10.5 Using Parallel and Perpendicular Lines

Essential Question How can you find the distance between two parallel lines?

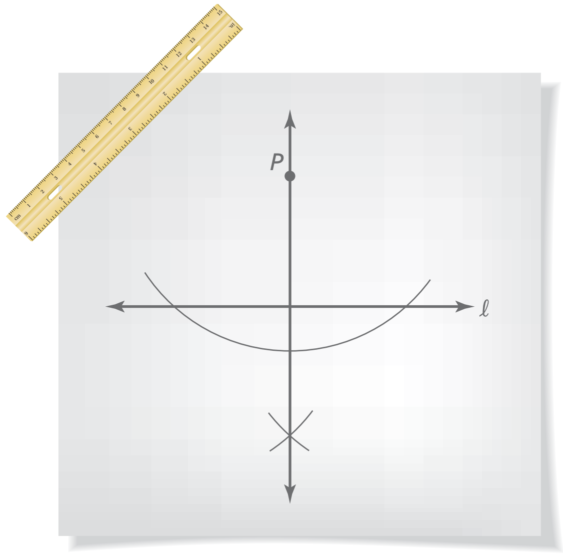
EXPLORATION 1 **Finding the Distance Between Two Parallel Lines**

Work with a partner.

a. Draw a line and label it ℓ. Draw a point not on line ℓ and label it P.

b. Construct a line through point P perpendicular to line ℓ.

c. Use a centimeter ruler to measure the distance from point P to line ℓ.

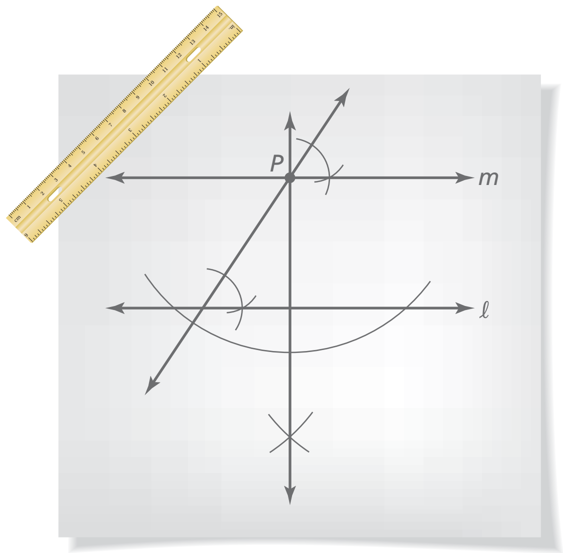


d. Construct a line through point P parallel to line ℓ and label it m.

e. Choose any point except point P on line ℓ or line m and label it Q. Describe how to find the distance from point Q to the other line.

f. Find the distance from point Q to the other line. Compare this distance to the distance from point P to line ℓ.

g. Is the distance from any point on line ℓ to line m constant? Explain your reasoning.



CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to justify your conclusions and communicate them to others.

Communicate Your Answer

2. How can you find the distance between two parallel lines?

3. Use centimeter graph paper and a centimeter ruler to find the distance between the two parallel lines.

a. $y = 2x + 2$
$y = 2x - 7$

b. $y = -x + 4$
$y = -x - 5$

10.5 Lesson

Core Vocabulary

Previous
parallel lines
perpendicular lines

What You Will Learn

- Prove the slope criteria for parallel lines.
- Find the distance from a point to a line.
- Find the distance between two parallel lines.

Proving the Slope Criteria for Parallel Lines

In the coordinate plane, the *x*-axis and the *y*-axis are perpendicular. Horizontal lines are parallel to the *x*-axis, and vertical lines are parallel to the *y*-axis.

CONNECTIONS TO ALGEBRA

The information you learned about parallel and perpendicular lines in Section 4.3 can be stated as theorems.

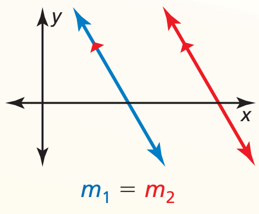
Theorems

Slopes of Parallel Lines

In a coordinate plane, two distinct nonvertical lines are parallel if and only if they have the same slope.

Any two vertical lines are parallel.

Proof p. 528; Ex. 26, p. 532

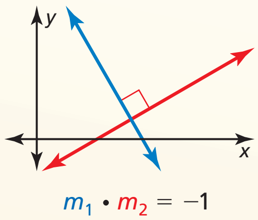


Slopes of Perpendicular Lines

In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

Horizontal lines are perpendicular to vertical lines.

