) of the graph of $f(x) = |x|$. *(Section 1.1)*

4. horizontal translation 9 units right

5. vertical translation 4.5 units up

6. horizontal stretch by a factor of 5

7. vertical shrink by a factor of $\frac{1}{4}$ and a reflection in the x-axis

Graph the function. Describe the domain and range. *(Section 1.2)*

8. $y = \begin{cases} \frac{3}{4}x + 1, & \text{if } x < -4 \\ -2x - 3, & \text{if } x \geq -4 \end{cases}$

9. $y = \begin{cases} -3, & \text{if } -4 \leq x < -2 \\ -2, & \text{if } -2 \leq x < -1 \\ -1, & \text{if } -1 \leq x < 1 \\ 0, & \text{if } 1 \leq x < 2 \end{cases}$

10. Write a piecewise function represented by the graph shown. *(Section 1.2)*

11. Write the function $y = -4|x + 1|$ as a piecewise function. *(Section 1.2)*

12. Find the inverse of the relation shown in the table. *(Section 1.3)*

Input	−4	−2	0	2	4
Output	6	3	0	−3	−6
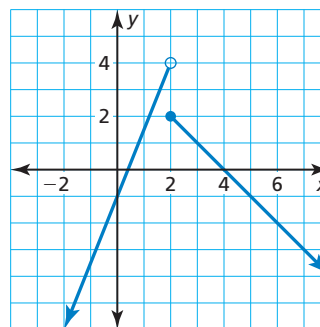
Find the inverse of the function. Then graph the function and its inverse.
(Section 1.3)

13. $k(x) = -\frac{1}{2}x$

14. $p(x) = 3x - 5$

15. $r(x) = 6 - 0.2x$

16. Write two absolute value functions whose graphs form a square in a coordinate plane. *(Section 1.1)*

17. Describe a real-life situation that can be represented by the graph shown. *(Section 1.2)*

18. You are ordering customized bracelets for a fundraiser. The total cost (in dollars) of ordering x bracelets is represented by the function below. *(Section 1.2)*

$$f(x) = \begin{cases} 0.9x + 25, & \text{if } 0 \leq x < 100 \\ 0.65x + 25, & \text{if } 100 \leq x < 500 \\ 0.4x + 25, & \text{if } 500 \leq x < 1000 \\ 0.1x + 25, & \text{if } x \geq 1000 \end{cases}$$

   a. Find and interpret $f(35)$.

   b. Determine the costs of ordering 100, 500, and 2500 bracelets.

   c. What do you think the 25s in the function represent?
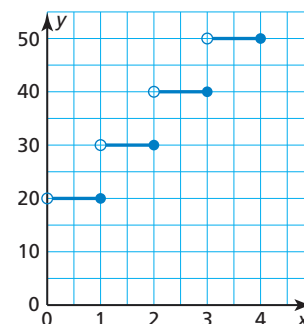
Essential Question How can you write general rules involving properties of exponents?

EXPLORATION 1 Writing Rules for Properties of Exponents

Work with a partner.

a. What happens when you multiply two powers with the same base? Write the product of the two powers as a single power. Then write a *general rule* for finding the product of two powers with the same base.

   i. $(2^2)(2^3) =$ **ii.** $(4^1)(4^5) =$

   iii. $(5^3)(5^5) =$ **iv.** $(x^2)(x^6) =$

b. What happens when you divide two powers with the same base? Write the quotient of the two powers as a single power. Then write a *general rule* for finding the quotient of two powers with the same base.

   i. $\dfrac{4^3}{4^2} =$ **ii.** $\dfrac{2^5}{2^2} =$

   iii. $\dfrac{x^6}{x^3} =$ **iv.** $\dfrac{3^4}{3^4} =$

c. What happens when you find a power of a power? Write the expression as a single power. Then write a *general rule* for finding a power of a power.

   i. $(2^2)^4 =$ **ii.** $(7^3)^2 =$

   iii. $(y^3)^3 =$ **iv.** $(x^4)^2 =$

d. What happens when you find a power of a product? Write the expression as the product of two powers. Then write a *general rule* for finding a power of a product.

   i. $(2 \cdot 5)^2 =$ **ii.** $(5 \cdot 4)^3 =$

   iii. $(6a)^2 =$ **iv.** $(3x)^2 =$

e. What happens when you find a power of a quotient? Write the expression as the quotient of two powers. Then write a *general rule* for finding a power of a quotient.

   i. $\left(\dfrac{2}{3}\right)^2 =$ **ii.** $\left(\dfrac{4}{3}\right)^3 =$

   iii. $\left(\dfrac{x}{2}\right)^3 =$ **iv.** $\left(\dfrac{a}{b}\right)^4 =$

Communicate Your Answer

2. How can you write general rules involving properties of exponents?

3. There are 3^3 small cubes in the cube below. Write an expression for the number of small cubes in the large cube at the right.
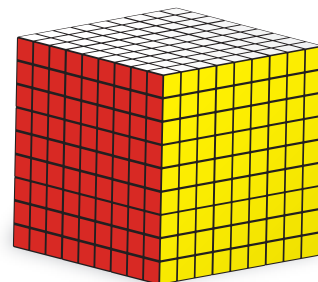
Core Vocabulary

Previous
power
exponent
base
scientific notation

What You Will Learn

▶ Use zero and negative exponents.
▶ Use the properties of exponents.
▶ Solve real-life problems involving exponents.

Using Zero and Negative Exponents

Core Concept

Zero Exponent

Words For any nonzero number a, $a^0 = 1$. The power 0^0 is undefined.

Numbers $4^0 = 1$ **Algebra** $a^0 = 1$, where $a \neq 0$

Negative Exponents

Words For any integer n and any nonzero number a, a^{-n} is the reciprocal of a^n.

Numbers $4^{-2} = \dfrac{1}{4^2}$ **Algebra** $a^{-n} = \dfrac{1}{a^n}$, where $a \neq 0$

EXAMPLE 1   **Using Zero and Negative Exponents**

Evaluate each expression.

a. 6.7^0 **b.** $(-2)^{-4}$

SOLUTION

a. $6.7^0 = 1$ Definition of zero exponent

b. $(-2)^{-4} = \dfrac{1}{(-2)^4}$ Definition of negative exponent

             $= \dfrac{1}{16}$ Simplify.

EXAMPLE 2   **Simplifying an Expression**

Simplify the expression $\dfrac{4x^0}{y^{-3}}$. Write your answer using only positive exponents.

SOLUTION

$$\dfrac{4x^0}{y^{-3}} = 4x^0 y^3 \qquad \text{Definition of negative exponent}$$

$$= 4y^3 \qquad \text{Definition of zero exponent}$$

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Evaluate the expression.

1. $(-9)^0$ **2.** 3^{-3} **3.** $\dfrac{-5^0}{2^{-2}}$

4. Simplify the expression $\dfrac{3^{-2}x^{-5}}{y^0}$. Write your answer using only positive exponents.

Using the Properties of Exponents

Core Concept

Product of Powers Property

Let a be a real number, and let m and n be integers.

Words To multiply powers with the same base, add their exponents.

Numbers $4^6 \cdot 4^3 = 4^{6+3} = 4^9$ **Algebra** $a^m \cdot a^n = a^{m+n}$

Quotient of Powers Property

Let a be a nonzero real number, and let m and n be integers.

Words To divide powers with the same base, subtract their exponents.

Numbers $\dfrac{4^6}{4^3} = 4^{6-3} = 4^3$ **Algebra** $\dfrac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$

Power of a Power Property

Let a be a real number, and let m and n be integers.

Words To find a power of a power, multiply the exponents.

Numbers $(4^6)^3 = 4^{6 \cdot 3} = 4^{18}$ **Algebra** $(a^m)^n = a^{mn}$

EXAMPLE 3 **Using Properties of Exponents**

Simplify each expression. Write your answer using only positive exponents.

a. $3^2 \cdot 3^6$ **b.** $\dfrac{(-4)^2}{(-4)^7}$ **c.** $(z^4)^{-3}$

SOLUTION

a. $3^2 \cdot 3^6 = 3^{2+6}$ Product of Powers Property

$\qquad = 3^8 = 6561$ Simplify.

b. $\dfrac{(-4)^2}{(-4)^7} = (-4)^{2-7}$ Quotient of Powers Property

$\qquad = (-4)^{-5}$ Simplify.

$\qquad = \dfrac{1}{(-4)^5} = -\dfrac{1}{1024}$ Definition of negative exponent

c. $(z^4)^{-3} = z^{4 \cdot (-3)}$ Power of a Power Property

$\qquad = z^{-12}$ Simplify.

$\qquad = \dfrac{1}{z^{12}}$ Definition of negative exponent

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Simplify the expression. Write your answer using only positive exponents.

5. $10^4 \cdot 10^{-6}$ **6.** $x^9 \cdot x^{-9}$ **7.** $\dfrac{-5^8}{-5^4}$

8. $\dfrac{y^6}{y^7}$ **9.** $(6^{-2})^{-1}$ **10.** $(w^{12})^5$

Core Concept

Power of a Product Property

Let a and b be real numbers, and let m be an integer.

Words To find a power of a product, find the power of each factor and multiply.

Numbers $(3 \cdot 2)^5 = 3^5 \cdot 2^5$ **Algebra** $(ab)^m = a^m b^m$

Power of a Quotient Property

Let a and b be real numbers with $b \neq 0$, and let m be an integer.

Words To find the power of a quotient, find the power of the numerator and the power of the denominator and divide.

Numbers $\left(\dfrac{3}{2}\right)^5 = \dfrac{3^5}{2^5}$ **Algebra** $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$, where $b \neq 0$

EXAMPLE 4 Using Properties of Exponents

Simplify each expression. Write your answer using only positive exponents.

a. $(-1.5y)^2$ **b.** $\left(\dfrac{a}{-10}\right)^3$ **c.** $\left(\dfrac{3d}{2}\right)^4$ **d.** $\left(\dfrac{2x}{3}\right)^{-5}$

SOLUTION

a. $(-1.5y)^2 = (-1.5)^2 \cdot y^2$ Power of a Product Property

$\qquad\qquad = 2.25y^2$ Simplify.

b. $\left(\dfrac{a}{-10}\right)^3 = \dfrac{a^3}{(-10)^3}$ Power of a Quotient Property

$\qquad\qquad = -\dfrac{a^3}{1000}$ Simplify.

ANOTHER WAY

Because the exponent is negative, you could find the reciprocal of the base first. Then simplify.

$\left(\dfrac{2x}{3}\right)^{-5} = \left(\dfrac{3}{2x}\right)^5 = \dfrac{243}{32x^5}$

c. $\left(\dfrac{3d}{2}\right)^4 = \dfrac{(3d)^4}{2^4}$ Power of a Quotient Property

$\qquad\qquad = \dfrac{3^4 d^4}{2^4}$ Power of a Product Property

$\qquad\qquad = \dfrac{81d^4}{16}$ Simplify.

d. $\left(\dfrac{2x}{3}\right)^{-5} = \dfrac{(2x)^{-5}}{3^{-5}}$ Power of a Quotient Property

$\qquad\qquad = \dfrac{3^5}{(2x)^5}$ Definition of negative exponent

$\qquad\qquad = \dfrac{3^5}{2^5 x^5}$ Power of a Product Property

$\qquad\qquad = \dfrac{243}{32x^5}$ Simplify.
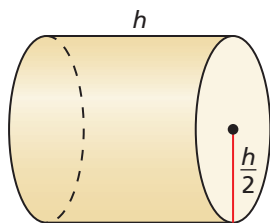

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Simplify the expression. Write your answer using only positive exponents.

