6.1 Exponential Functions

Essential Question What are some of the characteristics of the graph of an exponential function?

EXPLORATION 1 **Exploring an Exponential Function**

Work with a partner. Copy and complete each table for the *exponential function* $y = 16(2)^x$. In each table, what do you notice about the values of x? What do you notice about the values of y?

x	$y = 16(2)^x$
0	
1	
2	
3	
4	
5	

x	$y = 16(2)^x$
0	
2	
4	
6	
8	
10	

JUSTIFYING CONCLUSIONS

To be proficient in math, you need to justify your conclusions and communicate them to others.

EXPLORATION 2 **Exploring an Exponential Function**

Work with a partner. Repeat Exploration 1 for the exponential function $y = 16\left(\frac{1}{2}\right)^x$. Do you think the statement below is true for *any* exponential function? Justify your answer.

> *"As the independent variable x changes by a constant amount, the dependent variable y is multiplied by a constant factor."*

EXPLORATION 3 **Graphing Exponential Functions**

Work with a partner. Sketch the graphs of the functions given in Explorations 1 and 2. How are the graphs similar? How are they different?

Communicate Your Answer

4. What are some of the characteristics of the graph of an exponential function?

5. Sketch the graph of each exponential function. Does each graph have the characteristics you described in Question 4? Explain your reasoning.

a. $y = 2^x$ **b.** $y = 2(3)^x$ **c.** $y = 3(1.5)^x$

d. $y = \left(\frac{1}{2}\right)^x$ **e.** $y = 3\left(\frac{1}{2}\right)^x$ **f.** $y = 2\left(\frac{3}{4}\right)^x$

6.1 Lesson

Core Vocabulary

exponential function, *p. 274*

Previous
independent variable
dependent variable
parent function

What You Will Learn

- Identify and evaluate exponential functions.
- Graph exponential functions.
- Solve real-life problems involving exponential functions.

Identifying and Evaluating Exponential Functions

An **exponential function** is a nonlinear function of the form $y = ab^x$, where $a \neq 0$, $b \neq 1$, and $b > 0$. As the independent variable x changes by a constant amount, the dependent variable y is multiplied by a constant factor, which means consecutive y-values form a constant ratio.

EXAMPLE 1 Identifying Functions

Does each table represent an exponential function? Explain.

a.

x	0	1	2	3
y	2	4	12	48

b.

x	0	1	2	3
y	4	8	16	32

SOLUTION

a.

+ 1 + 1 + 1

x	0	1	2	3
y	2	4	12	48

× 2 × 3 × 4

As x increases by 1, y is not multiplied by a constant factor. So, the function is *not* exponential.

b.

+ 1 + 1 + 1

x	0	1	2	3
y	4	8	16	32

× 2 × 2 × 2

As x increases by 1, y is multiplied by 2. So, the function is exponential.

STUDY TIP

In Example 1b, consecutive y-values form a constant ratio.

$$\frac{8}{4} = 2, \frac{16}{8} = 2, \frac{32}{16} = 2$$

EXAMPLE 2 Evaluating Exponential Functions

Evaluate each function for the given value of x.

a. $y = -2(5)^x$; $x = 3$

b. $y = 3(0.5)^x$; $x = -2$

SOLUTION

a.

$y = -2(5)^x$	Write the function.
$= -2(5)^3$	Substitute for x.
$= -2(125)$	Evaluate the power.
$= -250$	Multiply.

b.

$y = 3(0.5)^x$
$= 3(0.5)^{-2}$
$= 3(4)$
$= 12$

Monitoring Progress

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Does the table represent an exponential function? Explain.

1.

x	0	1	2	3
y	8	4	2	1

2.

x	−4	0	4	8
y	1	0	−1	−2

Evaluate the function when $x = -2$, 0, and 3.