Proof Ex. 28, p. 532; p. 636



PROOF **Part of Slopes of Parallel Lines Theorem**

Given Two distinct nonvertical lines in a coordinate plane

Prove The lines are parallel if and only if they have the same slope.

Paragraph Proof Consider the system of linear equations at the right, where Equations 1 and 2 represent two distinct nonvertical lines in a coordinate plane. The lines are parallel if and only if the system has no solution.

$y = m_1x + b_1$ Equation 1

$y = m_2x + b_2$ Equation 2

CONNECTIONS TO ALGEBRA

In Section 5.4, you determined the number of solutions of a linear system by looking at its graph.

One solution: The lines intersect.

No solution: The lines are parallel.

Infinitely many solutions: The lines are the same.

Substitute $m_2x + b_2$ for y in Equation 1.

$y = m_1x + b_1$ Equation 1

$m_2x + b_2 = m_1x + b_1$ Substitute $m_2x + b_2$ for y.

$m_2x - m_1x = b_1 - b_2$ Isolate like terms.

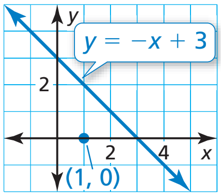
$(m_2 - m_1)x = b_1 - b_2$ Factor.

When $m_1 \neq m_2$, you can divide each side of the equation above by $m_2 - m_1$ to find the value of x and then substitute the value of x into Equation 1 or 2 to find the value of y. So, the system has a solution when $m_1 \neq m_2$. When $m_1 = m_2$, the equation simplifies to $b_2 = b_1$ for all values of x. This is false because b_1 and b_2 must be different for the lines to be distinct. So, the system has no solution when $m_1 = m_2$. Because two distinct nonvertical lines are parallel if and only if their system has no solution and their system has no solution if and only if they have the same slope, two distinct nonvertical lines in a coordinate plane are parallel if and only if they have the same slope.

Finding the Distance from a Point to a Line

Recall that the distance from a point to a line is the length of the perpendicular segment from the point to the line.

EXAMPLE 1 Finding the Distance from a Point to a Line



Find the distance from the point (1, 0) to the line $y = -x + 3$.

SOLUTION

Step 1 Find an equation of the line perpendicular to the line $y = -x + 3$ that passes through the point (1, 0).

First, find the slope m of the perpendicular line. The line $y = -x + 3$ has a slope of -1. Use the Slopes of Perpendicular Lines Theorem.

$-1 \cdot m = -1$	The product of the slopes of ⊥ lines is -1.
$m = 1$	Divide each side by -1.

Then find the y-intercept b by using $m = 1$ and $(x, y) = (1, 0)$.

$y = mx + b$	Use slope-intercept form.
$0 = 1(1) + b$	Substitute for x, y, and m.
$-1 = b$	Solve for b.

Because $m = 1$ and $b = -1$, an equation of the line is $y = x - 1$.

Step 2 Use the two equations to write and solve a system of equations to find the point where the two lines intersect.

$y = -x + 3$	Equation 1
$y = x - 1$	Equation 2

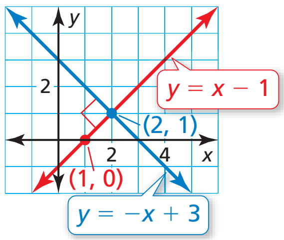
Substitute $-x + 3$ for y in Equation 2.

$y = x - 1$	Equation 2
$-x + 3 = x - 1$	Substitute $-x + 3$ for y.
$x = 2$	Solve for x.

Substitute 2 for x in Equation 1 and solve for y.

$y = -x + 3$	Equation 1
$y = -2 + 3$	Substitute 2 for x.
$y = 1$	Simplify.

So, the perpendicular lines intersect at (2, 1).



Step 3 Use the Distance Formula to find the distance from (1, 0) to (2, 1).

$$\text{distance} = \sqrt{(1 - 2)^2 + (0 - 1)^2} \approx 1.4$$

▶ So, the distance from the point (1, 0) to the line $y = -x + 3$ is about 1.4 units.

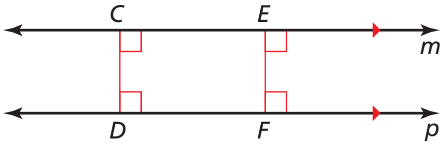
Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

1. Find the distance from the point (6, 4) to the line $y = x + 4$.
2. Find the distance from the point $(-1, 6)$ to the line $y = -2x$.