11. $(10y)^{-3}$ **12.** $\left(-\dfrac{4}{n}\right)^5$ **13.** $\left(\dfrac{1}{2k^2}\right)^5$ **14.** $\left(\dfrac{6c}{7}\right)^{-2}$

Solving Real-Life Problems

EXAMPLE 5 **Simplifying a Real-Life Expression**

h

$\frac{h}{2}$

Volume = ?

$2\pi r^3$	$\pi h^3 2^{-2}$	$\pi h 4^{-1}$
$\dfrac{\pi h^2}{4}$	$\dfrac{\pi h^3}{4}$	$\dfrac{\pi h^3}{2}$

Which of the expressions shown represent the volume of the cylinder, where r is the radius and h is the height?

SOLUTION

$V = \pi r^2 h$ — Formula for the volume of a cylinder

$= \pi\left(\dfrac{h}{2}\right)^2 (h)$ — Substitute $\dfrac{h}{2}$ for r.

$= \pi\left(\dfrac{h^2}{2^2}\right)(h)$ — Power of a Quotient Property

$= \dfrac{\pi h^3}{4}$ — Simplify.

Any expression equivalent to $\dfrac{\pi h^3}{4}$ represents the volume of the cylinder.

- You can use the properties of exponents to write $\pi h^3 2^{-2}$ as $\dfrac{\pi h^3}{4}$.

- Note $h = 2r$. When you substitute $2r$ for h in $\dfrac{\pi h^3}{4}$, you can write $\dfrac{\pi(2r)^3}{4}$ as $2\pi r^3$.

- None of the other expressions are equivalent to $\dfrac{\pi h^3}{4}$.

▶ The expressions $2\pi r^3$, $\pi h^3 2^{-2}$, and $\dfrac{\pi h^3}{4}$ represent the volume of the cylinder.

EXAMPLE 6 **Solving a Real-Life Problem**

A jellyfish emits about 1.25×10^8 particles of light, or photons, in 6.25×10^{-4} second. How many photons does the jellyfish emit each second? Write your answer in scientific notation and in standard form.

REMEMBER

A number is written in scientific notation when it is of the form $a \times 10^b$, where $1 \le a < 10$ and b is an integer.

SOLUTION

Divide to find the unit rate.

$\dfrac{1.25 \times 10^8}{6.25 \times 10^{-4}}$ ← photons / ← seconds — Write the rate.

$= \dfrac{1.25}{6.25} \times \dfrac{10^8}{10^{-4}}$ — Rewrite.

$= 0.2 \times 10^{12}$ — Simplify.

$= 2 \times 10^{11}$ — Write in scientific notation.

▶ The jellyfish emits 2×10^{11}, or 200,000,000,000 photons per second.

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15. Write two expressions that represent the area of a base of the cylinder in Example 5.

16. It takes the Sun about 2.3×10^8 years to orbit the center of the Milky Way. It takes Pluto about 2.5×10^2 years to orbit the Sun. How many times does Pluto orbit the Sun while the Sun completes one orbit around the center of the Milky Way? Write your answer in scientific notation.

Vocabulary and Core Concept Check

1. **VOCABULARY** Which definitions or properties would you use to simplify the expression $(4^8 \cdot 4^{-4})^{-2}$? Explain.

2. **WRITING** Explain when and how to use the Power of a Product Property.

3. **WRITING** Explain when and how to use the Quotient of Powers Property.

4. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

| Simplify $3^3 \cdot 3^6$. | Simplify 3^{3+6}. | Simplify $3^6 \cdot 3$. | Simplify $3^6 \cdot 3^3$. |

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In Exercises 5–12, evaluate the expression. *(See Example 1.)*

5. $(-7)^0$

6. 4^0

7. 5^{-4}

8. $(-2)^{-5}$

9. $\dfrac{2^{-4}}{4^0}$

10. $\dfrac{5^{-1}}{-9^0}$

11. $\dfrac{-3^{-3}}{6^{-2}}$

12. $\dfrac{(-8)^{-2}}{3^{-4}}$

In Exercises 13–22, simplify the expression. Write your answer using only positive exponents. *(See Example 2.)*

13. x^{-7}

14. y^0

15. $9x^0y^{-3}$

16. $15c^{-8}d^0$

17. $\dfrac{2^{-2}m^{-3}}{n^0}$

18. $\dfrac{10^0r^{-11}s}{3^2}$

19. $\dfrac{4^{-3}a^0}{b^{-7}}$

20. $\dfrac{p^{-8}}{7^{-2}q^{-9}}$

21. $\dfrac{2^2y^{-6}}{8^{-1}z^0x^{-7}}$

22. $\dfrac{13x^{-5}y^0}{5^{-3}z^{-10}}$

In Exercises 23–32, simplify the expression. Write your answer using only positive exponents. *(See Example 3.)*

23. $\dfrac{5^6}{5^2}$

24. $\dfrac{(-6)^8}{(-6)^5}$

25. $(-9)^2 \cdot (-9)^2$

26. $4^{-5} \cdot 4^5$

27. $(p^6)^4$

28. $(s^{-5})^3$

29. $6^{-8} \cdot 6^5$

30. $-7 \cdot (-7)^{-4}$

31. $\dfrac{x^5}{x^4} \cdot x$

32. $\dfrac{z^8 \cdot z^2}{z^5}$

33. **USING PROPERTIES** A microscope magnifies an object 10^5 times. The length of an object is 10^{-7} meter. What is its magnified length?

34. **USING PROPERTIES** The area of the rectangular computer chip is $112a^3b^2$ square microns. What is the length?

width = 8*ab* microns

ERROR ANALYSIS In Exercises 35 and 36, describe and correct the error in simplifying the expression.

35.
$$2^4 \cdot 2^5 = (2 \cdot 2)^{4+5}$$
$$= 4^9$$

36.
$$\frac{x^5 \cdot x^3}{x^4} = \frac{x^8}{x^4}$$
$$= x^{8/4}$$
$$= x^2$$

In Exercises 37–44, simplify the expression. Write your answer using only positive exponents. *(See Example 4.)*
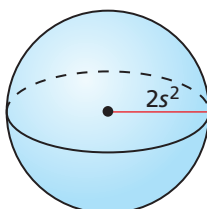
37. $(-5z)^3$

38. $(4x)^{-4}$

39. $\left(\dfrac{6}{n}\right)^{-2}$

40. $\left(\dfrac{-t}{3}\right)^2$

41. $(3s^8)^{-5}$

42. $(-5p^3)^3$

43. $\left(-\dfrac{w^3}{6}\right)^{-2}$

44. $\left(\dfrac{1}{2r^6}\right)^{-6}$

45. USING PROPERTIES
Which of the expressions represent the volume of the sphere? Explain. *(See Example 5.)*

2s²

Ⓐ $\left(\dfrac{3s^2}{2^4\pi s^8}\right)^{-1}$

Ⓑ $(2^5\pi s^6)(3^{-1})$

Ⓒ $\dfrac{32\pi s^6}{3}$

Ⓓ $(2s)^5 \cdot \dfrac{\pi s}{3}$

Ⓔ $\left(\dfrac{3\pi s^6}{32}\right)^{-1}$

Ⓕ $\dfrac{32}{3}\pi s^5$

46. MODELING WITH MATHEMATICS Diffusion is the movement of molecules from one location to another. The time t (in seconds) it takes molecules to diffuse a distance of x centimeters is given by $t = \dfrac{x^2}{2D}$, where D is the diffusion coefficient. The diffusion coefficient for a drop of ink in water is about 10^{-5} square centimeters per second. How long will it take the ink to diffuse 1 micrometer (10^{-4} centimeter)?

In Exercises 47–50, simplify the expression. Write your answer using only positive exponents.

47. $\left(\dfrac{2x^{-2}y^3}{3xy^{-4}}\right)^4$

48. $\left(\dfrac{4s^5t^{-7}}{-2s^{-2}t^4}\right)^3$

49. $\left(\dfrac{3m^{-5}n^2}{4m^{-2}n^0}\right)^2 \cdot \left(\dfrac{mn^4}{9n}\right)^2$

50. $\left(\dfrac{3x^3y^0}{x^{-2}}\right)^4 \cdot \left(\dfrac{y^2x^{-4}}{5xy^{-8}}\right)^3$

In Exercises 51–54, evaluate the expression. Write your answer in scientific notation and standard form.

51. $(3 \times 10^2)(1.5 \times 10^{-5})$

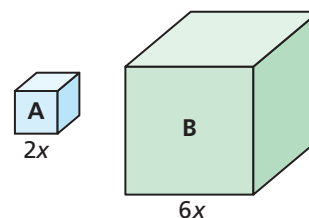**52.** $(6.1 \times 10^{-3})(8 \times 10^9)$

53. $\dfrac{(6.4 \times 10^7)}{(1.6 \times 10^5)}$

54. $\dfrac{(3.9 \times 10^{-5})}{(7.8 \times 10^{-8})}$

55. PROBLEM SOLVING In 2012, on average, about 9.46×10^{-1} pound of potatoes was produced for every 2.3×10^{-5} acre harvested. How many pounds of potatoes on average were produced for each acre harvested? Write your answer in scientific notation and in standard form. *(See Example 6.)*

56. PROBLEM SOLVING The speed of light is approximately 3×10^5 kilometers per second. How long does it take sunlight to reach Jupiter? Write your answer in scientific notation and in standard form.

OUR SOLAR SYSTEM
NEPTUNE · URANUS · SATURN · JUPITER · MARS · EARTH · VENUS · MERCURY · SUN
Average Distance: 7.8×10^8 kilometers

57. MATHEMATICAL CONNECTIONS Consider Cube A and Cube B.

A 2x
B 6x

a. Which property of exponents should you use to simplify an expression for the volume of each cube?

b. How can you use the Power of a Quotient Property to find how many times greater the volume of Cube B is than the volume of Cube A?

58. PROBLEM SOLVING A byte is a unit used to measure a computer's memory. The table shows the numbers of bytes in several units of measure.

Unit	kilobyte	megabyte	gigabyte	terabyte
Number of bytes	2^{10}	2^{20}	2^{30}	2^{40}

a. How many kilobytes are in 1 terabyte?

b. How many megabytes are in 16 gigabytes?

c. Another unit used to measure a computer's memory is a bit. There are 8 bits in a byte. How can you convert the number of bytes in each unit of measure given in the table to bits? Can you still use a base of 2? Explain.

REWRITING EXPRESSIONS In Exercises 59–62, rewrite the expression as a power of a product.

59. $8a^3b^3$

60. $16r^2s^2$

61. $64w^{18}z^{12}$

62. $81x^4y^8$

63. **USING STRUCTURE** The probability of rolling a 6 on a number cube is $\frac{1}{6}$. The probability of rolling a 6 twice in a row is $\left(\frac{1}{6}\right)^2 = \frac{1}{36}$.

  a. Write an expression that represents the probability of rolling a 6 n times in a row.

  b. What is the probability of rolling a 6 four times in a row?

 **c.** What is the probability of flipping heads on a coin five times in a row? Explain.

64. **HOW DO YOU SEE IT?** The shaded part of Figure n represents the portion of a piece of paper visible after folding the paper in half n times.