3. $y = 2(9)^x$

4. $y = 1.5(2)^x$

Graphing Exponential Functions

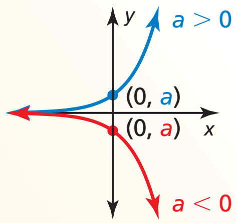
The graph of a function $y = ab^x$ is a vertical stretch or shrink by a factor of $|a|$ of the graph of the parent function $y = b^x$. When $a < 0$, the graph is also reflected in the x-axis. The y-intercept of the graph of $y = ab^x$ is a.

STUDY TIP

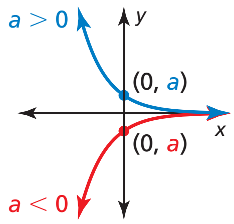
The graph of $y = ab^x$ approaches the x-axis but never intersects it.

Core Concept

Graphing $y = ab^x$ When $b > 1$



Graphing $y = ab^x$ When $0 < b < 1$



EXAMPLE 3 Graphing $y = ab^x$ When $b > 1$

Graph $f(x) = 4(2)^x$. Compare the graph to the graph of the parent function. Describe the domain and range of f.

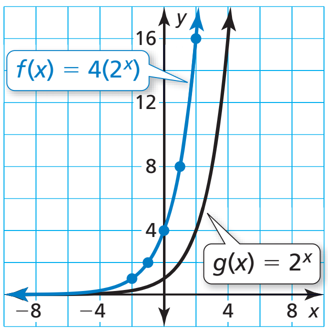
SOLUTION

Step 1 Make a table of values.

Step 2 Plot the ordered pairs.

Step 3 Draw a smooth curve through the points.

x	-2	-1	0	1	2
$f(x)$	1	2	4	8	16



▶ The parent function is $g(x) = 2^x$. The graph of f is a vertical stretch by a factor of 4 of the graph of g. The y-intercept of the graph of f, 4, is above the y-intercept of the graph of g, 1. From the graph of f, you can see that the domain is all real numbers and the range is $y > 0$.

EXAMPLE 4 Graphing $y = ab^x$ When $0 < b < 1$

Graph $f(x) = -\left(\frac{1}{2}\right)^x$. Compare the graph to the graph of the parent function. Describe the domain and range of f.

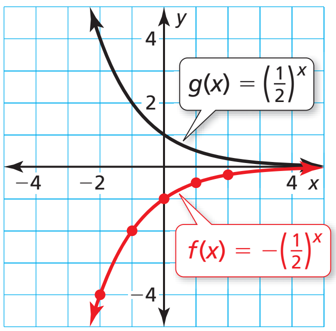
SOLUTION

Step 1 Make a table of values.

Step 2 Plot the ordered pairs.

Step 3 Draw a smooth curve through the points.

x	-2	-1	0	1	2
$f(x)$	-4	-2	-1	$-\frac{1}{2}$	$-\frac{1}{4}$



▶ The parent function is $g(x) = \left(\frac{1}{2}\right)^x$. The graph of f is a reflection in the x-axis of the graph of g. The y-intercept of the graph of f, -1, is below the y-intercept of the graph of g, 1. From the graph of f, you can see that the domain is all real numbers and the range is $y < 0$.

Monitoring Progress

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Graph the function. Compare the graph to the graph of the parent function. Describe the domain and range of *f*.

5. $f(x) = -2(4)^x$

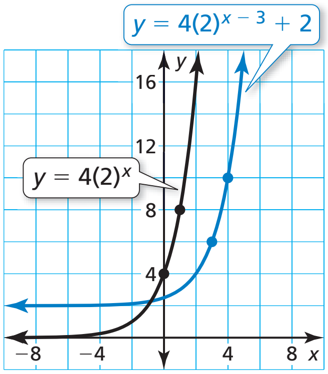
6. $f(x) = 2\left(\frac{1}{4}\right)^x$

To graph a function of the form $y = ab^{x-h} + k$, begin by graphing $y = ab^x$. Then translate the graph horizontally h units and vertically k units.