Finding the Distance Between Two Parallel Lines

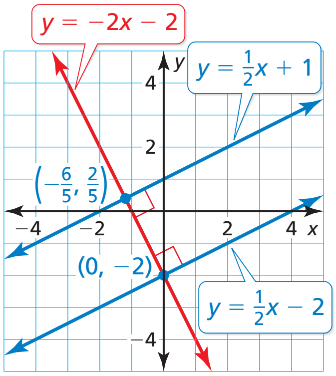
The *distance between two parallel lines* is the length of any perpendicular segment joining the two lines. For instance, the distance between line p and line m below is CD or EF.



EXAMPLE 2 **Finding the Distance Between Two Parallel Lines**

Find the distance between $y = \frac{1}{2}x - 2$ and $y = \frac{1}{2}x + 1$.

SOLUTION



Step 1 Find an equation of a line perpendicular to the two parallel lines. The slopes of the two parallel lines are both $\frac{1}{2}$. Use the Slopes of Perpendicular Lines Theorem.

$$\frac{1}{2} \cdot m = -1 \qquad \text{The product of the slopes of } \perp \text{ lines is } -1.$$

$$m = -2 \qquad \text{Multiply each side by 2.}$$

Any line with a slope of -2 is perpendicular to the two parallel lines. Use $y = -2x - 2$ because it has the same y-intercept, $(0, -2)$, as one of the two parallel lines.

Step 2 The distance between the points where $y = -2x - 2$ intersects the parallel lines is the distance between the lines. You already know that one point of intersection is $(0, -2)$. Use substitution to find the other point of intersection.

$$y = \frac{1}{2}x + 1 \qquad \text{Write original equation.}$$

$$-2x - 2 = \frac{1}{2}x + 1 \qquad \text{Substitute } -2x - 2 \text{ for } y.$$

$$x = -\frac{6}{5} \qquad \text{Solve for } x.$$

Substituting $x = -\frac{6}{5}$ into $y = -2x - 2$ and solving for y gives the y-value of the other point of intersection, which is $\frac{2}{5}$. So, the points of intersection are $(0, -2)$ and $\left(-\frac{6}{5}, \frac{2}{5}\right)$.

Step 3 Use the Distance Formula to find the distance from $(0, -2)$ to $\left(-\frac{6}{5}, \frac{2}{5}\right)$.

$$\text{distance} = \sqrt{\left(-\frac{6}{5} - 0\right)^2 + \left(\frac{2}{5} - (-2)\right)^2} \approx 2.7$$

▶ So, the distance between $y = \frac{1}{2}x - 2$ and $y = \frac{1}{2}x + 1$ is about 2.7 units.

Monitoring Progress

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Find the distance between the parallel lines.

3. $y = 2x - 4$, $y = 2x + 1$

4. $y = -3x - 2$, $y = -3x - 4$

5. $y = \frac{3}{4}x - 3$, $y = \frac{3}{4}x + 2$

6. $y = -\frac{3}{2}x - 6$, $y = -\frac{3}{2}x + 5$

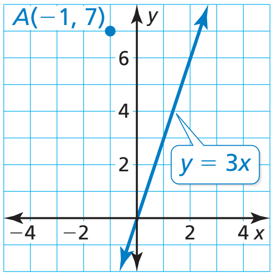
Vocabulary and Core Concept Check

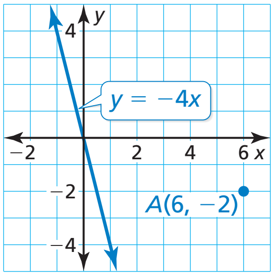
1. **COMPLETE THE SENTENCE** The distance between two parallel lines is the length of any ________ segment joining the two lines.

2. **WRITING** How is the distance between two parallel lines related to the distance between two points?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, find the distance from point A to the given line. *(See Example 1.)*

3. 

4. 

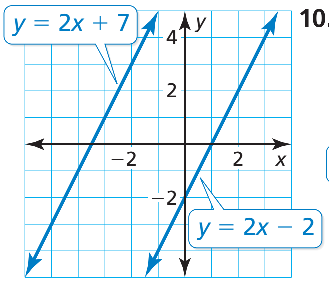
5. $A(-9, -3)$, $y = x - 6$

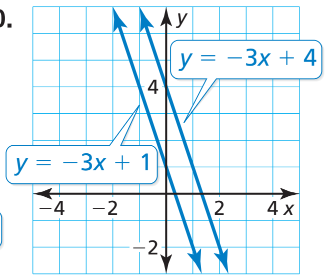
6. $A(-4, 4)$, $y = -2x + 1$

7. $A(-1, 4)$, line with a slope of -3 that passes through $(2, -4)$

8. $A(15, -21)$, line for which $f(4) = -8$ and $f(8) = -18$

In Exercises 9–14, find the distance between the parallel lines. *(See Example 2.)*

9. 