Figure 1   Figure 2

Figure 3   Figure 4
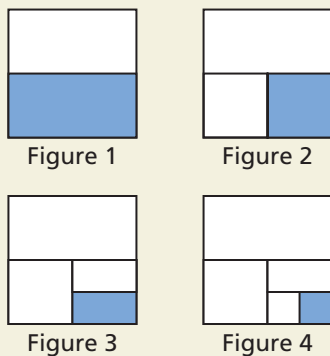
  a. What fraction of the original piece of paper is each shaded part?

  b. Rewrite each fraction from part (a) in the form 2^x.

65. **REASONING** Find x and y when $\dfrac{b^x}{b^y} = b^9$ and $\dfrac{b^x \cdot b^2}{b^{3y}} = b^{13}$. Explain how you found your answer.

66. **THOUGHT PROVOKING** Write expressions for r and h so that the volume of the cone can be represented by the expression $27\pi x^8$. Find r and h.
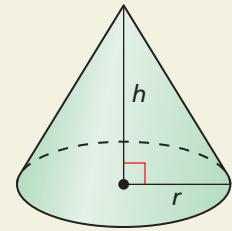
67. **MAKING AN ARGUMENT** One of the smallest plant seeds comes from an orchid, and one of the largest plant seeds comes from a double coconut palm. A seed from an orchid has a mass of 10^{-6} gram. The mass of a seed from a double coconut palm is 10^{10} times the mass of the seed from the orchid. Your friend says that the seed from the double coconut palm has a mass of about 1 kilogram. Is your friend correct? Explain.

68. **CRITICAL THINKING** Your school is conducting a survey. Students can answer the questions in either part with "agree" or "disagree."

Part 1: 13 questions		
Part 2: 10 questions		
Part 1: Classroom	Agree	Disagree
1. I come prepared for class.	O	O
2. I enjoy my assignments.	O	O

  a. What power of 2 represents the number of different ways that a student can answer all the questions in Part 1?

  b. What power of 2 represents the number of different ways that a student can answer all the questions on the entire survey?

  c. The survey changes, and students can now answer "agree," "disagree," or "no opinion." How does this affect your answers in parts (a) and (b)?

69. **ABSTRACT REASONING** Compare the values of a^n and a^{-n} when $n < 0$, when $n = 0$, and when $n > 0$ for (a) $a > 1$ and (b) $0 < a < 1$. Explain your reasoning.

Maintaining Mathematical Proficiency
Reviewing what you learned in previous grades and lessons

Find the square root(s). *(Skills Review Handbook)*

70. $\sqrt{25}$

71. $-\sqrt{100}$

72. $\pm\sqrt{\dfrac{1}{64}}$

Classify the real number in as many ways as possible. *(Skills Review Handbook)*

73. 12

74. $\dfrac{65}{9}$

75. $\dfrac{\pi}{4}$

Essential Question
How can you write and evaluate an *n*th root of a number?

Recall that you cube a number as follows.

3rd power

$$2^3 = 2 \cdot 2 \cdot 2 = 8 \qquad \text{2 cubed is 8.}$$

To "undo" cubing a number, take the cube root of the number.

Symbol for cube root is $\sqrt[3]{}$.

$$\sqrt[3]{8} = \sqrt[3]{2^3} = 2 \qquad \text{The cube root of 8 is 2.}$$

EXPLORATION 1   Finding Cube Roots

Work with a partner. Use a cube root symbol to write the side length of each cube. Then find the cube root. Check your answers by multiplying. Which cube is the largest? Which two cubes are the same size? Explain your reasoning.
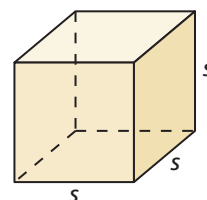
a. Volume = 27 ft³   **b.** Volume = 125 cm³   **c.** Volume = 3375 in.³

d. Volume = 3.375 m³   **e.** Volume = 1 yd³   **f.** Volume $= \dfrac{125}{8}$ mm³

JUSTIFYING CONCLUSIONS

To be proficient in math, you need to justify your conclusions and communicate them to others.

EXPLORATION 2   Estimating *n*th Roots

Work with a partner. Estimate each positive *n*th root. Then match each *n*th root with the point on the number line. Justify your answers.

a. $\sqrt[4]{25}$ **b.** $\sqrt{0.5}$ **c.** $\sqrt[5]{2.5}$

d. $\sqrt[3]{65}$ **e.** $\sqrt[3]{55}$ **f.** $\sqrt[6]{20{,}000}$
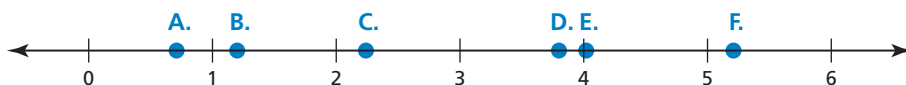
Communicate Your Answer

3. How can you write and evaluate an *n*th root of a number?

4. The body mass *m* (in kilograms) of a dinosaur that walked on two feet can be modeled by

$$m = (0.00016)C^{2.73}$$

where *C* is the circumference (in millimeters) of the dinosaur's femur. The mass of a *Tyrannosaurus rex* was 4000 kilograms. Use a calculator to approximate the circumference of its femur.

Core Vocabulary

nth root of a, *p. 36*
radical, *p. 36*
index of a radical, *p. 36*

Previous
square root

What You Will Learn

▶ Find *n*th roots.
▶ Evaluate expressions with rational exponents.
▶ Solve real-life problems involving rational exponents.

Finding *n*th Roots

You can extend the concept of a square root to other types of roots. For example, 2 is a cube root of 8 because $2^3 = 8$, and 3 is a fourth root of 81 because $3^4 = 81$. In general, for an integer n greater than 1, if $b^n = a$, then b is an **nth root of a**. An nth root of a is written as $\sqrt[n]{a}$, where the expression $\sqrt[n]{a}$ is called a **radical** and n is the **index** of the radical.

You can also write an nth root of a as a power of a. If you assume the Power of a Power Property applies to rational exponents, then the following is true.

$$(a^{1/2})^2 = a^{(1/2) \cdot 2} = a^1 = a$$
$$(a^{1/3})^3 = a^{(1/3) \cdot 3} = a^1 = a$$
$$(a^{1/4})^4 = a^{(1/4) \cdot 4} = a^1 = a$$

Because $a^{1/2}$ is a number whose square is a, you can write $\sqrt{a} = a^{1/2}$. Similarly, $\sqrt[3]{a} = a^{1/3}$ and $\sqrt[4]{a} = a^{1/4}$. In general, $\sqrt[n]{a} = a^{1/n}$ for any integer n greater than 1.

🄶 Core Concept

READING

$\pm \sqrt[n]{a}$ represents both the positive and negative nth roots of a.

Real *n*th Roots of *a*

Let n be an integer greater than 1, and let a be a real number.

- If n is odd, then a has one real nth root: $\sqrt[n]{a} = a^{1/n}$

- If n is even and $a > 0$, then a has two real nth roots: $\pm \sqrt[n]{a} = \pm a^{1/n}$

- If n is even and $a = 0$, then a has one real nth root: $\sqrt[n]{0} = 0$

- If n is even and $a < 0$, then a has no real nth roots.

The nth roots of a number may be real numbers or *imaginary numbers*. You will study imaginary numbers in Chapter 4.

EXAMPLE 1   Finding *n*th Roots

Find the indicated real nth root(s) of a.

a. $n = 3, a = -27$

b. $n = 4, a = 16$

SOLUTION

a. The index $n = 3$ is odd, so -27 has one real cube root. Because $(-3)^3 = -27$, the cube root of -27 is $\sqrt[3]{-27} = -3$, or $(-27)^{1/3} = -3$.

b. The index $n = 4$ is even, and $a > 0$. So, 16 has two real fourth roots. Because $2^4 = 16$ and $(-2)^4 = 16$, the fourth roots of 16 are $\pm\sqrt[4]{16} = \pm 2$, or $\pm 16^{1/4} = \pm 2$.

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Find the indicated real *n*th root(s) of *a*.

1. $n = 3, a = -125$

2. $n = 6, a = 64$

Evaluating Expressions with Rational Exponents

Recall that the radical $\sqrt{a}$ indicates the positive square root of a. Similarly, an nth root of a, $\sqrt[n]{a}$, with an *even* index indicates the positive nth root of a.

REMEMBER

The expression under the radical sign is the radicand.

EXAMPLE 2   **Evaluating nth Root Expressions**

Evaluate each expression.

a. $\sqrt[3]{-8}$ **b.** $-\sqrt[3]{8}$ **c.** $16^{1/4}$ **d.** $(-16)^{1/4}$

SOLUTION

a. $\sqrt[3]{-8} = \sqrt[3]{(-2) \cdot (-2) \cdot (-2)}$ Rewrite the expression showing factors.

        $= -2$ Evaluate the cube root.

b. $-\sqrt[3]{8} = -\left(\sqrt[3]{2 \cdot 2 \cdot 2}\right)$ Rewrite the expression showing factors.

        $= -(2)$ Evaluate the cube root.

        $= -2$ Simplify.

c. $16^{1/4} = \sqrt[4]{16}$ Rewrite the expression in radical form.

        $= \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2}$ Rewrite the expression showing factors.

        $= 2$ Evaluate the fourth root.

d. $(-16)^{1/4}$ is not a real number because there is no real number that can be multiplied by itself four times to produce -16.

A rational exponent does not have to be of the form $1/n$. Other rational numbers such as 3/2 can also be used as exponents. You can use the properties of exponents to evaluate or simplify expressions involving rational exponents.

STUDY TIP

You can rewrite $27^{2/3}$ as $27^{(1/3) \cdot 2}$ and then use the Power of a Power Property to show that

$27^{(1/3) \cdot 2} = (27^{1/3})^2$.

🅖 Core Concept

Rational Exponents

Let $a^{1/n}$ be an nth root of a, and let m be a positive integer.

Algebra $a^{m/n} = (a^{1/n})^m = \left(\sqrt[n]{a}\right)^m$

Numbers $27^{2/3} = (27^{1/3})^2 = \left(\sqrt[3]{27}\right)^2$

EXAMPLE 3   **Evaluating Expressions with Rational Exponents**

Evaluate (a) $16^{3/4}$ and (b) $27^{4/3}$.

SOLUTION

a. $16^{3/4} = (16^{1/4})^3$ Rational exponents   **b.** $27^{4/3} = (27^{1/3})^4$

       $= 2^3$ Evaluate the nth root. $= 3^4$

       $= 8$ Evaluate the power. $= 81$

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Evaluate the expression.

3. $\sqrt[3]{-125}$ **4.** $(-64)^{2/3}$ **5.** $9^{5/2}$ **6.** $256^{3/4}$

Section 1.5   Radicals and Rational Exponents   **37**

Solving Real-Life Problems

EXAMPLE 4 Solving a Real-Life Problem

Volume = 113 cubic feet

The radius r of a sphere is given by the equation $r = \left(\dfrac{3V}{4\pi}\right)^{1/3}$, where V is the volume of the sphere. Find the radius of the beach ball to the nearest foot. Use 3.14 for π.

SOLUTION

1. **Understand the Problem** You know the equation that represents the radius of a sphere in terms of its volume. You are asked to find the radius for a given volume.