EXAMPLE 5 Graphing $y = ab^{x-h} + k$

Graph $y = 4(2)^{x-3} + 2$. Describe the domain and range.

SOLUTION



Step 1 Graph $y = 4(2)^x$. This is the same function that is in Example 3, which passes through (0, 4) and (1, 8).

Step 2 Translate the graph 3 units right and 2 units up. The graph passes through (3, 6) and (4, 10).

Notice that the graph approaches the line $y = 2$ but does not intersect it.

▶ From the graph, you can see that the domain is all real numbers and the range is $y > 2$.

EXAMPLE 6 Comparing Exponential Functions

An exponential function g models a relationship in which the dependent variable is multiplied by 1.5 for every 1 unit the independent variable x increases. Graph g when $g(0) = 4$. Compare g and the function f from Example 3 over the interval $x = 0$ to $x = 2$.

SOLUTION

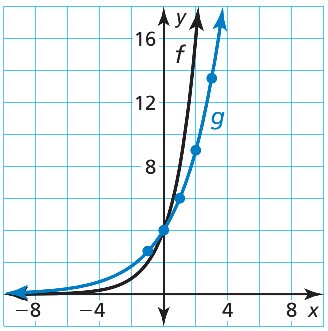
You know (0, 4) is on the graph of g. To find points to the right of (0, 4), multiply $g(x)$ by 1.5 for every 1 unit increase in x. To find points to the left of (0, 4), divide $g(x)$ by 1.5 for every 1 unit decrease in x.

Step 1 Make a table of values.

x	−1	0	1	2	3
$g(x)$	2.7	4	6	9	13.5

Step 2 Plot the ordered pairs.

Step 3 Draw a smooth curve through the points.



STUDY TIP

Note that f is increasing faster than g to the right of $x = 0$.

▶ Both functions have the same value when $x = 0$, but the value of f is greater than the value of g over the rest of the interval.

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Graph the function. Describe the domain and range.

7. $y = -2(3)^{x+2} - 1$

8. $f(x) = (0.25)^x + 3$

9. **WHAT IF?** In Example 6, the dependent variable of g is multiplied by 3 for every 1 unit the independent variable x increases. Graph g when $g(0) = 4$. Compare g and the function f from Example 3 over the interval $x = 0$ to $x = 2$.

Solving Real-Life Problems

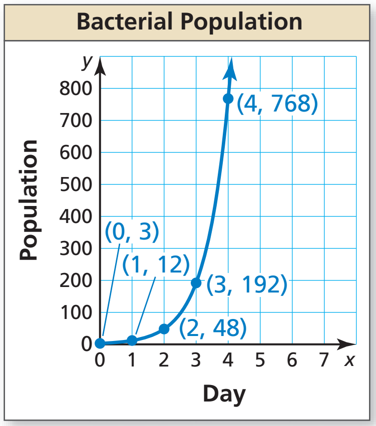
For an exponential function of the form $y = ab^x$, the y-values change by a factor of b as x increases by 1. You can use this fact to write an exponential function when you know the y-intercept, a. The table represents the exponential function $y = 2(5)^x$.

x	0	1	2	3	4
y	2	10	50	250	1250

+1 +1 +1 +1

×5 ×5 ×5 ×5

EXAMPLE 7 **Modeling with Mathematics**



The graph represents a bacterial population y after x days.

a. Write an exponential function that represents the population.

b. Find the population after 5 days.

SOLUTION

1. Understand the Problem You have a graph of the population that shows some data points. You are asked to write an exponential function that represents the population and find the population after a given amount of time.

2. Make a Plan Use the graph to make a table of values. Use the table and the y-intercept to write an exponential function. Then evaluate the function to find the population.

3. Solve the Problem

a. Use the graph to make a table of values.

x	0	1	2	3	4
y	3	12	48	192	768

+1 +1 +1 +1

×4 ×4 ×4 ×4

The y-intercept is 3. The y-values increase by a factor of 4 as x increases by 1.

