# 8.3 Using Midpoint and Distance Formulas

**Essential Question** How can you find the midpoint and length of a line segment in a coordinate plane?

**EXPLORATION 1** **Finding the Midpoint of a Line Segment**

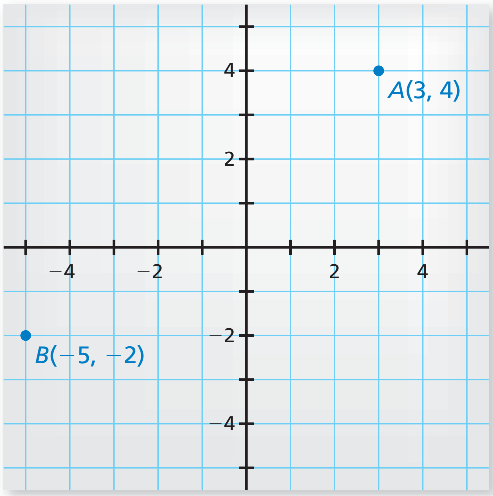
**Work with a partner.** Use centimeter graph paper.

**a.** Graph $\overline{AB}$, where the points $A$ and $B$ are as shown.

**b.** Explain how to *bisect* $\overline{AB}$, that is, to divide $\overline{AB}$ into two congruent line segments. Then bisect $\overline{AB}$ and use the result to find the *midpoint M* of $\overline{AB}$.

**c.** What are the coordinates of the midpoint $M$?

**d.** Compare the $x$-coordinates of $A$, $B$, and $M$. Compare the $y$-coordinates of $A$, $B$, and $M$. How are the coordinates of the midpoint $M$ related to the coordinates of $A$ and $B$?




**MAKING SENSE OF PROBLEMS**

To be proficient in math, you need to check your answers and continually ask yourself, "Does this make sense?"

**EXPLORATION 2** **Finding the Length of a Line Segment**

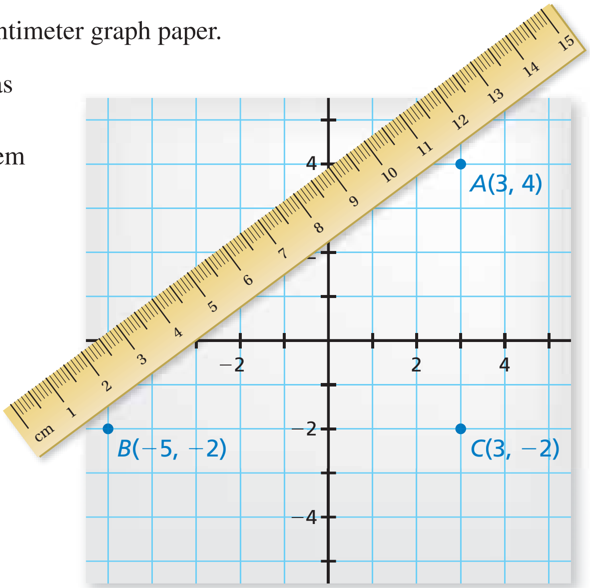
**Work with a partner.** Use centimeter graph paper.

**a.** Add point $C$ to your graph as shown.

**b.** Use the Pythagorean Theorem to find the length of $\overline{AB}$.

**c.** Use a centimeter ruler to verify the length you found in part (b).

**d.** Use the Pythagorean Theorem and point $M$ from Exploration 1 to find the lengths of $\overline{AM}$ and $\overline{MB}$. What can you conclude?




## Communicate Your Answer

**3.** How can you find the midpoint and length of a line segment in a coordinate plane?

**4.** Find the coordinates of the midpoint $M$ and the length of the line segment whose endpoints are given.

**a.** $D(-10, -4)$, $E(14, 6)$ **b.** $F(-4, 8)$, $G(9, 0)$

# 8.3 Lesson

## Core Vocabulary

midpoint, *p. 396*
segment bisector, *p. 396*

## What You Will Learn

- Find segment lengths using midpoints and segment bisectors.
- Use the Midpoint Formula.
- Use the Distance Formula.