2. **Make a Plan** Substitute the given volume into the equation. Then evaluate to find the radius.

3. **Solve the Problem**

$r = \left(\dfrac{3V}{4\pi}\right)^{1/3}$ Write the equation.

$= \left(\dfrac{3(113)}{4(3.14)}\right)^{1/3}$ Substitute 113 for V and 3.14 for π.

$= \left(\dfrac{339}{12.56}\right)^{1/3}$ Multiply.

≈ 3 Use a calculator.

▶ The radius of the beach ball is about 3 feet.

4. **Look Back** To check that your answer is reasonable, compare the size of the ball to the size of the woman pushing the ball. The ball appears to be slightly taller than the woman. The average height of a woman is between 5 and 6 feet. So, a radius of 3 feet, or height of 6 feet, seems reasonable for the beach ball.

EXAMPLE 5 Solving a Real-Life Problem

To calculate the annual inflation rate r (in decimal form) of an item that increases in value from P to F over a period of n years, you can use the equation $r = \left(\dfrac{F}{P}\right)^{1/n} - 1$.

Find the annual inflation rate to the nearest tenth of a percent of a house that increases in value from \$200,000 to \$235,000 over a period of 5 years.

SOLUTION

$r = \left(\dfrac{F}{P}\right)^{1/n} - 1$ Write the equation.

$= \left(\dfrac{235,000}{200,000}\right)^{1/5} - 1$ Substitute 235,000 for F, 200,000 for P, and 5 for n.

$= 1.175^{1/5} - 1$ Divide.

≈ 0.03278 Use a calculator.

REMEMBER

To write a decimal as a percent, move the decimal point two places to the right. Then add a percent symbol.

▶ The annual inflation rate is about 3.3%.

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7. **WHAT IF?** In Example 4, the volume of the beach ball is 17,000 cubic inches. Find the radius to the nearest inch. Use 3.14 for π.

8. The average cost of college tuition increases from \$8500 to \$13,500 over a period of 8 years. Find the annual inflation rate to the nearest tenth of a percent.

Vocabulary and Core Concept Check

1. **WRITING** Explain how to evaluate $81^{1/4}$.

2. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$$\left(\sqrt[3]{27}\right)^2 \qquad 27^{2/3} \qquad 3^2 \qquad \left(\sqrt[2]{27}\right)^3$$

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In Exercises 3 and 4, rewrite the expression in rational exponent form.

3. $\sqrt{10}$

4. $\sqrt[5]{34}$

In Exercises 5 and 6, rewrite the expression in radical form.

5. $15^{1/3}$

6. $140^{1/8}$

In Exercises 7–10, find the indicated real *n*th root(s) of *a*. *(See Example 1.)*
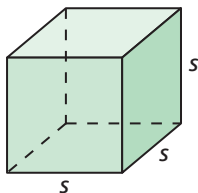
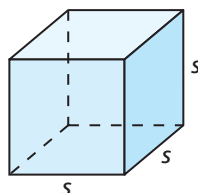7. $n = 2, a = 36$

8. $n = 4, a = 81$

9. $n = 3, a = 1000$

10. $n = 9, a = -512$

MATHEMATICAL CONNECTIONS In Exercises 11 and 12, find the dimensions of the cube. Check your answer.

11. Volume = 64 in.^{3}

12. Volume = 216 cm^3

In Exercises 13–18, evaluate the expression. *(See Example 2.)*

13. $\sqrt[4]{256}$

14. $\sqrt[3]{-216}$

15. $\sqrt[3]{-343}$

16. $-\sqrt[5]{1024}$

17. $128^{1/7}$

18. $(-64)^{1/2}$

In Exercises 19 and 20, rewrite the expression in rational exponent form.

19. $\left(\sqrt[5]{8}\right)^4$

20. $\left(\sqrt[5]{-21}\right)^6$

In Exercises 21 and 22, rewrite the expression in radical form.

21. $(-4)^{2/7}$

22. $9^{5/2}$

In Exercises 23–28, evaluate the expression. *(See Example 3.)*

23. $32^{3/5}$

24. $125^{2/3}$

25. $(-36)^{3/2}$

26. $(-243)^{2/5}$

27. $(-128)^{5/7}$

28. $343^{4/3}$

29. **ERROR ANALYSIS** Describe and correct the error in rewriting the expression in rational exponent form.

$$\text{✗} \quad \left(\sqrt[3]{2}\right)^4 = 2^{3/4}$$

30. **ERROR ANALYSIS** Describe and correct the error in evaluating the expression.

$$\text{✗} \quad \begin{aligned} (-81)^{3/4} &= [(-81)^{1/4}]^3 \\ &= (-3)^3 \\ &= -27 \end{aligned}$$

In Exercises 31–34, evaluate the expression.

31. $\left(\dfrac{1}{1000}\right)^{1/3}$

32. $\left(\dfrac{1}{64}\right)^{1/6}$

33. $(27)^{-2/3}$

34. $(9)^{-5/2}$

35. PROBLEM SOLVING A math club is having a bake sale. Find the area of the bake sale sign.
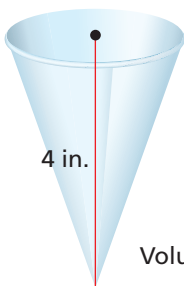

$4^{1/2}$ ft

$\sqrt[6]{729}$ ft

36. PROBLEM SOLVING The volume of a cube-shaped box is 27^5 cubic millimeters. Find the length of one side of the box.

37. MODELING WITH MATHEMATICS The radius r of the base of a cone is given by the equation

$$r = \left(\frac{3V}{\pi h}\right)^{1/2}$$

where V is the volume of the cone and h is the height of the cone. Find the radius of the paper cup to the nearest inch. Use 3.14 for π. *(See Example 4.)*

4 in.

Volume = 5 in.³

38. MODELING WITH MATHEMATICS The volume of a sphere is given by the equation $V = \dfrac{1}{6\sqrt{\pi}}S^{3/2}$, where S is the surface area of the sphere. Find the volume of a sphere, to the nearest cubic meter, that has a surface area of 60 square meters. Use 3.14 for π.

39. WRITING Explain how to write $(\sqrt[n]{a})^m$ in rational exponent form.

40. HOW DO YOU SEE IT? Write an expression in rational exponent form that represents the side length of the square.

Area = x in.²
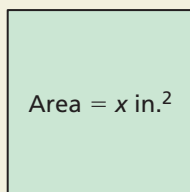
In Exercises 41 and 42, use the formula $r = \left(\dfrac{F}{P}\right)^{1/n} - 1$ to find the annual inflation rate to the nearest tenth of a percent. *(See Example 5.)*

41. A farm increases in value from \$800,000 to \$1,100,000 over a period of 6 years.

42. The cost of a gallon of gas increases from \$1.46 to \$3.53 over a period of 10 years.

43. REASONING For what values of x is $x = x^{1/5}$?

44. MAKING AN ARGUMENT Your friend says that for a real number a and a positive integer n, the value of $\sqrt[n]{a}$ is always positive and the value of $-\sqrt[n]{a}$ is always negative. Is your friend correct? Explain.

In Exercises 45–48, simplify the expression.

45. $(y^{1/6})^3 \cdot \sqrt{x}$

46. $(y \cdot y^{1/3})^{3/2}$

47. $x \cdot \sqrt[3]{y^6} + y^2 \cdot \sqrt[3]{x^3}$

48. $(x^{1/3} \cdot y^{1/2})^9 \cdot \sqrt{y}$

49. PROBLEM SOLVING The formula for the volume of a regular dodecahedron is $V \approx 7.66\,\ell^3$, where ℓ is the length of an edge. The volume of the dodecahedron is 20 cubic feet. Estimate the edge length.

ℓ
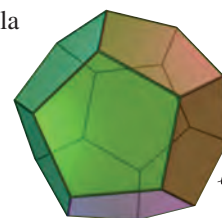
50. THOUGHT PROVOKING To find the arithmetic mean of n numbers, divide the sum of the numbers by n. To find the geometric mean of n numbers $a_1, a_2, a_3, \ldots, a_n$, take the nth root of the product of the numbers.

$$\text{geometric mean} = \sqrt[n]{a_1 \cdot a_2 \cdot a_3 \cdot \ldots \cdot a_n}$$

Compare the arithmetic mean to the geometric mean of n numbers.

ABSTRACT REASONING In Exercises 51–56, let x be a nonnegative real number. Determine whether the statement is *always*, *sometimes*, or *never* true. Justify your answer.

51. $(x^{1/3})^3 = x$

52. $x^{1/3} = x^{-3}$

53. $x^{1/3} = \sqrt[3]{x}$

54. $x^{1/3} = x^3$

55. $\dfrac{x^{2/3}}{x^{1/3}} = \sqrt[3]{x}$

56. $x = x^{1/3} \cdot x^3$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Evaluate the function when $x = -3, 0,$ and $8.$ *(Skills Review Handbook)*

57. $f(x) = 2x - 10$

58. $w(x) = -5x - 1$

59. $h(x) = 13 - x$

60. $g(x) = 8x + 16$

Essential Question What are some of the characteristics of the graph of an exponential function?

You can use a graphing calculator to evaluate an exponential function. For example, consider the exponential function $f(x) = 2^x$.

Function Value	Graphing Calculator Keystrokes	Display
$f(-3.1) = 2^{-3.1}$	2 [^] [(−)] 3.1 [ENTER]	0.1166291
$f\left(\frac{2}{3}\right) = 2^{2/3}$	2 [^] [(] 2 [÷] 3 [)] [ENTER]	1.5874011

EXPLORATION 1 Identifying Graphs of Exponential Functions

Work with a partner. Match each exponential function with its graph. Use a table of values to sketch the graph of the function, if necessary.

a. $f(x) = 2^x$ **b.** $f(x) = 3^x$ **c.** $f(x) = 4^x$

d. $f(x) = \left(\frac{1}{2}\right)^x$ **e.** $f(x) = \left(\frac{1}{3}\right)^x$ **f.** $f(x) = \left(\frac{1}{4}\right)^x$

A.

B.

C.

D.

E.

F.
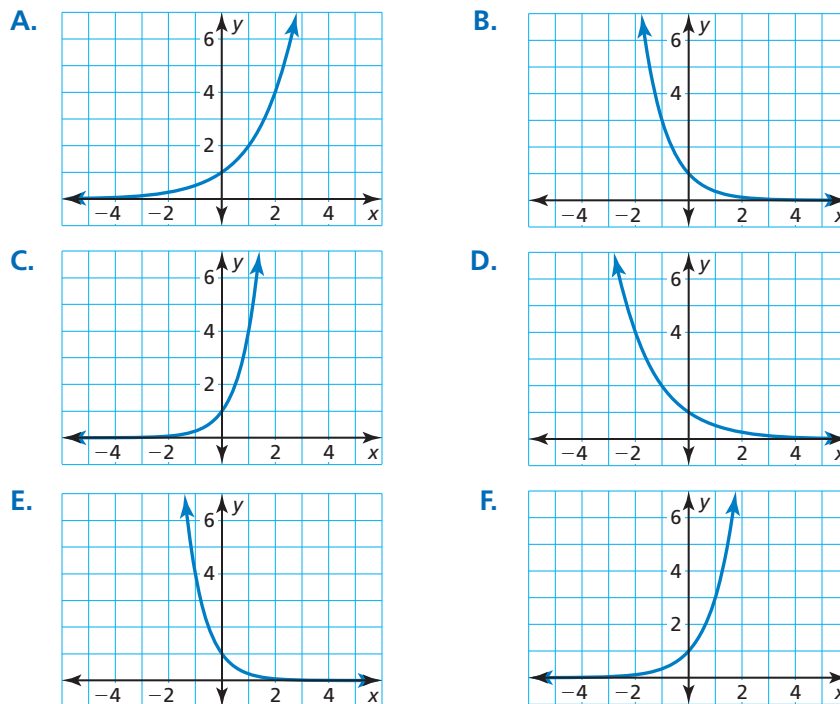
CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to justify your conclusions and communicate them to others.