▶ So, the population can be modeled by $y = 3(4)^x$.

b. To find the population after 5 days, evaluate the function when $x = 5$.

$y = 3(4)^x$ — Write the function.

$= 3(4)^5$ — Substitute 5 for x.

$= 3(1024)$ — Evaluate the power.

$= 3072$ — Multiply.

▶ There are 3072 bacteria after 5 days.

4. Look Back The graph resembles an exponential function of the form $y = ab^x$, where $b > 1$ and $a > 0$. So, the exponential function $y = 3(4)^x$ is reasonable.

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10. A bacterial population y after x days can be represented by an exponential function whose graph passes through (0, 100) and (1, 200). (a) Write a function that represents the population. (b) Find the population after 6 days. (c) Does this bacterial population grow faster than the bacterial population in Example 7? Explain.

6.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **OPEN-ENDED** Sketch an increasing exponential function whose graph has a y-intercept of 2.

2. **REASONING** Why is a the y-intercept of the graph of the function $y = ab^x$?

3. **WRITING** Compare the graph of $y = 2(5)^x$ with the graph of $y = 5^x$.

4. **WHICH ONE DOESN'T BELONG?** Which equation does *not* belong with the other three? Explain your reasoning.

$y = 3^x$	$f(x) = 2(4)^x$	$f(x) = (-3)^x$	$y = 5(3)^x$

Monitoring Progress and Modeling with Mathematics

In Exercises 5–10, determine whether the equation represents an exponential function. Explain.

5. $y = 4(7)^x$
6. $y = -6x$
7. $y = 2x^3$
8. $y = -3^x$
9. $y = 9(-5)^x$
10. $y = \frac{1}{2}(1)^x$

In Exercises 11–14, determine whether the table represents an exponential function. Explain. *(See Example 1.)*

11.

x	y
1	−2
2	−10
3	−40
4	−120

12.

x	y
1	6
2	12
3	24
4	48

13.

x	−1	0	1	2	3
y	0.25	1	4	16	64

14.

x	−3	0	3	6	9
y	10	1	−8	−17	−26

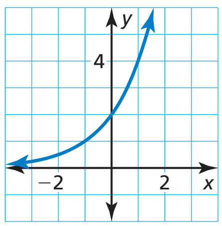
In Exercises 15–20, evaluate the function for the given value of x. *(See Example 2.)*

15. $y = 3^x;\ x = 2$
16. $f(x) = 3(2)^x;\ x = -1$
17. $y = -4(5)^x;\ x = 2$
18. $f(x) = 0.5^x;\ x = -3$
19. $f(x) = \frac{1}{3}(6)^x;\ x = 3$
20. $y = \frac{1}{4}(4)^x;\ x = 5$

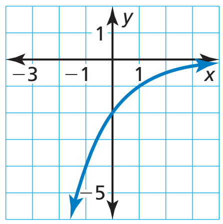
USING STRUCTURE In Exercises 21–24, match the function with its graph.

21. $f(x) = 2(0.5)^x$
22. $y = -2(0.5)^x$
23. $y = 2(2)^x$
24. $f(x) = -2(2)^x$

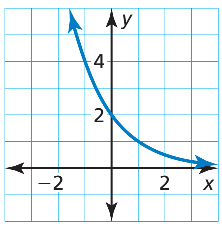
A.



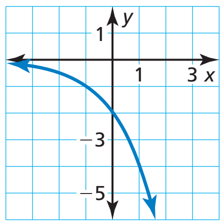
B.



C.



D.