10. 

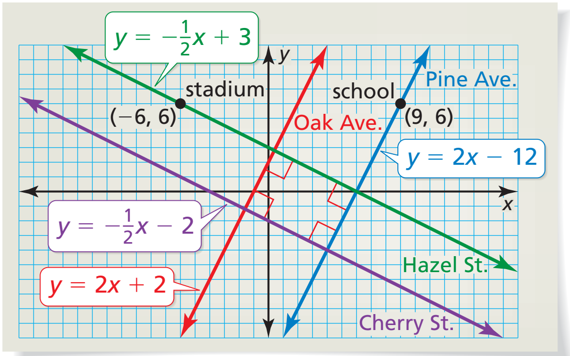
11. $y = -4x + 5$, $y = -4x + 7$

12. $y = 3x + 6$, $y = 3x - 2$

13. $y = -\frac{2}{3}x + 8$, parallel line that passes through $(0, 0)$

14. $y = -\frac{5}{6}x - 1$, parallel line that passes through $(6, -4)$

In Exercises 15–18, use the map shown. Each unit in the coordinate plane corresponds to 1 mile.



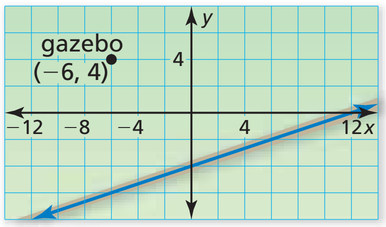
15. Find the distance from the school to Cherry Street.

16. Find the distance from the stadium to Pine Avenue.

17. Find the distance between Oak Avenue and Pine Avenue.

18. Find the distance between Hazel Street and Cherry Street.

19. **PROBLEM SOLVING** A gazebo is being built near a nature trail. An equation of the line representing the nature trail is $y = \frac{1}{3}x - 4$. Each unit in the coordinate plane corresponds to 10 feet. Approximately how far is the gazebo from the nature trail?



20. **PROBLEM SOLVING** A bike path is being constructed perpendicular to Washington Boulevard starting at point $P(2, 2)$. An equation of the line representing Washington Boulevard is $y = -\frac{2}{3}x$. Each unit in the coordinate plane corresponds to 10 feet. Approximately how far is the starting point from Washington Boulevard?

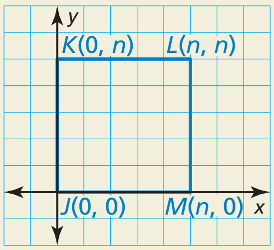
21. **ERROR ANALYSIS** Describe and correct the error in finding the distance between $y = x + 2$ and $y = x + 1$.

✗

$$\text{distance} = (x + 2) - (x + 1)$$
$$= x + 2 - x - 1$$
$$= 1$$

The distance between $y = x + 2$ and $y = x + 1$ is 1 unit.

22. **HOW DO YOU SEE IT?** Determine whether quadrilateral *JKLM* is a square. Explain your reasoning.



23. **MAKING AN ARGUMENT** Your classmate claims that no two nonvertical parallel lines can have the same y-intercept. Is your classmate correct? Explain.

PROVING A THEOREM **In Exercises 24 and 25, use the slopes of lines to write a paragraph proof of the theorem.**

24. Lines Perpendicular to a Transversal Theorem: In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

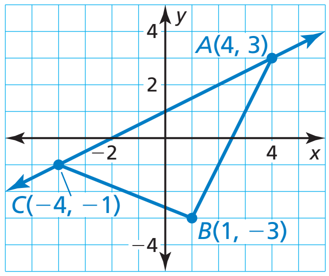
25. Transitive Property of Parallel Lines Theorem: If two lines are parallel to the same line, then they are parallel to each other.

26. **PROOF** Prove the statement: If two lines are vertical, then they are parallel.

27. **PROOF** Prove the statement: If two lines are horizontal, then they are parallel.

28. **PROOF** Prove that horizontal lines are perpendicular to vertical lines.

29. **MATHEMATICAL CONNECTIONS** Find the area of $\triangle ABC$.



30. **THOUGHT PROVOKING** Find a formula for the distance from the point (x_0, y_0) to the line $ax + by = 0$. Verify your formula using a point and a line.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Plot the point in a coordinate plane. *(Skills Review Handbook)*

31. $A(3, 6)$

32. $B(0, -4)$

33. $C(5, 0)$

34. $D(-1, -2)$

Copy and complete the table. *(Skills Review Handbook)*

35.

x	−2	−1	0	1	2
$y = x + 9$					

36.

x	−2	−1	0	1	2
$y = x - \frac{3}{4}$					