EXPLORATION 2 Characteristics of Graphs of Exponential Functions

Work with a partner. Use the graphs in Exploration 1 to determine the domain, range, and y-intercept of the graph of $f(x) = b^x$, where b is a positive real number other than 1. Explain your reasoning.

Communicate Your Answer

3. What are some of the characteristics of the graph of an exponential function?

4. In Exploration 2, is it possible for the graph of $f(x) = b^x$ to have an x-intercept? Explain your reasoning.

What You Will Learn

▶ Graph exponential growth and decay functions.

▶ Write exponential models and recursive rules.
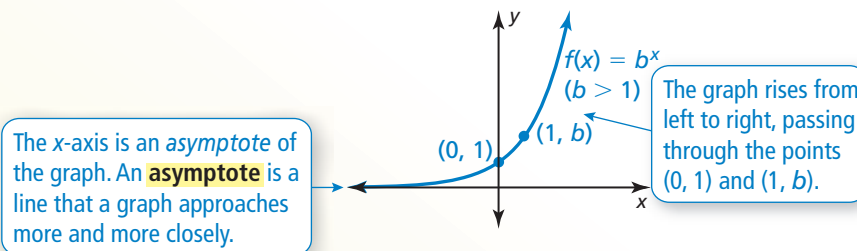
▶ Rewrite exponential functions.

Exponential Growth and Decay Functions

An **exponential function** has the form $y = ab^x$, where $a \neq 0$ and the base b is a positive real number other than 1. If $a > 0$ and $b > 1$, then $y = ab^x$ is an **exponential growth function**, and b is called the **growth factor**. The simplest type of exponential growth function has the form $y = b^x$.

🔄 Core Concept

Parent Function for Exponential Growth Functions

The function $f(x) = b^x$, where $b > 1$, is the parent function for the family of exponential growth functions with base b. The graph shows the general shape of an exponential growth function.

The *x*-axis is an *asymptote* of the graph. An **asymptote** is a line that a graph approaches more and more closely.

$f(x) = b^x$
$(b > 1)$

The graph rises from left to right, passing through the points $(0, 1)$ and $(1, b)$.
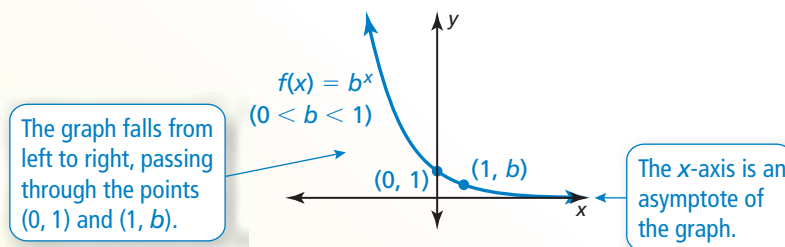
$(0, 1)$ $(1, b)$

The domain of $f(x) = b^x$ is all real numbers. The range is $y > 0$.

If $a > 0$ and $0 < b < 1$, then $y = ab^x$ is an **exponential decay function**, and b is called the **decay factor**.

🔄 Core Concept

Parent Function for Exponential Decay Functions

The function $f(x) = b^x$, where $0 < b < 1$, is the parent function for the family of exponential decay functions with base b. The graph shows the general shape of an exponential decay function.

The graph falls from left to right, passing through the points $(0, 1)$ and $(1, b)$.

$f(x) = b^x$
$(0 < b < 1)$

$(0, 1)$ $(1, b)$

The *x*-axis is an asymptote of the graph.

The domain of $f(x) = b^x$ is all real numbers. The range is $y > 0$.

EXAMPLE 1 **Graphing Exponential Growth and Decay Functions**

Determine whether each function represents *exponential growth* or *exponential decay*. Then graph the function.

a. $y = 2^x$

b. $y = \left(\frac{1}{2}\right)^x$

SOLUTION

a. Step 1 Identify the value of the base. The base, 2, is greater than 1, so the function represents exponential growth.

Step 2 Make a table of values.

x	−2	−1	0	1	2	3
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

Step 3 Plot the points from the table.

Step 4 Draw, from *left to right*, a smooth curve that begins just above the x-axis, passes through the plotted points, and moves up to the right.
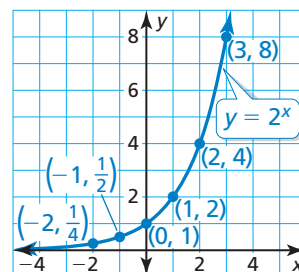
b. Step 1 Identify the value of the base. The base, $\frac{1}{2}$, is greater than 0 and less than 1, so the function represents exponential decay.

Step 2 Make a table of values.

x	−3	−2	−1	0	1	2
y	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$

Step 3 Plot the points from the table.

Step 4 Draw, from *right to left*, a smooth curve that begins just above the x-axis, passes through the plotted points, and moves up to the left.

When graphing exponential functions, you connect the points with a smooth curve because $y = ab^x$ is defined for rational x-values. This should make sense from your study of rational exponents. For instance, in Example 1(a), when $x = \frac{1}{2}$,
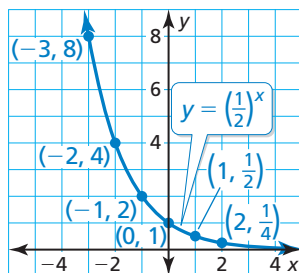
$$y = 2^{1/2} = \sqrt{2} \approx 1.4.$$

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Determine whether the function represents *exponential growth* or *exponential decay*. Then graph the function.

1. $y = 4^x$

2. $y = \left(\frac{2}{3}\right)^x$

3. $f(x) = (0.25)^x$

4. $f(x) = (1.5)^x$

Writing Exponential Models and Recursive Rules

Some real-life quantities increase or decrease by a fixed percent each year (or some other time period). The amount y of such a quantity after t years can be modeled by one of these equations.

Exponential Growth Model

$$y = a(1 + r)^t$$

Exponential Decay Model

$$y = a(1 - r)^t$$

Note that a is the initial amount and r is the percent increase or decrease written as a decimal. The quantity $1 + r$ is the growth factor, and $1 - r$ is the decay factor.

EXAMPLE 2   **Writing an Exponential Model**

The population of a country was about 6.09 million on January 1, 2000. The population at the beginning of each subsequent year increased by about 1.18%.

a. Write an exponential growth model that represents the population y (in millions) t years after January 1, 2000. Find and interpret the y-value when $t = 7.5$.

b. Estimate when the population was 7 million.

SOLUTION

a. The initial amount is $a = 6.09$, and the percent increase is $r = 0.0118$. So, the exponential growth model is

$y = a(1 + r)^t$		Write exponential growth model.
$= 6.09(1 + 0.0118)^t$		Substitute 6.09 for a and 0.0118 for r.
$= 6.09(1.0118)^t$.		Simplify.

Using this model, you can estimate the *midyear* population in 2007 ($t = 7.5$) to be $y = 6.09(1.0118)^{7.5} \approx 6.65$ million.

b. Use the *table* feature of a graphing calculator to determine that $y \approx 7$ when $t \approx 11.9$. So, the population was about 7 million near the end of 2011.

X	Y1
11.4	6.9614
11.5	6.9696
11.6	6.9778
11.7	6.9859
11.8	6.9941
11.9	7.0024
12	7.0106

X=11.9

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5. WHAT IF? Assume the population increased by 1.5% each year. Write an equation to model this situation. Estimate when the population was 7 million.

In real-life situations, you can also show exponential relationships using *recursive rules.*

A **recursive rule** for an exponential function gives the initial value of the function $f(0)$, and a recursive equation that tells how a value $f(n)$ is related to a preceding value $f(n - 1)$.

REMEMBER

Recall that for a sequence, a recursive rule gives the beginning term(s) of the sequence and a recursive equation that tells how a_n is related to one or more preceding terms.
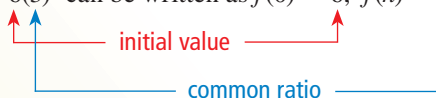
🅖 Core Concept

Writing Recursive Rules for Exponential Functions

An exponential function of the form $f(x) = ab^x$ is written using a recursive rule as follows.

Recursive Rule $f(0) = a, \; f(n) = r \cdot f(n - 1)$

where $a \neq 0$, r is the common ratio, and n is a natural number

Example $y = 6(3)^x$ can be written as $f(0) = 6, \; f(n) = 3 \cdot f(n - 1)$

initial value

common ratio

Notice that the base b of the exponential function is the common ratio r in the recursive equation. Also, notice the value of a in the exponential function is the initial value of the recursive rule.

EXAMPLE 3 **Writing a Recursive Rule for an Exponential Function**

Write a recursive rule for the function you wrote in Example 2.

SOLUTION

The function $y = 6.09(1.0118)^t$ is exponential with initial value $f(0) = 6.09$ and common ratio $r = 1.0118$. So, a recursive equation is

$f(n) = r \cdot f(n - 1)$ Recursive equation for exponential functions

$\quad = 1.0118 \cdot f(n - 1).$ Substitute 1.0118 for *r*.

▶ A recursive rule for the exponential function is $f(0) = 6.09$, $f(n) = 1.0118 \cdot f(n - 1)$.

STUDY TIP

Notice that the domain consists of the natural numbers when written recursively.

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Write an recursive rule for the exponential function.

6. $f(x) = 4(7)^x$

7. $y = 9\left(\dfrac{1}{3}\right)^t$

Rewriting Exponential Functions

EXAMPLE 4 **Rewriting Exponential Functions**

Rewrite each function to determine whether it represents *exponential growth* or *exponential decay*. Then identify the percent rate of change.

a. $y = 100(0.96)^{t/4}$

b. $f(t) = (1.1)^{t-3}$

SOLUTION

a. $y = 100(0.96)^{t/4}$ Write the function.

$\quad = 100(0.96^{1/4})^t$ Power of a Power Property

$\quad \approx 100(0.99)^t$ Evaluate the power.

▶ So, the function represents exponential decay. Use the decay factor $1 - r \approx 0.99$ to find the rate of decay $r \approx 0.01$, or about 1%.

STUDY TIP

You can rewrite exponential expressions and functions using the properties of exponents. Changing the form of an exponential function can reveal important attributes of the function.

b. $f(t) = (1.1)^{t-3}$ Write the function.

$\quad = \dfrac{(1.1)^t}{(1.1)^3}$ Quotient of Powers Property

$\quad \approx 0.75(1.1)^t$ Evaluate the power and simplify.

▶ So, the function represents exponential growth. Use the growth factor $1 + r = 1.1$ to find the rate of growth $r = 0.1$, or 10%.

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Rewrite the function to determine whether it represents *exponential growth* or *exponential decay*. Then identify the percent rate of change.