In Exercises 25–30, graph the function. Compare the graph to the graph of the parent function. Describe the domain and range of f. *(See Examples 3 and 4.)*

25. $f(x) = 3(0.5)^x$
26. $f(x) = -4^x$
27. $f(x) = -2(7)^x$
28. $f(x) = 6\left(\frac{1}{3}\right)^x$
29. $f(x) = \frac{1}{2}(8)^x$
30. $f(x) = \frac{3}{2}(0.25)^x$

In Exercises 31–36, graph the function. Describe the domain and range. *(See Example 5.)*

31. $f(x) = 3^x - 1$
32. $f(x) = 4^{x+3}$

33. $y = 5^{x-2} + 7$

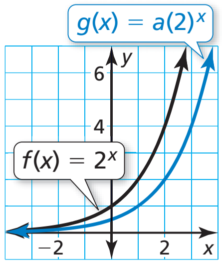
34. $y = -\left(\frac{1}{2}\right)^{x+1} - 3$

35. $y = -8(0.75)^{x+2} - 2$

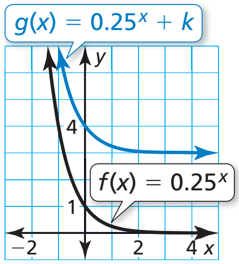
36. $f(x) = 3(6)^{x-1} - 5$

In Exercises 37–40, compare the graphs. Find the value of h, k, or a.

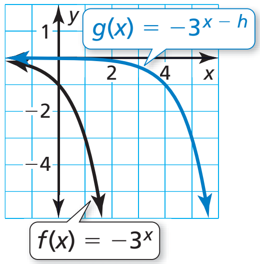
37.



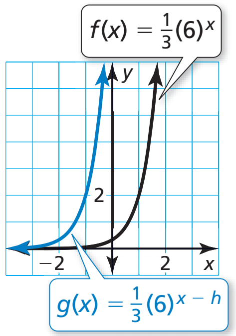
38.



39.



40.



41. **ERROR ANALYSIS** Describe and correct the error in evaluating the function.

$g(x) = 6(0.5)^x;\ x = -2$

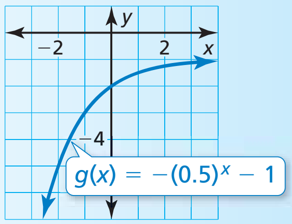
$g(-2) = 6(0.5)^{-2}$

$= 3^{-2}$

$= \frac{1}{9}$

42. **ERROR ANALYSIS** Describe and correct the error in finding the domain and range of the function.

The domain is all real numbers, and the range is $y < 0$.



In Exercises 43 and 44, graph the function with the given description. Compare the function to $f(x) = 0.5(4)^x$ over the interval $x = 0$ to $x = 2$. *(See Example 6.)*

43. An exponential function g models a relationship in which the dependent variable is multiplied by 2.5 for every 1 unit the independent variable x increases. The value of the function at 0 is 8.

44. An exponential function h models a relationship in which the dependent variable is multiplied by $\frac{1}{2}$ for every 1 unit the independent variable x increases. The value of the function at 0 is 32.

45. **MODELING WITH MATHEMATICS** You graph an exponential function on a calculator. You zoom in repeatedly to 25% of the screen size. The function $y = 0.25^x$ represents the percent (in decimal form) of the original screen display that you see, where x is the number of times you zoom in.

a. Graph the function. Describe the domain and range.

b. Find and interpret the y-intercept.

c. You zoom in twice. What percent of the original screen do you see?

46. **MODELING WITH MATHEMATICS** A population y of coyotes in a national park triples every 20 years. The function $y = 15(3)^x$ represents the population, where x is the number of 20-year periods.

a. Graph the function. Describe the domain and range.

b. Find and interpret the y-intercept.

c. How many coyotes are in the national park in 40 years?