## Midpoints and Segment Bisectors

**READING**
The word *bisect* means "to cut into two equal parts."

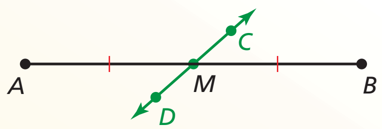
### Core Concept

**Midpoints and Segment Bisectors**

The **midpoint** of a segment is the point that divides the segment into two congruent segments.


$M$ is the midpoint of $\overline{AB}$.
So, $\overline{AM} \cong \overline{MB}$ and $AM = MB$.

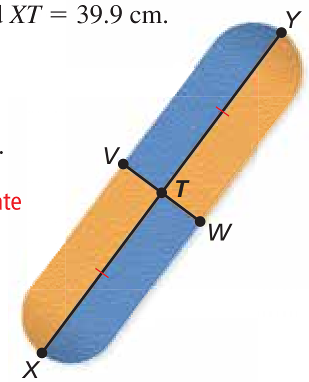
A **segment bisector** is a point, ray, line, line segment, or plane that intersects the segment at its midpoint. A midpoint or a segment bisector *bisects* a segment.




$\overleftrightarrow{CD}$ is a segment bisector of $\overline{AB}$.
So, $\overline{AM} \cong \overline{MB}$ and $AM = MB$.

**EXAMPLE 1** **Finding Segment Lengths**

In the skateboard design, $\overline{VW}$ bisects $\overline{XY}$ at point $T$, and $XT = 39.9$ cm. Find $XY$.




**SOLUTION**

Point $T$ is the midpoint of $\overline{XY}$. So, $XT = TY = 39.9$ cm.

| | |
|---|---|
| $XY = XT + TY$ | Segment Addition Postulate |
| $= 39.9 + 39.9$ | Substitute. |
| $= 79.8$ | Add. |

So, the length of $\overline{XY}$ is 79.8 centimeters.

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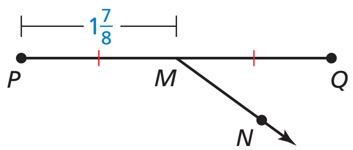
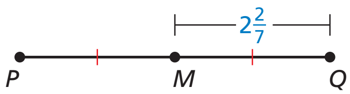
**Identify the segment bisector of $\overline{PQ}$. Then find $PQ$.**

**1.**

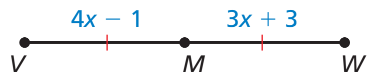



**2.**



## EXAMPLE 2 Using Algebra with Segment Lengths

Point $M$ is the midpoint of $\overline{VW}$. Find the length of $\overline{VM}$.




### SOLUTION

**Step 1** Write and solve an equation. Use the fact that $VM = MW$.

| | |
|---|---|
| $VM = MW$ | Write the equation. |
| $4x - 1 = 3x + 3$ | Substitute. |
| $x - 1 = 3$ | Subtract $3x$ from each side. |
| $x = 4$ | Add 1 to each side. |

**Step 2** Evaluate the expression for $VM$ when $x = 4$.

$$VM = 4x - 1 = 4(4) - 1 = 15$$

▶ So, the length of $\overline{VM}$ is 15.

**Check**

Because $VM = MW$, the length of $\overline{MW}$ should be 15.

$MW = 3x + 3 = 3(4) + 3 = 15$ ✓

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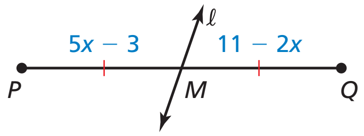
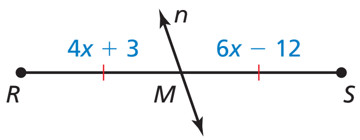
**3.** Identify the segment bisector of $\overline{PQ}$. Then find $MQ$.