8. $f(t) = 3(1.02)^{10t}$

9. $y = (0.95)^{t+2}$

EXAMPLE 5   Solving a Real-Life Problem

The value of a car is $21,500. It loses 12% of its value every year. (a) Write a function that represents the value y (in dollars) of the car after t years. (b) Find the approximate monthly percent decrease in value. (c) Graph the function from part (a). Use the graph to estimate the value of the car after 6 years.

SOLUTION

1. **Understand the Problem** You know the value of the car and its annual percent decrease in value. You are asked to write a function that represents the value of the car over time and approximate the monthly percent decrease in value. Then graph the function and use the graph to estimate the value of the car in the future.

REASONING QUANTITATIVELY

The decay factor, 0.88, tells you what fraction of the car's value *remains* each year. The rate of decay, 12%, tells you how much value the car *loses* each year. In real life, the percent decrease in value of an asset is the *depreciation rate*.

2. **Make a Plan** Use the initial amount and the annual percent decrease in value to write an exponential decay function. Note that the annual percent decrease represents the rate of decay. Rewrite the function using the properties of exponents to approximate the monthly percent decrease (rate of decay). Then graph the original function and use the graph to estimate the y-value when the t-value is 6.

3. **Solve the Problem**

   a. The initial value is $21,500, and the rate of decay is 12%, or 0.12.

   $$y = a(1 - r)^t \qquad \text{Write exponential decay model.}$$
   $$= 21{,}500(1 - 0.12)^t \qquad \text{Substitute 21,500 for } a \text{ and 0.12 for } r.$$
   $$= 21{,}500(0.88)^t \qquad \text{Simplify.}$$

   ▶ The value of the car can be represented by $y = 21{,}500(0.88)^t$.

   b. Use the fact that $t = \frac{1}{12}(12t)$ and the properties of exponents to rewrite the function in a form that reveals the monthly rate of decay.

   $$y = 21{,}500(0.88)^t \qquad \text{Write the original function.}$$
   $$= 21{,}500(0.88)^{(1/12)(12t)} \qquad \text{Rewrite the exponent.}$$
   $$= 21{,}500(0.88^{1/12})^{12t} \qquad \text{Power of a Power Property}$$
   $$\approx 21{,}500(0.989)^{12t} \qquad \text{Evaluate the power.}$$

   Use the decay factor $1 - r \approx 0.989$ to find the rate of decay $r \approx 0.011$.

   ▶ So, the monthly percent decrease is about 1.1%.


Value of a Car

$y = 21{,}500(0.88)^t$

   c. From the graph, you can see that the y-value is about 10,000 when $t = 6$.

   ▶ So, the value of the car is about $10,000 after 6 years.

4. **Look Back** To check that the monthly percent decrease is reasonable, multiply it by 12 to see if it is close in value to the annual percent decrease of 12%.

   $$1.1\% \times 12 = 13.2\% \qquad \text{13.2\% is close to 12\%, so 1.1\% is reasonable.}$$

   When you evaluate $y = 21{,}500(0.88)^t$ for $t = 6$, you get about $9985. So, $10,000 is a reasonable estimation.
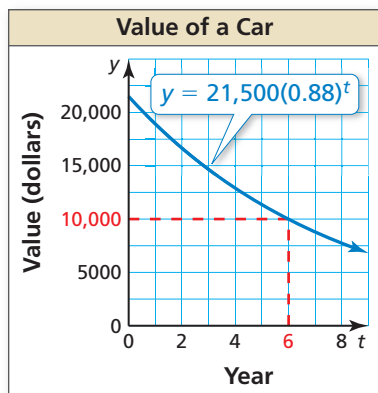
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10. **WHAT IF?** The car loses 9% of its value every year. (a) Write a function that represents the value y (in dollars) of the car after t years. (b) Find the approximate monthly percent decrease in value. (c) Graph the function from part (a). Use the graph to estimate the value of the car after 12 years. Round your answer to the nearest thousand.

Vocabulary and Core Concept Check

1. **VOCABULARY** In the exponential growth model $y = 2.4(1.5)^x$, identify the initial amount, the growth factor, and the percent increase.

2. **WHICH ONE DOESN'T BELONG?** Which characteristic of an exponential decay function does *not* belong with the other three? Explain your reasoning.

base of 0.8	decay factor of 0.8
decay rate of 20%	80% decrease

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In Exercises 3–8, evaluate the expression for (a) $x = -2$ **and (b)** $x = 3$.

3. 2^x

4. 4^x

5. $8 \cdot 3^x$

6. $6 \cdot 2^x$

7. $5 + 3^x$

8. $2^x - 2$

In Exercises 9–18, determine whether the function represents *exponential growth* or *exponential decay*. Then graph the function. *(See Example 1.)*

9. $y = 6^x$

10. $y = 7^x$

11. $y = \left(\dfrac{1}{6}\right)^x$

12. $y = \left(\dfrac{1}{8}\right)^x$

13. $y = \left(\dfrac{4}{3}\right)^x$

14. $y = \left(\dfrac{2}{5}\right)^x$

15. $y = (1.2)^x$

16. $y = (0.75)^x$

17. $y = (0.6)^x$

18. $y = (1.8)^x$

ANALYZING RELATIONSHIPS In Exercises 19 and 20, use the graph of $f(x) = b^x$ **to identify the value of the base** b**.**

19.

20.
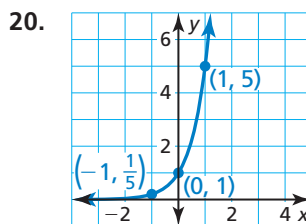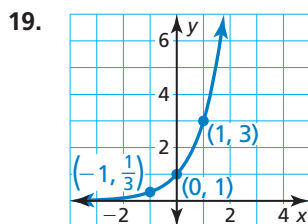
21. **MODELING WITH MATHEMATICS** The population of Austin, Texas, was about 494,000 at the beginning of a decade. The population increased by about 3% each year. *(See Example 2.)*

    a. Write an exponential growth model that represents the population y (in thousands) t years after the beginning of the decade. Find and interpret the y-value when $t = 10$.

    b. Estimate when the population was about 590,000.

22. **MODELING WITH MATHEMATICS** You take a 325 milligram dosage of ibuprofen. During each subsequent hour, the amount of medication in your bloodstream decreases by about 29% each hour.

    a. Write an exponential decay model giving the amount y (in milligrams) of ibuprofen in your bloodstream t hours after the initial dose. Find and interpret the y-value when $t = 1.5$.

    b. Estimate how long it takes for you to have 100 milligrams of ibuprofen in your bloodstream.

23. **ERROR ANALYSIS** You invest $500 in the stock of a company. The value of the stock decreases 2% each year. Describe and correct the error in writing a model for the value of the stock after t years.

$$\times \quad y = \left(\begin{matrix}\text{Initial} \\ \text{amount}\end{matrix}\right)\left(\begin{matrix}\text{Decay} \\ \text{factor}\end{matrix}\right)^t$$

$$y = 500(0.02)^t$$

24. USING EQUATIONS Complete a table of values for $0 \le n \le 5$ using the given recursive rule of an exponential function.

$$f(0) = 4, \ f(n) = 3 \cdot f(n-1)$$

In Exercises 25–32, write a recursive rule for the exponential function. *(See Example 3.)*

25. $y = 3(7)^x$

26. $y = 25(0.2)^x$

27. $y = 12(0.5)^t$

28. $y = 19(4)^t$

29. $g(x) = 0.5(3)^x$

30. $m(t) = \frac{1}{3}(2)^t$

31. $f(x) = 4\left(\frac{1}{6}\right)^x$

32. $f(t) = 0.25\left(\frac{2}{3}\right)^t$

In Exercises 33–36, show that an exponential model fits the data. Then write a recursive rule that models the data.

33.

n	0	1	2	3	4	5
f(n)	0.75	1.5	3	6	12	24

34.

n	0	1	2	3	4
f(n)	2	8	32	128	512

35.

n	0	1	2	3	4	5
f(n)	96	48	24	12	6	3

36.

| n	0	1	2	3	4	5
f(n)	162	54	18	6	2	$\frac{2}{3}$

In Exercises 37 and 38, write an exponential function for the recursive rule.

37. $f(0) = 24, f(n) = 0.1 \cdot f(n-1)$

38. $f(0) = \frac{1}{2}, f(n) = \frac{5}{2} \cdot f(n-1)$

39. PROBLEM SOLVING Describe a real-life situation that can be represented by the graph. Write a recursive rule that models the data. Then find and interpret $f(6)$.
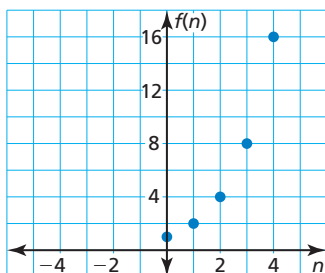
40. PROBLEM SOLVING The cross-sectional area of a tree 4.5 feet from the ground is called its *basal area*. The table shows the basal areas (in square inches) of Tree A over time.

Year	0	1	2	3	4
Basal area	120	132	145.2	159.7	175.7

Tree B

Growth rate: 6%

Initial basal area: 154 in.^{2}
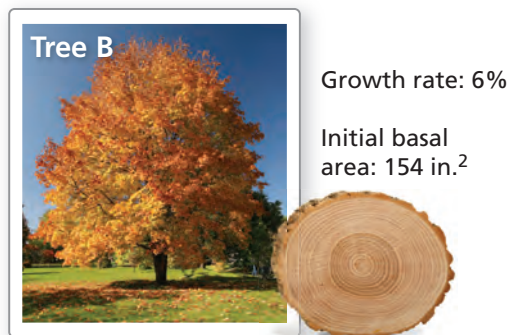
a. Write recursive rules that represent the basal areas of the trees after t years.

b. Which tree has a greater initial basal area? a greater basal area growth rate? Use the recursive rules you wrote in part (a) to justify your answers.

c. Use a graph to represent the growth of the trees over the interval $0 \le t \le 10$. Does the graph support your answers in part (b)? Explain. Then make an additional observation from the graph.

In Exercises 41–44, determine whether the function represents *exponential growth* or *exponential decay*. Then identify the percent rate of change.

41. $y = 5(0.6)^t$

42. $y = 10(1.07)^t$

43. $f(t) = 200\left(\frac{4}{3}\right)^t$

44. $f(t) = 0.8\left(\frac{1}{4}\right)^t$

45. PROBLEM SOLVING A website recorded the number y of referrals it received from social media websites over a 10-year period. The results can be modeled by $y = 2500(1.50)^t$, where t is the year and $0 \le t \le 9$.

a. Describe the real-life meaning of 2500 and 1.50 in the model.

b. What is the annual percent increase? Explain.

46. PROBLEM SOLVING The population p of a small town after x years can be modeled by the function $p = 6850(0.97)^x$.

a. What is the annual percent decrease? Explain.

b. What is the average rate of change in the population over the first 6 years? Justify your answer.

JUSTIFYING STEPS In Exercises 47 and 48, justify each step in rewriting the exponential function.

47. $y = a(3)^{t/14}$ Write original function.

$= a[(3)^{1/14}]^t$ [____]

$\approx a(1.0816)^t$ [____]

$= a(1 + 0.0816)^t$ [____]

48. $y = a(0.1)^{t/3}$ Write original function.