In Exercises 47–50, write an exponential function represented by the table or graph. *(See Example 7.)*

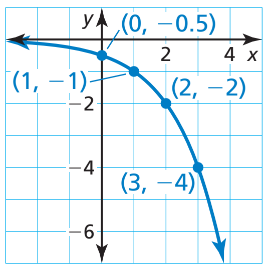
47.

x	0	1	2	3
y	2	14	98	686

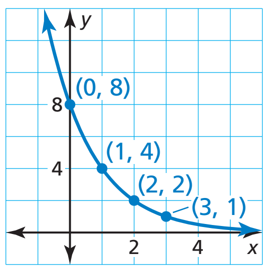
48.

x	0	1	2	3
y	−50	−10	−2	−0.4

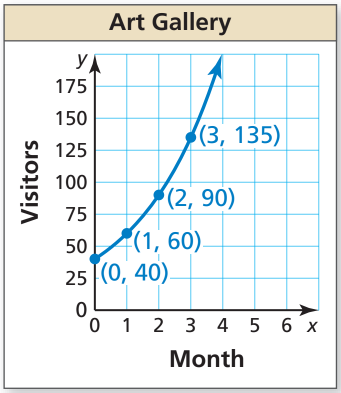
49.



50.



51. MODELING WITH MATHEMATICS The graph represents the number y of visitors to a new art gallery after x months.



a. Write an exponential function that represents this situation.

b. Approximate the number of visitors after 5 months.

52. PROBLEM SOLVING A sales report shows that 3300 gas grills were purchased from a chain of hardware stores last year. The store expects grill sales to increase 6% each year. About how many grills does the store expect to sell in Year 6? Use an equation to justify your answer.

53. WRITING Graph the function $f(x) = -2^x$. Then graph $g(x) = -2^x - 3$. How are the y-intercept, domain, and range affected by the translation?

54. MAKING AN ARGUMENT Your friend says that the table represents an exponential function because y is multiplied by a constant factor. Is your friend correct? Explain.

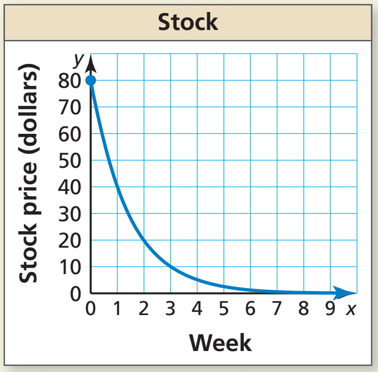
x	0	1	3	6
y	2	10	50	250

55. WRITING Describe the effect of a on the graph of $y = a \cdot 2^x$ when a is positive and when a is negative.

56. OPEN-ENDED Write a function whose graph is a horizontal translation of the graph of $h(x) = 4^x$.

57. USING STRUCTURE The graph of g is a translation 4 units up and 3 units right of the graph of $f(x) = 5^x$. Write an equation for g.

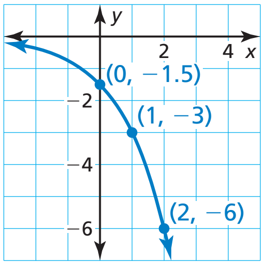
58. HOW DO YOU SEE IT? The exponential function $y = V(x)$ represents the projected value of a stock x weeks after a corporation loses an important legal battle. The graph of the function is shown.



a. After how many weeks will the stock be worth $20?

b. Describe the change in the stock price from Week 1 to Week 3.

59. USING GRAPHS The graph represents the exponential function f. Find $f(7)$.



60. THOUGHT PROVOKING Write a function of the form $y = ab^x$ that represents a real-life population. Explain the meaning of each of the constants a and b in the real-life context.

61. REASONING Let $f(x) = ab^x$. Show that when x is increased by a constant k, the quotient $\frac{f(x+k)}{f(x)}$ is always the same regardless of the value of x.

62. PROBLEM SOLVING A function g models a relationship in which the dependent variable is multiplied by 4 for every 2 units the independent variable increases. The value of the function at 0 is 5. Write an equation that represents the function.

63. PROBLEM SOLVING Write an exponential function f so that the slope from the point $(0, f(0))$ to the point $(2, f(2))$ is equal to 12.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write the percent as a decimal. *(Skills Review Handbook)*

64. 4% **65.** 35% **66.** 128% **67.** 250%