**4.** Identify the segment bisector of $\overline{RS}$. Then find $RS$.

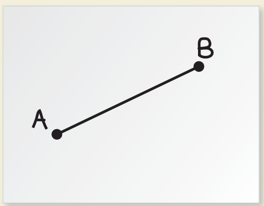



## CONSTRUCTION Bisecting a Segment

Construct a segment bisector of $\overline{AB}$ by paper folding. Then find the midpoint $M$ of $\overline{AB}$.

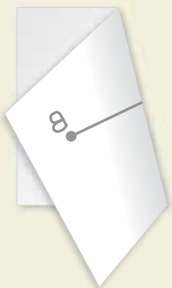
### SOLUTION

**Step 1**



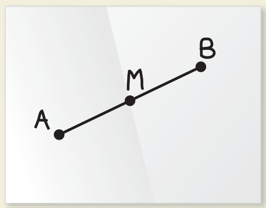

**Draw the segment**
Draw $\overline{AB}$ on a piece of paper.

**Step 2**



**Fold the paper**
Fold the paper so that $B$ is on top of $A$.

**Step 3**




**Label the midpoint**
Label point $M$. Compare $AM$, $MB$, and $AB$.

$$AM = MB = \tfrac{1}{2}AB$$

## Using the Midpoint Formula

You can use the coordinates of the endpoints of a segment to find the coordinates of the midpoint.

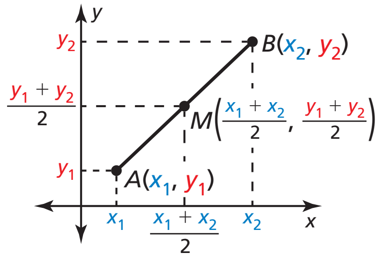
## Core Concept

### The Midpoint Formula

The coordinates of the midpoint of a segment are the averages of the $x$-coordinates and of the $y$-coordinates of the endpoints.

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the midpoint $M$ of $\overline{AB}$ has coordinates

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$




### EXAMPLE 3 Using the Midpoint Formula

**a.** The endpoints of $\overline{RS}$ are $R(1, -3)$ and $S(4, 2)$. Find the coordinates of the midpoint $M$.

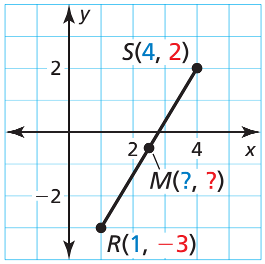
**b.** The midpoint of $\overline{JK}$ is $M(2, 1)$. One endpoint is $J(1, 4)$. Find the coordinates of endpoint $K$.

### SOLUTION

**a.** Use the Midpoint Formula.

$$M\left(\frac{1 + 4}{2}, \frac{-3 + 2}{2}\right) = M\left(\frac{5}{2}, -\frac{1}{2}\right)$$

▶ The coordinates of the midpoint $M$ are $\left(\frac{5}{2}, -\frac{1}{2}\right)$.




**b.** Let $(x, y)$ be the coordinates of endpoint $K$. Use the Midpoint Formula.

**Step 1** Find $x$.

$$\frac{1 + x}{2} = 2$$

$$1 + x = 4$$

$$x = 3$$

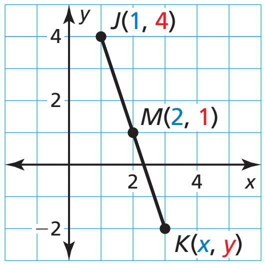
**Step 2** Find $y$.

$$\frac{4 + y}{2} = 1$$

$$4 + y = 2$$

$$y = -2$$

▶ The coordinates of endpoint $K$ are $(3, -2)$.

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**5.** The endpoints of $\overline{AB}$ are $A(1, 2)$ and $B(7, 8)$. Find the coordinates of the midpoint $M$.