$= a[(0.1)^{1/3}]^t$ [____]

$\approx a(0.4642)^t$ [____]

$= a(1 - 0.5358)^t$ [____]

In Exercises 49–56, rewrite the function to determine whether it represents *exponential growth* or *exponential decay*. Then identify the percent rate of change. *(See Example 4.)*

49. $y = (0.9)^{t-4}$ **50.** $y = (1.4)^{t+8}$

51. $y = 2(1.06)^{9t}$ **52.** $y = 5(0.82)^{t/5}$

53. $x(t) = (1.45)^{t/2}$ **54.** $f(t) = 0.4(1.16)^{t-1}$

55. $b(t) = 4(0.55)^{t+3}$ **56.** $r(t) = (0.88)^{4t}$

In Exercises 57–62, rewrite the function in the form $y = a(1 + r)^t$ or $y = a(1 - r)^t$. Then state the growth or decay rate.

57. $y = a(2)^{t/3}$ **58.** $y = a(0.5)^{t/12}$

59. $y = a\left(\frac{2}{3}\right)^{t/10}$ **60.** $y = a\left(\frac{5}{4}\right)^{t/22}$

61. $y = a(2)^{8t}$ **62.** $y = a\left(\frac{1}{3}\right)^{3t}$

63. PROBLEM SOLVING When a plant or animal dies, it stops acquiring carbon-14 from the atmosphere. The amount y (in grams) of carbon-14 in the body of an organism after t years is $y = a(0.5)^{t/5730}$, where a is the initial amount (in grams). What percent of the carbon-14 is released each year?

64. PROBLEM SOLVING The number y of duckweed fronds in a pond after t days is $y = a(1230.25)^{t/16}$, where a is the initial number of fronds. By what percent does the duckweed increase each day?


65. PROBLEM SOLVING A city has a population of 25,000. The population is expected to increase by 5.5% annually for the next decade. *(See Example 5.)*


a. Write a function that represents the population y after t years.

b. Find the approximate monthly percent increase in population.

c. Graph the function from part (a). Use the graph to estimate the population after 4 years.

66. PROBLEM SOLVING Plutonium-238 is a material that generates steady heat due to decay and is used in power systems for some spacecraft. The function $y = a(0.5)^{t/x}$ represents the amount y of a substance remaining after t years, where a is the initial amount and x is the length of the half-life (in years).


Plutonium-238
Half-life ≈ 88 years
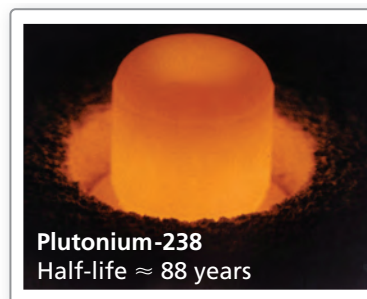
a. A scientist is studying a 3-gram sample. Write a function that represents the amount y of plutonium-238 after t years.

b. What is the yearly percent decrease of plutonium-238?

c. Graph the function from part (a). Use the graph to estimate the amount remaining after 12 years.

67. MAKING AN ARGUMENT Your friend says the graph of $f(x) = 2^x$ increases at a faster rate than the graph of $g(x) = 4x$ when $x \geq 0$. Is your friend correct? Explain your reasoning.
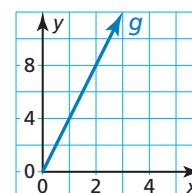
68. HOW DO YOU SEE IT? Consider the graph of an exponential function of the form $f(x) = ab^x$.
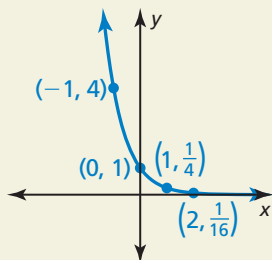
a. Determine whether the graph of f represents exponential growth or exponential decay.

b. What are the domain and range of the function? Explain.

69. COMPARING FUNCTIONS The two given functions describe the amount y of ibuprofen (in milligrams) in a person's bloodstream t hours after taking the dosage.

$$y \approx 325(0.9943)^{60t} \qquad y \approx 325(0.843)^{2t}$$

a. Show that these models are approximately equivalent to the model you wrote in Exercise 22.

b. Describe the information given by each of the models above.

70. DRAWING CONCLUSIONS The amount A in an account after t years with principal P, annual interest rate r (expressed as a decimal), and compounded n times per year is given by

$$A = P\left(1 + \frac{r}{n}\right)^{nt}.$$

You deposit $1000 into three separate bank accounts that each pay 3% annual interest. For each account, evaluate $\left(1 + \frac{r}{n}\right)^n$. Interpret this quantity in the context of the problem. Then complete the table.

Account	Compounding	Balance after 1 year
1	quarterly	
2	monthly	
3	daily	

71. REASONING Consider the exponential function $f(x) = ab^x$.

a. Show that $\dfrac{f(x + 1)}{f(x)} = b$.

b. Use the equation in part (a) to explain why there is no exponential function of the form $f(x) = ab^x$ whose graph passes through the points in the table below.

x	0	1	2	3	4
y	4	4	8	24	72

72. THOUGHT PROVOKING The function $f(x) = b^x$ represents an exponential decay function. Write a second exponential decay function in terms of b and x.

73. PROBLEM SOLVING The number E of eggs a Leghorn chicken produces per year can be modeled by the equation $E = 179.2(0.89)^{w/52}$, where w is the age (in weeks) of the chicken and $w \geq 22$.

a. Identify the decay factor and the percent decrease.

b. Graph the model.

c. Estimate the egg production of a chicken that is 2.5 years old.

d. Explain how you can rewrite the model so that time is measured in years rather than in weeks.

74. CRITICAL THINKING You buy a new stereo for $1300 and are able to sell it 4 years later for $275. Assume that the resale value of the stereo decays exponentially with time. Write an equation giving the resale value V (in dollars) of the stereo as a function of the time t (in years) since you bought it.

Maintaining Mathematical Proficiency Reviewing what you learned in previous grades and lessons

Simplify the expression. *(Skills Review Handbook)*

75. $x + 3x$

76. $8y - 21y$

77. $13z + 9 - 8z$

78. $-9w + w - 5$

Simplify the expression. Write your answer using only positive exponents. *(Section 1.4)*

79. $x^{-9} \cdot x^2$

80. $\dfrac{x^4}{x^3}$

81. $(-6x)^2$

82. $\left(\dfrac{4x^8}{2x^6}\right)^4$

Core Vocabulary

Core Concepts

Section 1.4

Section 1.5

Section 1.6

Mathematical Practices

1. How did you apply what you know to simplify the complicated situation in Exercise 56 on page 33?

2. How can you use previously established results to construct an argument in Exercise 44 on page 40?

Performance Task:

Pool Shots

How can mathematics help you become a better pool player? What type of function could you use to sink a pool ball? What aspects of the function will help the shot be successful?

To explore the answers to these questions and more, check out the Performance Task and Real-Life STEM video at *BigIdeasMath.com*.

1.1 Absolute Value Functions *(pp. 3–10)*

Let $g(x) = -3|x + 1| + 2$. (a) Describe the transformations from the graph of $f(x) = |x|$ to the graph of g. (b) Graph g.

a. Step 1 Translate the graph of f horizontally 1 unit left to get the graph of $t(x) = |x + 1|$.

  Step 2 Stretch the graph of t vertically by a factor of 3 to get the graph of $h(x) = 3|x + 1|$.

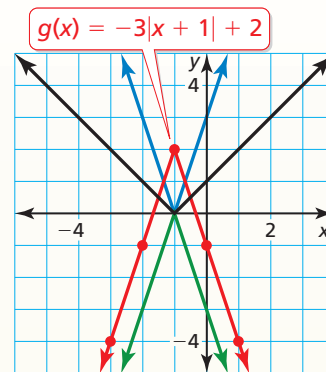 **Step 3** Reflect the graph of h in the x-axis to get the graph of $r(x) = -3|x + 1|$.

  Step 4 Translate the graph of r vertically 2 units up to get the graph of $g(x) = -3|x + 1| + 2$.

b. Step 1 Make a table of values.

x	-3	-2	-1	0	1
$g(x)$	-4	-1	2	-1	-4

$g(x) = -3|x + 1| + 2$

  Step 2 Plot the ordered pairs.

  Step 3 Draw the V-shaped graph.

Graph the function. Compare the graph to the graph of $f(x) = |x|$. Describe the domain and range.

1. $m(x) = |x| + 6$ **2.** $p(x) = |x - 4|$ **3.** $q(x) = 4|x|$ **4.** $r(x) = -\frac{1}{4}|x|$

5. Graph $f(x) = |x - 2| + 4$ and $g(x) = |3x - 2| + 4$. Compare the graph of g to the graph of f.

6. Describe another method you can use to graph g in the example above.

7. Let $g(x) = \frac{1}{3}|x - 1| - 2$. (a) Describe the transformations from the graph of $f(x) = |x|$ to the graph of g. (b) Graph g.
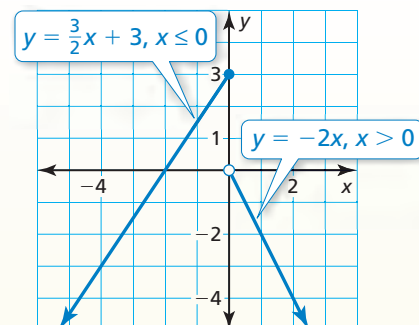
1.2 Piecewise Functions *(pp. 11–18)*

Graph $y = \begin{cases} \frac{3}{2}x + 3, & \text{if } x \leq 0 \\ -2x, & \text{if } x > 0 \end{cases}$. Describe the domain and range.

Step 1 Graph $y = \frac{3}{2}x + 3$ for $x \leq 0$. Because x is less than or equal to 0, use a closed circle at $(0, 3)$.

Step 2 Graph $y = -2x$ for $x > 0$. Because x is not equal to 0, use an open circle at $(0, 0)$.

▶ The domain is all real numbers. The range is $y \leq 3$.

$y = \frac{3}{2}x + 3, x \leq 0$

$y = -2x, x > 0$

8. Evaluate the function in the example when (a) $x = 0$ and (b) $x = 5$.

Graph the function. Describe the domain and range.

9. $y = \begin{cases} x + 6, & \text{if } x \le 0 \\ -3x, & \text{if } x > 0 \end{cases}$

10. $y = \begin{cases} 4x + 2, & \text{if } x < -4 \\ 2x - 6, & \text{if } x \ge -4 \end{cases}$

Write the absolute value function as a piecewise function.

11. $y = |x| + 15$

12. $y = 4|x + 5|$

13. $y = 2|x + 2| - 3$

14. You are organizing a school fair and rent a popcorn machine for 3 days. The rental company charges \$65 for the first day and \$35 for each additional day. Write and graph a step function that represents the relationship between the number x of days and the total cost y (in dollars) of renting the popcorn machine.

1.3 **Inverse of a Function** *(pp. 19–24)*

Find the inverse of $f(x) = \frac{1}{3}x - 2$. Then graph the function and its inverse.

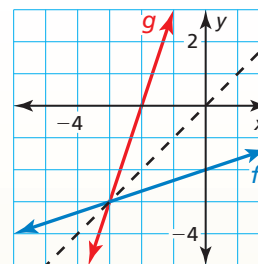|	
$y = \frac{1}{3}x - 2$	Set y equal to $f(x)$.
$x = \frac{1}{3}y - 2$	Switch x and y in the equation.
$x + 2 = \frac{1}{3}y$	Add 2 to each side.
$3x + 6 = y$	Multiply each side by 3.