**6.** The endpoints of $\overline{CD}$ are $C(-4, 3)$ and $D(-6, 5)$. Find the coordinates of the midpoint $M$.

**7.** The midpoint of $\overline{TU}$ is $M(2, 4)$. One endpoint is $T(1, 1)$. Find the coordinates of endpoint $U$.

**8.** The midpoint of $\overline{VW}$ is $M(-1, -2)$. One endpoint is $W(4, 4)$. Find the coordinates of endpoint $V$.

## Using the Distance Formula

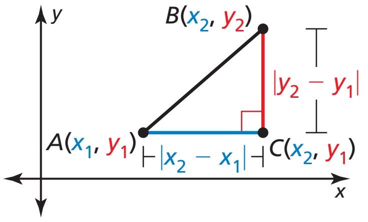
You can use the Distance Formula to find the distance between two points in a coordinate plane.

### Core Concept

**The Distance Formula**

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the distance between $A$ and $B$ is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

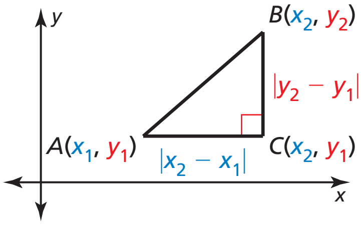


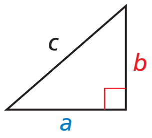

**READING**

The red mark at the corner of the triangle that makes a right angle indicates a right triangle.

The Distance Formula is related to the *Pythagorean Theorem*, which you will see again when you work with right triangles in a future course.

| Distance Formula | Pythagorean Theorem |
|---|---|
| $(AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ | $c^2 = a^2 + b^2$ |



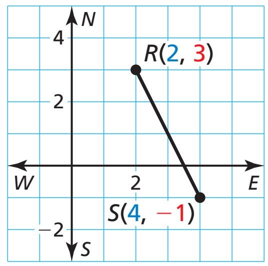

### EXAMPLE 4 Using the Distance Formula

Your school is 4 miles east and 1 mile south of your apartment. A recycling center, where your class is going on a field trip, is 2 miles east and 3 miles north of your apartment. Estimate the distance between the recycling center and your school.

**SOLUTION**

You can model the situation using a coordinate plane with your apartment at the origin (0, 0). The coordinates of the recycling center and the school are $R(2, 3)$ and $S(4, -1)$, respectively. Use the Distance Formula. Let $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (4, -1)$.

$$RS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$
$$= \sqrt{(4 - 2)^2 + (-1 - 3)^2} \quad \text{Substitute.}$$
$$= \sqrt{2^2 + (-4)^2} \quad \text{Subtract.}$$
$$= \sqrt{4 + 16} \quad \text{Evaluate powers.}$$
$$= \sqrt{20} \quad \text{Add.}$$
$$\approx 4.5 \quad \text{Use a calculator.}$$




**READING**

The symbol ≈ means "is approximately equal to."

▶ So, the distance between the recycling center and your school is about 4.5 miles.

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**9.** In Example 4, a park is 3 miles east and 4 miles south of your apartment. Find the distance between the park and your school.

# 8.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

## Vocabulary and Core Concept Check

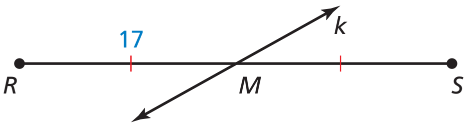
1. **VOCABULARY** If a point, ray, line, line segment, or plane intersects a segment at its midpoint, then what does it do to the segment?

2. **COMPLETE THE SENTENCE** To find the length of $\overline{AB}$, with endpoints $A(-7, 5)$ and $B(4, -6)$, you can use the ______________.

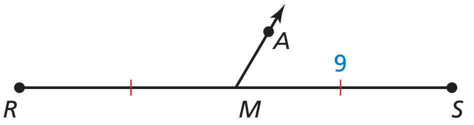
## Monitoring Progress and Modeling with Mathematics

**In Exercises 3–6, identify the segment bisector of $\overline{RS}$. Then find $RS$.** *(See Example 1.)*

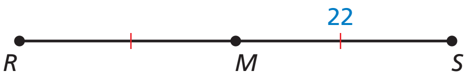
**3.**



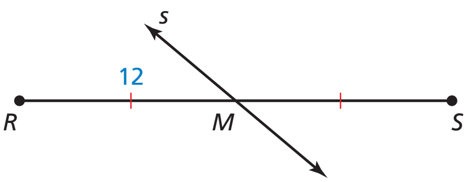

**4.**




**5.**

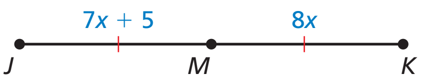



**6.**

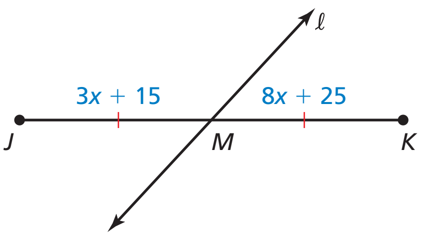



**In Exercises 7 and 8, identify the segment bisector of $\overline{JK}$. Then find $JM$.** *(See Example 2.)*

**7.**

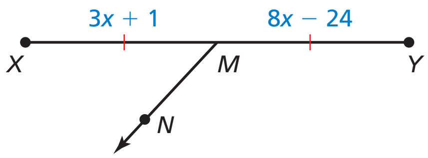



**8.**

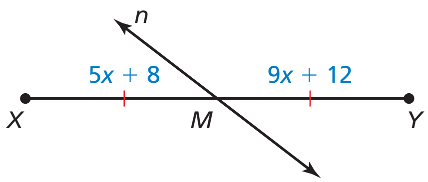



**In Exercises 9 and 10, identify the segment bisector of $\overline{XY}$. Then find $XY$.** *(See Example 2.)*

**9.**




**10.**

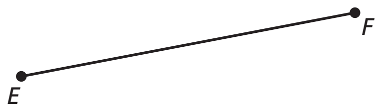



**CONSTRUCTION** **In Exercises 11–14, copy the segment and construct a segment bisector by paper folding. Then label the midpoint $M$.**

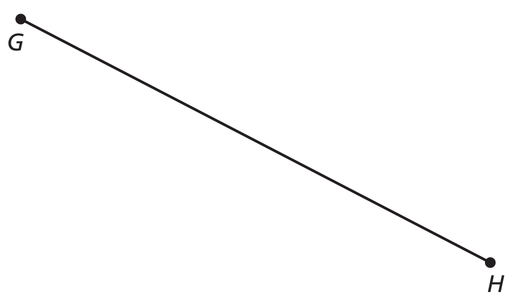
**11.**


**12.**


**13.**




**14.**



In Exercises 15–18, the endpoints of $\overline{CD}$ are given. Find the coordinates of the midpoint $M$. *(See Example 3.)*

**15.** $C(3, -5)$ and $D(7, 9)$

**16.** $C(-4, 7)$ and $D(0, -3)$

**17.** $C(-2, 0)$ and $D(4, 9)$

**18.** $C(-8, -6)$ and $D(-4, 10)$

In Exercises 19–22, the midpoint $M$ and one endpoint of $\overline{GH}$ are given. Find the coordinates of the other endpoint. *(See Example 3.)*

**19.** $G(5, -6)$ and $M(4, 3)$

**20.** $H(-3, 7)$ and $M(-2, 5)$

**21.** $H(-2, 9)$ and $M(8, 0)$

**22.** $G(-4, 1)$ and $M\left(-\frac{13}{2}, -6\right)$

In Exercises 23–30, find the distance between the two points. *(See Example 4.)*

**23.** $A(13, 2)$ and $B(7, 10)$

**24.** $C(-6, 5)$ and $D(-3, 1)$

**25.** $E(3, 7)$ and $F(6, 5)$

**26.** $G(-5, 4)$ and $H(2, 6)$

**27.** $J(-8, 0)$ and $K(1, 4)$

**28.** $L(7, -1)$ and $M(-2, 4)$

**29.** $R(0, 1)$ and $S(6, 3.5)$

**30.** $T(13, 1.6)$ and $V(5.4, 3.7)$

**ERROR ANALYSIS** In Exercises 31 and 32, describe and correct the error in finding the distance between $A(6, 2)$ and $B(1, -4)$.