▶ The inverse of f is $g(x) = 3x + 6$.

15. Find the inverse of the relation: $(1, -10), (3, -4), (5, 4), (7, 14), (9, 26)$.

Find the inverse of the function. Then graph the function and its inverse.

16. $f(x) = \frac{3}{4}x$

17. $f(x) = -5x + 10$

18. $f(x) = -\frac{2}{5}x + 6$

19. In bowling, a handicap is an adjustment to a bowler's score to even out differences in ability levels. In a particular league, you can find a bowler's handicap h by using the formula $h = 0.8(210 - a)$, where a is the bowler's average. Solve the formula for a. Then find a bowler's average when the bowler's handicap is 28.

1.4 **Properties of Exponents** *(pp. 27–34)*

Simplify $\left(\frac{x}{4}\right)^{-4}$. Write your answer using only positive exponents.

$\left(\frac{x}{4}\right)^{-4} = \dfrac{x^{-4}}{4^{-4}}$	Power of a Quotient Property
$= \dfrac{4^4}{x^4}$	Definition of negative exponent
$= \dfrac{256}{x^4}$	Simplify.

Simplify the expression. Write your answer using only positive exponents.

20. $y^3 \cdot y^{-5}$

21. $\dfrac{x^4}{x^7}$

22. $(x^0y^2)^3$

23. $\left(\dfrac{2x^2}{5y^4}\right)^{-2}$

Evaluate $512^{1/3}$.

$$512^{1/3} = \sqrt[3]{512}$$ Rewrite the expression in radical form.

$$= \sqrt[3]{8 \cdot 8 \cdot 8}$$ Rewrite the expression showing factors.

$$= 8$$ Evaluate the cube root.

Evaluate the expression.

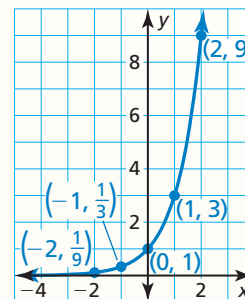**24.** $\sqrt[3]{8}$ **25.** $\sqrt[5]{-243}$ **26.** $625^{3/4}$ **27.** $(-25)^{1/2}$

1.6 **Exponential Functions** *(pp. 41–50)*

a. **Determine whether the function $y = 3^x$ represents *exponential growth* or *exponential decay*. Then graph the function.**

   Step 1 Identify the value of the base. The base, 3, is greater than 1, so the function represents exponential growth.

   Step 2 Make a table of values.

x	-2	-1	0	1	2
y	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9

   Step 3 Plot the points from the table.

   Step 4 Draw, from *left to right*, a smooth curve that begins just above the x-axis, passes through the plotted points, and moves up to the right.

b. **Rewrite the function $y = 10(0.65)^{t/8}$ to determine whether it represents *exponential growth* or *exponential decay*. Then identify the percent rate of change.**

$$y = 10(0.65)^{t/8}$$ Write the function.

$$= 10(0.65^{1/8})^t$$ Power of a Power Property

$$\approx 10(0.95)^t$$ Evaluate the power.

▶ So, the function represents exponential decay. Use the decay factor $1 - r \approx 0.95$ to find the rate of decay $r \approx 0.05$, or about 5%.

Determine whether the function represents exponential growth or exponential decay. Identify the percent rate of change. Then graph the function.

28. $f(x) = \left(\frac{1}{3}\right)^x$ **29.** $y = 5^x$ **30.** $f(x) = (0.2)^x$

Rewrite the function to determine whether it represents *exponential growth* or *exponential decay*. Then identify the percent rate of change.

31. $f(t) = 4(1.25)^{t+3}$ **32.** $y = (1.06)^{8t}$ **33.** $f(t) = 6(0.84)^{t-4}$

Evaluate the expression.

1. $-\sqrt[4]{16}$

2. $729^{1/6}$

3. $(-32)^{7/5}$

Simplify the expression. Write your answer using only positive exponents.

4. $z^{-2} \cdot z^4$

5. $\dfrac{b^{-5}}{a^0 b^{-8}}$

6. $\left(\dfrac{2c^4}{5}\right)^{-3}$

Graph the function. Describe the domain and range.

7. $h(x) = -|x - 1| - 4$

8. $p(x) = \left|\dfrac{1}{2}x + 3\right| + 2$

9. $y = \begin{cases} 2x + 4, & \text{if } x \le -1 \\ \frac{1}{3}x - 1, & \text{if } x > -1 \end{cases}$

10. $y = \begin{cases} 1, & \text{if } 0 \le x < 3 \\ 0, & \text{if } 3 \le x < 6 \\ -1, & \text{if } 6 \le x < 9 \\ -2, & \text{if } 9 \le x < 12 \end{cases}$

Find the inverse of the function. Then graph the function and its inverse.

11. $g(x) = -4x$

12. $r(x) = 3x + 5$

13. $v(x) = -\dfrac{3}{4}x - \dfrac{1}{3}$

Use the equation to complete the statement "a ▢ b" with the symbol <, >, or =. Do not attempt to solve the equation.

14. $\dfrac{5^a}{5^b} = 5^{-3}$

15. $9^a \cdot 9^{-b} = 1$

16. Write a piecewise function defined by three equations that has a domain of all real numbers and a range of $-3 < y \le 1$.

17. A rock band releases a new single. Weekly sales s (in thousands of dollars) increase and then decrease as described by the function $s(t) = -2|t - 20| + 40$, where t is the time (in weeks).

    a. Graph s. Describe the transformations from the graph of $f(x) = |x|$ to the graph of s.

    b. Identify and interpret the vertex of the graph of s in the context of the problem.

18. The speed of light is approximately 3×10^5 kilometers per second. The average distance from the Sun to Neptune is about 4.5 billion kilometers. How long does it take sunlight to reach Neptune? Write your answer in scientific notation and in standard form.

19. The value of a mountain bike y (in dollars) can be approximated by the model $y = 200(0.75)^t$, where t is the number of years since the bike was new.

    a. Determine whether the model represents exponential growth or exponential decay.

    b. Identify the annual percent increase or decrease in the value of the bike.

    c. Estimate when the value of the bike will be $50.

20. The amount y (in grams) of the radioactive isotope iodine-123 remaining after t hours is $y = a(0.5)^{t/13}$, where a is the initial amount (in grams). What percent of the iodine-123 decays each hour?
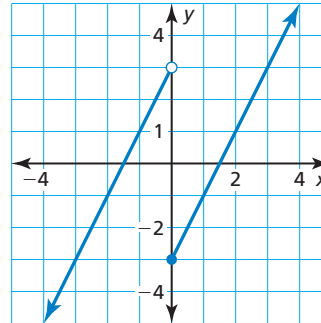
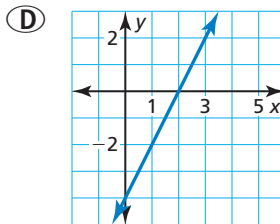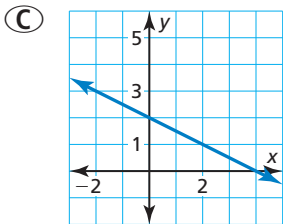**1.** Fill in the exponent of x with a number to simplify the expression.
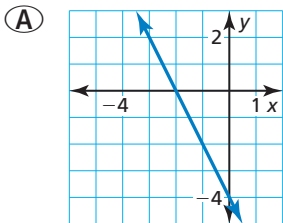
$$\frac{x^{5/3} \cdot x^{-1} \cdot \sqrt[3]{x}}{x^{-2} \cdot x^{0}} = x^{\boxed{}}$$

2. Fill in the piecewise function with $-$, $+$, $<$, $\leq$, $>$, or $\geq$ so that the function is represented by the graph.

$$y = \begin{cases} 2x \,\boxed{}\, 3, & \text{if } x \,\boxed{}\, 0 \\ 2x \,\boxed{}\, 3, & \text{if } x \,\boxed{}\, 0 \end{cases}$$

3. Which graph represents the inverse of the function $f(x) = 2x + 4$?

(A)

(B)

(C)

(D)
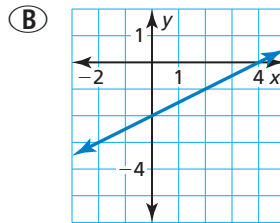
4. Let $f(x) = |x|$ and $g(x) = -4|x + 11|$. Select the possible transformations of the graph of f represented by the function g.

(A) reflection in the x-axis   (B) reflection in the y-axis

(C) horizontal translation 11 units right   (D) horizontal translation 11 units left

(E) horizontal shrink by a factor of $\frac{1}{4}$ (F) vertical stretch by a factor of 4

5. Select all the numbers that are in the range of the function shown.

$$y = \begin{cases} |x + 2| + 3, & \text{if } x \leq -2 \\ \frac{1}{2}x + 3, & \text{if } x > -2 \end{cases}$$
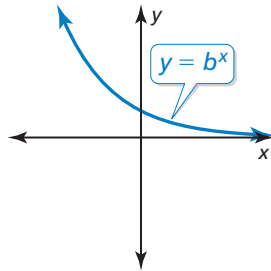
| 0 | $\frac{1}{2}$ | 1 | $1\frac{1}{2}$ | 2 | $2\frac{1}{2}$ | 3 | $3\frac{1}{2}$ | 4 |

6. Select every value of b for the equation $y = b^x$ that could result in the graph shown.
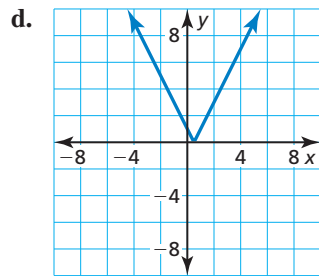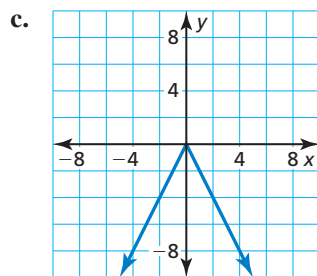

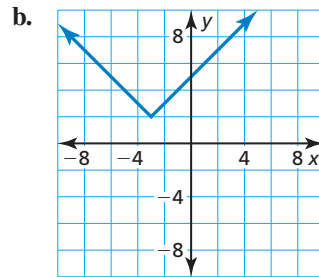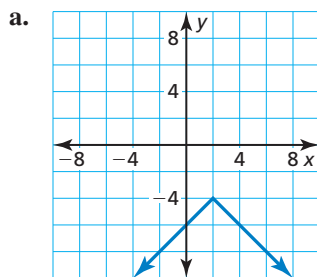| 1.08 | 0.94 | $1 + 0.85$ |

| 2.04 | $1 - 0.6$ | $\dfrac{5}{4}$ |

7. Pair each function with its inverse.

| $y = -\dfrac{1}{3}x$ | $y = -x + 3$ | $y = 3x - 12$ | $y = 3x + 3$ |

| $y = \dfrac{1}{3}x + 4$ | $y = -3x$ | $y = \dfrac{1}{3}x - 1$ | $y = -x + 3$ |

8. Describe the transformation of the graph of $f(x) = |x|$ represented in each graph.

a.

b.

c.

d.

9. Which of the expressions are equivalent?

| $32^{3/5}$ | $(32^{1/5})^3$ | 2^5 | $(32^{1/3})^5$ | 8 | $\sqrt[3]{32^5}$ |