**31.** ✗

$$\begin{aligned} AB &= (6-1)^2 + [2-(-4)]^2 \\ &= 5^2 + 6^2 \\ &= 25 + 36 \\ &= 61 \end{aligned}$$

**32.** ✗

$$\begin{aligned} AB &= \sqrt{(6-2)^2 + [1-(-4)]^2} \\ &= \sqrt{4^2 + 5^2} \\ &= \sqrt{16 + 25} \\ &= \sqrt{41} \\ &\approx 6.4 \end{aligned}$$

**COMPARING SEGMENTS** In Exercises 33 and 34, the endpoints of two segments are given. Find each segment length. Tell whether the segments are congruent. If they are not congruent, state which segment length is greater.

**33.** $\overline{AB}$: $A(0, 2)$, $B(-3, 8)$ and $\overline{CD}$: $C(-2, 2)$, $D(0, -4)$

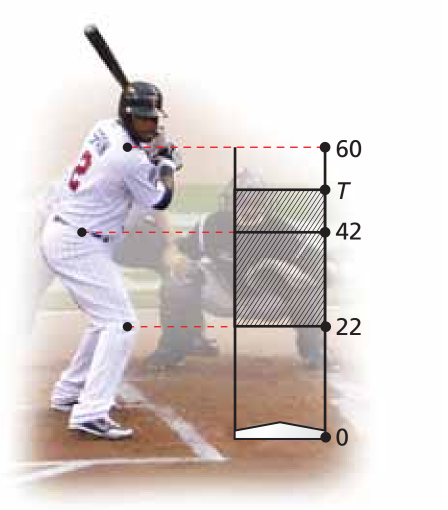
**34.** $\overline{EF}$: $E(1, 4)$, $F(5, 1)$ and $\overline{GH}$: $G(-3, 1)$, $H(1, 6)$

**35.** **WRITING** Your friend is having trouble understanding the Midpoint Formula.

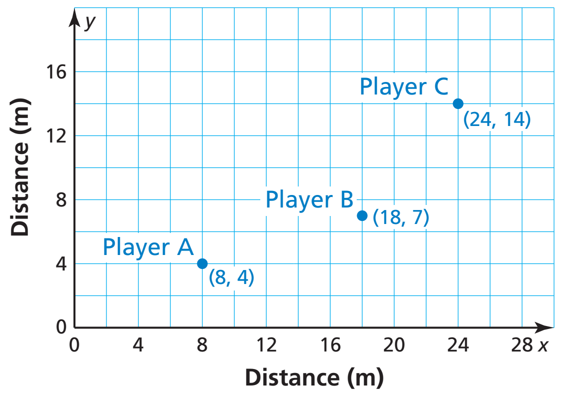
a. Explain how to find the midpoint when given the two endpoints in your own words.

b. Explain how to find the other endpoint when given one endpoint and the midpoint in your own words.

**36.** **PROBLEM SOLVING** In baseball, the strike zone is the region a baseball needs to pass through for the umpire to declare it a strike when the batter does not swing. The top of the strike zone is a horizontal plane passing through the midpoint of the top of the batter's shoulders and the top of the uniform pants when the player is in a batting stance. Find the height of $T$. (*Note:* All heights are in inches.)




**37.** **MODELING WITH MATHEMATICS** The figure shows the position of three players during part of a water polo match. Player A throws the ball to Player B, who then throws the ball to Player C.

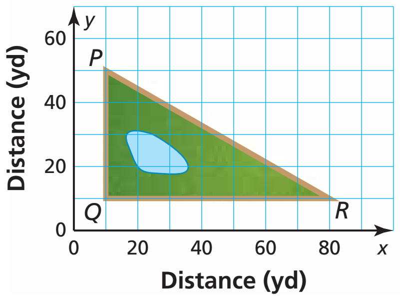



a. How far did Player A throw the ball? Player B?

b. How far would Player A have to throw the ball to throw it directly to Player C?

**38. MODELING WITH MATHEMATICS** Your school is 20 blocks east and 12 blocks south of your house. The mall is 10 blocks north and 7 blocks west of your house. You plan on going to the mall right after school. Find the distance between your school and the mall assuming there is a road directly connecting the school and the mall. One block is 0.1 mile.

**39. PROBLEM SOLVING** A path goes around a triangular park, as shown.




**a.** Find the distance around the park to the nearest yard.

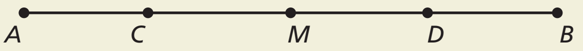
**b.** A new path and a bridge are constructed from point $Q$ to the midpoint $M$ of $\overline{PR}$. Find $QM$ to the nearest yard.

**c.** A man jogs from $P$ to $Q$ to $M$ to $R$ to $Q$ and back to $P$ at an average speed of 150 yards per minute. About how many minutes does it take? Explain your reasoning.

**40. MAKING AN ARGUMENT** Your friend claims there is an easier way to find the length of a segment than the Distance Formula when the $x$-coordinates of the endpoints are equal. He claims all you have to do is subtract the $y$-coordinates. Do you agree with his statement? Explain your reasoning.

**41. MATHEMATICAL CONNECTIONS** Two points are located at $(a, c)$ and $(b, c)$. Find the midpoint and the distance between the two points.

**42. HOW DO YOU SEE IT?** $\overline{AB}$ contains midpoint $M$ and points $C$ and $D$, as shown. Compare the lengths. If you cannot draw a conclusion, write *impossible to tell*. Explain your reasoning.




**a.** $AM$ and $MB$

**b.** $AC$ and $MB$

**c.** $MC$ and $MD$

**d.** $MB$ and $DB$

**43. ABSTRACT REASONING** Use the diagram in Exercise 42. The points on $\overline{AB}$ represent locations you pass on your commute to work. You travel from your home at location $A$ to location $M$ before realizing that you left your lunch at home. You could turn around to get your lunch and then continue to work at location $B$. Or you could go to work and go to location $D$ for lunch today. You want to choose the option that involves the least distance you must travel. Which option should you choose? Explain your reasoning.

**44. THOUGHT PROVOKING** Describe three ways to divide a rectangle into two congruent regions. Do the regions have to be triangles? Use a diagram to support your answer.

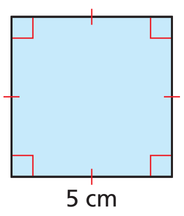
**45. ANALYZING RELATIONSHIPS** The length of $\overline{XY}$ is 24 centimeters. The midpoint of $\overline{XY}$ is $M$, and $C$ is on $\overline{XM}$ so that $XC$ is $\frac{2}{3}$ of $XM$. Point $D$ is on $\overline{MY}$ so that $MD$ is $\frac{3}{4}$ of $MY$. What is the length of $\overline{CD}$?

## Maintaining Mathematical Proficiency

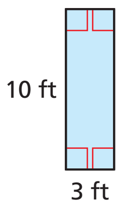
Reviewing what you learned in previous grades and lessons

**Find the perimeter and area of the figure.** *(Skills Review Handbook)*

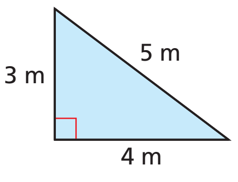
**46.**



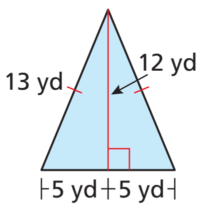

**47.**




**48.**




**49.**




**Solve the inequality. Graph the solution.** *(Section 2.2 and Section 2.3)*

**50.** $a + 18 < 7$

**51.** $y - 5 \geq 8$

**52.** $-3x > 24$

**53.** $\frac{z}{4} \leq 12$